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Methane Emissions From Gassy Coals in Storage Silos



UNITED STATES DEPARTMENT OF THE INTERIOR

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By J. E. Matta, J. C. LaScola, and Fred N. Kissell



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METHANE EMISSIONS FROM GASSY COALS IN STORAGE SILOS

by

J. E. Matta,¹ J. C. LaScola,² and Fred N. Kissell³

ABSTRACT

The methane gas emitted from coal samples collected from the conveyor belts dumping into silos was measured by the Bureau of Mines. Approximately 50 pct of the total gas desorbed into a sealed can within 1 week was released during the first 24 hours. No simple correlation between the gassiness of the coal stored and the methane concentration in the silo open space above the coal was found. This was probably because the gassier coals were stored in open-top silos, which were better ventilated. Although the methane concentration in the open space above the coal pile was less than 1 pct for all 34 silos investigated, a methane measurement in the coal pile showed that high methane concentrations can exist. The methane released in the pile appears to accumulate and not liberate freely into the open space above the coal pile. Probably this gas is released during reclaiming operations, but further research is required to substantiate this claim.

INTRODUCTION

Coal can be stored in a number of ways (1).⁴ It can be stored under water, in open pits, bins, bunkers, stockpiles, and silos. The current trend is to store coal in stockpiles or silos at the mine site, so that a predetermined quantity can be stored and easily loaded into railroad cars. In the past, coal was generally loaded into railroad cars the day it was mined; however, the use of the unit train, which affords reduced transportation rates for trainloads or partial trainloads of coal, has made temporary stockpiling at the mine a necessity (7). In addition to convenience in handling the coal, the use of silos lessens the possibility of physical and chemical changes in the coal, thereby preserving the utilization value of the fuel.

¹Research physicist.

²Mathematician.

³Physical research scientist.

All authors are with Pittsburgh Mining and Safety Research Center, Bureau of Mines, Pittsburgh, Pa.

⁴Underlined numbers in parentheses refer to items in the list of references preceding the appendix.

Subbituminous coal can be stored without much degradation in a closed bin with minimum air circulation, a slow rate of temperature change of the bin interior, and a low temperature (5). However, when gaseous bituminous coal is stored in a confined space such as a silo, methane accumulation can become an explosion hazard. The English were aware of this problem as early as 1875 when the Royal Commission inquired into the spontaneous combustion of coal and gas explosions in ships (6). Current Federal regulations (8) requires the methane content in the air of any structure, enclosure, or other facility to be less than 1.0 vol-pct. As deeper and gasier coals are mined, problems associated with methane accumulations in silos will increase.

The purpose of the present investigation was to study the possibility of a hazard resulting from methane accumulations in silos. Methane concentrations were measured in the open space above the coal and in the coal pile. Coal samples were collected from the conveyor belt that dumps the coal into the silo. The gas emitted from the samples was measured to determine the relationship between the gassiness of the coal stored and the methane concentration in the silo. This information is relevant because improper storage may result in high methane concentrations. Now, with even deeper and more gassy coalbeds being mined and with the growing practice of short- and long-term storage of fine coal, the explosion hazard due to methane emission is worthy of investigation. Most of the silos investigated in this study store large quantities of coal extracted from mines that emit large volumes of methane (table 1) and, therefore, methane accumulations in these silos are possible if sufficient ventilation is not maintained.

