VARIATION OF HORIZONTAL STRESSES AND STRAINS IN MINES IN BEDDED DEPOSITS IN THE EASTERN AND MIDWESTERN UNITED STATES

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ABSTRACT

In general, the direction of the maximum horizontal stress in the eastern United States is fairly well defined. However, the variation of the magnitudes of the horizontal stresses is not very well understood. Because the horizontal stresses cause severe ground control problems in underground coal and limestone mines throughout the eastern United States a more complete understanding of how the magnitudes vary would be useful for developing mine design strategies to combat horizontal stress related ground control problems. Therefore, in this National Institute for Occupational Safety and Health (NIOSH) study, the variation of the magnitude of the horizon stresses in sedimentary deposits in the eastern and Midwestern United States are examined with respect to two factors, the elastic modulus of the rock and the site depth. Stress measurements from thirty-seven sites are used in the evaluation.

Examining the applied excess strains indicates that the eastern United States can be, separated into high and low strain zones. For most of the eastern United States the maximum applied excess or tectonic strain ranges from only 300 to 550 micro strains. However, there is one area, a portion of the Beckley seam in the central Appalachian region where the strains are significantly higher than the other regions. In this higher strain zone, the maximum applied tectonic strains range from 700 to 1,000 micro strains. Regression models for each zone based on the elastic modulus can explain between 83 to 85% of the variation of the maximum horizontal stress. Because one region, the northern Appalachian district, has strains that are about 20% higher than the other regions in the low strain zone, multiple strain models based on geographic regions were developed for the low strain zone that can explain 87 to 91% of the maximum horizontal stress variation with the elastic modulus.

Depth was found not to be a significant causal factor in any increase in the horizontal stress even though the site depths ranged from 275 to 2,500 ft. Beyond a theoretical increase, based on Poisson's effect and gravity, no other increase in the horizontal stress with depth can be justified with this data. The most significant factor controlling the variation of the maximum horizontal stress is the elastic modulus of the rock, not the overburden depth.

INTRODUCTION

In coal mines in the eastern United States, the horizontal stress direction and pattern has been recognized and is fairly well understood with the direction of the maximum horizontal stresses apparently related to plate tectonics (1). From a standpoint of mine design and layout to control ground control problems due to horizontal stress, the direction of the maximum horizontal stress is very important (2). However, other important horizontal stress parameters, such as the magnitude and the magnitude variation, are not as well understood. The magnitudes, in general, from the stress measurements used in this analysis though fit the stress model for the North American mid-plate region developed from the World Stress Map Project (3). Essentially, the largest stress component is the maximum horizontal stress. Therefore, the resultant maximum horizontal stress magnitude is caused, to a large degree, by applied loads at the tectonic plate boundaries.

In the past, some linkage has been shown between the depth and an increase in the magnitude of the horizontal stress in North America (4, 5). In a recent study, equations have been developed for the increase in the maximum horizontal stress with depth in coal mines in the eastern United States (6). However, although there is a statistical relationship between the maximum horizontal stress and depth, the correlation is weak.

Recent studies in the United Kingdom have shown that a very strong correlation exists between the maximum horizontal stress and the elastic properties of the rock (7). Coefficients of determination as high as 0.95 indicate that there is a strong linear relationship between the elastic modulus and the maximum horizontal stress. Obviously, the theory of elasticity provides a direct relationship between the elastic modulus and the stress at a point. The implication of such a strong linear correlation as seen in the United Kingdom, is that the region is being subjected to a fairly uniform strain field at least from the maximum horizontal stress component. However, the linear correlation between the minimum horizontal stress and the elastic modulus is much weaker (8). Site depths in these studies ranged from 300 to 2,500 ft and therefore the results cover the general zone of mining. A relationship between the elastic modulus and the horizontal stress has also been observed in China (9).

In this study, stress measurements in the eastern United States are examined to determine if there is a relationship between the elastic modulus and the horizontal stress. Further, the effect of depth on the magnitude of the horizontal stress is reexamined. Besides the stresses, the horizontal strains are also evaluated. This analysis uses only those sites from sedimentary deposits including both coal and limestone.

