THE STATE-OF-THE-ART IN COAL PILLAR DESIGN

C. Mark

NIOSH
Pittsburgh, PA

ABSTRACT

By 1980, the U.S. mining community had reached a broad consensus regarding coal pillar design. The pillar load could be estimated from tributary area theory, and the pillar strength from empirical formulas and laboratory coal strength testing. Then the growth of longwall mining required new thinking. Recently, powerful design methods have emerged from analysis of large data bases of real-world pillar successes and failures. These include the Analysis of Retreat Mining Pillar Stability (ARMPS), the Analysis of Longwall Pillar Stability (ALPS), the Mark-Bieniawski rectangular pillar strength formula, and guidelines for preventing massive pillar collapses. Sophisticated numerical models have also helped transform the pillar design landscape. In the process, our understanding of pillar mechanics has been greatly enriched. A new paradigm divides pillar failure into three categories:

- **Slender pillars (w/h<3.0)**, which are subject to sudden collapse;
- **Squat pillars (w/h>10)**, which are dominated by entry failure (rib, roof, or floor) and coal bumps, and;
- **Intermediate**, in which pillar squeezes seem to be the most common failure mode.

INTRODUCTION

The science of pillar design in the U.S. goes back nearly a century. One early pioneer noted that "to mine without adequate pillar support will result, sooner or later, in a squeeze; the inherent effects of which are crushing of the pillars, caving of the roof, and heaving of the bottom" (Bunting, 1911). Various pillar design formulas were proposed in the early days, based upon laboratory testing, full-scale pillar testing, and back-analysis of in-mine case histories (see Mark and Barton, 1996). They were developed for an industry that relied almost exclusively on room-and-pillar mining at relatively shallow depth.

The energy crisis of the 1970's and 1980's saw a revival of interest in coal pillar design. A number of ambitious field studies were undertaken, many of them funded or conducted by the U.S. Bureau of Mines. By 1980, the "classic" pillar design methodology had fully matured. It consisted of three steps:

1. Estimating the *pillar load* using tributary area theory;
2. Estimating the *pillar strength* using a pillar strength formula, and;
3. Calculating the pillar *"safety factor"* (SF).

Several formulas were available for step 2. Each estimated the pillar strength as a function of two variables, the pillar's width-to-height ratio (w/h) and the coal seam strength determined from laboratory testing (Bieniawski, 1984).

The growth of longwall mining exposed some serious shortcomings in this classic pillar design methodology. Most obvious was the need to go beyond tributary area and consider the abutment loads brought about by full-extraction mining. More serious was that longwall mining raised the issue of what constituted pillar "failure." The classic approach contended that "pillars will fail when the applied load reaches the compressive strength of the pillars" and that "the load-bearing capacity of the pillar reduces to zero the moment the ultimate strength is exceeded" (Bieniawski, 1992). This model was clearly inappropriate for the squat (large w/h) pillars used in longwall mining. When longwall pillars "failed," their load-bearing capacity did not disappear. Rather, the gate roads became unservicable.

Arthur Wilson of the British National Coal Board was the first to take a radically different approach to pillar design. His analytic method treated the pillar as a complex structure, with a non-uniform stress gradient, a build-up of confinement around a high-stress core, and progressive pillar failure. Although his mathematics were seriously flawed (Mark, 1987; Salamon, 1992), Wilson's basic concepts are now broadly accepted and underlie nearly all modern numerical models.

By 1990, the number of pillar strength formulas and numerical models had proliferated, but their predictions for squat pillars varied widely. One study compared 10 formulas, and found that some predicted that pillar strength would increase exponentially as the w/h ratio increased, others predicted it would tend towards a maximum limiting value, and still others predicted an intermediate, linear increase (figure 1). Stress measurements from 34 coal pillars were also analyzed, but were no help in narrowing the field (Mark and Iannacchione, 1992).

