Statistical Notes for Health Planners

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INFANT MORTALITY

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INTRODUCTION

Infant mortality rates have often been used as health status indicators, particularly for international comparisons.¹ However, it is generally recognized that infant mortality is not a valid indicator of the overall health status of a nation or community, although it is very useful in pointing to problems with the health status of infants and mothers and possible problems in the delivery of health care and related services to these groups in a community. The use of infant mortality rates at the local planning level is unfortunately limited by yearly fluctuations in the rates which are purely random (i.e., unrelated to characteristics of the infants, mothers, environment, etc.). This stability problem will be discussed in more detail below.

The factors which influence infant mortality rates² are many: race, sex, residence, birth weight, gestation age, age of mother, birth order, prior pregnancy outcome, socioeconomic status,³ and prenatal care.⁴

Infant mortality rates for white and all other births have declined rapidly since the early part of the 20th century. After a period of reduced decline in the 1950's to mid-1960's, an accelerated decline occurred through 1974, the most recent year for which data are available. However, wide variations within the United States still exist. For example the U.S. mortality rate in 1974 for infants other than white (24.6 per 1,000 live births) was 67 percent higher than the corresponding rate for white infants $(14.7 \text{ per } 1,000).^5$ In addition, substantial geographic variation exists across the United States² and within small areas of large cities.⁶ This variation is especially important for planning purposes.

DEFINITIONS

NCHS

In order to provide comparable reporting of birth and death data, precise definitions of terms need to be agreed upon. The National Center for Health Statistics has recommended the World Health Organization's (WHO) definition of a live birth. According to WHO, "every product of conception that gives signs of life after birth, regardless of the length of the pregnancy, is considered a live birth."7 This definition thus distinguishes a live birth from a fetal death, which is defined as "death prior to the complete expulsion or extraction from its mother of a product of conception irrespective of the duration of pregnancy."8 Approximately half the States have adopted this definition, the remainder use minimum periods of gestation to define fetal deaths and live births.⁸ Table 1 shows the fetal death registration requirements for each State.

Infant deaths are usually divided into two categories according to age:

- Neonatal deaths are those which occur during the first 27 days of life.
- Postneonatal deaths are those which occur between 28 days and 1 year of age.

Mortality rates are then defined as follows (in the definitions, "period" refers to a calendar year or combination of more than 1 calendar year):

Neonatal mortality _	number of neonatal deaths during a period		1 0 0 0
rate per 1,000 (NMR)	number of live births during the same period	х	1,000
Postneonatal mortality	number of postneonatal deaths during a period	v	1,000
rate per 1,000 (PNMR)	number of live births during the same period	X	
Infant mortality rate per 1,000 (IMR)	number of infant deaths during a period	v	1 000
	number of live births during the same period	X	1,000

(Note that the infant mortality rate is the sum of the neonatal and postneonatal mortality rates, i.e., IMR = NMR + PNMR)

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State	Minmum period of gestation for which fetal-death registration is required	State	Minmum period of gestation for which fetal-death registration is required
Alabama	Advanced to 5th month (20 weeks) 20 weeks or more ¹ Advanced to 20th week All periods of gestation Advanced to 20th week ²	Missouri Montana Nebraska Nevada New Hampshire	After 20 weeks After 20 weeks Advanced to 20th week After 20 weeks 20 weeks or more
Colorado Connecticut Delaware District of Columbia . Florida	All periods of gestation 20 weeks or more 20 weeks or more After 5th month 20 weeks or more	New Jersey New Mexico New York North Carolina North Dakota	20 weeks or more 20 weeks or more All periods of gestation Advanced to 20th week 20 weeks or more
Georgia Hawaii Idaho Illinois Indiana	All periods of gestation All periods of gestation After 20 weeks 20 weeks or more 20 weeks or more	Ohio Oklahoma Oregon Pennsylvania Rhode Island	20 weeks or more 20 weeks or more All periods of gestation After 16 weeks 20 weeks or more ¹
Iowa Kansas Kentucky Louisiana Maine	Advanced to 5th month (20 weeks) Weight in excess of 350 grams 20 weeks or more After 20 weeks All periods of gestation	South Carolina South Dakota Tennessee Texas	After 20 weeks 20 weeks or more 20 weeks or more 20 weeks or more After 20 weeks
Maryland Massachusetts Michigan Minnesota Mississippi	20 weeks or more ³ 20 weeks or more Advanced through 20th week After 20 weeks All periods of gestation	Vermont	All periods of gestation All periods of gestation 20 weeks or more Advanced to 5th month ⁴ After 20 weeks After 20 weeks

