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Electromagnetic Surface Fields Due to a Magnetic Dipole Buried in a Three-Layered Earth

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UNITED STATES DEPARTMENT OF THE INTERIOR

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CONTENTS

ILLUSTRATIONS

ELECTROMAGNETIC SURFACE FIELDS DUE TO A MAGNETIC DIPOLE BURIED IN A THREE-LAYERED EARTH

By Steven M. Shope[1]

ABSTRACT

The Bureau of Mines electromagnetic trapped miner location and communications system requires a thorough understanding of through-the-earth electromagnetic wave propagation. An earth model incorporating a magnetic dipole buried in a three-layered earth has been developed. The dipole source is located in the second subsurface layer. By application of proper limiting values, the three-layered model is reduced to two different two-layered models and eventually to the homogeneous half-space model. The solutions are in the form of infinite integrals. A numerical analysis was carried out and a computer program written to evaluate the surface magnetic fields. Numerical values for the magnetic field at the point above the source are presented in a variety of models.

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INTRODUCTION

The Bureau of Mines electromagnetic trapped miner location and communication program requires a thorough understanding of subsurface wave propagation since the effective design of such a system requires a quantitative knowledge of both the propagation characteristics of the earth, and the radiational characteristics of the subsurface magnetic dipole. To gain this understanding of the propagation, it is necessary to combine this theoretical analysis with the results of field measurements.

Subsurface transmission experiments have been conducted at a diverse collection of coal mines throughout the United States under Bureau contract (8).[2] This has provided the Bureau with a large data base upon which the analytical propagational models could be validated. The data from these field tests present an opportunity to determine the electrical conductivity structure at each mine. The data consist of the surface vertical magnetic field strength at four frequencies. The receiver and source location were fixed, with the receiver on the surface directly above the source. Recent analysis of these data by the Bureau and a Bureau contractor (1) has suggested that a model employing the dipole embedded in a homogeneous half-space does not sufficiently describe the experimental data.

It is the purpose of this discussion to formulate analytical models that include a layered subsurface. This will accommodate the surface field effects due to major overburden stratification. The development begins with the formulation of a three-layered model from Maxwell's equations. The source is located in the second layer. By applying proper limiting values to appropriate variables, the three-layered model reduces to less complex stratifications.

A series of computer programs were written that numerically evaluate the integrals which represent the solutions. The programs allow fields to be computed at arbitrary points on or above the earth's surface. In this report results for a number of examples are given. In each case the vertical magnetic field on the surface directly above the source is compared with the field strength that would result for a homogeneous earth with the electrical conductivity of the source layer. Field strengths were computed directly above the source to allow initial comparison with field data.

ACKNOWLEDGMENTS

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DEFINITION OF THE PROBLEM

The half-space model representing the earth is comprised of three regions of varying conductivity. The boundaries are located at $z = h$, b, and $-c$, as seen in figure 1. For the purpose of this derivation, each region is horizontally infinite, isotropic, and homogeneous. The electrical characteristics of each region as expressed by σ and ε, the conductivity and dielectric constant, respectively, are known to be frequency independent for most geological materials aside from certain metallic ores and

[2]Underlined numbers in parentheses refer to the list of references preceding the appendix.

frequencies above 1 GHz. Also, the specific magnetic permeability μ/μ_o will be assumed unity for the subsurface materials (2, 4).

The buried antenna will be considered a circular current loop possessing a magnetic moment entirely in the $\hat{z}$ direction. The geometry of this problem lends itself to the use of a circular cylindrical coordinate system, with an origin at the loop axes. As noted in figure 1, the radial loop axis divides the second layer into region 2^+ and 2^-. The two regions are electrically identical, and the purpose of this additional boundary is to readily include the source by a boundary conditional at this interface.

Throughout this paper, the rationalized MKSA (meter, kilogram, second, ampere) unit system is employed.

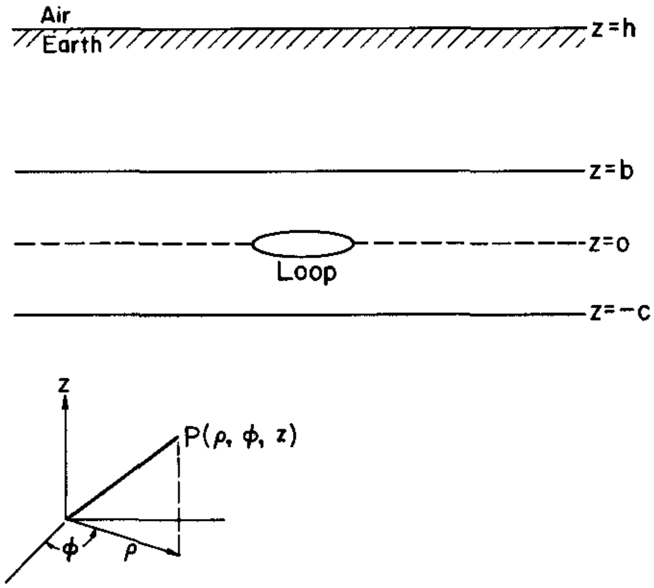

FIGURE 1. - Magnetic dipole buried in a three-layered earth, case A.

DERIVATION OF FIELD SOLUTIONS

The proper field solutions will be obtained from Maxwell's equations via Hertz potentials and application of the appropriate boundary conditions and charge distribution. Maxwell's equations for a homogeneous conducting space are

$$\bar{\nabla} \times \bar{E} + \frac{\partial \bar{B}}{\partial t} = 0 \qquad \bar{\nabla} \bullet \bar{D} = \rho \; ; \tag{1}$$

$$\bar{\nabla} \times \bar{H} - \frac{\partial \bar{D}}{\partial t} - \sigma \bar{E} = 0 \qquad \bar{\nabla} \bullet \bar{B} = 0 \; . \tag{2}$$

The analysis of certain electromagnetic fields is often facilitated by use of potentials. The derivation of field solutions for the problem at hand will utilize Hertzian potentials. Hertz showed it possible to define an electromagnetic field in terms of a single vector quantity $\bar{\Pi}^*$ by the following relationships:

$$\bar{E} = \mu \frac{\partial}{\partial t} (\bar{\nabla} \times \bar{\Pi}^*) \tag{3}$$

and

$$\bar{H} = - \bar{\nabla} \; (\bar{\nabla} \bullet \bar{\Pi}^*) + \mu\sigma \frac{\partial \bar{\Pi}^*}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{\Pi}^*}{\partial t^2} \; . \tag{4}$$

$\bar{\Pi}^*$ must be a solution to the following second-order differential equation:

$$\nabla^2\bar{\Pi}^* - \mu\varepsilon \frac{\partial^2\bar{\Pi}^*}{\partial t^2} - \mu\sigma \frac{\partial\bar{\Pi}^*}{\partial t} = 0 \ . \tag{5}$$

Without any loss of generality, $\bar{\Pi}^*$ can be assumed to be a harmonic function with a time dependence of $e^{-i\omega t}$. Denoting the solution in each region by subscript i, i = 0, 1, 2, 3, Π^* must satisfy

$$\nabla^2\bar{\Pi}_i^* + \mu\omega\ (\omega\varepsilon_i + i\sigma_i)\ \bar{\Pi}_i^* = 0 \tag{6}$$

in each region.

