RI 8869

Bureau of Mines Report of Investigations/1984

Apparent Earth Conductivity Over Coal Mines as Estimated From Through-the-Earth Electromagnetic Transmission Tests

By John Durkin

UNITED STATES DEPARTMENT OF THE INTERIOR

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Library of Congress Cataloging in Publication Data:

Durkin, John

Apparent earth conductivity over coal mines as estimated from through-the-earth electromagnetic transmission tests.

(Report of investigations / United States Department of the Interior, Bureau of Mines ; 8869)

Bibliography: p. 16.

Supt. of Docs. no.: I 28.23:8869.

1. Earth resistance—Measurement. 2. Electromagnetic waves—Transmission—Measurement. 3. Coal mines and mining—United States. I. Title. II. Series: Report of investigations (United States. Bureau of Mines) ; 8869.

QC809.E15D87 1984 551.1'4 83-600385

UNIT OF MEASURE ABBREVIATIONS USED IN THIS REPORT

A	ampere	m^2	meter squared
A/m	ampere per meter	μA	microampere
$A\text{-}m^2$	ampere-meter squared	μA/m	microampere per meter
dB	decibel	Ω	ohm
ft	foot	pct	percent
H/m	henry per meter	rad/s	radian per second
Hz	hertz	s	second
m	meter		

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APPARENT EARTH CONDUCTIVITY OVER COAL MINES AS ESTIMATED FROM THROUGH-THE-EARTH ELECTROMAGNETIC TRANSMISSION TESTS

By John Durkin [1]

ABSTRACT

Electromagnetic narrow-band signals were transmitted through the earth at 27 coal mines located throughout the United States. From those Bureau of Mines tests, apparent earth conductivity values were derived based upon a homogeneous half-space model of the earth. The derived conductivity values were found to be inversely proportional to the transmitted frequency and mine depth. A linear regression model relating the logarithm of the conductivity to the mine depth was formulated, and the results indicate that the mine depth can be an adequate predictor of the apparent earth conductivity above coal mines. Apparent earth conductivity was found to decrease with mine depth.

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INTRODUCTION

Earth conductivity (or earth resistance) has been the topic of many studies over the years. Earth resistance is the resistance of the earth to the passage of an electrical current. For the most part, the earth is not a good conductor of current, and in comparison with metallic conductors, it is extremely poor. The earth was once used as a conductor for power distribution, and for safety considerations, the earth resistance was a matter of considerable importance.

Surveying the conductivity structure of the earth has been a frequent practice for different applications. This work has enabled the civil engineer to determine the depth of bedrock or the geologist to find the depth of the water table. Conductivity surveying has been found useful in detecting and locating ore bodies or mineral lodes and has recently become popular among treasure hunters in locating lost items.

There are many factors that influence the conductivity of the earth; the principal ones are type of soil, moisture content, chemical composition and concentration of salts dissolved in the contained water, and temperature. These factors then give rise to earth conductivity variations both in time and location. Therefore, the only meaningful way of estimating the value of the earth conductivity is to measure it.

EARTH CONDUCTIVITY MEASUREMENT TECHNIQUES

There are numerous techniques for making earth conductivity measurements. These techniques can be broadly classified into three distinct approaches. The first approach calls for removing a sample of the media and making a conductivity measurement. In the second approach, a drillhole is made and instrumentation to make the conductivity measurement is then lowered into the hole. The third approach involves probing the earth electromagnetically with a transmitter and receiver where conductivity information is contained in the received signal.

The choice of the proper approach depends mostly on the manner in which the conductivity information will be applied. In this report, the application is for predicting the influence of the conducting earth on a through-the-earth (TTE) communications system. The third approach, electromagnetic probing, appears to be the most closely related to this application.

A number of techniques are available for this type of probing, including the well-known four-electrode method (1)[2] in which an electrical potential is applied between probes on the surface, and the resulting potential between another spaced pair is observed. Other techniques employ current-carrying coils of wire as magnetic dipole probes (2). In addition, several passive techniques are employed that make use of the natural electric and magnetic fields existing at the earth's surface (3). Informative summary texts can be found in Keller and Frischknect (4, chapters 4-7) or Grant and West (5). Excellent theoretical background work can be found in Wait (3). There is also an Institute of Electrical and Electronics Engineers guide that describes the application and interpretation of these techniques (6).

In addition to all of these techniques, there is another basic approach that is more aligned with applications for TTE communications. This technique can be described as transmission path

[2]Underlined numbers in parentheses refer to items in the list of references preceding the appendixes.

attenuation tests. By transmitting a signal of known level and frequency from a transmitter located within the earth to a receiver located on the surface, information on the signal's path conductivity can be inferred from the received signal strength. This approach was taken in this study, and the results are discussed in the next section.

TTE TRANSMISSION TESTS

MINE SELECTION

Since experimental results were intended to be representative of a large number of active mines, each with a unique physical and operational characteristic, it was necessary to utilize some type of sampling process to determine specific test locations for the program. By involving statistical sampling theory in the selection process, it was possible to (1) assure the validity of estimated performance measures subsequently derived from test results, (2) increase the precision of these estimates, and (3) reduce or estimate the possibility of built-in biases in interpreting test results.

The choice of the mines could have been done on a random basis, in which case each mine would have an equal probability of being selected; however, it was decided that the sampling plan should consider two other points. First, since TTE communications are believed to be very effective at small depths and less effective at deeper depths, the distribution of mines selected should take into account the greater variability in test results anticipated at greater depths. Also, within each depth interval of mine selection, emphasis should be placed on selecting the mines having the largest number of miners employed, thus giving rise to a selection that covers the most miners possible.

The population considered was identified from a computer listing of 1,222 active mines in the United States. This computer listing was constructed by the Bureau of Mines Pittsburgh Research Center (PRC) from two independent computer data bases, one from the Mine Safety and Health Administration (MSHA), which contained data on the number of miners at each mine, and one from the Bureau's Eastern Field Operations Center, which contained data on the maximum overburden depth at each mine. The compiled listing included the mine name, address, maximum overburden depth, number of miners employed, and MSHA identification number. All mines were subsequently stratified according to maximum mine overburden depth values into 15 different depth intervals. Total miners employed at all mines contained within each depth interval were also tabulated. Although the sample could have been allocated so that the ratio of the number of mines sampled to the total number of mines would be constant within each interval, it was decided to vary the sampling fraction based upon the first consideration above: that a greater variability in results was anticipated at greater depths.