TABLE 1. - General information concerning silos investigated

Mine	Seam	Number of silos	Coal stored	Type of top	Capacity of each silo, tons	Daily mine production, tons	Mine methane emission, MM cu ft/day
Loveridge No. 22...	Pittsburgh.....	1	Clean..	Closed	10,000	9,200	11.6
Federal No. 2.....do.....	2	Raw....	..do..	2,500	4,000	8.1
Valley Camp No. 3..do.....	1	Clean..	..do..	10,000	4,200	2.0
Allison.....do.....	1	Raw....	..do..	6,800	4,500	.2
Do.....do.....	1	Clean..	..do..	17,400	4,500	.2
Nelms No. 1.....	Lower Freeport.	1	Raw....	..do..	5,000	4,500	2.1
Nelms No. 2.....do.....	1	Raw....	..do..	2,500	-	.4
Do.....do.....	6	Crushed	Open..	2,000	-	.4
Beatrice.....	Pocahontas No. 3.	1	Raw....	..do..	2,000	3,300	5.6
Virginia-Pocahontas No. 1.do.....	1	Raw....	..do..	2,000	3,000	3.9
Virginia-Pocahontas No. 2.do.....	1	Raw....	..do..	2,000	1,100	3.4
Virginia-Pocahontas No. 3.do.....	1	Raw....	..do..	5,000	2,400	3.3
Virginia-Pocahontas No. 4.do.....	1	Raw....	..do..	5,000	850	1.9
Oak Grove.....	Mary Lee.....	1	Raw....	Closed with fan.	6,000	800	1.3

SILO VENTILATION

Silos can be divided into two types according to their type of ventilation--open top and closed top. Open-top silos allow large air movements above the coal pile, thus reducing the hazard of a methane explosion. However, open-top silos provide no protection against precipitation, and wetting of some coals increases the chance of spontaneous combustion. Furthermore, the movement of large volumes of air through a silo can readily entrain dust into the atmosphere.

Although closed-top silos provide the protection not afforded by open-top silos, methane may accumulate above the coal pile during storage of gassy coals. Ventilation is provided by various types of openings at the top of the silo. Naturally ventilated silos depend upon wind to move air through the openings, although some silos employ exhaust fans to insure sufficient air movement.

Typically, ventilation of closed-top silos is through rectangular holes approximately 1 by 2 feet, spaced along the outer edge of the silo immediately below the concrete roof. This scheme is subject to slight variations. For example, in the silo at Nelms No. 1 mine, the side vent holes are expanded into a single 18-inch-high slit, starting at the point where the conveyor belt crosses the side of the silo, to a point halfway around the circumference. Some silos have round, covered ducts, 4 to 10 inches in diameter, that protrude through the roof to provide additional openings for ventilation. Openings on the roof such as access holes, inspection holes, and open space around the transfer point and conveyor belt also enhance air circulation.

When air movement due to the prevailing wind is insufficient or methane emission is high, exhaust fans are mounted on top of the silo. In a design used at the Oak Grove mine, eight I-beams support the concrete roof. To insure adequate ventilation, 8- by 16-inch vent holes appear on one side of the silo only, one for each of the nine spaces between the I-beams. On the other side of the silo, a metal duct passes up through the roof from each space and manifolds into a 30-inch exhaust fan powered by a 13.4-bhp motor. Air sweeps through the silo and flows parallel along the I-beams at a total rate of 16,600 cu ft/min.

A compromise between an open- and closed-top arrangement is used at the Nelms No. 2 mine. Here, a cluster of six open-top silos is covered by a slanted metal roof mounted high above the top edge. This allows free air movement while providing fair protection against precipitation.

SILO USAGE

Coal freshly mined from a working face may be temporarily stored underground in small batches and then brought to the surface. In a typical processing operation where silos are used, coal is conveyed by belt and dropped into the top of the raw coal silo during each mining shift. Coal is unloaded from the raw-coal silo through feeders in the silo base, onto a conveyor belt, and into the preparation plant at a continuous rate determined by the capacity

of the plant. Since the plant capacity is usually large enough to handle a full day's mine production in one plant shift, coal does not normally remain in the raw coal silo more than 24 hours. After processing, the coal is conveyed by belt to the clean-coal silo where it will be loaded onto a unit train. Unit trains are loaded as often as daily or as seldom as weekly.