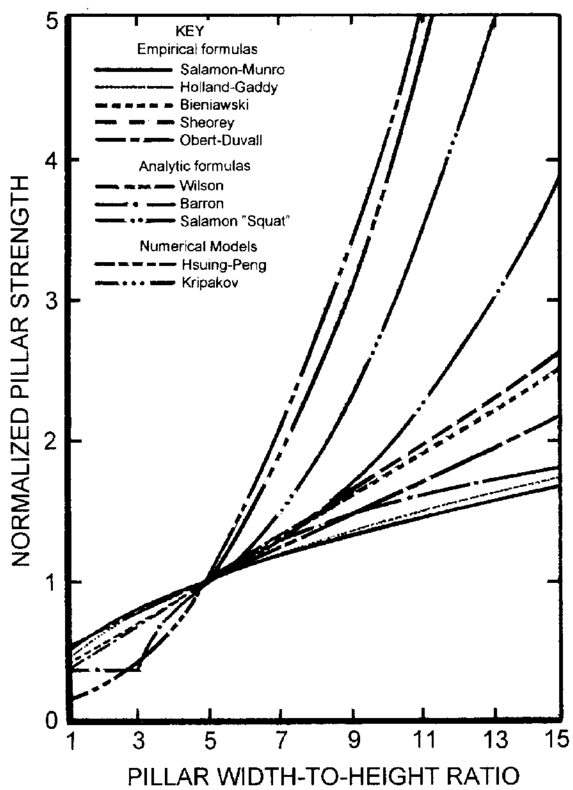

Figure 1. Comparison of pillar strength predictions from various formulas.

The need for material properties is another Achilles heel shared by both the classic formulas and numerical methods. Relating laboratory uniaxial compressive strength tests to full-size coal pillars has been the subject of controversy since the turn of the century. Numerical models compound the problem by requiring friction angles and post-failure properties in addition to the in situ coal strength. It is difficult to have faith in the accuracy of even the most sophisticated numerical models when critical material properties and structural features are either guessed or ignored.

Fortunately, another approach is available to help solve complex ground control problems. The empirical, or statistical, approach relies instead on the scientific interpretation of actual mining experience. Hundreds of longwall and room-and-pillar panels are mined each year, and each one is a full-scale test of a pillar design. Once data has been collected on enough of these case histories, statistical techniques can be used to determine those combinations of factors most likely to result in pillar failure.

The empirical approach is widely used in other fields, such as medicine, where the scientific understanding of the physical problem is incomplete, but a large quantity of data is available. Because the solutions are so firmly linked to reality, they are particularly well suited for solving practical problems like pillar design. Perhaps the best example of the approach is the Salamon and Munro pillar strength formula, which has been so convincing it has been used to design more than one million South African pillars (Salamon and Wagner, 1985).

Effective use of the empirical technique requires, as Salamon (1989) pointed out, "a reasonably clear understanding of the physical phenomenon in question." Without prudent simplification, the complexity of the problem will overwhelm the method's ability to discern relationships between the important variables. But a key advantage is that critical variables may be included even if they are difficult to measure directly, through the use of "rating scales."

This paper will focus on a number of practical pillar design methods that have been developed through the use of the empirical technique.

Recent Examples of Pillar Failure

Given today's high-production, advanced technology underground coal industry, do pillar failures still happen? Unfortunately, the answer is yes. Some recent examples:

Massive Collapses: In 1992, miners were splitting pillars at a southern West Virginia mine when the fenders in a 2.3 ha area suddenly collapsed. The miners were knocked to floor by the resulting air blast, and 103 ventilation stoppings were destroyed. At least 12 similar events have occurred in recent years, miraculously without a fatality (Mark et al., 1997b).

Pillar Squeezes: At a Kentucky coal mine, pillars were being extracted in the main entries under 270 m of cover. The pillars began to crush in response to the vertical load, resulting in a roof fall that killed two miners (MSHA, 1993). This incident is an extreme example of hazardous conditions that can be associated with slow pillar failure. At least 45 recent instances of pillar squeezes in room-and-pillar mines have been identified (Mark and Chase, 1997).

Longwall Tailgate Blockages: In 1984, 26 miners at the Wilberg Mine in Utah could not escape a deadly fire because of a tailgate roof fall. Similar blockages were common in the 1980's, and 50 cases have been documented (Mark, 1992).

Pillar Bumps: Extracting the initial lift from a standing pillar at a deep, eastern Kentucky operation resulted in a bump that killed two miners (MSHA, 1996). Bumps are not confined to pillars, however--another fatal bump occurred at a Utah longwall face just days later.