In determining the number of mines to be sampled in a given depth interval, a technique known as optimum allocation was used. This technique is based upon the principle that larger samples are required in depth intervals that exhibit greater variability. This principle can be expressed as follows:

$$n_h = \frac{n\ N_h S_h}{\Sigma\ N_h S_h}, \qquad (1)$$

where n_h = sample size for the h-th depth interval,

N_h = total number of mines in the h-th depth interval,

S_h = variance of the characteristic being measured in the h-th stratum (e.g., the estimated probability of successful transmission at a specific frequency),

and n = total sample size.

By estimating the relative variability S_h, the number of mines to be tested in a given depth interval was determined. Also, as stated above, the sample selection within each depth interval was based on the number of miners employed at each mine.

The important features of the sampling procedure used in this study are summarized below.

- Each mine had a chance of being selected for this test.

- The chance (i.e., the probability of selection) was known beforehand and was based on the relative size of the mine in terms of miners employed.

- The selection process was random.

- All depth intervals were represented.

- Test results could be used to make valid inferences about all mines.

However, as in any field test program of this size, what is initially hoped for and planned and what is actually performed are often two different things. Although a reasonable attempt was made to visit the mines selected, necessity, mine availability, and practical travel schedule constraints introduced deviations from the originally selected sample of mines. However, it is believed that these deviations do not significantly affect or bias the results derived from the data. Care was taken to adhere to the spirit and form of the original selection list while coping with the realities imposed on the practical implementation of such an extensive field measurement study.

At the completion of the test program, measurements had been performed at 27 mine sites well distributed throughout the United States coalfields. The names of the mines tested can be found in appendix A.

TEST DESCRIPTION AND DATA

The primary objective of this field test was to obtain a data base on the transmission loss incurred by a signal as transmitted through the earth. At each mine site, a minimum of two test personnel were used, one underground and one on the surface, positioned approximately above the underground individual.

Underground, a loop of wire of known length, in a configuration that can be described loosely as being between a circle and a rectangle, was deployed around one or two coal pillars. The ends of this wire loop were attached to a small and lightweight electromagnetic (EM) narrow-band transmitter. This transmitter produces a pulsed square wave output voltage with an on-time of 0.1 s and an off-time of 0.9 s, and a frequency of 630, 1,050, 1,950 or 3,030 Hz.

On the surface, recordings were made of received signals. The signal recordings were returned to the laboratory where signal levels were reduced using a fast Fourier transform (FFT).

Table 1 gives the number of the mine tested, mine depth, underground antenna area, offset parameter D (which is equal to the ratio of the distance between the vertical axis of the underground antenna and the observation point to the mine depth), and parameter A (which is equal to the ratio of the radius of the underground antenna, with the assumption that the antenna may be represented as a circle, to the mine depth).

TABLE 1. - Mine depth and physical arrangement for TTE transmission tests

Mine	Mine depth, m	Transmit antenna area, m^2	D	A
1.......	143	279	0	0.07
2.......	58	195	.29	.14
3.......	473	929	0	.04
4.......	116	126	.06	.05
5.......	122	335	0	.08
6.......	279	543	0	.05
7.......	131	418	.11	.09
8.......	201	547	.01	.07
9.......	128	520	.14	.10
10......	198	256	.11	.05
11......	88	522	.03	.15
12......	165	223	.14	.05
13......	171	282	0	.06
14......	198	565	.02	.07
15......	21	28	.48	.14
16......	79	335	.04	.13
17......	366	929	.03	.05
18......	152	455	0	.08
19......	183	130	0	.04
20......	153	232	.16	.06
21......	80	455	0	.15
22......	365	344	.03	.03
23......	140	238	.02	.06
24......	219	557	.00	.06
25......	86	204	.35	.09
26......	67	79	.14	.07
27......	46	132	1.70	.14

Table 2 gives the mine tested, transmitter magnetic moment (M), and received surface vertical magnetic signal (H_z). The antenna current, which was used in calculating the magnetic moment, was measured by placing a 0.1-Ω resistor in series with the transmit antenna and measuring the voltage across it.

The mine depths were provided by the mining companies when the tests were performed. This information is obtained from topographical maps that are very accurate, and the depth estimate of the in-mine transmitter can be obtained within an uncertainty of approximately ±1 pct when the surface position is near a survey point. However, owing to the natural variation in the terrain above coal mines, this estimate of uncertainty in depth may be larger for shallow mines if the surface position is not near a survey point.

The underground antenna length was measured using a conventional tape-measuring rule, and the estimated uncertainty in this measurement is expected to be much less than ±1 pct. Using the mine maps coupled with the approximate deployment configuration and knowledge of the length of the loop, the underground antenna area was calculated, and this estimate is believed to be accurate within ±10 pct.

It should also be mentioned that the position of the underground transmitter was determined by measuring the horizontal signal field at different locations and, by finding the null in the field, finding the underground location by triangulation. This location measurement technique had the added benefit of assuring that the transmitted signal was actually traversing the earth medium and was not conducted along pipes, wires, or other conductors.

The A parameter was defined as the ratio of the radius of the underground loop to the mine depth. However, the underground loop was deployed into a configuration that was between a circle and a rectangle. Therefore, the estimate of the radius, which assumes the antenna length represents the circumference of a circle, is in error. In later estimates of conductivity, this point will need to be considered if the A value becomes large, becoming less of a dipole situation.

Assuming that the transmit current measurement is accurate, the accuracy of the estimate of the magnetic moment is related to the accuracy of the antenna area estimate, discussed earlier. It is concluded that the uncertainty in the magnetic moment is the same as the uncertainty of the area estimate, ±10 pct.