Table 1. Minimum period of gestation for which fetal-death registration is required, by State.

¹Further specifies that certificates for fetal deaths of less than 20 weeks of gestation may be filed although registration is not

mandatory. ²Futher stipulated that "when the length of gestation is not determinable the following criteria may be used: A. weight of both of the above criteria are met the fetus should be 400 grams or more; B. crown-heel length of 28 centimeters or more. If both of the above criteria are met the fetus should be

registered as a fetal death." ³Further specifies that "if the gestation period is unknown but the fetus weights 400 grams or more, a fetal death certificate must be filed."

⁴Further specified "or total length of 10 inches."

The definition of fetal deaths presents some problem due to different practices among the States. The Guide to Data for Health Systems Planners⁹ recommends the following definitions of rates related to fetal mortality:

•	Fetal death	fetal deaths of 20 weeks or more gestation during a period		1.000
rate (FDR)	number of live births plus fetal deaths of 20 weeks or more gestation during the same period	X 1,000		
Perinatal • mortality = rate (PMR)	fetal deaths of 28 weeks or more gestation plus infant deaths under 7 days during a period	x 1,000		
	number of live births plus fetal deaths of 28 weeks or more gestation during the same period			

However, the data needed to compute the latter rate are not published in the Vital Statistics volume.⁸ County data in the volume refer only to infant, neonatal, and fetal deaths of 20 weeks or more gestation. If other data sources are unavailable, the following alternative definition of perinatal mortality rate may be used (Guide to Data for Health Systems Planners⁹ "method 2"):



This perinatal mortality rate can be obtained from the neonatal mortality and fetal death rates by the following formula:

PMR2 = NMR + FDR - (NMR)(FDR)/1,000

CAUSE-SPECIFIC AND RACE-SPECIFIC RATES

Since deaths are classified by cause and particular causes may point to problems amenable to planning intervention, certain cause-specific death rates are sometimes of interest. An infant mortality rate specific to a particular cause (X) is defined as

$$IMR(X) = \frac{\begin{array}{c} Number of deaths due to cause \\ X during a period \\ \hline Number of live births in the \\ same period \end{array}} \times 1,000$$

Of course, the rate may be limited to a certain age group in which case it becomes both

cause- and age-specific. The crucial point is that the denominator be defined as the population at risk of death (i.e., every case counted in the denominator should have a chance of appearing in the numerator).

Since the infant mortality rate (and related measures) varies by many characteristics of the mother and infant, an area's infant mortality rate will depend upon the composition of its childbearing population. One of the most important of these characteristics is race. Figure 1 defines race-specific rates and gives an example of how an area's total infant mortality rate may be misleading. Thus, when comparing areas or planning for services to meet the needs of high-risk groups, it is important to examine race-specific rates.

Numb	er of live births of speci	period	
The following are race-sp	ecific infant mortality ra	tes for the United States	in 1973:
Race	Number of births	Number of infant deaths	Infant mortality rate per 1,000 live births
Total	3,136,965	55,581	17.7
Chinese	7,425	39	5,3
Japanese	7,599	73	9.6
Vhite	2,551,030	40,239	15,8
ndian	26,464	501	18.9
Negro	512,597	14,411	28.1
All other	31,850	318	10.0
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Figure 1. Race-specific infant mortality rates.