The above will simplify to

$$\nabla^2\bar{\Pi}_i^* - \gamma_i{}^2\bar{\Pi}_i^* = 0 \tag{7}$$

with the substitution of

$$-\gamma_i^2 = \mu\omega\ (\omega\varepsilon_i + i\sigma_i). \tag{8}$$

Solutions to this equation are obtained by the separation-of-variables technique with solutions of the form

$$\bar{\Pi}_i^* = \left[J_m\ (\lambda\rho) + Y_m\ (\lambda\rho)\right]\left[\cos m\phi + \sin m\phi\right]\left[e^{(\lambda^2 + \gamma_i{}^2)^{1/2}z} + e^{-(\lambda^2 + \gamma_i{}^2)^{1/2}z}\right]\hat{z} \ . \tag{9}$$

Immediately, the symmetry of the problem will reduce this solution. Since $\bar{\Pi}_i^*$ must remain finite as $\rho \to 0$, the Neumann functions Y_m $(\lambda\rho)$ must be excluded from the solution owing to their infinite behavior at $\rho = 0$. The angular homogeneity of all regions dictates no angular field variation; thus,

$$\frac{\partial\bar{\Pi}^*}{\partial\phi} = o \text{ for all i.} \tag{10}$$

This demands periodic functions; hence only integral values are permitted for m. The above condition is satisfied for

$$m \cos(m\ \phi) = m \sin\ (m\phi)\ ;$$

hence

$$m = 0\ .$$

The above conditions reduce $\bar{\Pi}_i^*$ to the general form

$$\Pi_i^* = J_o\ (\lambda\rho)\ e^{\pm(\lambda^2 + \gamma_i{}^2)^{1/2} z} \ . \tag{11}$$

Since $\bar{\Pi}_l^*$ has only a $\hat{z}$ component, the vector notation will be eliminated. Allowing the substitution $k_l{}^2 = \lambda^2 + \gamma_l^2$, the solution takes the well-known form

$$\Pi_l^* = J_o\,(\lambda\rho)\; e^{\pm k_l z} \;. \tag{12}$$

k_l can be associated with a complex wave number. Since the fields propagating in the +z and $-\hat{z}$ direction may not be of equal amplitudes, the introduction of amplitude coefficients $\Psi_l^+(\lambda)$ and $\Psi_l^-(\lambda)$ will generalize the solution:

$$\Pi_l^* = J_o\,(\lambda\rho)\;\{\Psi_l^-\,(\lambda)e^{-k_l z} + \Psi_l^+\,(\lambda)\;e^{k_l z}\}\;. \tag{13}$$

The complete form is the superposition of all solutions:

$$\Pi_l = \sum_{\lambda=o}^{\infty} J_o\,(\lambda\rho)\;\{\Psi_l^-\,(\lambda)e^{-k_l z} + \Psi_l^+\,(\lambda)e^{k_l z}\}\;. \tag{14}$$

Since λ is a continuum over the region $o \to \infty$, the summation may be replaced by an integral:

$$\Pi_l = \int_o^{\infty} J_o\,(\lambda\rho)\;\{\Psi_l^-\,(\lambda)e^{-k_l z} + \Psi_l^+\,(\lambda)e^{k_l z}\}d\lambda\;. \tag{15}$$

It was previously shown that the electric and magnetic fields could be expressed in terms of $\bar{\Pi}_l^*$. Since $\bar{\Pi}_l^*$ has only a $\hat{z}$ component, the electric field is

$$\bar{E}_l = i\mu\omega\,\frac{\partial\Pi_l^*}{\partial\rho}\;\hat{\phi}\;, \tag{16}$$

and the magnetic field is

$$\bar{H}_l = -\,\frac{\partial^2\Pi_l^*}{\partial\rho\partial z}\;\hat{\rho} + (\gamma_l^2 - \frac{\partial^2}{\partial z^2})\;\Pi_l^*\;\hat{z}\;. \tag{17}$$

It is noted that the electric field has only a $\hat{\phi}$ component, while the magnetic field contains only $\hat{\rho}$ and $\hat{z}$ terms. These are the expected field orientations produced by a magnetic dipole with a $\hat{z}$ moment.

To obtain explicit expressions for E and H, one must first derive an explicit expression for Π_l^*. This requires determination of the coefficients $\Psi_l^-\,(\lambda)$ and $\Psi_l^+\,(\lambda)$ for the region i of observation. This will be accomplished by proper application of boundary conditions at all boundaries. The boundary condition that tangential $\bar{E}$ and $\bar{H}$ must be continuous will be sufficient to solve for all unknowns. From the previous expressions for $\bar{E}$ and $\bar{H}$, only the $\hat{\phi}$ component of $\bar{E}$ and the $\hat{\rho}$ component of $\bar{H}$ are needed to apply the boundary conditions. Expressing these fields as

$$E_{\phi l} = i\mu\omega\,\frac{\partial}{\partial\rho}\int_o^{\infty} J_o\,(\lambda\rho)\;\{\Psi_l^-\,(\lambda)e^{-k_l z} + \Psi_l^+\,(\lambda)e^{k_l z}\}\;d\lambda \tag{18}$$

and

$$H_{\rho l} = -\frac{\partial^2}{\partial\rho\,\partial z} \int_0^\infty J_0\,(\lambda\rho)\,\{\Psi_l^-\,(\lambda)e^{-k_l z} + \Psi_l^+\,(\lambda)e^{k_l z}\}\,d\lambda\ , \tag{19}$$

and utilizing the identity

$$\frac{\partial}{\partial\rho} J_0\,(\lambda\rho) = -\lambda J_1\,(\lambda\rho)\ , \tag{20}$$

the above expressions for E_ϕ and H_ρ reduce to

$$E_{\phi l} = i\mu\omega \int_0^\infty \lambda J_1\,(\lambda\rho)\,\{\Psi_l^-\,(\lambda)e^{-k_l z} + \Psi_l^+\,(\lambda)e^{k_l z}\}\,d\lambda \tag{21}$$

and

$$H_{\rho l} = \int_0^\infty k_l\,\lambda\,J_1\,(\lambda\rho)\,\{\Psi_l^-\,(\lambda)e^{-k_l z} - \Psi_l^+\,(\lambda)e^{k_l z}\}d\lambda\ , \tag{22}$$

where $\Psi_l^-\,(\lambda)$ and $\Psi_l^+\,(\lambda)$ are as-yet-undetermined expansion coefficients.

The set of boundary conditions requiring tangential $\bar{E}$ and $\bar{H}$ to be continuous across all boundaries follows:

$$H_{\rho_3} - H_{\rho_{2-}} = 0 \qquad @\ z = -c \tag{23}$$

$$E_{\phi_3} - E_{\phi_{2-}} = 0 \qquad @\ z = -c \tag{24}$$

$$H_{\rho_{2+}} - H_{\rho_{2-}} = j_\phi(\rho) \qquad @\ z = 0 \tag{25}$$

$$E_{\phi_{2+}} - E_{\phi_{2-}} = 0 \qquad @\ z = 0 \tag{26}$$

$$H_{\rho_1} - H_{\rho_{2+}} = 0 \qquad @\ z = b \tag{27}$$

$$E_{\phi_1} - E_{\phi_{2+}} = 0 \qquad @\ z = b \tag{28}$$

$$H_{\rho_0} - H_{\rho_1} = 0 \qquad @\ z = h \tag{29}$$

$$E_{\phi_0} - E_{\phi_1} = 0 \qquad @\ z = h \tag{30}$$

It can be noted in equation 27, at the 2^+, 2^- boundary, that H_ρ is discontinuous across this boundary by an amount equal to the surface current density $j_\phi\,(\rho)$. Since the current loop (antenna) has been modeled as an infinitesimally thin ring of current, $j\phi(\rho)$ will be the source term for the field. No bound charges are present anywhere in the model; thus, there will be no electric source terms present.

The above set of 8 equations contains 10 unknown coefficients:

$$\Psi_o^{\mp}(\lambda),\ \Psi_1^{\mp}(\lambda),\ \Psi_{2+}^{\mp}(\lambda),\ \Psi_{2-}^{\mp}(\lambda),\ \Psi_3^{\mp}(\lambda)$$

However, some immediate simplifications can be made: $\Psi_3^-(\lambda) \equiv 0$ since there are no sources below $z = -c$; also $\Psi_o^+(\lambda) \equiv 0$ since there are no sources above $z = h$. This reduces the set to eight equations and eight unknowns.