The signal strength measurements were made by playing the tapes into an FFT instrument. Signal averaging was done with 128 independent samples. The

TABLE 2. - Surface vertical magnetic signal (H_z) (in decibels re 1.0 μA/m-RMS) and underground transmit moment (M) (in ampere-meters squared), at different frequencies

Mine	630 Hz		1,050 Hz		1,950 Hz		3,030 Hz	
	H_z	M	H_z	M	H_z	M	H_z	M
1..................	7.1	923	4.9	769	-1.3	515	-11.6	326
2..................	34.1	747	31.9	625	24.6	407	20.4	288
3..................	-4.9	2,031	-10	1,573	-19.3	928	-26.5	620
4..................	27.1	532	21.9	456	17.6	326	13.4	231
5..................	30.1	1,093	28.0	868	21.7	565	16.4	385
6..................	ND	ND	ND	ND	-8.4	684	ND	ND
7..................	ND	ND	ND	ND	29.5	611	22.1	412
8..................	24.1	1,477	20.0	1,142	ND	722	ND	478
9..................	34.1	1,434	32.0	1,123	ND	698	ND	476
10.................	ND	903	ND	739	6.6	496	.4	343
11.................	43.1	1,444	39.9	1,120	29.7	705	23.4	477
12.................	ND	835	ND	661	ND	469	ND	333
13.................	22.1	935	11.9	785	5.7	521	-1.6	349
14.................	ND	1,524	ND	1,186	ND	734	ND	ND
15.................	49.1	196	48.0	141	41.7	87	38.4	58
16.................	41.1	1,092	39.9	871	28.7	570	29.4	387
17.................	8.1	2,053	3.9	1,532	-6.3	923	-11.6	619
18.................	24.1	1,334	20.9	1,028	11.6	649	6.4	442
19.................	ND	555	10.9	488	-1.3	346	-9.5	271
20.................	ND	ND	10.4	698	ND	453	-9.9	332
21.................	40.1	1,315	38.9	1,046	33.6	662	26.4	448
22.................	ND	ND	ND	818	ND	ND	ND	ND
23.................	27.1	852	22.0	713	12.4	466	ND	320
24.................	21.5	1,392	16.6	1,167	9.2	745	2.7	553
25.................	31.7	549	29.3	471	26.1	316	24.6	212
26.................	30.7	211	ND	ND	28.3	148	25.5	104
27.................	32.0	324	ND	ND	28.5	212	25.5	150

ND No data.

information needed was the root-mean-square (RMS) value of the signal during transmit time. Since the transmitted signal had a duty cycle associated with it, this had to be first measured for each transmitter and used with the FFT measurement to derive the on-time signal level. It was then found that the 95-pct confidence interval for the signal strength measurement was -1.44 to 1.56 dB. From these field tests, data on earth transmission loss can be obtained. Implicit in these transmission loss data is information on apparent earth conductivity. It is then the objective of this study to derive estimates of apparent earth conductivities from the signal transmission data.

EARTH CONDUCTIVITY ESTIMATES

As an electrostatic EM signal is transmitted TTE, it experiences attenuation from two sources. The first is the attenuation from geometric spreading, which falls off as the distance cubed. The second is the attenuation caused by the conducting earth, which can be used in TTE transmission experiments to gather information on the earth conductivity.

The vertical magnetic signal received on the surface from an underground transmitter can be expressed as

$$H_z = \frac{M}{2\pi d^3} Q, \qquad (2)$$

where H_z = surface vertical magnetic field, A/m,

M = magnetic mount, A-m^2

(I = transmit current, A,

N = number of turns of the transmit antenna),

d = mine depth, m,

and Q = attenuation factor of the conducting earth.

The functional form of Q depends upon the conductivity model of the earth. The value of Q would be 1 for a nonconducting earth and, in general, decreases with increases in earth conductivity. Therefore, though Q can be used to determine the value of the earth conductivity, the appropriate model must be used. Choosing a particular model and using the data from the field tests, conductivity values can be obtained, and these values, in turn, can be studied to verify the choice of the model. If the model appears acceptable, a statistical study of the conductivity data can be done to obtain information on expected or variations in earth conductivity.

Wait and Spies (7) give the functional form of Q for a homogeneous half-space model of the earth as

$$Q = \int_0^\infty \frac{J_1(Ax)}{(Ax/2)} \frac{x^3}{(x^2+iH^2)^{1/2}+x}$$

$$\exp\left[-(x^2+id^2)^{1/2}\right] J_0(Dx)dx, \qquad (3)$$

where $H = (\sigma\mu_0\omega)^{1/2}d$,

σ = earth conductivity, mho/m,

μ_0 = free space permeability, H/m,

ω = angular frequency, rad/s,

d = mine depth, m,

A = ratio of the loop radius to mine depth,

and D = ratio of the offset to mine depth.

From the field test values given in tables 1 and 2, apparent earth conductivity values were found using this form of Q. These values are given in table 3 for each mine and for each frequency tested. Also given in this table are Q values derived from equation 2. As discussed in the previous section, an expected uncertainty existed in the measurement of the signal, magnetic moment, and mine depth. Assuming the simplest case of a coaxial dipole measurement scheme (A = D = 0), an uncertainty expression for Q can be derived over small regions of Q where linearity can be assumed. This can be written as

$$\sigma_Q^2 = \left(\frac{\partial Q}{\partial H_z}\right)^2 \sigma_{H_z}^2 + \left(\frac{\partial Q}{\partial d}\right)^2 \sigma_d^2 + \left(\frac{\partial Q}{\partial M}\right) \sigma_M^2 \qquad (4)$$

or

$$\left(\frac{\sigma_Q}{Q}\right)^2 = \left(\frac{\sigma_{H_z}}{H_z}\right)^2 + 9\left(\frac{\sigma_d}{d}\right)^2 + \left(\frac{\sigma_M}{M}\right)^2. \qquad (5)$$

From the earlier stated values of expected error in these parameters, the resultant expected error in Q would then be approximately ±20 pct.