Thus the total infant mortality rate depends upon both the race-specific rate and the proportion of all live births which occur in each race. Thus, when comparing two areas with different racial compositions the results could be misleading. For example, suppose an area has 40,000 white births with a white infant mortality rate of 13 per 1,000 and 10,000 Negro births with a Negro infant mortality rate of 35 per 1,000 (well above the national average). The total infant mortality rate for that area (assuming no births for other races) is $\frac{13 \times 40,000 + 35 \times 10,000}{17.4} = 17.4$.

50,000

If only the total infant mortality rate is examined, the area seems to be very similar to the national average, and the high mortality rate for Negro infants will be missed.

SOURCES OF DATA

As is evident from the definitions in the previous section, two items are needed in order to produce infant mortality data: deaths (neonatal, postneonatal, and fetal) and births. Birth data can serve alone as useful health status indicators and will be discussed in that context in a future issue. For the purpose of providing denominators for infant mortality, it is sufficient to know that birth data are available on the same geographic basis as mortality data.

A complete description of the sources and classification of mortality data is given in the *Guide to Data for Health Systems Planners*⁹ and *Vital Statistics of the United States,* 1971.⁸ The basic information is obtained from the death certificate. Various forms of mortality data for the United States can be obtained on magnetic tape from the National Center for Health Statistics.¹⁰

Published annual tabulations (most recently tor 1973) are also available for counties and urban places within counties.¹¹ More recent data and data for subcity units (e.g., census tracts) may be available from State and local health departments.⁶

LIMITATIONS OF THE DATA

Registration and Classification

Since the original sources of the data are the persons completing the birth and death certificates, there is potential for variations in registration of births and deaths, completeness and validity of certain items on the certificate, and classification of cause of death. As far as completeness of registration of births and infant deaths is concerned, it is generally believed that this presents no problems.² Registration of fetal deaths, however, does pose a problem.⁸ State requirements for registration of fetal deaths vary according to the minimum period of gestation used. Due to difficulties in determining the period of gestation it is likely that underregistration occurs, especially for fetal deaths near the lower limit of required registration. In addition some live-born infants who die very shortly after birth may be erroneously recorded as fetal deaths.⁸ Recent changes in abortion laws have resulted in an increase in legal abortions and

in problems with the classification of fetal deaths. Some States include fetal deaths due to induced abortions while others do not.

Cause-of-death classification also presents problems because a large portion of infant deaths occurs during the first few days of life. Nearly half of all infant deaths are classified in general "catchall" categories. Thus, one should be very cautious when using cause-specific rates. This is especially true when comparing cause-specific rates among areas with very different provider characteristics, since the cause-of-death information varies among physicians by specialty, age, etc.

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Stability

An area's observed infant mortality rate should be considered an estimate of the true underlying mortality rate.^b As is the case with any estimate, the infant mortality rate is subject to chance variation. If the area has very few births, the observed infant mortality rate may be very different from the true rate. Thus if two areas are compared in a given year and one (or both) of the area's rates is based on a small number of births, it would not be unusual to find the comparison reversed the following year. An example is shown is figure 2 where Ionia County's infant mortality rate in 1965 (31.7) was more than 20 percent higher than Wayne County's rate (26.3). The next year Ionia's rate (22.3) was slightly lower than Wayne's (25.0). However, nothing has really changed. The small number of births in Ionia County resulted in very imprecise estimates of Ionia's infant mortality rate.

Thus, a method is needed to assess the adequacy of the observed infant mortality rate as an estimate of its true value. The most common method is the use of confidence intervals which is explained in detail in the appendix. Basically a 95-percent confidence interval is defined so that the probability is 95 percent that the true rate is included in the interval. If the interval is very wide, the true rate is not known with much precision. The

^bThe model implied here is that the number of infant deaths in an area varies by chance depending upon the number of births and the probability of infant death (the "true" infant mortality rate). As the number of births increases the chance component becomes less important and the observed infant mortality rate is a better estimate of the true rate.



Figure 2. Effects of chance variation on the estimation of infant mortality rates.

Source: Rita Zemach, Ph.D., Department of Public Health, State of Michigan.