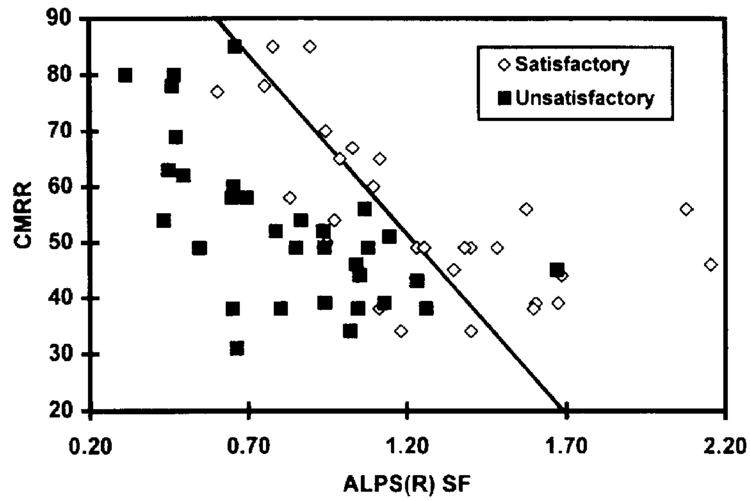

Figure 2. ALPS (R) SF and CMRR for longwall tailgate performance case histories.

Multiple seam interactions: Some studies indicate that most remaining coal reserves will experience multiple seam interactions. At a West Virginia mine where four seams had been previously extracted, one fatality occurred when the roof collapsed without warning beneath a remnant barrier pillar.

Abandoned mine subsidence: As suburban development expands into historic coal mining areas, unplanned subsidence has become an important issue. In one case, water caused floor failure in a mined-out section of an active mine, causing $1 million in damage to overlying structures. In another, residents above 50 yr old workings were disturbed by seismicity emanating from collapsing pillars (Iannacchione and Mark, 1989).

DESIGN OF LONGWALL GATE ENTRY SYSTEMS

In the fifteen years after 1972 the number of U.S. longwall faces grew from 32 to 118 (Barczak, 1992). The new technology created a host of operational and safety problems, including the maintenance of stable travelways on the tailgate side. Researchers initially viewed gate entry ground control primarily as a pillar design issue. The clear correlation between larger pillars and improved conditions that had been established by trial-and-error at many mines supported this approach.

Comparing longwall pillars to traditional coal pillars, the most obvious difference is the abutment loading. The major contribution of the original Analysis of Longwall Pillar Stability (ALPS) was a formula for estimating the longwall pillar load, based on numerous underground measurements (Mark, 1990).

It became clear, however, that tailgate stability required more than good pillar design. Other factors, such as roof quality and artificial support, were clearly important. Attacking this complex problem with analytical or numerical models would have been extremely difficult. On the other hand, the problem was ideal for the empirical approach.

Data were collected from approximately 55% of all U.S. longwall mines, selected to represent a geographic and geologic cross-section of the U.S. longwall experience. A total of 64 case histories were classified as "satisfactory" or "unsatisfactory" based on the conditions in the tailgate (Mark et al., 1994).

Each case history was described by the ALPS stability factor (SF), entry width, primary support rating, and the Coal Mine Roof Rating (CMRR). The CMRR then weighs the importance of all the geologic factors that determine roof competence, and combines them into a single rating on a scale from 0 to 100 (Molinda and Mark, 1994; Mark and Molinda, 1996).

Multi variate statistical analysis showed that when the roof is strong, smaller pillars can safely be used. For example, when the CMRR is 75, the an ALPS stability factor (SF) of 0.7 is adequate. When the CMRR drops to 35, the ALPS SF must be increased to 1.3. Significant correlations were also found between the CMRR and both entry width and the level of primary support (Mark et al., 1994).

The ALPS data base was recently revisited, with several new variables added. These included:

Rectangular pillar strength formula: All the SF were recalculated with the Mark-Bieniawski formula (see the Appendix to this paper) substituted for the original Bieniawski formula. The new result is designated as the **ALPS (R) SF.**

Uniaxial compressive strength: Nearly 4,000 laboratory tests were compiled from the literature into the Database of Uniaxial Coal Strength, or **DUCS** (Mark and Barton, 1996). From these data, typical seam strength values were obtained for 60 U.S. coalbeds.

Width-to height ratio (w/h): The w/h of the largest pillar in the gate entry system was included as an independent variable, to check if the pillar strength formula could be improved.

Depth of Cover (H): H was included as an independent variable primarily to check the loading formulation.