TABLE 3. - Q values and apparent earth conductivity values (σ) as derived from a half-space earth model, at different frequencies

Mine	630 Hz		1,050 Hz		1,950 Hz		3,030 Hz	
	σ	Q	σ	Q	σ	Q	σ	Q
1.........	0.589	0.045	0.366	0.042	0.225	0.031	0.190	0.015
2.........	1.961	.083	1.235	.077	.831	.051	.571	.045
3.........	.024	.186	.018	.133	.014	.077	.011	.051
4.........	.176	.418	.174	.266	.108	.227	.078	.198
5.........	.210	.334	.128	.328	.092	.245	.071	.196
6.........	ND	ND	ND	ND	.039	.076	ND	ND
7.........	ND	ND	ND	ND	.016	.687	.025	.436
8.........	.039	.552	.033	.442	ND	ND	ND	ND
9.........	.110	.466	.066	.465	ND	ND	ND	ND
10........	ND	ND	ND	ND	.038	.209	.032	.149
11........	.290	.424	.203	.376	.216	.185	.175	.133
12........	ND	ND	ND	ND	ND	ND	ND	ND
13........	.080	.428	.124	.157	.082	.116	.067	.075
14........	ND	ND	ND	ND	ND	ND	ND	ND
15........	9.476	.085	4.893	.103	3.143	.081	1.988	.084
16........	.494	.326	.271	.354	.311	.149	.139	.239
17........	.020	.381	.015	.313	.014	.161	.011	.131
18........	1.171	.265	.113	.237	.096	.129	.070	.104
19........	ND	ND	.069	.275	.080	.095	.073	.048
20........	ND	ND	.184	.105	ND	ND	.136	.022
21........	.633	.248	.352	.270	.218	.232	.195	.150
22........	ND	ND	ND	ND	ND	ND	ND	ND
23........	.108	.458	.107	.303	.101	.154	ND	ND
24........	.031	.567	.034	.382	.028	.255	.025	.164
25........	.282	.280	.196	.247	.102	.255	.049	.320
26........	.693	.308	ND	ND	.204	.334	.127	.344
27........	.001	.074	ND	ND	.0	.075	.0	.075

ND No data.

The error in estimated conductivity is related to this error in Q. However, this relationship is nonlinear and is dependent upon frequency, depth, and the value of Q. A typical relationship, one that will be of most interest, is shown in figure 1 where the error in conductivity estimate is normalized to the error in Q for a frequency of 3,030 Hz and a mine depth of 200 m. Relationships for other frequencies and depths are similar to that shown in this figure. As can be seen from figure 1, estimates of conductivity can be found with the same uncertainty in Q for values of Q around 0.3. For small values of Q, a situation that exists for the majority of the field data, the uncertainty in conductivity estimation is less than the uncertainty for the Q measurement (<20 pct). For values of Q greater than 0.3, the uncertainty of the conductivity estimate is greater than the uncertainty of the Q estimate (>20 pct).

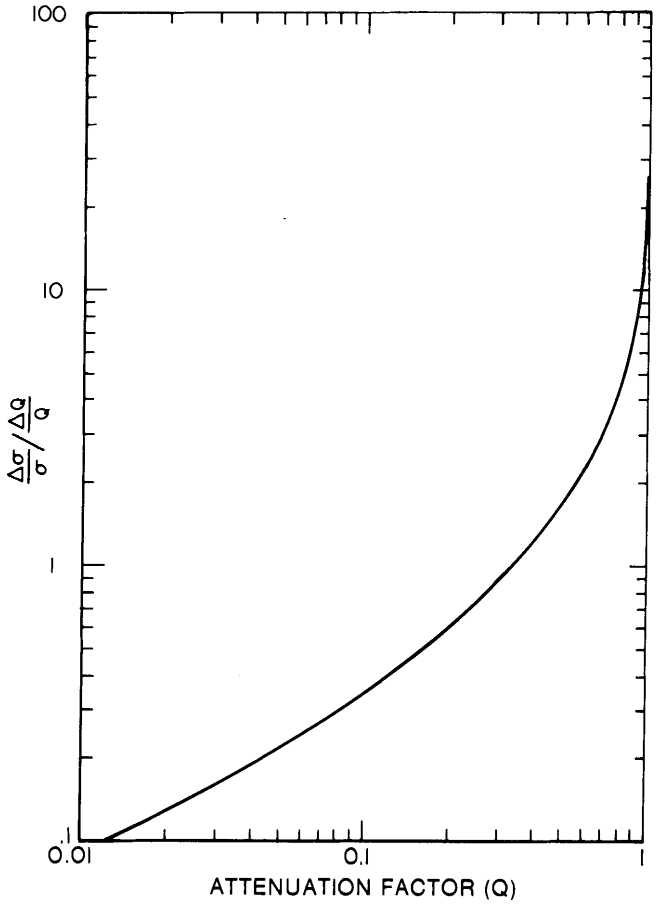

FIGURE 1. - Uncertainty in conductivity normalized to the uncertainty in Q for a depth of 200 m and a frequency of 3,030 Hz.

STUDY OF CONDUCTIVITY DATA

The preceding section described how apparent earth conductivity estimates were made based on a homogeneous half-space model of the earth. The data provide estimates for different frequencies and for mine sites having different depths. Conductivity estimates obtained for Q values greater than 0.3 must be checked for the possibility of large errors. Also, as discussed in the section "Test Description and Data," depth estimates contained errors on the order of 1 pct, and this error value might be valid only in cases where the mine depth was large enough (d > 100 m) that the natural contour of the land between survey points would not influence the depth estimate.

Considering these points, it was decided to choose a subset of the conductivity estimates that were obtained at mines exceeding a certain depth and where the Q estimates were less than some given value. The choice made was to consider only those mines where d > 100 m and Q < 0.5. It is felt that, with the choice of these data points, the conductivity estimate would be valid with an upper uncertainty of approximately ±23 pct.

An observation of the data from table 3 shows that the conductivity estimate at most mines tends to decrease with frequency, which contradicts the homogeneous half-space assumption. If all of the mine data for which d > 100 m and Q < 0.5 are pooled and segregated by frequency, approximate curves for the cumulative distributions of the conductivity data (fig. 2) also show that the conductivity tends to decrease with frequency.