Before going further, an explicit expression for the current density, $j_1(\rho)$, existing at $z = o$ must be obtained. The loop antenna may be described as a two-dimensional ring of current, expressed mathematically as

$$j_\phi(\rho) = I\ \delta(\rho - a), \tag{31}$$

which can be represented by a Fourier-Bessel integral:

$$j_\phi(\rho) = \int_o^\infty f(\lambda)\ J_n(\lambda\rho)\ \lambda d\lambda\ , \tag{32}$$

where the expansion coefficient $f(\lambda)$ is defined as

$$f(\lambda) = \int_o^\infty j_\phi(\rho) J_n(\lambda\rho)\rho d\rho \tag{33}$$

or

$$f(\lambda) = \int_o^\infty I\ \delta(\rho - a)\ J_n(\lambda\rho)\rho d\rho\ . \tag{34}$$

Owing to the delta function, the integral is readily evaluated to be

$$f(\lambda) = Ia\ J_n(\lambda a)\ . \tag{35}$$

Thus the source term may be represented by the following Fourier-Bessel integral:

$$j_\phi(\rho) = \int_o^\infty IaJ_1(\lambda\rho)\lambda d\lambda\ , \tag{36}$$

where $n = 1$ has been chosen to keep the Fourier-Bessel representation of the current density in the same functional space as that of the H_ρ representation.

The set of eight equations generated by the boundary conditions 23-30 is given in the appendix (equations A-1-A-8). For the purpose of this paper, the surface will be the only region for which fields are explicitly derived. Hence, an expression for $\overline{\Pi}_o^*$ must be obtained. This requires an explicit form for the coefficient $\Psi_o^-(\lambda)$, which entails simultaneous evaluation of the eight equations in the appendix. The lengthly process associated with this has not been included in this discussion; however, the final form of $\Psi_o^-(\lambda)$ is included in the appendix. Utilizing this, Π_o^* may be explicitly expressed, from which the surface electromagnetic fields will follow.

Explicit Field Forms

The form of the vertical magnetic field in region i is

$$H_{zi} = \gamma^2 - \frac{\partial^2}{\partial z^2} \Pi_i^* , \tag{37}$$

which in region o is

$$H_{zo} = {}_o\int^\infty \left[\gamma_o^2 - k_o^2 \right] J_o (\lambda\rho)\, \Psi_o^- (\lambda) e^{-k_o z} d\lambda . \tag{38}$$

Likewise, the form of the horizontal magnetic field is

$$H\rho_i = \frac{\partial \Pi_i^*}{\partial\rho\partial z} , \tag{39}$$

which yields the following in region o:

$$H_{\rho o} = {}_o\int^\infty k_o\, \lambda J_1 (\lambda\rho)\, \Psi_o^- (\lambda) e^{-k_o z} d\lambda . \tag{40}$$

By including of the expression for $\Psi_o^- (\lambda)$ into the above integrals, a set of complete solutions may be had for the vertical and radial magnetic fields. The integral forms for H_{zo} and $H_{\rho o}$ are

$$H_{zo} = {}_o\int^\infty \frac{\left[\gamma_o{}^2 - k_o{}^2\right] J_o(\lambda\rho)\; IaJ_1\,(\lambda a)\; T_{12} \left[1 + R_{23} e^{-2k_2 c} \right]}{(k_o + k_1)\; \{R_{23} \left[R_{01} + R_{12} e^{2k_1(h-b)} \right]} \cdots$$

$$\cdots \frac{e^{(k_o+k_1)h}\; e^{2k_2 c}\; {}^{(k_2-k_1)b\; -\; k_o z}\; d\lambda}{+\, e^{2k_2(b+c)} \left[R_{01}R_{12} + e^{2k_1(h-b)} \right] \}} \tag{41}$$

and

$$H\rho_o = {}_o\int^\infty \frac{k_o\lambda \;\; J_1(\lambda\rho)\; IaJ_1(\lambda a) T_{12} \left[1 + R_{23} {-2k_2 c} \right]}{(k_o + k_1)\; \{R_{23} \left[R_{01} + R_{12} e^{2k_1(h-b)} \right]} \cdots$$

$$\cdots \frac{e^{(k_o+k_1)h}\; e^{(2k_2-k_1)b}\; e^{-k_o z}\; d\lambda}{+\, e^{2k_2(b+c)} \left[R_{01}\; R_{12} + e^{2k_1(h-b)} \right] \}} , \tag{42}$$

where the R_{ij} and T_{ij} terms are the amplitude reflection and transmission coefficients, respectively, defined as

$$R_{ij} = \frac{k_i - k_j}{k_i + k_j} \quad \text{and} \quad T_{ij} = \frac{2k_i}{k_i + k_j} . \tag{43}$$

Recall that γ_i is defined as

$$\gamma_i^2 = -\mu\omega(\omega\varepsilon_i + i\sigma_i) . \tag{44}$$

For low frequencies (<1 MHz), the ohmic currents will be much greater than any displacement currents; thus γ_i^2 can be approximated by

$$\gamma_i^2 = -i\mu\omega\sigma_i . \tag{45}$$

Obviously $\gamma_o = o$; hence $k_o = \lambda$.

Utilizing a well-known convention (6), the fields may be expressed as

$$H_{zo} = -bQ_o \tag{46}$$

and

$$H_{\rho o} = bP_o , \tag{47}$$

where $b = \frac{M}{2\pi h^3}$ and M is the magnetic moment, INA. For observations directly above the buried loop, b is the vertical magnetic field strength for a nonconducting earth (free space condition). P_o and Q_o can be viewed as the transmission loss due to the presence of a conducting earth. Thus Q_o and P_o may be written as

$$Q_o = \frac{h^3}{z} \int_o^\infty \frac{\lambda^2 T_{01}\, J_o(\lambda\rho) \left[\left[\frac{2\, J_1(\lambda a)}{\lambda a}\right]\right] T_{12} \left[1 + R_{23} e^{-2k_2 c}\right]}{R_{23}{}^{-2k_2 c} \left[R_{10} e^{-2k_1 h} + R_{12} e^{-2k_2 b}\right]} \cdots \tag{48}$$

$$\cdots \frac{e^{-k_1(h+b)}\, e^{k_2 b}\, e^{k_o(h-z)}\, d\lambda}{+ e^{2k_2 b}\left[R_{01}\, R_{12} e^{-2k_1 h} + e^{-2k_1 b}\right]}$$

and

$$P_o = \frac{h^3}{2} \frac{\int_0^\infty \lambda^2 \, T_{01} \, J_1(\lambda\rho) \left[\left[\frac{2\,J_1(\lambda a)}{\lambda a} \right]\right] T_{12} \left[1 + R_{23} e^{-2k_2 c} \right]}{R_{23}^{-2k_2 c} \left[R_{01} e^{-2k_1 h} + R_{12} e^{-2k_1 b} \right] + e^{2k_2 b}} \cdots \tag{49}$$

$$\frac{e^{-k_1(h+b)\,k_2 b\,k_0(h-z)} \, d\lambda}{\left[R_{01}\, R_{12} e^{-2k_1 h} + e^{-2k_1 b} \right]}$$

The above terms in double brackets may be replaced by 1 for small a, this is the dipole approximation.[3] For $J_1(\lambda a)$, when $\lambda a \ll 1$, $\frac{2J_1(\lambda a)}{\lambda a} \rightarrow 1$.

Numerical Evaulation

The above forms simplify for numerical evaluation if the dimensional variables are normalized with respect to depth (h). The following new variables are introduced:

$$D = \rho/h \qquad B = b/h \qquad C = c/h \qquad Z = z/h$$

Also,

$$H = (\mu\omega\sigma_2)^{1/2}\, h\ , \tag{51}$$

H is the ratio of source depth to skin depth. In the low-frequency limit the skin depth becomes infinite; thus $H = 0$, and Q_o has the value for free space. For a fixed source depth, H is proportional to the square root of frequency.