CONVEYOR BELT GRAB SAMPLES

To ascertain the gas emission potential of gassy coals stored in a silo, random coal samples of a few pounds each were collected in sealed canisters. All of the coal samples tested originated from coalbeds with a high gas content, as indicated by the Bureau of Mines direct method for determining gas content (3) and presented in table 2, and also the daily mine methane emissions (table 1). The samples were collected from the conveyor belt emptying into the silo. Gas emission from the grab samples was measured periodically by bleeding the gas into an inverted water-filled graduated cylinder. Chromatograph analyses indicated that the gas released from the coal was essentially methane. Figure 1 shows some typical gas emission curves. Table 2 lists all the samples gathered plus the total gas emitted within various time intervals. As indicated from the figure and table, the gas emission rate decreases significantly with time. In fact, for the grab samples collected, approximately 50 pct of the total gas released within 1 week was emitted during the first 24 hours.

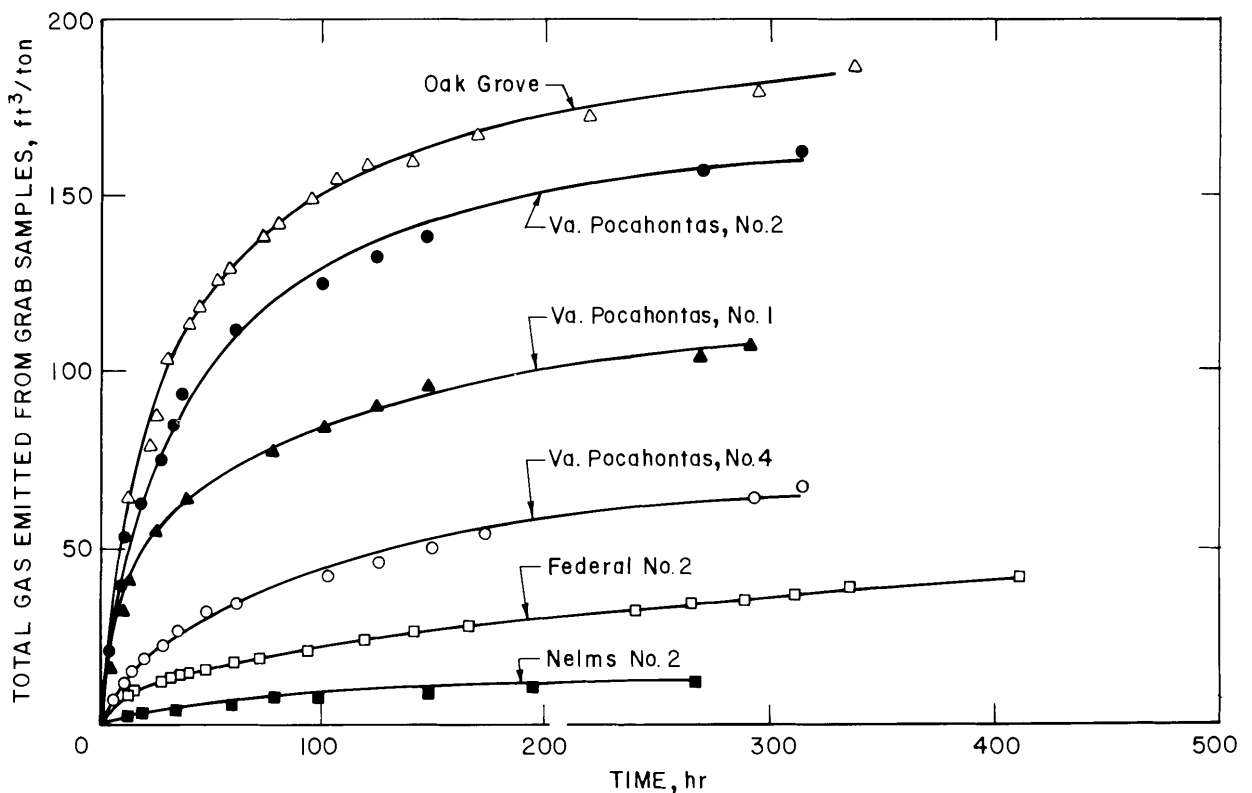


FIGURE 1. - Gas emitted from conveyor belt grab samples.