STRESS MEASUREMENTS

Stress measurements from 37 sites are used in this study with figure 1 showing the site locations. The stress measurements are from the northern (7 sites) and central (19 sites) Appalachian regions as well as from the eastern Mid-Continent region (11 sites). Most of the stress measurement results have been previously reported (1, 10). The site depths range from 275 to 2,500 feet and encompass the range of coal and limestone mining depths in the eastern United States.

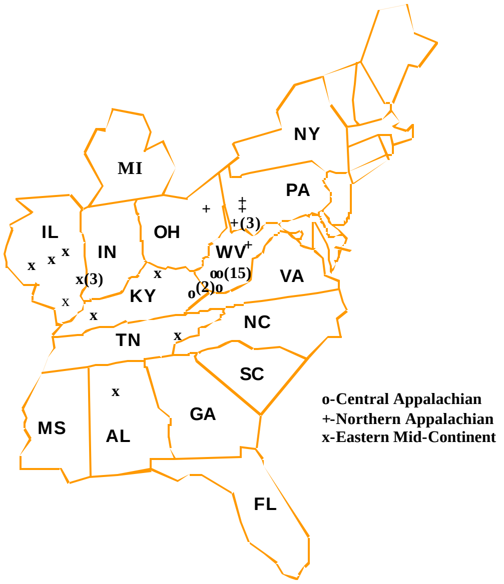

Figure 1. Location of stress measurement sites used in this study, numbers in parentheses indicate multiple sites.

A requirement for the inclusion of a stress measurement in the analysis was that not only the magnitudes of the horizontal stress had to be known but also the site depth and the elastic modulus of the rock. Requiring the elastic properties of the rock limited the published stress measurement data that could be used in this analysis.

APPLIED EXCESS HORIZONTAL STRAIN VARIATION FOR THE EASTERN UNITED STATES

For a strong relationship or significant correlation to exist between the horizontal stress and the elastic modulus, a necessary condition is that the strain field must be sufficiently uniform. Therefore, the applied excess strains for the sites are examined. Excess strains are the strains after the effects of gravity have been removed and are calculated from the excess horizontal stresses. Using the excess strains minimizes the expected influence of depth in the comparisons between sites. These strains are used to evaluate the strain variation regionally and across the eastern United States.

Excess horizontal stresses are calculated by subtracting the expected effects of gravity from the measured horizontal stresses. The following equations can be used to calculate the excess stresses (10, 11):

$$Q_e = Q - 1.1\left[\frac{n}{(1-n)}\right] \text{ x } D \qquad (1a)$$

and

$$P_e = P - \left[1.1\left(\frac{n}{1-n}\right)\right] \text{ x } D \qquad (1b)$$

where Pe = maximum excess horizontal stress, psi,
P = maximum horizontal stress, psi,
í = Poisson’s ratio,
D = depth, ft,
Qe = minimum excess horizontal stress, psi, and
Q = minimum horizontal stress, psi.

These stresses have also been refered to as tectonic stresses. To calculate the excess stress, a Poisson’s ratio of 0.25 was used for all sites.

The applied excess horizontal strains can then be calculated from the excess stresses by the following equations (12).

$$\mathring{a}_{pA} = \frac{(P_e - n\,Q_e)}{E} \qquad (2a)$$

and

$$\mathring{a}_{qA} = \frac{(Q_e - n\,P_e)}{E} \qquad (2b)$$

where åpA = maximum excess applied strain, micro strain,
åqA = minimum excess applied strain, micro strain, and
E = elastic modulus of rock, million psi.

Again, a Poisson’s ratio of 0.25 is used to calculate the strains. The strain components in equation 2 are the actual strains that are being applied in excess of the gravity load.

Figure 2 shows a histogram of the maximum excess applied horizontal strain by site across the eastern United States. Fifteen of the sites are between 300 to 400 micro strains and 23 of the 37 sites have a maximum excess strain between 300 and 550 micro strains. These sites are spread across all the main geographic regions. Therefore, the typical maximum excess strain field across the eastern United States appears be between 300 to 550 micro strains. The maximum excess strains above 550 micro strains were all measured in the central Appalachian region. There are 12 sites in this higher strain group. Those strains above 700 micro strains are all found in the Beckley seam where an extensive stress measurement program was conducted (13).