The conductivities of regions 1 and 3 are normalized to the conductivity of the source layer, region 2. These normalized conductivities are defined as

$$S_{12}^2 = \sigma_1/\sigma_2 \tag{52}$$

and

$$S_{23}^2 = \sigma_3/\sigma_2 \tag{53}$$

---

[3]The function $J_n(x)$ may be expanded as follows:

$$J_n(x) = \frac{x^n}{2^n \Gamma(n+1)} \left[1 - \frac{x^2}{2\,(2n+2)} + \frac{x^4}{2 \bullet 4(2n+2)(2n+4)} - \cdots \right] \tag{50}$$

It follows that the wave numbers may be expressed as

$$k_1 = \frac{1}{h} (x^2 + iH^2S_{12}^2)^{1/2} , \tag{54}$$

$$k_2 = \frac{1}{h} (x^2 + iH^2)^{1/2} , \tag{55}$$

and

$$k_3 = \frac{1}{h} (x^2 + iH^2S_{23}^2)^{1/2} . \tag{56}$$

Introducing a new variable of integration,

$$x = \lambda h,$$

the Q_o and P_o expressions for a dipole source become

$$Q_o = \frac{1}{2} \int_o^\infty \frac{x^2T_{01}T_{12}\left[1 + R_{23}e^{-2C(x^2 + iH^2)^{1/2}}\right]e^{-(x^2+iH^2S_{12}^2)\,(1 + B)}}{R_{23}^{-2(x^2 + iH^2)^{1/2}C}\left[R_{01}e^{-2(x^2+iH^2S_{12}^2\;)^{1/2}} + R_{12}e^{2(x^2+iH^2)^{1/2}B}\right]} \cdots$$

$$\cdots \frac{e^{(x^2 + iH^2)^{1/2}B}\,J_o(xD)e^{x(1-Z)}\,dx}{} \tag{57}$$

and

$$P_o = \frac{1}{2} \int_o^\infty \frac{T_{01}T_{12}\; x^2\left[1 + R_{23}e^{-2(x^2+iH^2)^{1/2}C}\right]}{R_{23}e^{-2(x^2+iH^2)^{1/2}C}\left[R_{01}e^{-2(x^2+iH^2S_{12}^2\;)^{1/2}} + R_{12}e^{-2(x^2+iH^2S_{12}^2)^{1/2}B}\right]} \cdots$$

$$\cdots \frac{e^{-(x^2+iH^2S_{12}^2)^{1/2}(1+B)\; +\; (x^2+iH^2)^{1/2}B}J_1(xD)e^{x(1-Z)}dx}{+\; e^{2(x^2+iH^2)^{1/2}B}\left[R_{01}R_{12}e^{-2(x^2+iH^2S_{12}{}^2)^{1/2}} - e^{-2(x^2+iH^2S_{12}{}^2)^{1/2}B}\right]} . \tag{58}$$

Since the coefficients R_{ij} and T_{ij} are dimensionless ratios, they remain numerically unchanged by the variable transformations. The infinite integrals contained in Q_o and P_o have been numerically evaluated by means of 12-point gaussian quadrature.

The results of the integration are presented graphically, as Q_o versus H for various numeric values of the parameters S_{12}, S_{23}, B, and C. These graphs may be seen in figure 2. Only the fields observed coaxially above the source have been presented, D = 0; here the radial fields are zero. The dashed curve on each plot is Q versus H for a homogeneous half-space with conductivity σ_2. Comparison of the other curves with the dashed one shows how the more complicated structures affect the variation of Q_o with H and how the variation differs from that of the homogeneous half-space model.

As seen in the graphs, the presence of layering may actually produce larger fields than a homogeneous half-space would. This is in part due to the reflecting layer below the source.

Limiting Cases

From the three-layered half-space solution, field expressions for less complex layering can be derived by inspection. Figures 1 and 3-5 characterize the cases for which solutions will be obtained; "case A" is the three-layer-earth model already discussed. These limiting solutions are obtained either by reducing a layer's vertical dimension to zero or by letting the conductivity of two adjacent layers be identical; for example, $\sigma_2 = \sigma_3$.

Case B: Two-Layered Half-Space

In this model the half-space has two layers, with the interface located at z = b. In this case, the solution is obtained by allowing $\sigma_3 = \sigma_2$; thus $S_{23} \to 1$ or $C \to \infty$ in the three-layered expression. The dipole solutions are

$$Q_o = \frac{h^3}{2}\int_0^\infty \frac{\lambda^2 T_{01} T_{12} e^{-k_1(h+b)} e^{k_o(h-z)} J_o(\lambda\rho)d\lambda}{e^{k_2 b}\left[R_{01}R_{12}e^{-2k_1 h} + e^{-2k_1 b}\right]} \tag{59}$$

$$P_o = \frac{h^3}{2}\int_0^\infty \frac{\lambda^2 T_{01} T_{12} e^{-k_1(h+b)} e^{k_o(h-z)} J_1(\lambda\rho)d\lambda}{e^{k_2 b}\left[R_{01}R_{12}e^{-2k_1 h} + e^{-2k_1 b}\right]} \tag{60}$$

The above expressions agree with an earlier derivation of a dipole buried in a two-layered earth by Wait and Spies (7). However, before a direct comparison with their work can be made, the following coordinate transformation would have to be made: $z \to z + h$, and $b \to h - b$.

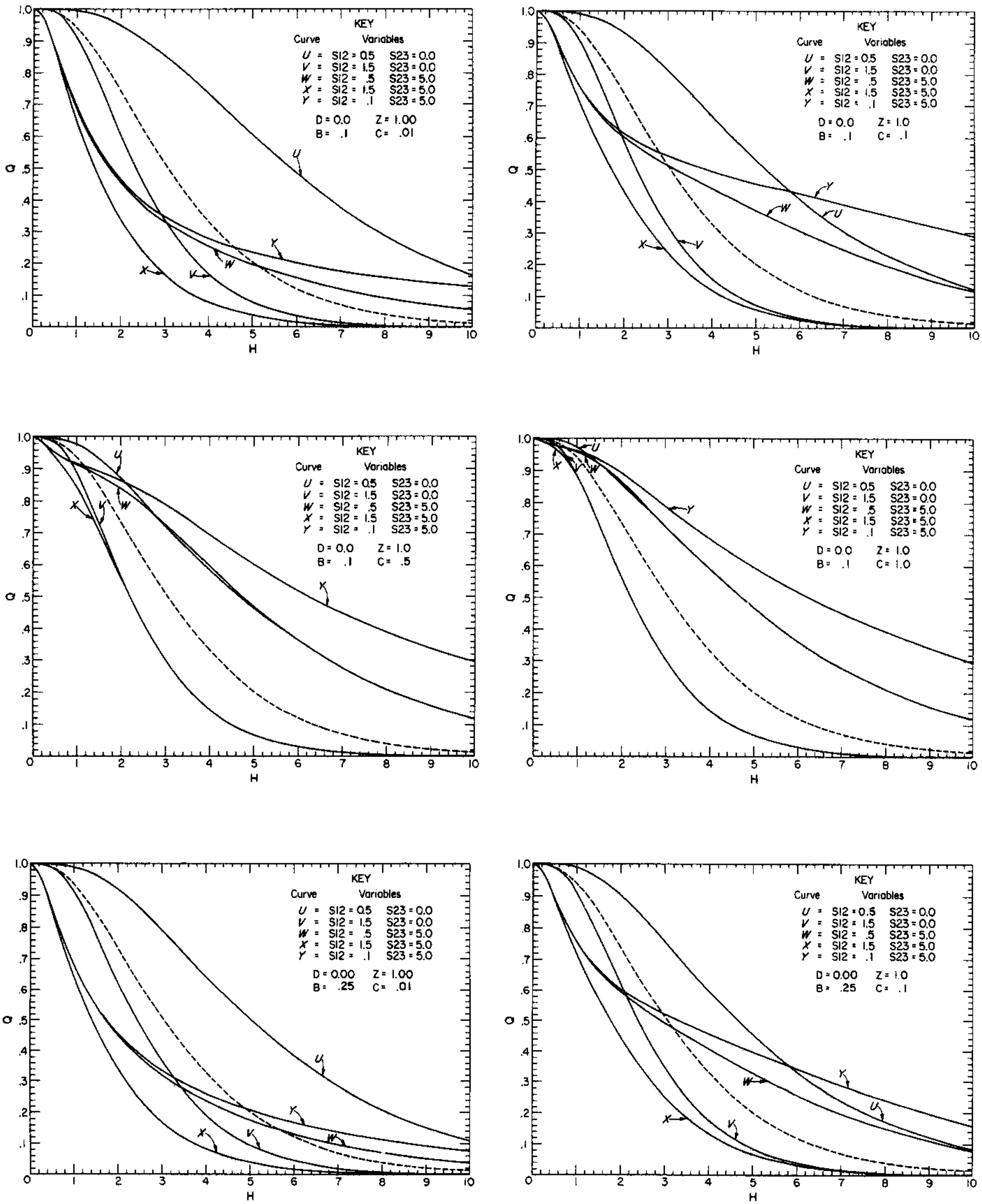

FIGURE 2. - Vertical component of normalized magnetic field for various field layer parameters, case A.