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interval generally becomes narrower as the number of births upon which the rate is based increases. Two common methods of increasing the numbers of births are to combine years and to combine smaller areas into larger ones. This will almost always be necessary. For example, during the 5-year period 1969-73, 35 percent of 3,073 U.S. counties had confidence intervals which were so wide that their infant mortality rates were meaningless.

Although aggregation over years and areas permits the computation of stable rates, there is a loss of information. Combining heterogeneous areas to obtain a stable rate may be more misleading than helpful. Combining years involves the assumption that in each of the years the ranking of the areas is the same, i.e., annual changes in the rates are the same for all areas.

The stability issue is especially important when comparing areas or determining whether "real" changes have occurred over time within an area. In these situations confidence limits should be used to assesss the magnitude of the differences. Two areas (or two time periods for one area) can be compared by using the absolute difference in their rates $(D=r_1-r_2)$ or by the ratio of their rates $(R = r_1/r_2)$. The latter measure of change is usually preferred since it allows for comparison of areas or time over a wide range of rates. For example, in 1935 the U.S. white infant mortality rate was 51.9 per 1,000. By 1955 the rate had dropped to 23.6, a decrease of nearly 30 per 1,000. Clearly the rate cannot be expected to drop by 30 per 1,000 again by 1975. However, it does make sense to express the 1935-55 decrease as 51.9/23.6=2.20 (i.e., the 1935 rate was 2.2 times the 1955 rate), and compare the 1955-75 ratio with the 1935-55 ratio. In fact the 1974 provisional rate is 14.7, which gives a ratio of 23.6/14.7 = 1.61, not quite as high as 2.20 but more comparable than the two differences (28.3 = 51.9 - 23.6 and 8.9 =23.6-14.7). The ratio of two rates is closely related to the proportional difference which is defined as either $D/r_1 = 1 \cdot 1/R$ or $D/r_2 = R \cdot 1$.

Another indication of the usefulness of the ratio (or relative risk as it is sometimes called) occurs when rates are being used for evaluation. If the infant mortality rate is being monitored over time as additional resources are being phased into a community, the absolute changes in infant mortality rate will almost always decrease, suggesting diminishing returns to the investment of resources. However, the relative changes in mortality may even be increasing.^c

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Differences Between Numerator and Denominator Data

Since infant mortality rates are calculated by comparing infant deaths to live births during the same year instead of by following the cohort of births to determine the cohort's mortality experience, the numerator and denominator in the rate may relate to different populations. For example, an urban renewal project may result in a rapid, sudden change in the characteristics of an area's population. In this case the infant deaths during the year would be compared to a very different population of births for that year and the rate could be misleading. These types of population shifts should be kept in mind when analyzing small area mortality rates.

In addition, the source of information for the numerator is the death certificate while the source for the denominator is the birth certificate. Lack of comparability in certain items has been noted in special studies which match birth and death certificates for the same infant. For example, a study in Minnesota¹² showed that 2.4 percent of the 2,120births with deaths under 1 day had a different county of residence on the death certificate. Discrepancies in designation of race were also noted-of 117 infant deaths with Negro recorded on the birth certificate, 7 (6 percent) had another race indicated on the death certificate and of 97 Indian infant deaths, 28 (29 percent) were recorded as white on the death certificate.

^cThis could happen if the rate begins at 50 per 1,000, decreases to 30 at a second time period, and then to 15 at a third period. The absolute changes are -20 and -15 while the relative changes are 1.67 and 2. An advantage of the absolute change is that it is easily translatable into excess deaths or lives saved and this may be a more appropriate measure (especially when dealing with allocation of scarce resources). If there were 10,000 births at each period in the example cited above, the decrease from 50 to 30 resulted in 200 lives saved while the decrease from 30 to 15 resulted in 150 lives saved.

RELEVANCE

Planning

Although an area's infant mortality rate (and its components) may be "explained" by the demographic and socioeconomic characteristics of the area, the need for maternal and infant care programs in an area with high rates is evident. The form which these programs take should be determined in part by infant mortality and other related data. Take, for example, an area with unusually high neonatal mortality. If the area had no intensive care nurseries, it may be in need of such services. On the other hand, if the area already had sufficient intensive care nurseries, the need might be more closely related to making prenatal care more accessible or to emphasizing the educational component of prenatal care. Similarly high fetal mortality rates would indicate the need for more prenatal or even preconception care (although the completeness of fetal deaths registration should be kept in mind as a possible explanation). Causespecific perinatal mortality data might be especially useful in suggesting genetic counseling linked to family planning.