From the histogram, the strain field is skewed with only two sites below 300 micro strains. Both these sites are in the Illinois basin of the eastern Mid-Continent region. Because the mode is between 300 and 400 micro strains with the sites in this group from all the regions and from a range of depths, those sites below 300 micro strains may be in partial strain relief. One site is near a fault and the other is at a depth of only 275 ft. This is the shallowest site in the Illinois basin located in a region with valleys filled with unconsolidated material. This shallow site was excluded from further analysis in this study. Essentially, the distribution is skewed because the strains cannot realistically fall below zero and the location of the mode indicates that those sites below about 300 micro strains are possibly in partial strain relief.

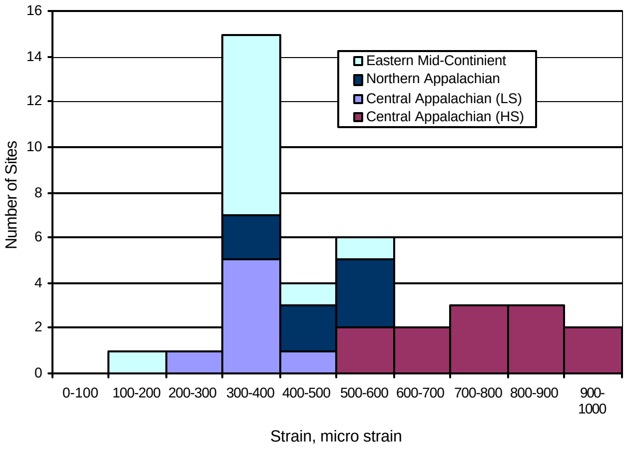

Figure 2. Average maximum excess applied horizontal strain distribution across eastern United States, by site.

For the central Appalachian region the strain field appears to be bimodal. Because of the strain distribution and the geographic location of the sites, the sites can be grouped into high and low strain zones. Those sites with the maximum strain above 550 micro-strains are considered in the high strain group. In the Beckley seam, eight of the high strain sites are in adjoining areas of three mines (Bonny mine, Beckley Mining Co. mine, and northeast section of Maple Meadows mine) covering a distance of about 18 miles (figure 3). Five of the low strain sites are in a separate but adjacent geographic area to the high strain zone stretching across two mines (southwest section of Maple Meadows Mine and Beckley No. 1 Mine) for a distance of about 6 miles. Therefore, geographically distinct strain zones exist in the Beckley seam encompassing areas that are tens to hundreds of square miles. Two other high strain sites, including one in the Beckley No. 2 mine, are adjacent to the low strain zone but are geographically separated from the high strain zone.

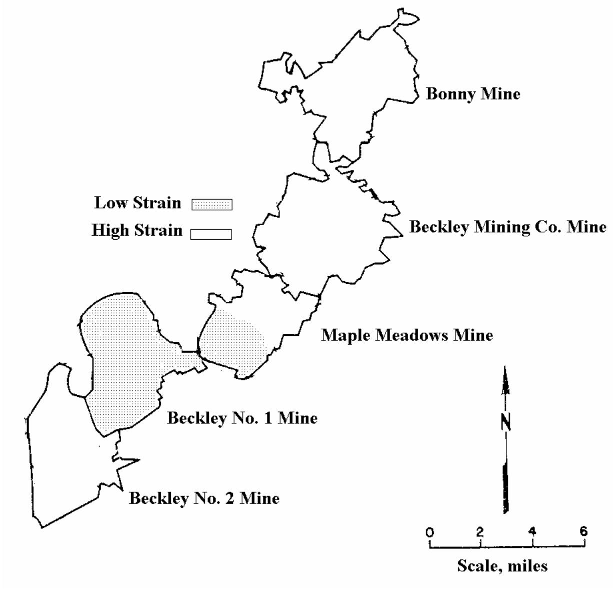

Figure 3. Beckley seam mines indicating the high and low strain zones, the low strain zone is shaded.

In the rest of the central Appalachian region, the two remaining high stain sites are in a mine 25 miles to the southwest of the Beckley seam study area. The remaining two low strain sites are 10 miles to the west and 100 miles to the southwest of the Beckley study area. The relationship of these high and low strain sites to those in the Beckley area and the extent of the high and low strain zones is not known. However, because of their geographic locations these sites are grouped in the central Appalachian region with the Beckley sites.

For both the eastern Mid-Continent and low strain central Appalachian regions, the sites are concentrated between 300 to 400 micro strains with only one or two sites for each region between 400 to 550 micro strains. For the northern Appalachian region, the sites are distributed fairly evenly across this 300 to 550 micro strain range.