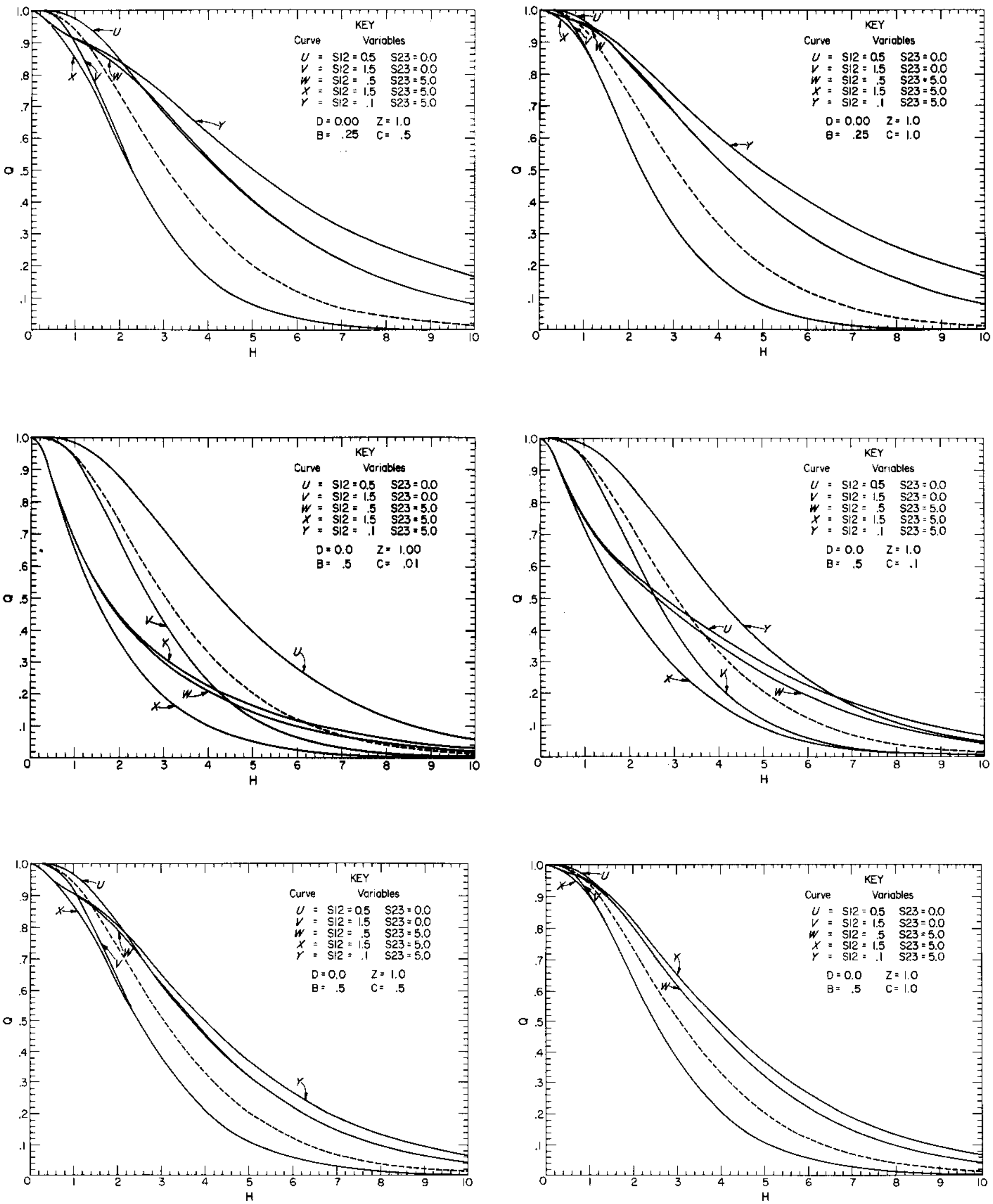

FIGURE 2. - Vertical component of normalized magnetic field for various field layer parameters, case A—Continued.

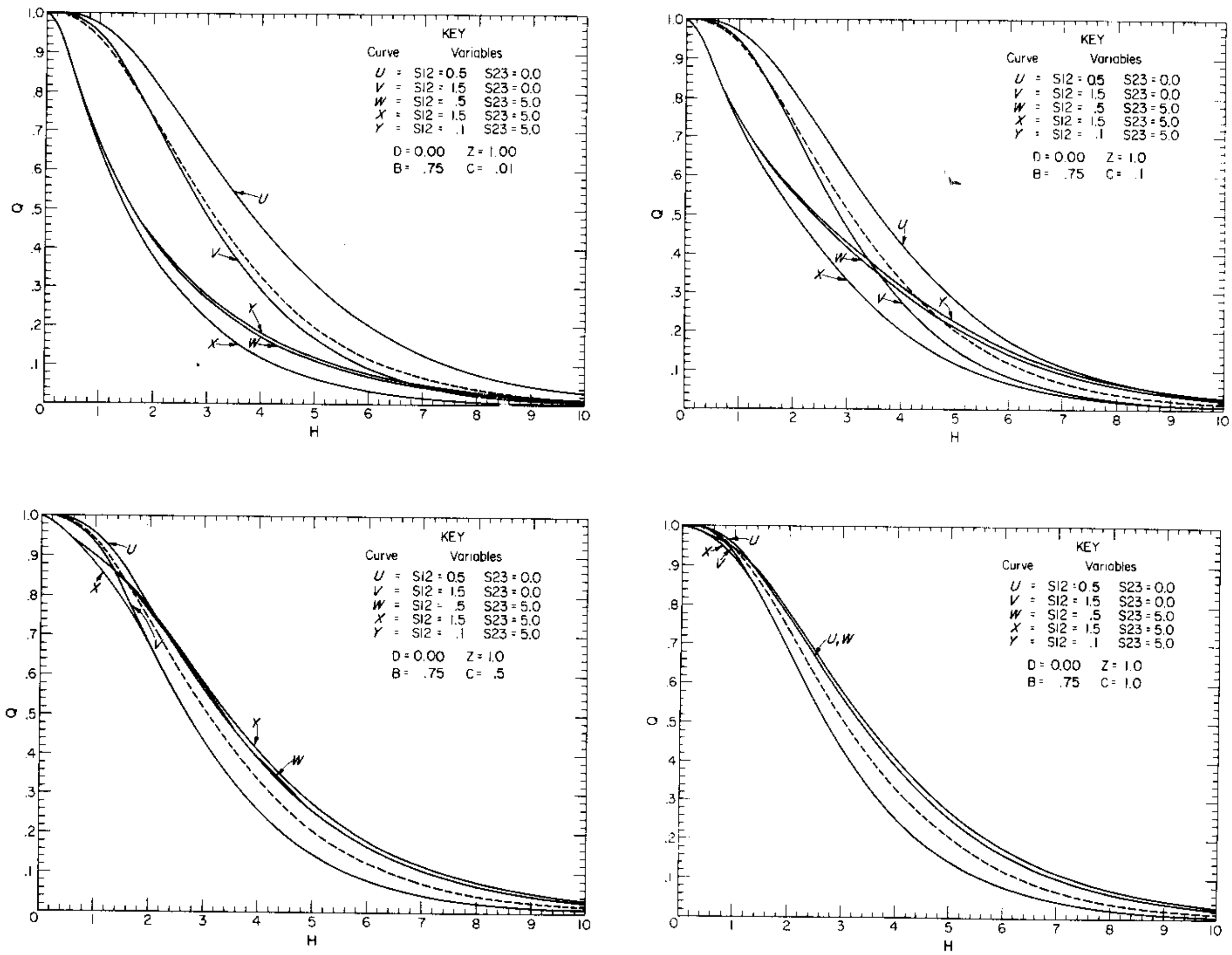

FIGURE 2. - Vertical component of normalized magnetic field for various field layer parameters, case A—Continued.