TABLE 2. - Gas emission from grab samples

Silo	Gas content, ¹ cu ft/ton	Sam- ple No.	Total gas emitted, cu ft/ton after--					Coal stored
			24 hr	48 hr	1 week	2 weeks ²	3 weeks ²	
PITTSBURGH								
Loveridge, West Virginia.	190	1	11.2	13.8	23.4	-	-	} Clean.
	190	2	10.2	12.9	22.8	-	-	
Federal No. 2, West Virginia.	190	³ 1	11.5	15.7	28.2	38.4	46.4	} Raw.
	190	2	5.8	7.5	12.2	16.7	19.8	
Valley Camp No. 3..	60-130	1	9.0	11.6	23.6	28.8	-	} Clean.
	60-130	2	9.1	11.3	22.0	26.6	-	
	60-130	3	7.5	9.7	20.3	24.6	-	
	60-130	4	12.3	14.9	26.7	31.4	35.4	
Allison.....	60-130	1	2.0	2.0	2.8	-	-	} Raw.
	60-130	2	2.8	2.9	4.0	-	-	
	60-130	3	1.0	1.7	4.5	-	-	} Clean.
LOWER FREEPORT								
Nelms No. 1.....	60-130	1	3.3	4.2	5.7	7.5	-	} Raw.
	60-130	2	3.6	4.5	6.8	8.7	-	
	60-130	3	4.2	4.5	6.2	7.9	-	
	60-130	4	3.8	4.2	6.1	8.2	-	
Nelms No. 2.....	60-130	1	3.6	5.2	10.6	-	-	} Raw.
	60-130	2	3.1	5.9	11.3	-	-	
	60-130	3	3.3	4.8	8.7	-	-	
	60-130	4	3.3	4.6	9.1	-	-	
	60-130	5	3.7	5.2	9.7	-	-	} Raw (crushed).
	60-130	6	3.4	4.8	9.6	-	-	
	60-130	7	2.8	4.1	8.4	-	-	
	60-130	³ 8	3.3	4.8	9.7	-	-	
POCAHONTAS								
Beatrice.....	350-600	1	85.8	178.1	197.9	230.9	242.6	} Raw.
	350-600	2	63.9	95.3	150.5	179.6	189.8	
Virginia-Pocahontas No. 1.	350-600	³ 1	55.0	67.6	93.0	109.2	115.0	} Raw.
Virginia-Pocahontas No. 2.	350-600	³ 1	71.1	106.3	145.8	163.6	170.4	
	350-600	2	30.7	33.6	47.0	43.7	44.2	
Virginia-Pocahontas No. 3.	350-600	1	14.2	18.4	26.0	32.8	35.5	} Raw.
Virginia-Pocahontas No. 4.	350-600	³ 1	20.5	31.8	53.0	68.2	75.5	
MARY LEE								
Oak Grove.....	500	³ 1	85.8	121.8	166.6	186.2	198.9	} Raw.
	500	2	85.9	120.0	161.5	179.2	191.3	
	500	3	67.4	101.7	145.9	165.5	178.2	
	500	4	31.1	45.9	64.0	72.1	77.4	

¹Direct-method results.²Dashes indicate that measurements were terminated since gas was no longer being emitted.³Emission curves for these samples are plotted in figure 1.

According to Müller (4), the total gas emitted, Q , can be expressed as follows:

$$Q = aFt^x, \quad (1)$$

where a is a constant, F is the coal tonnage, t is the time, and the exponent x ranges from 0.3 for coal size under 8 mm to 0.7 for grain size between 25 to 120 mm. Figure 2 is a log-log plot of the gas emitted per ton during the first 48 hours for the Federal No. 2 sample shown in figure 1. Here, the initial time, $t = 0$, is the time when the coal is taken from the silo belt. The slope of the dashed line, 0.54, corresponds to the exponent x for this coal. The best fit to the other five emission curves shown in figure 1 resulted in exponents that ranged from 0.4 to 0.8. Although these values are consistent with those obtained by Müller, it should be noted that Müller chose the time of cutting the coal as $t = 0$. The choice of initial time does affect the exponent. For example, ignoring the first 2 hours of gas emission from the Federal No. 2 grab sample, which would be equivalent to an additional 2-hour delay in transporting the coal to the silo belt altered the exponent to 0.68.