Figure 4 shows a histogram of the minimum excess applied horizontal strains across the eastern United States. Most of the sites (22) have strains between 100 to 400 micro strains. Nine of the sites have strains that are below 100 micro strains and almost all are found in the eastern Mid-Continent region. All the sites with strains above 400 micro strains are found in the Beckley seam and central Appalachian region. However, although the distribution is again skewed, it is not as well defined as that for the maximum horizontal strain. Further, because several of the sites have relatively low strains, some of these sites could be in a partial strain relief situation regarding the minimum strain.

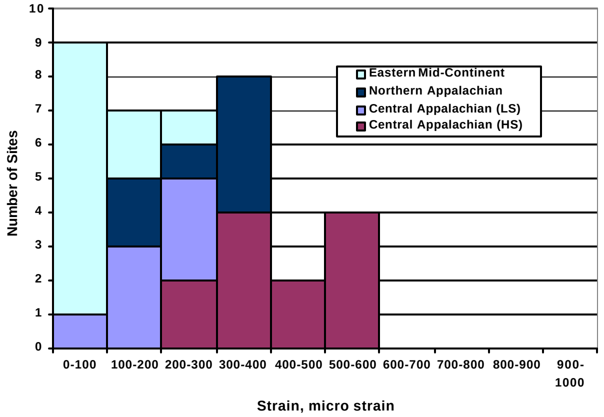

Figure 4. Average minimum excess applied horizontal strain distribution across eastern United States, by site.

Table 1 gives the regional average excess applied strain. The data from the central Appalachian region is separated into a low and high strain zones. The average strain for the high strain zone is statistically greater than the average for any of the low stain groups at a 0.05 significance level. Clearly, the central Appalachian region has areas with substantially higher maximum excess applied strains than the other regions. In the Beckley seam the low and high strain sites are in distinct geographic groups. However, there is not sufficient data from the rest of the central Appalachian region to establish a clear pattern to the regional strain field.

Table 1. Average excess applied horizontal strain by region for the eastern United States.

Region	Number of Sites	Maximum Strain Micro strain	Standard Deviation	Range of Site Maximum Strain Micro strain	Minimum Strain Micro strain	Standard Deviation	Strain Ratio Max/Min
Northern Appalachian	7	440	90	300-540	260	70	1.69
Central Appalachian							
Low Strain Zone	7	370	60	300-480	170	70	2.17
High Strain Zone	12	760	130	550-970	410	130	1.85
Eastern Mid-Continent	10	370	90	240-530	80	90	4.6

Table 2. Coefficients and regression statistics for maximum and minimum stress models (linear regressions fit through zero).

Region	Number of Sites	Maximum Stress		Minimum Stress	
		Coefficient, K_P	Coefficient of Determination	Coefficient, K_Q	Coefficient of Determination
Low Strain Models					
Northern Appalachian/Central Appalachian/ Eastern Mid-Continent	24	460	0.83	260	0.48
Eastern Mid-Continent/Central Appalachian	17	410	0.88	200	0.33
Northern Appalachian	7	530	0.87	350	0.73
Eastern Mid-Continent	10	390	0.91	120	0.02
High Strain Model					
Central Appalachian	12	920	0.85	610	0.43

Except for the central Appalachian high strain zone, the other regions appear to have similar strain fields. The northern Appalachian region though has strains that are about 20 percent higher than those in the other low strain regions. This has broadened the range of the lower strain zones as seen in the histograms. Further, the strain fields in the eastern Mid-Continent region are more biaxial than the other regions as signified by the maximum to minimum strain ratio. The standard deviations are about 20% or less of the average regional strains indicating the uniformity of the strain fields across these geographic areas.

HORIZONTAL STRESS VARIATION WITH THE ELASTIC MODULUS FOR THE EASTERN UNITED STATES

For the variation of the horizontal stress with the elastic modulus in the eastern Untied States, a separate analysis needs to be conducted for both the low and high strain sites. This can be done because the high and low strain sites can be grouped to a large extent geographically. Since the high strain sites are all found in the central Appalachian region, the high strain model is restricted to this region. However, the low strain model could be in general applicable to much of the eastern United States. Again, to minimize the expected depth effects, the excess horizontal stresses are used.