After a similar variable transformation, as done in case A, the above two expressions become

$$Q_o = \frac{1}{2}\int_o^\infty \frac{x^2 T_{12} T_{01} e^{(x^2+iH^2S_{12}^2)^{1/2}(1+B)} J_o(xD) e^{x(1-Z)}\,dx}{e^{(x^2+iH)^{1/2}B}\left[R_{01}R_{12}e^{-2(x^2+iH^2S_{12}^2)^{1/2}} + e^{-2(x^2+iH^2S_{12}^2)^{1/2}B}\right]} \tag{61}$$

and

$$P_o = \frac{1}{2}\int_o^\infty \frac{x^2 T_{12} T_{01} e^{(x^2+H^2S_{12}^2)(1+B)} J_1(xD) e^{x(1-Z)}\,dx}{e^{(x^2+iH^2)^{1/2}B}\left[R_{01}R_{12}e^{-2(x^2+iH^2S_{12}^2)^{1/2}} + e^{-2(x^2+iH^2S_{12}^2)^{1/2}B}\right]} . \tag{62}$$

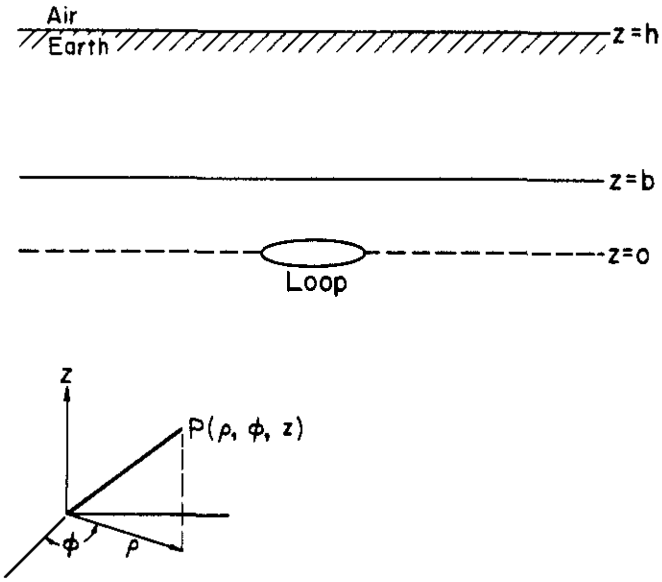

FIGURE 3. - Magnetic dipole buried in a two-layered earth, case B.

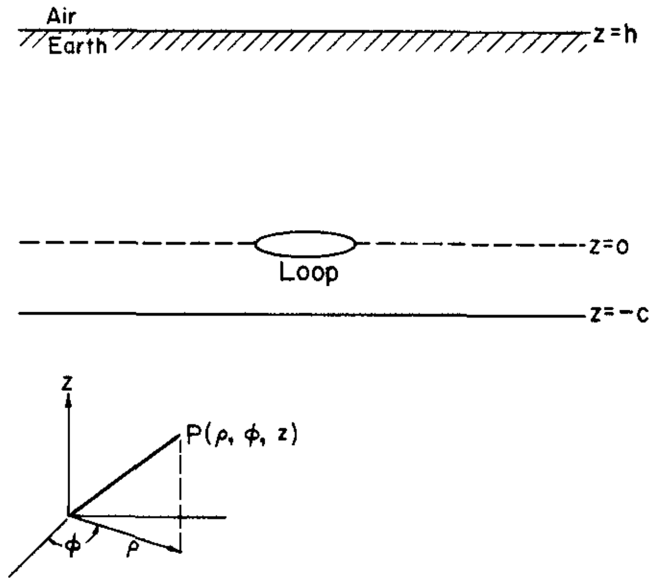

FIGURE 4. - Magnetic dipole buried in a two-layered earth, case C.

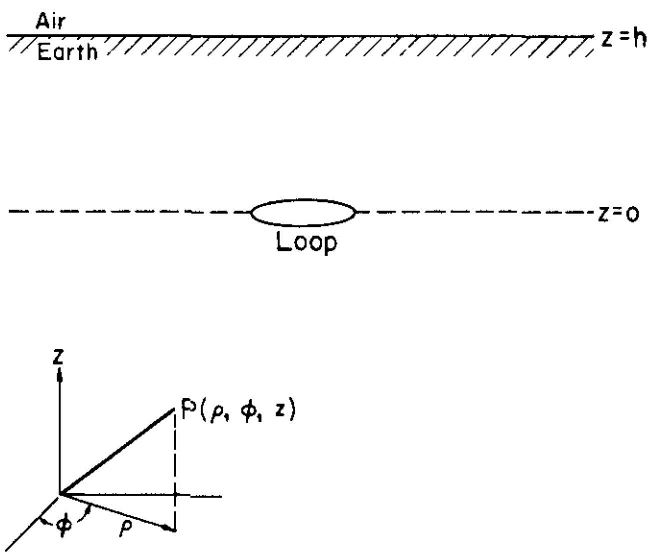

FIGURE 5. - Magnetic dipole buried in a homogeneous half-space, case D.

Graphs of the numerical integration may be seen in figure 6. Even without the presence of a reflector below the loop, it is possible to have surface fields larger than the homogeneous half-space situation. Conversely, the fields may also be smaller, depending upon the parameters of layer thickness and conductivity.

Case C: Two-Layered Half-Space

This model is comprised of an earth containing two layers, with the layer interface located at $z = -c$. The layer located below the loop should serve as a reflector, causing larger surface fields than would be expected with a homogeneous half-space.

This model is derived from the three-layered expression by allowing $\sigma_1 = \sigma_2$, hence $S_{12}{}^2 \to 1$, or by setting $B = 1$. The case C solutions for the vertical and radial magnetic fields are

$$Q_o = \frac{h^3}{2}\int_0^\infty \frac{\lambda^2 T_{02}\left[1 + R_{23}e^{-2k_2c}\right] e^{-k_2h}\, e^{k_o(h-z)}\, J_o(\lambda\rho)\,d\lambda}{\left[R_{23}{}^{-2k_2c}\, R_{02}e^{-2k_2h} + 1\right]} \tag{63}$$

and

$$P_o = \frac{h^3}{2} \int_0^\infty \frac{\lambda^2 T_{02}\left[1 + R_{23}e^{-2k_2c}\right] e^{-k_2h} e^{k_0(h-z)} J_1(\lambda\rho)d\lambda}{\left[R_{23}{}^{-2k_2c} R_{02}e^{-2k_2h} + 1\right]} \cdot \tag{64}$$