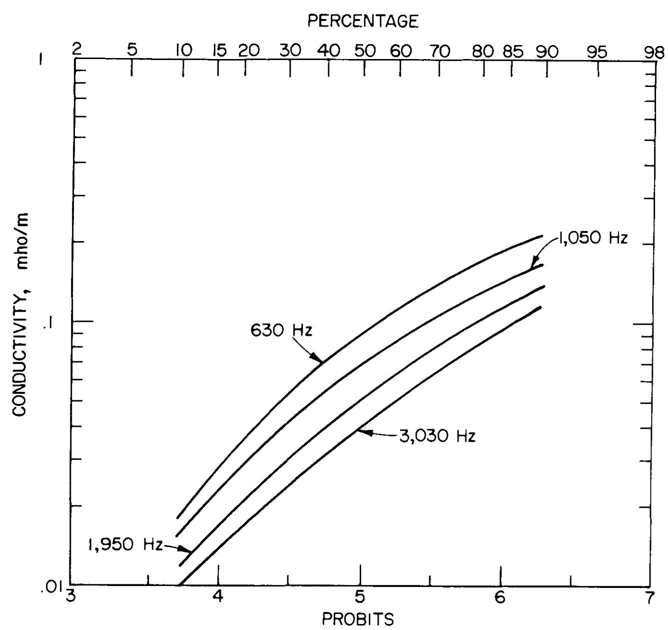

FIGURE 2. - Cumulative distributions of conductivity estimates segregated by frequency.

Besides the frequency dependency, the conductivity also appears to show a dependency on depth. This is illustrated in table 4 where mean and standard deviation values of the conductivity are given for different depth intervals. The conductivity appears to decrease with mine depth. It can be seen from table 4 that for a given frequency, the apparent conductivity decreases by about a factor of 10 over the depth range, with slightly smaller differences for an increase in frequency. Also, it can be seen that in a given depth interval, the apparent conductivity decreases by a factor of 2 with frequency over the frequency range.

Neither the frequency nor depth dependency of the conductivity values would be expected for a homogeneous half-space model of the earth. Geyer (8) found that conductivity was generally higher near the surface above a number of mines. A high conductivity surface layer could explain the trend of apparent conductivity with depth for the data reported here, since the weighted contribution of this layer would be proportionally higher the closer the transmitter was to the measurement point. However, the existence of this surface layer would not explain the frequency dependence of the conductivity data.

An alternate earth conductivity model proposed by Durkin[3] and studied by Shope (9) proposes an earth model having a thin, highly conducting surface layer over a lower conducting layer in which the transmit loop is buried, followed by a higher conducting half-space. Results by Shope (9) indicate that this model may explain both the frequency and depth dependency of the conductivity data, but studies are continuing. A related model proposed by Hill (10) consists of a magnetic dipole source buried in an earth consisting of a highly conducting thin sheet located over a homogeneous half-space. This model is interesting because it is simple, shows the conductivity depth dependency, and when the upper conductivity layer is considered extremely thin, demonstrates the general trend in the frequency dependency found in the conductivity data.

[3]Further information on this model is available upon request from J. Durkin, Pittsburgh Res. Cent., BuMines, Pittsburgh, PA.

TABLE 4. - Mean and standard deviation of conductivity values (in mho per meter) for each frequency within different depth intervals for estimates having Q < 0.5

Freq, Hz	100-150 m		151-200 m		201-300 m		Over 300 m	
	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev
630.......	0.238	0.201	0.125	0.064	ND	ND	0.022	0.003
1,050.....	.168	.117	.123	.047	0.034	ND	.017	.002
1,950.....	.131	.063	.074	.025	.033	0.008	.014	.001
3,030.....	.091	.070	.076	.038	.025	ND	.011	ND
All freq..	.161	.132	.094	.044	.032	.005	.016	.005

ND No data.

In the next section, following the work by Hill (10), a homogeneous conducting earth model containing a thin sheet is studied for its influence on apparent conductivity estimates for transmission with a finite loop transmitting antenna.

THIN CONDUCTING SHEET EARTH MODEL

Figure 3 shows a vertical magnetic loop buried in a homogeneous earth containing a thin conducting sheet at the surface. The homogeneous earth has a conductivity σ_0, and free space permeability μ_0 is assumed everywhere. The thin conducting sheet has a small thickness d and a conductivity-thickness product σd. The propagation constant-distance product

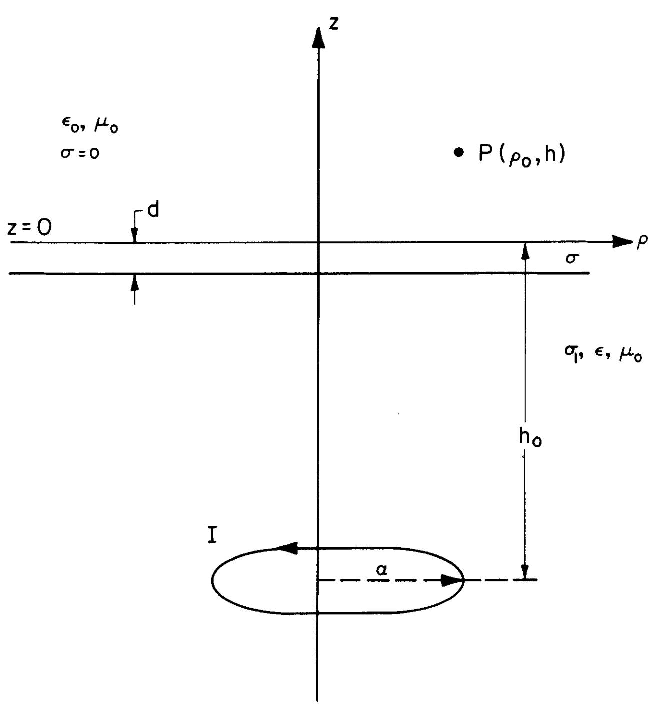

FIGURE 3. - Geometry for a finite magnetic loop of current buried in a conducting half-space earth containing a highly conducting thin sheet at the surface. (Symbols are identified in the text and in appendix B.)

(γd) of this thin sheet is assumed small. Displacement currents are neglected everywhere. This model is similar to the metal-cased borehole problem studied by Wait (11).

In the half-space, $z < 0$, the magnetic vector potential F satisfies the Helmholtz equation everywhere except at the source, and is given by

$$(\nabla^2 + \gamma_1^2)F_1 = 0, \tag{6}$$

where $\gamma_1 = (i\ \omega\ \mu_1\ \sigma_1)^{1/2}$.