To examine the variation of the excess stress with the elastic modulus, a linear regression can be fit through the data with a zero intercept. The resulting equation is

$$P_e = K_p \text{ x } E \qquad (3a)$$

and

$$Q_e = K_Q \text{ x } E \qquad (3b)$$

where K_P = maximum excess strain coefficient, micro strain, and
K_Q = minimum excess strain coefficient, micro strain.

Figures 5 and 6 show the site average maximum and minimum excess horizontal stress versus the elastic modulus for both the high and low strain sites. Table 2 gives the resulting coefficients for the elastic modulus in equation 3 for the low and high strain groups. These coefficients can be considered the strain from the designated excess or tectonic stress component.

The low strain group is further separated geographically because the northern Appalachian region has strains that are about 20 percent higher than the other regions. The results of these separate models based on geographic regions or groupings are also shown in table 2. The improved correlation when separate low strain models are used clearly reflects the higher strains in the northern Appalachian region. Although the low strain area of the central Appalachian region is geographically adjacent to the other two regions, the limitation is that the distribution of the low and high strain zones is not completely known across this region.

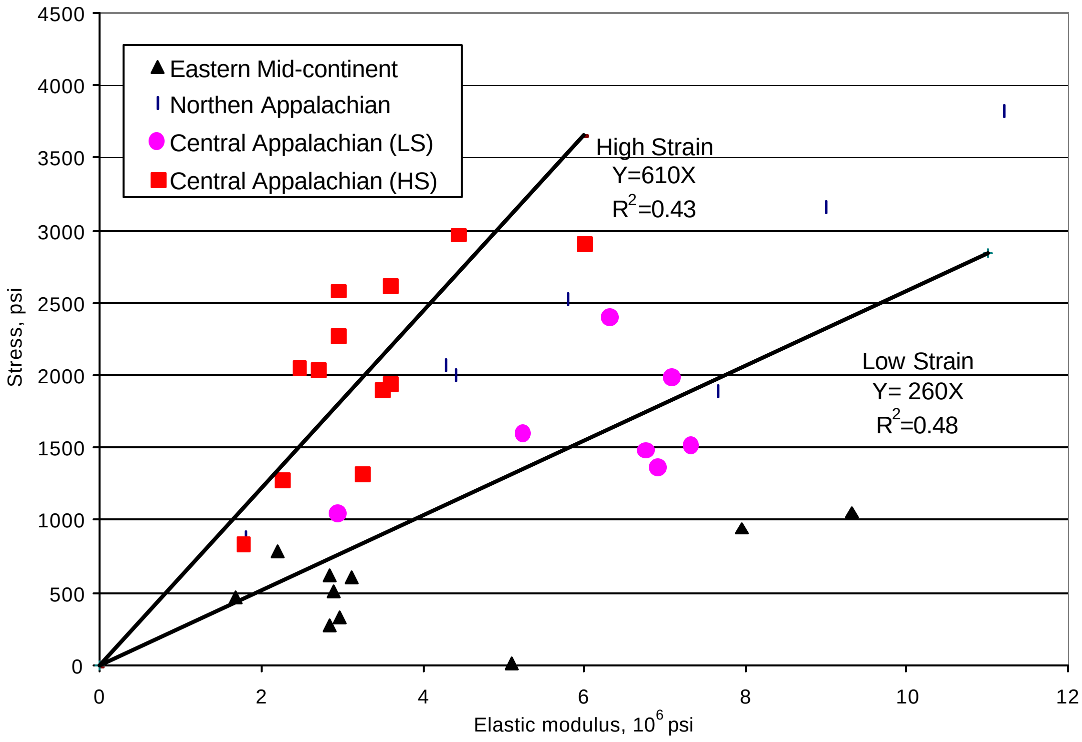

Figure 6. Minimum excess horizontal stress versus elastic modulus for the eastern United States.