The entry width and the primary support were included as before

The statistical analysis showed that the ALPS (R) SF and the CMRR still correctly predicted 85% of the outcome, including 94% of the failures. None of the other three new variables would be included even at the 50% confidence level (a 90% confidence level would be required for a covariate to be considered statistically significant). Figure 2 shows the distribution of the case histories, and the revised design equation:

$$\text{ALPS (R) SF} = 2.0 - 0.016\ \text{CMRR}$$

Since 1987, ALPS has become the most widely-used pillar design method in the U.S. The ALPS-CMRR method directly addresses gate entry performance, and makes U.S. longwall experience available to mine planners in a practical form. Tailgate blockages are far less common today than they were 10 years ago, and ALPS can surely claim some of the credit.

PILLAR DESIGN FOR RETREAT MINING

The classical empirical pillar strength formulas were all developed for room and pillar mining. However, none ever attempted to consider the abutment loads that occur during pillar recovery operations. The abutment load formulas used in ALPS provided a means to rectify that shortcoming.

The Analysis of Retreat Mining Pillar Stability (ARMPS) employs the same basic constructs as ALPS, adapted to more complex and varied mining geometries (Mark and Chase, 1997). The abutment load formulas have been adapted to three dimensions, to account for the presence of barrier pillars and previously-extracted panels. The Mark-Bieniawski pillar strength formula is used to estimate pillar strength. Features such as varied entry spacings, angled crosscuts, and slab cuts in the barrier can all be modeled.

To evaluate the validity of ARMPS, more than 200 retreat mining case histories have been obtained from field visits throughout the U.S. Variables included in the statistical analysis included the w/h, CMRR, UCS, and H. When the entire data set was evaluated, it was found that 77% of the outcomes could be correctly predicted simply by setting the ARMPS SF to 1.46. Including either the depth or the w/h increased the r^2 slightly, without improving the accuracy (figure 3). The depth and the w/h ratio were strongly correlated with each other within the data set.

The accuracy improved when the data set was divided into two parts. One group included only cases where cover was shallow (H less then 200 m (650 ft)) and where the pillars were not squat (w/h was less than 8). For this group, when ARMPS = 1.5 it successfully predicted 83% of the outcomes. However, for the deep cover/squat pillar group, only 58% of the cases were correctly predicted at ARMPS SF=0.93. No other variables could be included in either group at the 90% confidence level. The conclusion seems to be that ARMPS works quite well at shallow depth and moderate w/h ratios. It is of much less value for squat pillars at greater depth.

COAL STRENGTH TESTING AND PILLAR DESIGN

The classic pillar strength methods considered laboratory uniaxial compressive strength (UCS) tests an integral part of pillar design. However, neither the longwall or the room-and-pillar case history analyses described above found UCS to be of any help in pillar design. An earlier study (Mark and Barton, 1996) had reached a similar conclusion. That study explored DUCS, the largest and most complete data base of the uniaxial compressive strength of coal ever assembled. It found that the "size effect" varies dramatically from seam to seam, depending on the coal cleat structure. More importantly, there was no correlation between the UCS and the in situ pillar strength documented in case histories of failed pillars (figure 4). The conclusion was that pillar design was more reliable when a uniform coal strength was used.

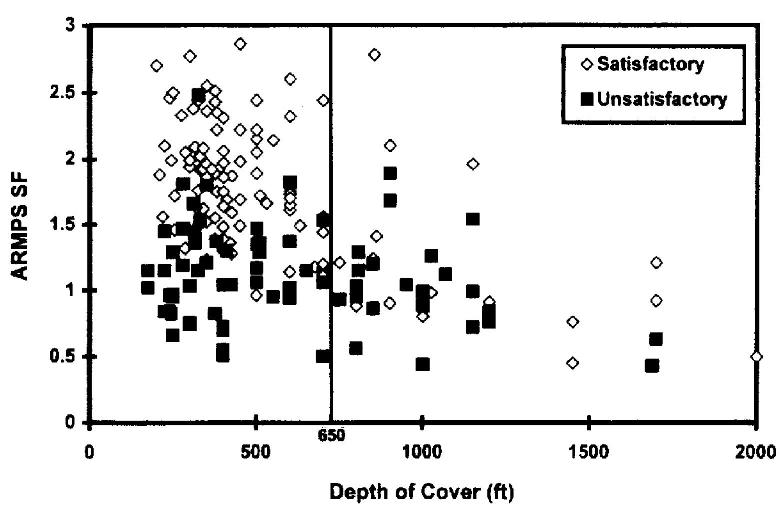

Figure 3. ARMPS SF and depth of over 200 retreat mining case histories.

