SUPPLEMENTARY MATERIAL

An example dataset to illustrate the RPC method all source code, data, and expected output are publicly available at `https://github.com/bjks10/RPC`.

README file: Text file overviewing MATLAB functions and files to perform RPC program

RPC source code: MATLAB code to perform the RPC method and supporting files.

Simulated data set: Simulated example data set to illustrate RPC method.

Expected Output: Saved output results from running RPC example set.

Posterior Computation: MCMC Gibbs sampler for posterior computation of RPC

1. Update the global component indicators $(G_{ij} \mid s_i = s) \sim \text{Bern}(p_{ij})$, where

$$p_{ij} = \frac{\nu_j^{(s)} \prod_{r=1}^d \Theta_{0jC_i,r}^{\mathbf{1}(y_{ij}=r)}}{\nu_j^{(s)} \prod_{r=1}^d \Theta_{0jC_i,r}^{\mathbf{1}(y_{ij}=r)} + (1-\nu_j^{(s)}) \prod_{r=1}^d (\Theta_{1jL_{ij},r}^{(s)})^{\mathbf{1}(y_{ij}=r)}}$$

   for each subject $i \in (1, \ldots, n)$ with respective subpopulation index s.

2. Update global cluster index C_i, $i = 1, \ldots, n$ from its multinomial distribution where

$$Pr(C_i = h) = \frac{\pi_h \prod_{j:G_{ij}=1} \prod_{r=1}^d \Theta_{0jh,r}^{\mathbf{1}(y_{ij}=r)}}{\sum_{l=1}^K \pi_l \prod_{j:G_{ij}=1} \prod_{r=1}^d \Theta_{0jl,r}^{\mathbf{1}(y_{ij}=r)}}.$$

3. Update local cluster index L_{ij} for all $i : s_i = s$ and $j = 1, \ldots, p$, repeating for each s, from its multinomial distribution conditional on $s_i = s$ where

$$Pr(L_{ij} = h) = \frac{\lambda_h^{(s)} \prod_{r=1}^d \left(\Theta_{1jh,r}^{(s)}\right)^{\mathbf{1}(y_{ij}=r, G_{ij}=0)}}{\sum_{l=1}^K \lambda_l^{(s)} \prod_{r=1}^d \left(\Theta_{1jl,r}^{(s)}\right)^{\mathbf{1}(y_{ij}=r, G_{ij}=0)}}.$$

4. Update the global clustering weights

$$\pi = (\pi_1, \ldots, \pi_K) \sim \text{Dir}\left(\frac{1}{K} + \sum_{i=1}^{n} \mathbf{1}(C_i = 1), \ldots, \frac{1}{K} + \sum_{i=1}^{n} \mathbf{1}(C_i = K)\right).$$

5. Update the local clustering weights in subpopulation s,

$$\lambda^{(s)} = \left(\lambda_1^{(s)}, \ldots, \lambda_K^{(s)}\right) \sim \text{Dir}\left(\frac{1}{K} + \sum_{i:s_i=s} \sum_{j=1}^{p} \mathbf{1}(L_{ij} = 1), \ldots, \frac{1}{K} + \sum_{i:s_i=s} \sum_{j=1}^{p} \mathbf{1}(L_{ij} = K)\right).$$

6. Update the multinomial parameters, where η is a flat, symmetric Dirichlet hyperparameter preset at 1

$$\theta_{0jh,\cdot} \sim \text{Dir}\left(\eta + \sum_{i:G_{ij}=1,C_i=h} \mathbf{1}(y_{ij} = 1), \ldots, \eta + \sum_{i:G_{ij}=1,C_i=h} \mathbf{1}(y_{ij} = d)\right)$$

$$\theta_{1jh,\cdot}^{(s)} \sim \text{Dir}\left(\eta + \sum_{i:G_{ij}=0,L_{ij}=h,s_i=s} \mathbf{1}(y_{ij} = 1), \ldots, \eta + \sum_{i:G_{ij}=0,L_{ij}=h,s_i=s} \mathbf{1}(y_{ij} = d)\right)$$

7. Update $\nu_j^{(s)} \sim \text{Be}(1 + \sum_{i:s_i=s} G_{ij}, \beta^{(s)} + \sum_{i:s_i=s}(1 - G_{ij}))$.
8. Update Beta-Bernoulli hyperparameter: $\beta^{(s)} \sim \text{Ga}(a + p, b - \sum_{j=1}^{p} \log(1 - \nu_j^{(s)}))$.