Many States routinely match their birth and infant death certificates. In such States the potential for using such data in planning is even greater because information about the infants who died is available from the birth certificate as well as the death certificate. For example birth certificates in 39 States and the District of Columbia include data on the month of pregnancy in which prenatal care began. By comparing this item for mothers of infants who died with other mothers, more direct information becomes available about whether prenatal care is accessible to high-risk mothers. It also enables the planner to determine for a particular area which maternal factors present the greatest potential for infant mortality. Programs directed at such women may then be initiated.

Evaluation

Monitoring infant mortality over time is a potential method for evaluating the outcome of health programs directed at mothers and infants. There are, however, a number of problems which must be considered in using mortality data. One of these, the stability problem (p. 4), requires that several years of data be combined. Thus, results of an evaluation will not be available for immediate feedback.

Comparison with areas which do not have programs but which are similar in other characteristics to the areas with programs are also helpful to rule out the possibility of something other than the program being responsible for any decrease.¹³ If such areas are not available, the rates for larger units (e.g., other cities, U.S. rural counties) may be used. In addition, some corrections to the raw rates for other variables (e.g., based on multiple regression analysis) are often necessary.

Another point which should be considered is the use of specific components of infant mortality to evaluate particular programs. For example, the postneonatal mortality rate should be used to evaluate a program designed to reduce postneonatal mortality. If the infant mortality rate is used, any effects of the program might be overlooked since postneonatal mortality accounts for only one quarter of infant mortality. REFERENCES

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APPENDIX

ASSESSING STABILITY OF RATES AND CHANGES IN RATES

As indicated on page 4, an area's infant mortality rate (or any other rate) cannot be taken as the true rate for that area. It is an estimate and as such its variability must be assessed. The simplest method for doing this is the computation of a 95-percent confidence interval. This interval is defined so that it has 95-percent probability of including the true rate. The formula and an example are given in figure 3. The computations shown are derived

under the assumption that the number of deaths in an area has a Poisson distribution.¹⁴

A useful rule is that any rate based on fewer than 20 cases in the numerator will have a 95-percent confidence interval which is about as wide as the rate itself (i.e., from 0.5rto 1.5r). Roughly speaking, this means all that can confidently be said about an area with 20 deaths out of, say, 1,000 live births is that the true rate is within 20 ± 10 per 1,000. Clearly

Figure 3. Confidence intervals for rates.



The interval (19.3, 40.7) is much narrower than the one in the first situation, but it still shows that the area's true rate is not known with much precision.

this is not very precise information. Of course for any rate based on more than 20 deaths, it is important to know the confidence limits in order to determine just how precisely the true rate is estimated.

On page 6, it was suggested that the ratio of two rates be used to compare areas or measure change in one area between two time periods. Figure 4 shows the formula for estimating 95-percent confidence limits for the ratio and gives an example. If the 95-percent confidence interval for the ratio of two rates does not include the value 1, the rates are significantly different at the 5-percent level.^d

The formula given in figure 4 is valid only when the two rates are independent. This means that they refer to completely different areas or time periods: no birth or death which is included in one rate should be included in the other. In addition, the formula is valid only when the rate in the denominator is based upon more than 100 deaths.

Figure 5 gives the formula for obtaining a 95-percent confidence interval for the difference between two independent rates. If this interval does not include zero, the rates are significantly different at the 5-percent level.^e Note that the significance tests based on the two methods will occasionally give different results, i.e., the confidence interval for the ratio might include one but the confidence interval for the difference might not include zero. If this happens it is safest to conclude that the two rates are not significantly different.

^dThis method of testing the significance of the difference between two rates will give slightly different results than the standard method (see Armitage,¹⁴ pp. 129-131). However, the ease with which confidence limits for the ratio are obtained by this method lead to the recommendation for its use.