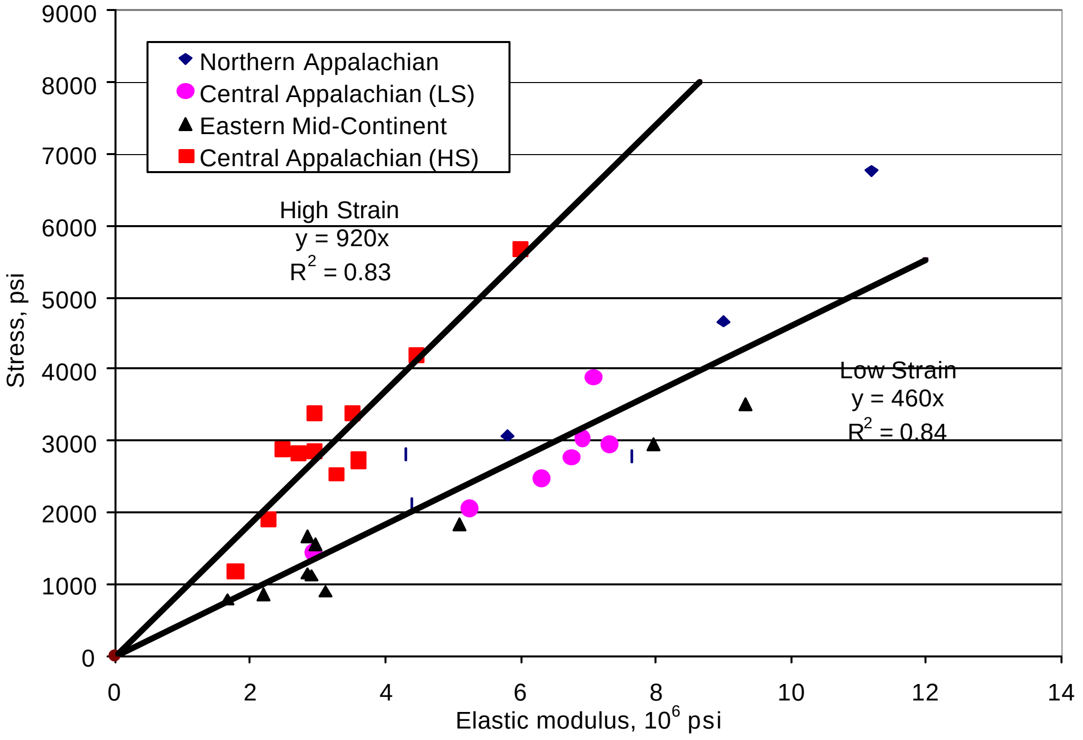

Figure 5. Maximum excess horizontal stress versus elastic modulus for the eastern United States.

For the maximum horizontal stress, there are clearly two groups of data, with no overlap between the low and high strain groups while the correlations for the models are extremely good. Therefore, the low strain model or models explain between 83 to

91% of the variation of the maximum excess horizontal stress across much of the eastern United States. The high strain model explains 85% of the variation in the high strain zones of the central Appalachian region. However, there are no definitive boundaries over where the model should be applied. In general, the low strain models reflect the strain field applied across most of the eastern United States, including a portion of the central Appalachian region, while the high strain model reflects a high strain region that has only been observed in the central Appalachian region.

For the minimum horizontal stress, the correlation with the elastic modulus is much lower. The elastic modulus explains only between 2 and 73% of the variation (table 2). This lower correlation results from the difference in the biaxial nature of the strain fields between regions (table 1). Essentially, to have a minimum stress that is as highly correlated to the elastic modulus as the maximum horizontal stress, the ratio of the maximum to minimum stress or strain must be similar between the sites and across the regions. Apparently there is a difference between the natures of the two stress components where there may be other factors that are controlling the minimum horizontal stress magnitude.

Based upon the calculated coefficients for the elastic modulus, equations or models for estimating the maximum horizontal stress that includes depth and the elastic modulus can be developed. Combining equation 3 with the expected increase of the horizontal stress with depth results in the following type of equation:

$$P = \left[1.1\left(\frac{n}{1-n}\right)\right] \text{ x } D + K_1 \text{ x } E \qquad (4)$$

This equation calculates the maximum horizontal stress in a given region or strain zone and explains between 83 to 91% of the variation of the maximum horizontal stress between sites.

A similar equation can be developed for the minimum horizontal stress. However, much lower correlations make estimations less reliable and subject to the regional and site differences in the biaxial nature of the stress field.