For $z > 0$, F satisfies Laplace's equation.

$$\nabla^2 F_0 = 0. \tag{7}$$

In the half-space, the EM fields can be derived from F by

$$H_{1\rho} = \frac{1}{i\omega\mu_0} \frac{\partial^2 F_1}{\partial\rho\partial_z}; \tag{8}$$

$$E_{1\phi} = \frac{\partial F_1}{\partial\rho}; \tag{9}$$

$$H_{1z} = -\sigma_1 + \frac{1}{i\omega\mu_0} \frac{\partial^2}{\partial_z^2} F_1, \tag{10}$$

where E is the elective field and H is the magnetic field.

For the region $z > 0$, the EM fields are derived from F by

$$H_{0\rho} = \frac{1}{i\omega\mu_0} \frac{\partial^2 F_0}{\partial\rho\partial_z}; \tag{11}$$

$$E_{0\phi} = \frac{\partial F_0}{\partial\rho}; \tag{12}$$

$$H_{0z} = \frac{1}{i\omega\mu_0} \frac{\partial^2 F_0}{\partial_z^2}. \tag{13}$$

The specific expressions for F_0 and F_1 are given by Wait (3) as

$$F_0 = \frac{i\mu_0\omega a^2 I}{2}\int_0^\infty \frac{J_1(\lambda a)}{\lambda a}\frac{\lambda}{k_1} T(\lambda)\exp(-k_1h_0-k_0z)J_0(\lambda\rho)d\lambda \qquad (14)$$

and

$$F_1 = \frac{i\mu_0\omega a^2 I}{2}\int_0^\infty \frac{J_1(\lambda a)}{\lambda a}\frac{\lambda}{k_1}\cdot\left[\exp-(k_1(z+h_0))+R(\lambda)\exp-(k_1(z-h_0))\ J_0(\lambda p)d\lambda\right], \qquad (15)$$

where the wave number $(k) = (\lambda^2 + \gamma^2)^{1/2}$,

and $R(\lambda)$ and $T(\lambda)$, the reflection and transmission terms, are unknown.

The boundary conditions are such that the tangential electric fields are continuous, and from Ampere's law, the tangential magnetic field is discontinuous by the amount of the longitudinal current per unit length carried by the thin sheet. Also, recalling that γd is very small, it can be assumed that

$$(E_{1\phi} - E_{0\phi})\Big|_{z=0} = 0 \qquad (16)$$

and

$$(H_{0\rho} - H_{1\rho})\Big|_{z=0} = \sigma dE_{0\phi}\Big|_{z=0}\ . \qquad (17)$$

Using these boundary conditions and equations 14 and 15, it follows that

$$R(\lambda) = \frac{k_1 - k_0 - i\sigma d\mu_0\omega}{k_0 + k_1 + i\sigma d\mu_0\omega}\ ;$$

$$T(\lambda) = \frac{2k_1}{k_0 + k_1 + \sigma di\mu_0\omega}\ . \qquad (18)$$

From equation 13 and noting that $k_0 = \lambda$, the vertical magnetic field above the earth is given by

$$H_{0z} = a^2 I\int_0^\infty \frac{J_1(\lambda a)}{\lambda a}\frac{\lambda^3 e^{-(k_1h_0 + \lambda z)}}{k_1 + \lambda + i\omega\mu_0\sigma d} J_0(\lambda\rho)d\lambda\ . \qquad (19)$$

It is convenient to write H_{1z} in the following form with $x = \lambda h_0$, where h_0 is the depth of the buried loop:

$$H_{1z} = \frac{M}{2\pi h_0^3} Q\ , \qquad (20)$$

where

$$Q = \int_0^\infty \frac{J_1(Ax)}{(Ax/2)}\frac{x^3}{x + (x^2 + iH^2)^{1/2} + iHS}\cdot\exp(-Zx)\exp\left\{-(x^2 + iH^2)^{1/2}\right\} J_0(Dx)dx, \qquad (21)$$

where $H = (\omega\mu_0\sigma_1)^{1/2} h_0$,

$D = \rho_0/h_0$,

$Z = z/h_0$, $A = a/h_0$,

and $S = \sigma d(\omega\mu_0/\sigma_1)^{1/2}$.

Q is dimensionless and approaches zero as H, A, D, and Z approach zero, and it can be considered a correction factor to the field of a static magnetic dipole. For S equal to zero ($\sigma d = 0$), Q reduces to a homogeneous half-space equation.

In general, the integral in equation 21 must be evaluated numerically. The case of an observer at the surface (Z = 0) directly above the source (D = 0), with an underground dipole source (A = 0), is of particular interest. By setting the products σd and σ_1, Q(H,S) values can be found for the field data. Further, by setting

$$|Q(H,S)| = |Q(H_a,0)|, \qquad (22)$$

where $Q(H_a,0)$ represents the homogeneous half-space factor, the corresponding H_a value can be found and the apparent half-space conductivity can be found from

$$\sigma_a = H_a^2 / (\omega\mu_0 h_0^2). \qquad (23)$$

These conductivity values can then be compared with those found from the field data shown in table 3 in order to check the appropriateness of the thin sheet model.

The choice of the σd product is not entirely arbitrary. It must be realistic and still maintain the assumption made in the formulation of the thin sheet model. The assumption stated earlier is that γd is small to avoid the exponential attenuation factor of the signal as it traverses the thin layer. If this product is set equal to $\gamma d = 0.1$, then it places a constraint on the product σd.

Typically in the literature, conductivity measurements on earth material rarely exceed 1 mho/m. If this value is considered an upper bound and the highest frequency of interest is 3,030 Hz, then it follows that $|\gamma|_{max} = 0.155$/m and d = 0.647 m and $\sigma = 0.647$ mho.

Computer results using this σd product with various values of σ_1 showed a decrease in apparent conductivity with depth but an increase in conductivity for an increase in frequency at a given depth.

A different choice of σd having a value of 20 with $\sigma_1 = 10^{-7}$ mho does show the trend of conductivity with depth and frequency as reflected in the field data. These results are shown in table 5. A comparison of the conductivity and Q values of table 5 with those shown in tables 3 and 4 shows that the thin sheet model does predict the qualitative behavior of the experimental values, but does not predict the magnitude of decrease in apparent conductivity with depth. Also, it should be noted that the choice of σd = 20 may not be realistic and may need further study.