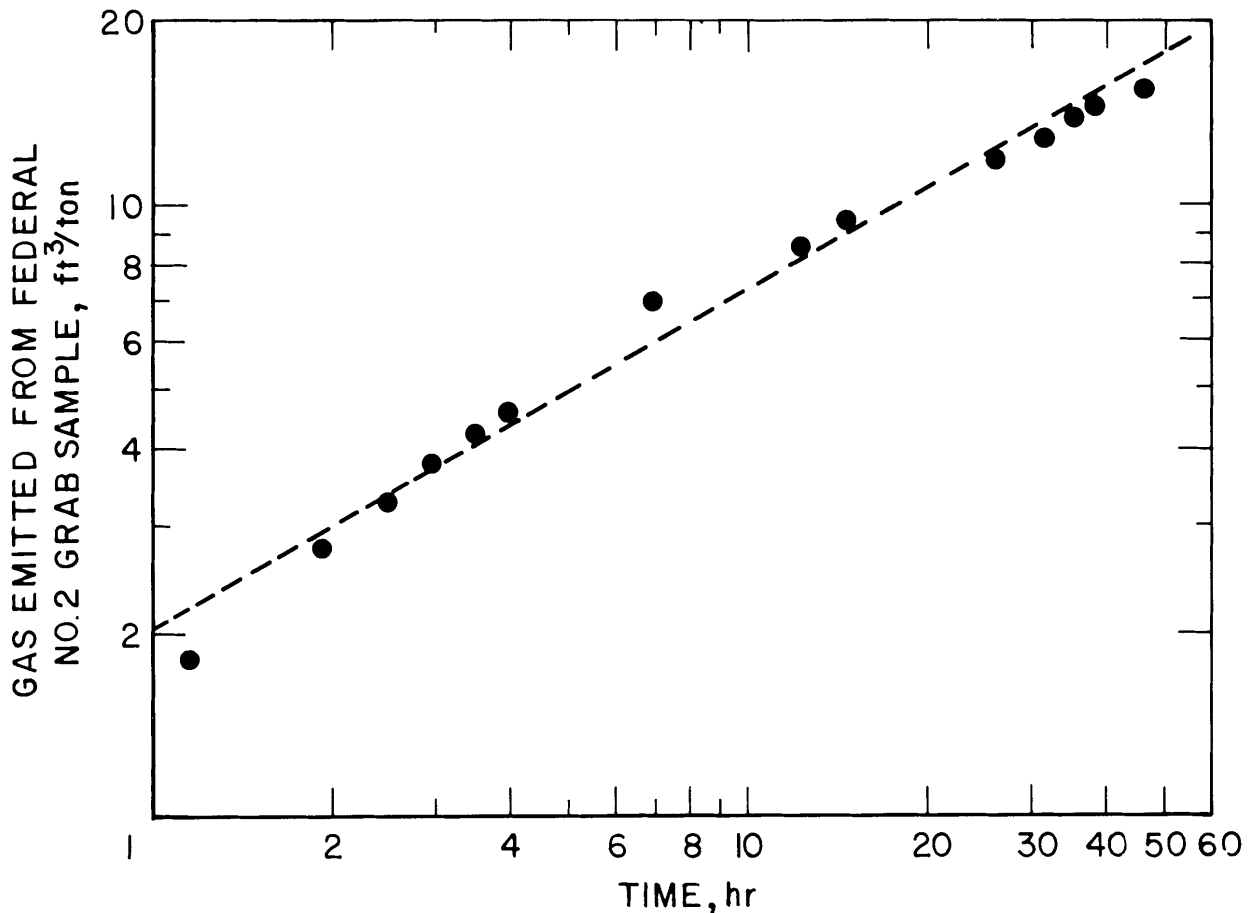


FIGURE 2. - Gas emitted from grab sample at Federal No. 2 mine (log-log plot).