The study also provided indirect evidence about which geologic factors do affect pillar strength. It does not seem likely that the in situ strength of U.S. coal seams really is uniform, because there is quite a bit of variability in the in situ data. However, the strength of the intact coal, which is what is measured in a laboratory test, is apparently largely irrelevant to the pillar strength. Instead, it seems likely that variations in seam strength are due to large-scale geologic features like bedding planes, clay bands, and rock partings. Such features probably determine the amount of confinement which can be generated within the pillar, and therefore the load-bearing capacity of the pillar core.

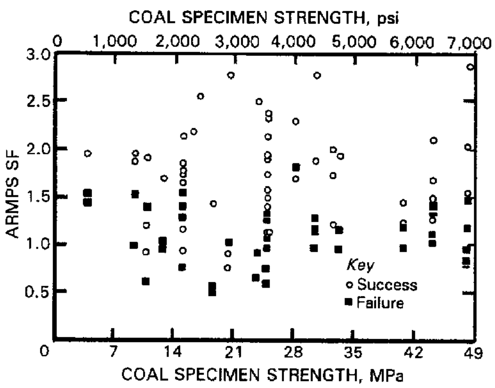

Figure 4. ARMPS SF compared with coal specimen strength.

MASSIVE PILLAR COLLAPSES

Most of the pillar failures included in the ARMPS data base are "squeezes" in which the section converged over hours, days or even weeks. Another important subset are 15 massive pillar collapses (Mark et al., 1997b). These occurred when undersized pillars failed and rapidly shed their load to adjacent pillars, which in turn failed. The consequences of such chain reaction-like failures typically include a powerful, destructive, and hazardous airblast.

Data collected at 12 massive collapse sites revealed that the ARMPS SF was less that 1.5 in every case, and was less that 1.2 in 81% of the cases (figure 5). What really distinguished the sudden collapses from the slow squeezes, however, was the pillar's w/h ratio. Every massive pillar collapse involved **slender** pillars whose w/h was less than 3. Laboratory tests have shown slender coal specimens typically have little residual strength, which means that they shed almost their entire load when they fail (figure 6). As the specimens become more squat, their residual strength increases, reducing the potential for a rapid domino failure. The mechanism of massive collapses has been replicated in a numerical model (Zipf and Mark, 1997).

Two alternative strategies were proposed to prevent massive pillar collapses:

- **Prevention**: With the prevention approach, the panel pillars are designed so that collapse is highly unlikely. This can be accomplished by increasing either the SF of the pillars, or their w/h ratio.

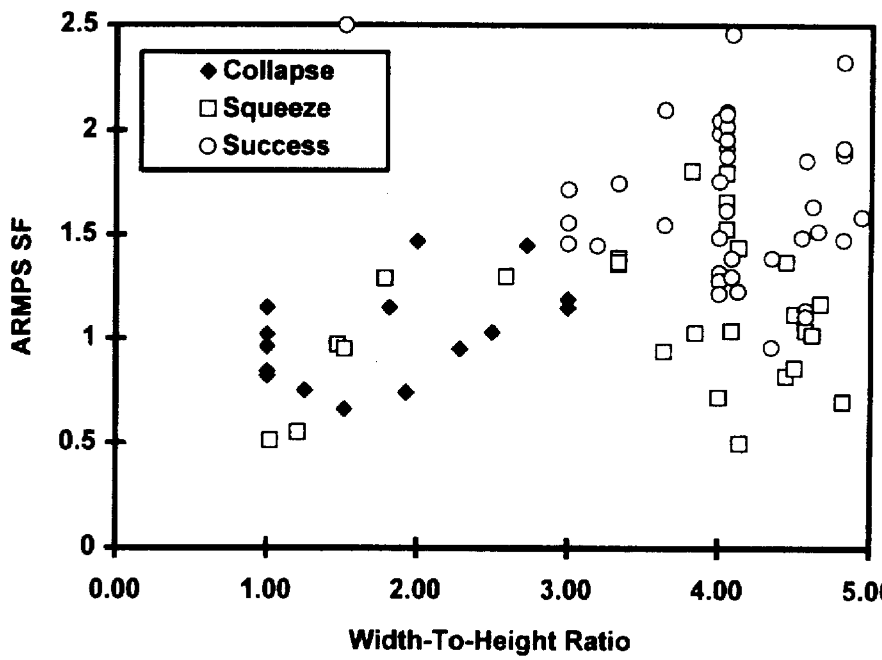

Figure 5. ARMPS SF and pillar width-to-height ratio for pillar collapses and other case histories.