^eThis method of testing the significance of the difference between two rates will also give slightly different results than the standard method (see footnote d). The difference between the two is that in the significance testing approach one begins with the null hypothesis assumption that the two rates are equal and uses a pooled estimate of error, while in the confidence interval approach no such assumption is made.

Figure 4. Confidence intervals for the ratio of two independent rates.

Let $r_1 = \text{rate for period 1 (or area 1)}$ $d_1 = \text{number of deaths for period 1 (or area 1)}$ $r_2 = \text{rate for period 2}$ $d_2 = \text{number of deaths for period 2}$ $R = r_1/r_2$

Then the limits of the 95-percent confidence interval are

upper limit:
$$R + 1.96R \sqrt{\frac{1}{d_1} + \frac{1}{d_2}}$$

lower limit: $R - 1.96R \sqrt{\frac{1}{d_1} + \frac{1}{d_2}}$

Consider the following example:

	Year	Number of infant deaths	Number of live births	Infant mortality rate per 1,000
1961-65		200	5,000	40
1966-70	********	100	4,000	25

$$R = \frac{40}{25} = 1.6$$

1.96R $\sqrt{\frac{1}{d_1} + \frac{1}{d_2}} = 1.96 (1.6) (.1225) = .384$

upper limit: 1.6 + .384 = 1.984

lower limit: 1.6 - .384 = 1.216

Thus the rate in 1961-65 is from 1.22 to 1.98 times the 1966-70 rate with 95-percent confidence. Since this interval does not include 1, there was a statistically significant (P < .05) decrease in the area's infant mortality rate.

The confidence interval for the ratio of two independent rates can also be easily obtained from the confidence intervals for each rate. If the confidence intervals for each rate are

$$r_1 \pm 61.981 \sqrt{\frac{r_1}{n_1}} = r_1 \pm CL_1$$

 $r_2 \pm 61.981 \sqrt{\frac{r_2}{n_2}} = r_2 \pm CL_2$

then the confidence interval for $R = \frac{r_1}{r_2}$ is

$$R \pm R \sqrt{\left(\frac{CL_1}{r_1}\right)^2 + \left(\frac{CL_2}{r_2}\right)^2}$$

 r_1 = rate for period 1 (or area 1) n_1 = denominator upon which r_1 is based r_2 = rate for period 2 (or area 2) n_2 = denominator upon which r_2 is based $D = r_1 - r_2$

The limits of the 95-percent confidence interval are

upper limit: D + 61.981 $\sqrt{\frac{r_1}{n_1} + \frac{r_2}{n_2}}$ lower limit: D - 61.981 $\sqrt{\frac{r_1}{n_1} + \frac{r_2}{n_2}}$

Using the same example as in figure 4,

$$D = 40 - 25 = 15$$

$$\frac{r_1}{n_1} = \frac{40}{5,000} = .008$$

$$\frac{r_2}{n_2} = \frac{25}{4,000} = .00625$$

$$\sqrt{\frac{r_1}{n_1} + \frac{r_2}{n_2}} = \sqrt{.01425} = .1194$$

$$61.981 \quad \sqrt{\frac{r_1}{n_1} + \frac{r_2}{n_2}} = 7.399$$
upper limit: 15 + 7.399 = 22.399
lower limit: 15 - 7.399 = 7.601

Thus the difference between the two rates is between 7.6 and 22.4 with 95-percent confidence. Since this interval does not include zero, the rates are significantly different at the 5-percent level.

The confidence interval for the difference between two independent rates can also be easily obtained from the confidence intervals for each rate. If the confidence intervals for each rate are

$$r_1 \pm 61.981 \sqrt{\frac{r_1}{n_1}} = r_1 \pm CL_1$$

 $r_2 \pm 61.981 \sqrt{\frac{r_2}{n_2}} = r_2 \pm CL_2$

then the confidence interval for $D = r_1 - r_2$ is

$$D \pm \sqrt{CL_1^2 + CL_2^2}$$