EFFECTS OF DEPTH ON HORIZONTAL STRESS FOR THE EASTERN UNITED STATES

Because there is such a strong correlation between the maximum horizontal stress and the elastic modulus, what, if any, of the remaining variation of the horizontal stress can be explained by the depth? Figure 7 shows a graph of the maximum horizontal stress versus depth for the sites in this study. The resulting linear regression equation is:

$$P = 1.48 \text{ x } D + 1700 \qquad (5)$$

The coefficient of determination is only 0.18 while the t-statistic indicates that the depth factor is significantly greater than zero at a 0.05 significance level. Therefore, in general there appears to be a significant increase in the maximum horizontal stress of about 1.5 psi per foot of depth in the eastern United States, though the depth can only explain about 18 percent of the variation. Further, depth cannot explain the high maximum stresses at shallow depths (figure 7). The highest maximum horizontal stress in the study is at a depth of only 400 ft. For the maximum stresses above 5,000 psi, the elastic modulus ranges from 6.32 to 11.2 million psi. These high stresses are explained by the elastic modulus in combination with the strain field and reflects the strong relationship with the elastic modulus.

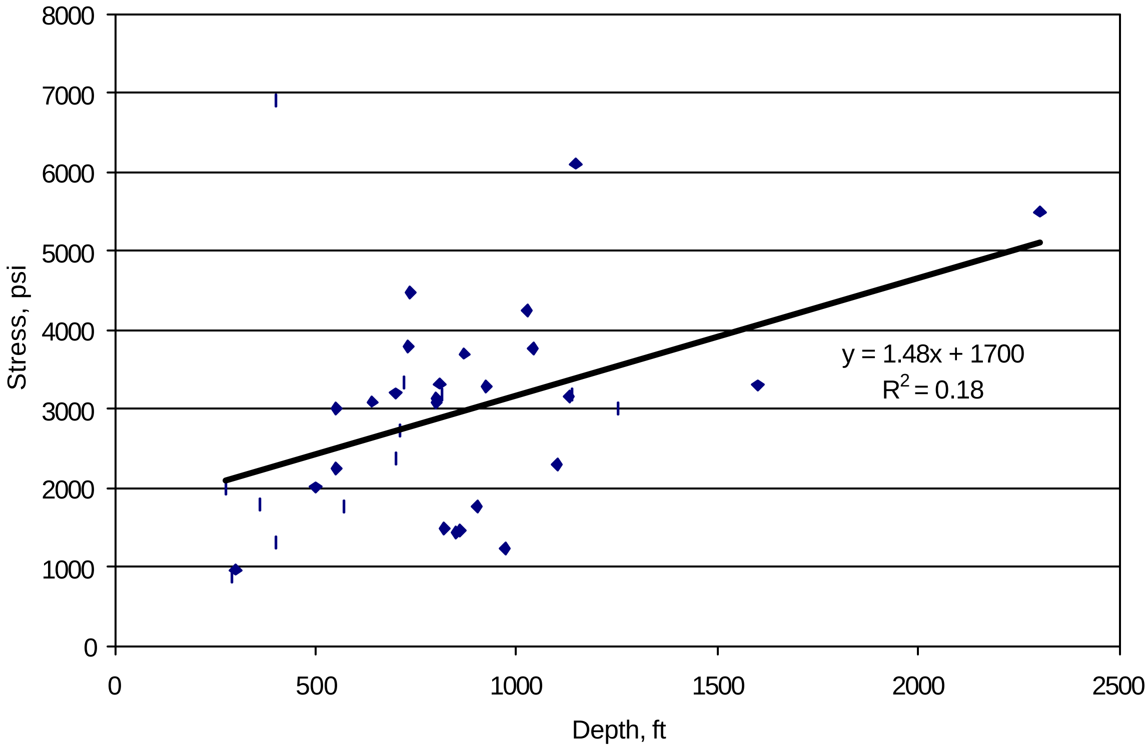

Figure 7. Maximum horizontal stress versus depth in eastern United States.

To more clearly evaluate the depth effects, the influence of the elastic properties can be removed by normalizing the maximum horizontal stress by the elastic modulus. This normalization produces the strain from the maximum horizontal stress. The resulting regression equation is

$$\varepsilon_p = 0.16 \text{ x } D + 570 \qquad (6)$$

where å$_p$ = strain from maximum horizontal stress, micro strain.