- **Containment:** In this approach, high-extraction is practiced within individual compartments that are separated by barriers. The small pillars may collapse within a compartment, but because the compartment size is limited, the consequences are not great. The barriers may be true barrier pillars, or they may be rows of development pillars that are not split on retreat. The containment approach has been likened to the use of compartments on a submarine.

Design charts have been developed for each approach, considering the width of the panel, the seam thickness, and the depth of cover (Mark et al., 1997b).

USE OF NUMERICAL MODELING IN PILLAR DESIGN

It is beyond the scope of this paper to discuss in detail the contributions made by numerical modeling to pillar design. However, two efforts are particularly worthy of note.

Su and Hasenfus (1996,1997) have employed finite element models (FEM) to explore the effect of various geologic conditions. They have shown, for example, that a rock parting may increase the pillar strength, while a clay parting can reduce it. A weak floor can reduce the pillar strength by as much as 50%. All of these effects are minimal for slender pillars, but become much more pronounced once the w/h exceeds 5. The models also indicated that varying the coal strength had almost no effect on pillar strength. It is remarkable that despite the differences in technique, many of the conclusions from Su and Hasenfus' FEM parallel those derived from empirical back-analysis of case history data.

Displacement-discontinuity (DD) methods have been used for many years to estimate pillar loadings in complex and multiple-seam geometries. When it was introduced in 1992, the DD program MULSIM/NL represented a significant advance because it incorporated realistic pillar and gob mechanics (Zipf, 1992). The next-generation DD model, called LAMODEL, was recently developed by Heasley and Salamon (1996a, 1996b). LAMODEL simulates the overburden as a stack of layers separated by frictionless interfaces, thus providing a realistic suppleness to the overburden that was not possible with older DD models. Input pillar strength parameters are typically derived from empirical formulas, using the stress gradient approach described in the Appendix (Heasley, 1998; Karabin and Evanto, 1994). Thus the modeled pillar strengths are closely linked to real-world behavior, while LAMODEL's analytical mechanics allow it to accurately analyze complex mining situations including multiple seams, random pillar layouts and/or variable topography. The result is a powerful synthesis of empirical and numerical approaches to pillar design.

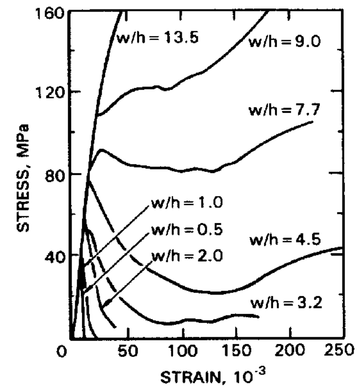

Figure 6. Complete stress-strain curves for coal specimens (after December 1986).

CONCLUSIONS

Recent years have seen significant advances in the state-of-the-art in coal pillar design. From a practical standpoint, the development of reliable empirical methods like ALPS and ARMPS has been particularly valuable. They have been widely accepted throughout the mining community because they have been verified by extensive data bases of real-world case histories, and because they have been readily available in user-friendly computerized formats. The DD models MULSIM/NL and LAMODEL have been another important success story.

The research has led to some other important conclusions, including:

- Laboratory testing of small coal samples, particularly uniaxial compressive strength tests, are not useful for predicting pillar strength;
- The strength becomes more difficult to predict as the pillar becomes more squat;
- The w/h ratio is important for predicting not just the pillar strength, but the mode of failure, and;
- Many ground control problems must be considered from the standpoint of entry stability, where pillar behavior is just one component.

Furthermore, old concepts of pillar "failure" have given way to a new paradigm that identifies three broad categories of pillar behavior:

- **Slender pillars** (w/h<3), which have little residual strength and are prone to massive collapse when used over a large area;

- **Intermediate pillars** (4<w/h<8), where "squeezes" are the dominant failure mode in room-and-pillar mining, and where empirical pillar strength formulas seem to be reasonably accurate, and;

- **Squat pillars** (w/h>10), which can carry very large loads and are strain-hardening, and which are dominated by entry failure (roof, rib, and floor) and by coal bumps.