TABLE 5. - Conductivity (σ) and Q values for different depths and frequencies for the thin sheet model of the earth with $\sigma d = 20$ and $\sigma_1 = 10^{-7}$ mho

Freq, Hz	125 m		175 m		250 m		400 m	
	σ	Q	σ	Q	σ	Q	σ	Q
630.......	0.167	0.397	0.114	0.306	0.074	0.226	0.040	0.147
1,050.....	.154	.265	.100	.197	.063	.141	.032	.089
1,950.....	.129	.151	.081	.110	.048	.077	.024	.049
3,030.....	.108	.099	.066	.072	.039	.050	.018	.031

REGRESSION MODEL

Since the apparent conductivity shows a depth and frequency dependency, it may be possible to obtain a linear model that would be acceptable for making statements of expected values of apparent conductivity for a given frequency and mine depth. Such an approach was taken through two parameter regression studies.

Several linear regression models were considered. The model found to best fit the behavior of the data is one in which the log of the apparent conductivity is linearly related to frequency and depth. This is shown by

$$\log(\sigma) = \alpha + \beta_1(\text{depth}) + \beta_2(\text{freq}) + \varepsilon. \quad (24)$$

The parameters α, β_1, and β_2 are numbers that are estimated from the data. The parameter ε represents a random variable that is normally distributed, with expected value of zero and uniform variance across the independent variables. Regression results for this model are shown in table 6. Data from mine 1 were deleted because they appeared as a large outlier to the rest of the data. Also, because of the expected uncertainty of the conductivity estimates, all data for $Q > 0.5$ or $d < 100$ m were excluded.

TABLE 6. - Regression results for log(σ) versus mine depth (in meters) and transmit frequency (in hertz)

Number observed	42
Intercept	-0.45818
Depth slope	-0.00276
Frequency slope	-0.00012
Correlation coefficient	0.88
Estimated standard error	0.1794

Comparing the two slopes of table 6, it can be seen that the expected conductivity value appears to be more sensitive to the depth parameter. This is consistent with the results of table 4, as previously discussed. However, for small depths and high frequencies this would not be the case.

Earth conductivity estimates are often used for estimating the transmission loss an EM signal incurs as it is transmitted TTE. As such, earth conductivity estimates can be used with an earth model for making predictions of transmitted signal strengths. For small depths, the signal loss will be small and exact knowledge of the earth conductivity value would not be as important as it is at larger depths. After reviewing the results of table 6, it can be argued that possibly the conductivity frequency dependency can be ignored and the depth parameter can be used as the sole predictor of conductivity.

A single parameter regression model was formed relating the log of conductivity to mine depth, and the results are shown in table 7. This model was formed without the data from mines 1 and 3 and the single data point from mine 7 because they appeared as large outliers to the balance of the data. Also, as before, all data for which $Q > 0.5$ and $d < 100$ m were excluded. The regression line for this model is shown in figure 4 along with a 95-pct confidence interval (CI). As can be seen from figure 4, a prediction of apparent earth conductivity can be obtained from a depth measurement with an uncertainty of around ±20 pct.

TABLE 7. - Regression results for log(σ) versus mine depth (in meters)

Number observed	37
Estimated intercept	-0.45131
Estimated slope	-0.00393
Correlation coefficient	0.87
Estimated standard error	0.1655
Standard deviation depth	74.27

The regression analysis assumes normally distributed residuals with a homogeneous variance across the independent variable. These assumptions were checked and found to be adequately met.

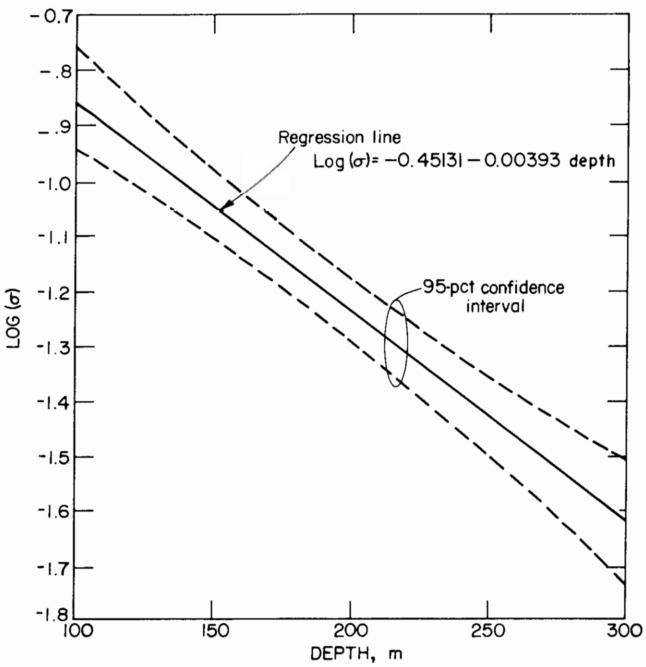

FIGURE 4. - Regression line and 95-pct confidence interval for log(σ) versus depth.

Also, an F-test was performed to see whether or not the linear model is appropriate. In general form,

$$H_0 : Y = \alpha + \beta x + \varepsilon \qquad (25)$$

is tested against

$$H_1 : Y \neq \alpha + \beta x + \varepsilon. \qquad (26)$$

The F-test statistic for this regression analysis was found to be F = 112. Comparing this with the critical value $F_{35,1,0.05} \approx 25\,0.5$, H_0 cannot be rejected and the linear model can be accepted.

From this study, it can be concluded that depth can be a good predictor of conductivity. However, since the objective of this study is to be able to predict the expected signal loss for TTE communications, the related uncertainty of expected received signal strength from an underground transmitter for this earth conductivity regression model must be determined.

Using the linear regression model of table 7, surface field strength calculations were made for an underground transmitter loop at different depths, having a transmit magnetic moment of 1 A-m². Calculations were made using the expected conductivity value for a given depth and for the conductivity values forming the 95-pct CI. The results are shown in figure 5 for each of the four frequencies.

The CI of figure 5 can be interpreted as a band within which one can expect to find, with 95-pct confidence, the mean value of surface signal strength for a number of tests having an underground transmitter with a magnetic moment of 1 A-m². It can be seen from figure 5 that the CI increases with frequency and depth; the uncertainty of expected field strength at the largest depths are on the order of ±3 dB. It should also be pointed out that the results of figure 5 are in good agreement with surface signal regression analysis by Lagace (12) where TTE transmission data were studied from a larger data base.