After transformation of variables to facilitate numerical evaluation

$$Q_o = \frac{1}{2} \int_0^\infty \frac{x^2 T_{02}\left[1 + R_{23}e^{-2(x^2+iH^2)^{1/2}C}\right] e^{-(x^2+iH^2)^{1/2}}}{\left[1 + R_{02}R_{23}e^{-2(x^2+iH^2)^{1/2}\,(C+1)}\right]} \cdots \tag{65}$$

$$\cdots \quad J_o(xD)e^{x(1-Z)}dx$$

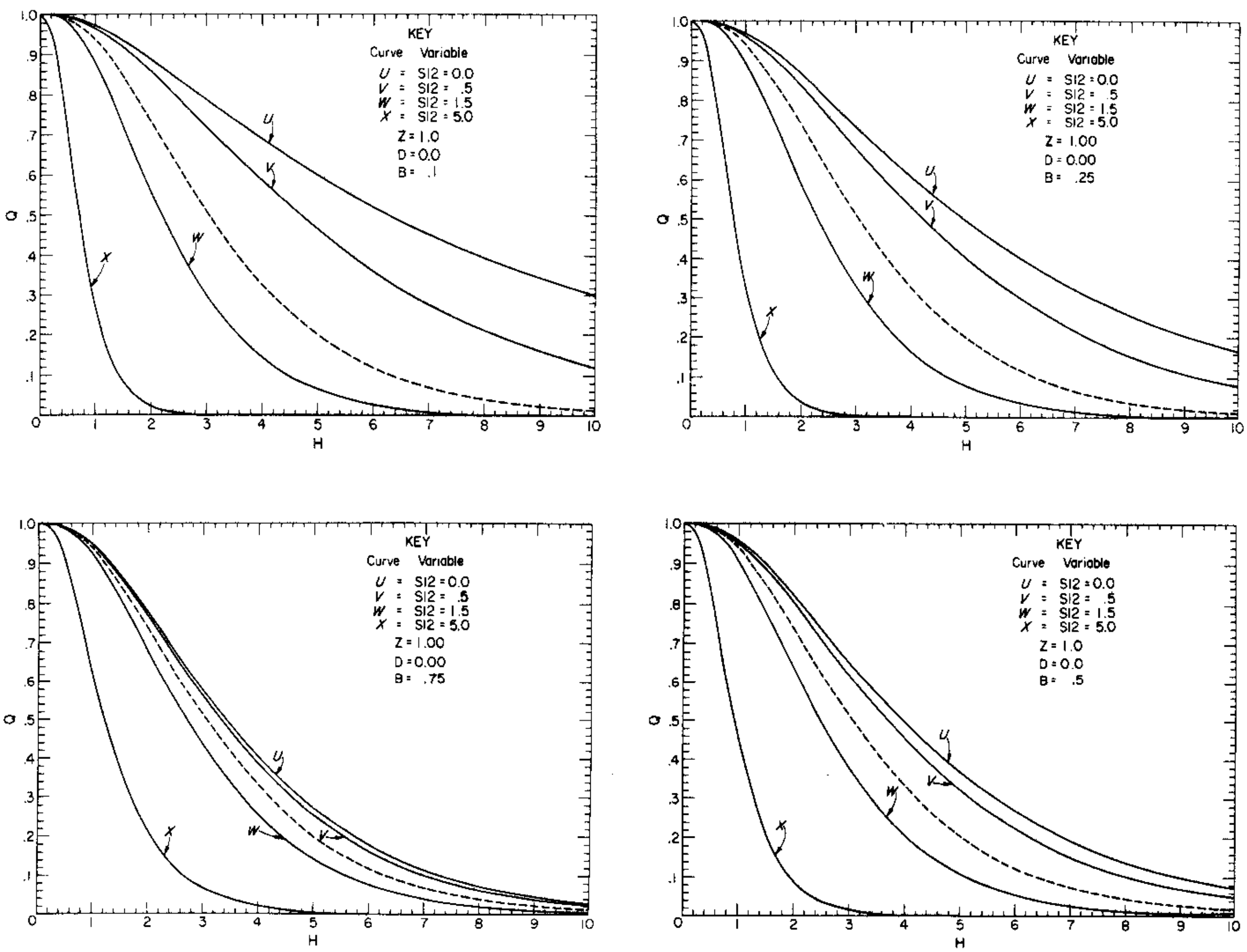

FIGURE 6. - Normalized H_z for a dipole buried in a two-layered earth, case B.

and

$$P_o = \frac{1}{2} \int_0^{\infty} \frac{x^2 T_{02} \left[1 + R_{23} e^{-2(x^2+iH^2)C} \right] e^{-(x^2+iH^2)^{1/2}} J_1(xD) e^{x(1-Z)} dx}{\left[1 + R_{02} R_{23} e^{-2(x^2+iH^2)^{1/2}(C+1)} \right]} . \quad (66)$$

Graphs created by the evaluation of the previous expressions may be seen in figure 7. Again, depending upon the value of the lower layer's conductivity and position, the surface field may be larger or smaller than the homogeneous half-space case. By examining the graphs, one can see that when σ_3 is greater than σ_2, the surface field decreases. The phase of the reflected field is reversed from that of the primary field; thus, there is a degree of cancellation at the surface.

Case D: Homogeneous Half-Space

As a final example, the homogeneous half-space expression may be derived from the case A (three-layered half-space) expressions. This is done by simply combining the limiting factors introduced in the previous two cases; that is, $S_{12}^2 = 1$ and $S_{23}^2 = 1$.

The case D solutions for the vertical and radial magnetic surface fields are

$$Q_o = h^3 \int_0^{\infty} \frac{\lambda^3 e^{-k_2 h} J_o(\lambda\rho) e^{k_o(h-z)} d\lambda}{(\lambda+k_2)} \quad (67)$$

and

$$P_o = h^3 \int_0^{\infty} \frac{\lambda^3 e^{-k_2 h} J_1(\lambda\rho) e^{k_o(h-z)} d\lambda}{(\lambda+k_2)} . \quad (68)$$

As in the previous examples, the variable of integration may be changed to facilitate numerical integration:

$$Q_o = \int_0^{\infty} \frac{x^3 e^{-(x^2+iH^2)^{1/2} + x(1-Z)} J_o(xD) dx}{x + (x^2+iH^2)^{1/2}} \quad (69)$$

and

$$P_o = \int_0^{\infty} \frac{x^3 e^{-(x^2+iH^2)^{1/2} + x(1-Z)} J_1(xD)\, dx}{x + (x^2+iH^2)^{1/2}} . \quad (70)$$

The above expressions for Q_o have been evaluated and superimposed as a dashed line on the graphs of the previous three cases; thus, no separate graph of the homogeneous half-space model is included.