Certainly, more work remains before the age-old questions of pillar design is finally solved. In particular, there is much more to learn about the mechanics of squat pillars and roof-pillar-floor interactions. Currently, there is no accepted way to determine the frictional characteristics of the contacts, bedding planes, and partings that are so crucial to pillar strength. It is similarly difficult to characterize the bearing capacity of the floor. Simple, meaningful field techniques for estimating these properties will be necessary for further progress with either numerical or empirical techniques. Indeed, the cross-pollination between the numerical and empirical methods that has characterized the recent past can be expected to bear further fruit in the future.

REFERENCES

Barczak, T. M, 1992, The History and Future of Longwall Mining in the United States, BuMines IC 9316, 26 pp.

Bieniawski, Z. T, 1984, Rock Mechanics Design in Mining and Tunneling, A. A. Balkema, 272 pp.

Bieniawski, Z. T, 1992, A Method Revisited: Coal Pillar Strength Formula Base on Field Investigations, Proceedings, Workshop on Coal Pillar Mechanics and Design, BuMines IC 9315, pp. 158-165.

Bunting, D, 1991, Chamber Pillars in Deep Anthracite Mines. Trans. AIME, vol. 42, pp. 236-245.

Heasley, Keith A., and Miklos D. G. Salamon, 1996a, "A New Laminated Displacement-Discontinuity Program," Proceedings of the Second North American Rock Mechanics Symposium, Montreal, Quebec, Canada, June 19-21, p. 1879-1886.

Heasley, Keith A., and Miklos D. G. Salamon, 1996b, "A New Laminated Displacement-Discontinuity Program: Fundamental Behavior," Proceedings of the Fifteenth International Conference on Ground Control in Mining, August 13-15. p. 111-125.

Heasley, Keith A. 1998, "Numerical Modeling of Coal Mines with a Laminated Displacement-Discontinuity Code", Ph.D. dissertation, Colorado School of Mines, in progress.

Iannacchione, A. T., and C. Mark, 1990, Possible Mechanism for Surface Vibrations Near Maxwell Hill, West Virginia, Bull. Assoc. Eng. Geol., vol. 27, no. 3, pp. 341-353.

Karabin, G. J., and M. A. Evanto, 1994, Experience with the Boundary Element Method of Numerical Modeling as a Tool to Resolve Complex Ground Control Problems, Paper in Proceedings, 13th Conference on Ground Control in Mining, Morgantown, WV, pp. 201-213.

Mark, C, 1987, Analysis of Longwall Pillar Stability, Ph.D. Thesis, PA State Univ., University Park, PA, 414 pp.

Mark, C, 1992, Analysis of Longwall Pillar Stability: An Update, Paper in the Proceedings of the Workshop on Coal Pillar Mechanics and Design, USBM IC 9315, pp. 238-249.

Mark, C, 1990, Pillar Design Methods for Longwall Mining, USBM IC 9247, 53 pp.

Mark C, and T. M. Barton, 1996, The Uniaxial Compressive Strength of Coal: Should it be Used to Design Pillars?, Proceedings, 15th International Conference on Ground Control in Mining, Golden, CO, August 13-15.

Mark, C., and F. Chase, 1997, Analysis of Retreat Mining Pillar Stability, Paper in New Technology for Ground Control in Retreat Mining, NIOSH IC 9446, pp. 17-34.

Mark, C., F. Chase, and R. K. Zipf, Jr, 1997b, Preventing Massive Pillar Collapses in Coal Mines, Paper in New Technology for Ground Control in Retreat Mining, NIOSH IC 9446, pp. 35-48.

Mark, C., F. E. Chase, and G. M. Molinda 1994, Design of Longwall Gate Entry Systems Using Roof Classification, Paper in New Technology for Longwall Ground Control: Proceedings of the USBM Technology Transfer Seminar, USBM SP 94-01, pp. 5-18.

Mark, C. and A. T. Iannacchione, 1992, Coal Pillar Mechanics: Theoretical Models and Field Measurements Compared, Paper in the Proceedings of the Workshop on Coal Pillar Mechanics and Design, USBM IC 9315, pp. 78-93.

Mark, C. and G. M. Molinda, 1996, Rating Coal Mine Roof Strength from Exploratory Drill Core, Proceedings, 15th International Conference on Ground Control in Mining, Golden, CO, August 13-15, pp. 415-428.

Molinda, G. and C. Mark, 1994, The Coal Mine Roof Rating (CMRR)--A Practical Rock Mass Classification for Coal Mines, USBM IC 9387, 83 pp.