The coefficient of determination is only 0.05 while the depth coefficient is not statistically greater than zero. Therefore, there is no correlation between strain and depth and no significant increase in strain with depth. This change in significance implies that there could be a relationship between the elastic modulus and the depth. The correlation between the elastic modulus and depth is only 0.2. Essentially there is very little correlation between the two parameters. However, there is sufficient relationship between the elastic modulus and the depth that the stresses normalized by the modulus show no significant increase with depth.

Both the increase in the maximum horizontal stress and the elastic modulus could be related to regional differences in the depth, elastic modulus and strain from the maximum horizontal stress. Table 3 shows various average parameters for each region. There are differences in the average depth, elastic modulus, and maximum average strain for each region. In this case the Illinois basin data has been separated from the eastern Mid-Continent region. Although the average strain in the Illinois basin is similar to the other regions except for the high strain zone, the elastic modulus is much lower and results in a much lower maximum horizontal stress than the other regions. Combine these lower stresses with the lowest regional average depth and some increase in the horizontal stress and elastic modulus will probably be observed across the eastern United States. Therefore the statistical increase may be due in part to regional differences where the cause may not be the depth.

Table 3. Summary of average parameters for each region.

Region	Number of Sites	Depth, ft	Elastic modulus, 10^6 psi	Maximum[1] train Micro strain
Northern Appalachian		839	6.31	600
Central Appalachian				
Low strain	7	831	6.09	490
High Strain	12	996	3.29	1040
Eastern Mid-Continent Region	10	636	4.09	480
Illinois Basin	7	600	2.60	510

[1] Strain from maximum horizontal stress.

Further, there are at least two strain fields in the eastern United States that are apparently related to the geographic location rather than depth. To eliminate the affects of this regional strain difference only those sites from the lower strain regions can be examined.

The resulting regression equation for the twenty-four low strain sites is

$$\varepsilon_p = -0.005 \text{ x } D + 520. \qquad (7).$$

The coefficient of determination is only 0.0004 while the strain actually decreases with depth though not significantly. Essentially, without the high strain zone in the central Appalachian region, there is no increase in the strain with depth. This is an indication of the effect of the high strain zone having the greatest average depth. However, development of the high strain zone does not appear to be related to an increase in depth but to the geographic location.

Regional differences result in a statistically significant relationship between the depth and the maximum horizontal stress. Because the strains do not increase, depth cannot be considered a major causal factor in this increase. However, a theoretical increase based on gravity and Poisson's ratio is used in this study. This effect, if it exists, is small and could easily be masked in the data.

CONCLUSIONS

In general, the strain field across the eastern United States can be separated into a high and low strain zones based on geographic location. The high strain zone is geographically limited to the Beckley coal seam and central Appalachian region though the exact extent of this zone or zones is not known. The high and low strain zone distribution in the central Appalachian region is also not fully recognized. The lower strain zone encompasses much of the remaining eastern United States. The lower strain zone itself can be further subdivided because of the higher strains in the northern Appalachian region. Within each zone or region there is still some variation in the maximum horizontal strains between sites, though the standard deviations are 20% or less of the average strain.

Even though there is some variation between sites, the maximum horizontal strains are sufficiently uniform in each zone where the elastic modulus is the main factor controlling the magnitude of the horizontal stress. The elastic modulus can explain between 83 to 91% of the variation in the maximum horizontal stress between sites for each region.

Developed regression models that include the elastic modulus and the effects of gravity and Poisson's ratio can estimate the maximum horizontal stress magnitude. The low strain models are applicable to much of the eastern United States. The high strain model is confined to the central Appalachian region, though the exact area of application is not known.

For the minimum horizontal stress, the correlation to the elastic modulus is much weaker. In general, the minimum horizontal stress magnitude is less dependent on the elastic properties of the rock. This indicates a difference in the nature of the two stress components and is reflected by the regional variations in the ratio between the maximum and minimum horizontal stress or strain.

For this data set, depth was found not to be a significant causal factor in any increase in the horizontal stress. The statistically significant increase apparently results from the regional differences between other parameters such as the strain, elastic modulus, and average depth and a small correlation between the elastic modulus and depth.

Results of this study for the eastern United States are similar to those found in the United Kingdom studies and can be summarized by the following statement "Unlike the vertical stress, the horizontal stress component is not related to depth but to the rock stiffness" (14). It must be emphasized that this conclusion applies only to the range of depths and the geographic areas investigated.

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