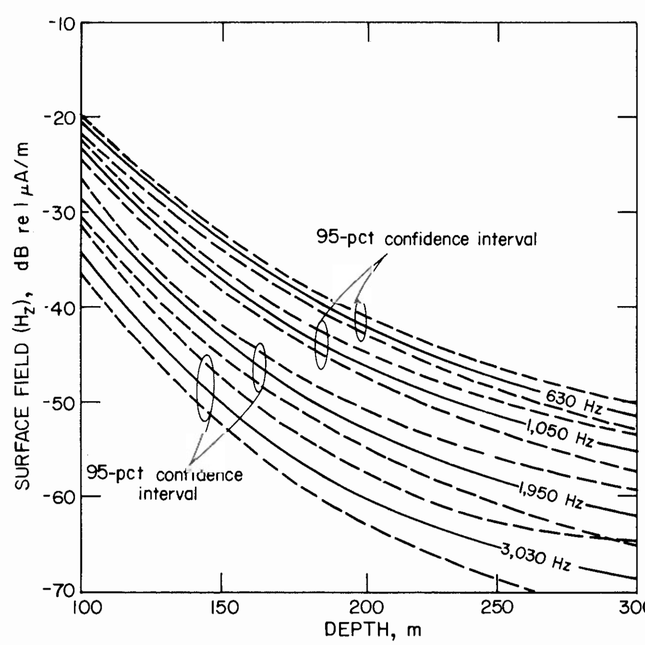

FIGURE 5. - Surface vertical magnetic field versus depth by frequency for an underground transmitter with a magnetic moment of 1 A/m.

SUMMARY

Field data from a number of narrow-band EM TTE communication tests were studied, and apparent earth conductivity estimates were made based on a homogeneous half-space earth model. The derived conductivity estimates were found to decrease with the transmitted frequency and mine depth. This result is not consistent with the assumption of a homogeneous half-space earth.

A different earth model was studied that contained a thin sheet of high conductivity in a lower conductivity homogeneous earth. Results of studies of this model show that it can predict the qualitative behavior of the conductivity estimates with frequency and depth, but falls short of predicting the quantitative results.

Linear regression analysis was performed, and it was found that the apparent earth conductivity could be estimated from a linear combination of the transmitted frequency and mine depth. Further regression studies demonstrated that an adequate predictor of the apparent earth conductivity could be the mine depth alone. The major value of this work is that an adequate statistical model of the apparent earth conductivity has been found, which can be used in future studies of TTE communications.

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APPENDIX A.--NAMES AND LOCATIONS OF MINES VISITED DURING FIELD TESTS

Mine	Depth, ft	Name and location
1	470	Youghiogheny and Ohio Coal, Allison; Beallsville, Belmont County, OH.
2	190	Peabody Coal Co., Alston No. 4; Centertown, Ohio County, KY.
3	1,550	Jim Walter Resources, Blue Creek No. 3; Adger, Jefferson County, AL.
4	381	Cal Glo, No. 21; Siler, Knox County, KY.
5	400	Eastover Mining Co., Highsplint No. 4; Highsplint, Harlan County, KY.
6	915	U.S. Steel, Gary No. 2; Wilcoe, McDowell County, WV.
7	430	U.S. Steel, Gary No. 9; Filbert, McDowell County, WV.
8	658	Gateway Coal Co., Gateway Mine; Clarksville, Greene County, PA.
9	420	Allied Chemical Corp., Harewood; Boomer, Fayette County, WV.
10	650	Alabama By-Products Corp., Mary Lee No. 1; Goodsprings, Walker County, AL.
11	289	Monterey Coal Co., Monterey No. 1; Carlinville, Macoupin County, IL.
12	540	Youghiogheny and Ohio Coal, Nelm's No. 2; Hopedale, Harrison County, OH.
13	560	Consolidation Coal, Oak Park No. 7; Cadiz, Harrison County, OH.
14	650	Old Ben Coal Co., Old Ben No. 26; Sesser, Franklin County, IL.
15	70	Owl Creek Corp., Sue-Jan; St. Charles, Hopkins County, KY.
16	260	Peter Cave, Mine No. 1; Lovely, Martin County, KY.
17	1,200	Plateau Mining Co., Star Point No. 2; Wattis, Carbon County, UT.
18	500	Pontika, No. 1; Lovely, Martin County, KY.
19	600	North American Coal, Powhatan No. 1; Powhatan Point, Belmont County, OH.
20	500	North American Coal, Powhatan No. 3; Powhatan Point, Belmont County, OH.
21	260	Peabody Coal Co., Sinclair No. 2; Drakesboro, Butler County, KY.
22	1,200	Kaiser Steel, Sunnyside No. 1; Sunnyside, Carbon County, UT.
23	460	Zeigler Coal Co., Mine No. 4; Johnston City, Williamson County, IL.
24	720	Helen Mining Co., Helen Mine; Homer City, Indiana County, PA.
25	282	Bureau of Mines, Lake Lynn; Fairchance, Fayette County, PA.
26	220	J-4 Cave; University Park, Centre County, PA.
27	150	Woodward Cave; University Park, Centre County, PA.

APPENDIX B.--ABBREVIATIONS AND SYMBOLS USED IN THIS REPORT

NOTE.--This list does not include the unit of measure abbreviations listed at the front of this report.

a	loop radius
A	ratio of radius of underground antenna to mine depth
CI	confidence interval
d	mine depth
D	ratio of distance between vertical axis of underground antenna and observation point to mine depth
E	electric field
EM	electromagnetic
F	vector potential
FFT	fast Fourier transform
H	magnetic field
h_0	depth of buried loop
H_z	surface vertical magnetic signal
I	current
k	wave number
M	magnetic moment
Q	attenuation factor of conducting earth
R	reflection coefficient
RMS	root-mean-square
T	transmission coefficient
TTE	through the earth
z	field measurement point
Z	ratio of distance of field measurement point above earth to mine depth
γ	propagation constant
λ	integration constant
μ_0	free space permeability
σ	earth conductivity
ω	angular frequency

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