MSHA, 1993, Coal Mine Fatal Accident Investigation Report, Fatal Coal Mine Roof Fall at South East Coal Mine.

MSHA, 1996, Coal Mine Fatal Accident Investigation Report, Fatal Coal Mine Burst at Harlan-Cumberland C-2 Mine.

Salamon, M. D. G., 1989, Significance of Strata Control to the Safety and Efficiency of Mining, Paper in Proceedings of the 8th International Strata Control Conference, Dusseldorf, FRG, 9 pp.

Salamon, M. D. G. and H. Wagner, 1985, Practical Experiences in the Design of Coal Pillars. Paper in Proceedings of the 21st International Conference of the Safety in Mines Research Institutes, Sydney, Australia, pp. 3-10.

Salamon, M. D. G., 1992, Strength and Stability of Coal Pillars, Paper in Proceedings, Workshop on Coal Pillar Mechanics and Design, BuMines IC 9315, pp. 94-121.

Su, D. W. H., and G. J. Hasenfus, 1996, Practical Coal Pillar Design Considerations Based on Numerical Modeling, Presented at 15th International Conference on Ground Control in Mining, Golden, CO, (handout).

Su, D. W. H., and G. J. Hasenfus, 1997, Effects of In-Seam and Near-Seam Conditions on Coal Pillar Strength, Presented at 16th International Conference on Ground Control in Mining, Morgantown, WV, (handout).

Zipf, R. K., 1992, MULSIM/NL Application and Practitioner's Manual, BuMines IC 9322, 48 pp.

Zipf K, and C. Mark, 1997, Design Methods to Control Violent Pillar Failures in Room and Pillar Mines. Transactions IMM, Vol. 106, Sept-Dec, pp. A124-A132.

APPENDIX. THE MARK-BIENIAWSKI RECTANGULAR PILLAR STRENGTH FORMULA

A major drawback of most of the classic pillar strength formulas is that they only apply to square pillars. They underestimate the strength of rectangular pillars that contain proportionately much more core area. The concept of the "stress gradient" provides the link which allows classic formulas to be extended to rectangular pillars. The stress gradient defines the vertical stress within the pillar at maximum load as a function of the distance from the nearest rib.

Although classic formulas do not explicitly consider the effect of internal pillar mechanics, it is apparent that they imply a non-uniform stress distribution because of the w/h effect. A derivation of the implied stress gradients was published by Mark and Iannacchione (1992). For example, the Bieniawski formula:

$$S_p = S_l (0.64 + 0.36\ w/h)$$

implies a stress gradient of:

$$S_v = S_l (0.64 + 2.16\ x/h)$$

where: S_p = Pillar strength
S_l = In situ coal strength
S_v = Vertical pillar stress
x = Distance from pillar rib

When this stress gradient is integrated over the load bearing area of a rectangular pillar, the Mark-Bieniawski pillar strength formula is obtained:

$$S_p = S_l (0.64 + 0.54\ w/h - 0.18\ (w^2/Lh)$$

Where L = pillar length. The approach is illustrated in figure A-1, and described in more detail by Mark and Chase (1997).

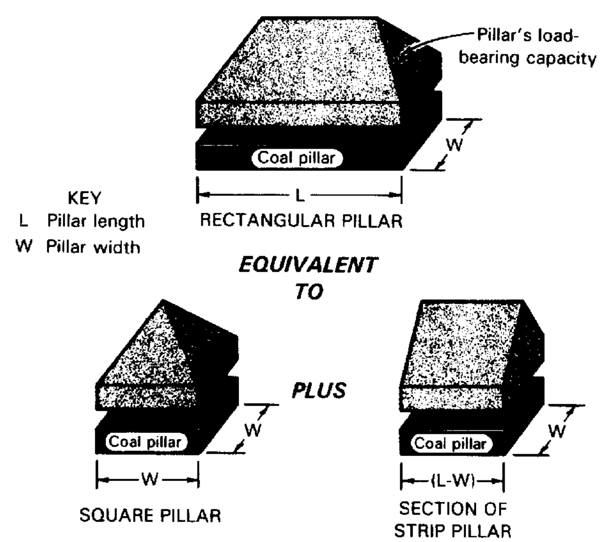

Figure A-1. Stress distribution concept used in the Mark-Bieniawski pillar strength formula.