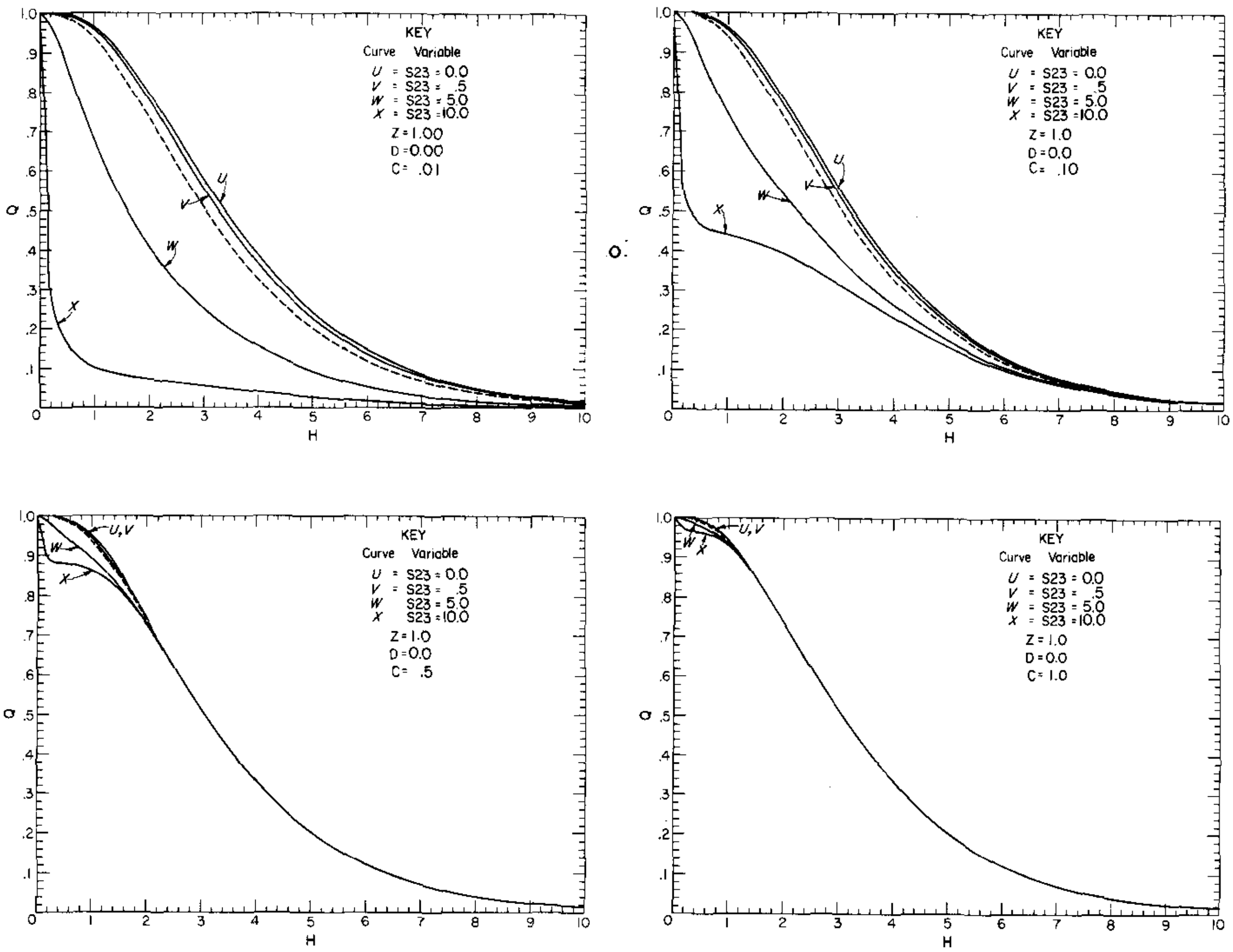

FIGURE 7. - Normalized H_z for a dipole buried in a two-layered earth, case C.

CONCLUSIONS

This discussion has shown the electromagnetic surface field solutions for a magnetic dipole buried in a three-layered earth as derived from Maxwell's equations. Two approximations were made, the first being that the displacement currents are negligible as compared with the ohmic currents; this is certainly valid in the frequency range of interest (less than 5 kHz). The other assumption is that the loop radius is small compared with the burial depth and may be considered a dipole source.

Even in instances when this may not be valid, the approximation was made late in the formulation and the final solutions may be readily adapted to account for a finite-size loop.

As seen in the three cases involving stratified half-space models, the effects of layering produce nontrivial changes in the surface fields as compared with the homogeneous half-space model. The magnitude of these effects is wholly dependent upon the characteristics of the layers as expressed by conductivity, thickness, frequency, and position. Not only are the surface fields diminished, but in some instances the stratification may cause larger fields than expected. The mechanisms responsible for altering the fields appear to be attenuation, reflection (transmission) effects of the interfaces, and constructive (destructive) addition of fields due to phase shifts caused by interface reflections.

The programs developed will be useful for several purposes--interpreting data from past field tests, planning search procedures, developing location procedures, and predicting the signal to noise ratios for calculation of signal detection probabilities. However, the primary objective of this work is to compare existing and ongoing experimental data with the various theoretical models. From this, it may be found that VF (voice frequency) through-the-earth propagation may be best represented by one particular model in a majority of cases.

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APPENDIX.--BOUNDARY EQUATION SET

The set of eight equations generated by application of tangential continuity boundary conditions follows:

$$\Psi_{2_-}^{+}(\lambda)e^{-k_2c} - \Psi_{2_-}^{-}(\lambda)e^{k_2c} - \frac{k_3}{k_2}\Psi_3^{+}(\lambda)e^{-k_3c} = 0 \tag{A-1}$$

$$\Psi_{2_-}^{+}(\lambda)e^{-k_2c} + \Psi_{2_-}^{-}(\lambda)e^{k_2c} - \frac{k_3}{k_2}\Psi_3^{+}(\lambda)e^{-k_3c} = 0 \tag{A-2}$$

$$\Psi_{2_+}^{-}(\lambda) - \Psi_{2_+}^{+}(\lambda) - \Psi_{2_-}^{-}(\lambda) + \Psi_{2_-}^{+}(\lambda) = \frac{I_a J_1(\lambda a)}{k_2} \tag{A-3}$$

$$\Psi_{2_+}^{-}(\lambda) + \Psi_{2_+}^{+}(\lambda) - \Psi_{2_-}^{-}(\lambda) - \Psi_{2_-}^{+}(\lambda) = 0 \tag{A-4}$$

$$\Psi_1^{-}(\lambda)e^{-k_1b} - \Psi_1^{+}(\lambda)e^{k_1b} - \frac{k_2}{k_1}\Psi_{2_+}^{-}(\lambda)e^{-k_2b} + \frac{k_2}{k_2}\Psi_{2+}^{+}(\lambda)e^{k_2b} = 0 \tag{A-5}$$

$$\Psi_1^{-}(\lambda)e^{-k_1b} + \Psi_1^{+}(\lambda)e^{k_1b} - \Psi_{2_+}^{-}(\lambda)e^{-k_2b} - \Psi_{2_+}^{+}(\lambda)e^{k_2b} = 0 \tag{A-6}$$

$$\frac{k_o}{k_1}\Psi_0^{-}(\lambda)e^{-k_oh} - \Psi_1^{-}(\lambda)e^{-k_1h} + \Psi_1^{+}(\lambda)e^{k_1h} = 0 \tag{A-7}$$

$$\Psi_0^{-}(\lambda)e^{-k_oh} - \Psi_1^{-}(\lambda)e^{-k_1h} - \Psi_1^{+}(\lambda)e^{k_1h} = 0 \tag{A-8}$$

By simultaneous evaluation of the above eight expressions, an explicit form for the coefficient of interest $\Psi_0^{-}(\lambda)$ may be found. Explicit forms of $\Psi_1^{-}(\lambda)$ and $\Psi_1^{+}(\lambda)$ have also been presented, which would be valuable in developing subsurface field expressions; this, however, is beyond the scope of this paper.

$$\Psi_0^{-}(\lambda) = \frac{Ia\, J_1(\lambda a)\, T_{12}\left[1 + R_{23}e^{-2k_2c}\right] e^{(k_o + k_1)} e^{2k_2c} e^{(k_2 - k_1)b}}{(k_o + k_1)\left\{R_{23}\left[R_{01} + R_{12}e^{2k_1(h-b)}\right] + e^{2k_2(b+c)}\left[R_{01}R_{12} + e^{2k_1(h-b)}\right]\right\}} \tag{A-9}$$

$$\Psi_I^{+}(\lambda) = \frac{(k_1 - k_0)\ Ia\ J_1\ (\lambda a)\left[1 + R_{23}e^{2k_2c}\right]e^{-2k_1h}\ e^{b(k_1+k_2)}}{(k_o + k_1)(k_1 + k_2)\ \{R_{12}\left[R_{01}R_{23}e^{2k_2(b+c)\ +\ 2k_2(b-h)} + 1\right]} \cdots$$

$$\cdots \frac{}{+ R_{01}e^{2k_1(b-h)} + R_{23}e^{2k_2(b+c)}\}} \tag{A-10}$$

$$\Psi_I^{-}(\lambda) = \frac{Ia\ J_1(\lambda a)\left[1+R_{23}e^{2k_2c}\right]\ e^{2k_1h}\ e^{b(k_1+k_2)}}{(k_1 + k_2)\ \{R_{12}\left[R_{01}R_{23}e^{2k_2(b+c)\ +\ 2k_1(b-h)} + 1\right]} \cdots \tag{A-11}$$

$$\cdots \frac{}{+ R_{01}e^{2k_1(b-h)} + R_{23}e^{2k_2(b+c)}\}}$$

It may be noted that

$$\Psi_I^{+}(\lambda) = \left[\frac{k_1 - k_o}{k_1 + k_o}\right]\Psi^{-}\ (\lambda) = -R_{01}\ \Psi_I^{-}(\lambda) \tag{A-12}$$