The grab-sample emission curve in figure 2 can be used to estimate gas emission from the Federal No. 2 silo. Using this emission rate for the coal and assuming that an empty silo is filled at a constant rate over an 8-hour shift, the methane emission rate from the stored coal is shown in figure 3. Also superimposed is the quantity of coal stored versus time. The derivation of this emission curve is presented in the appendix. The emission rate increases with time to the 0.54 power until the silo is filled, then it begins to decrease. The highest emission rate occurs when the silo is just filled; that is, when the coal is gassiest and the quantity stored is maximum. Although the maximum gas emission rate is only about 40 cfm, this could result in a large methane concentration in a short time if proper ventilation is not provided.

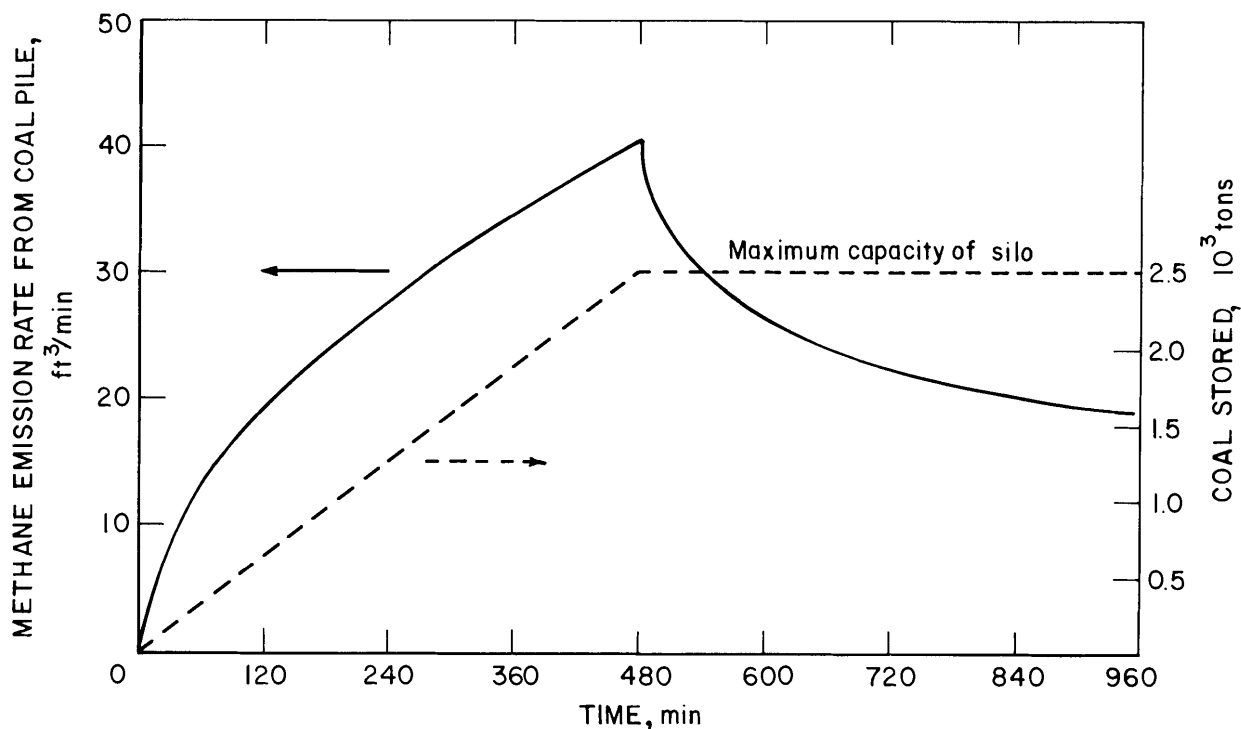


FIGURE 3. - Calculated gas emission from a coal silo using the Federal No. 2 grab sample and assuming a constant 8-hour filling rate.

METHANE CONCENTRATION ABOVE THE STORED PILE

Methane gas concentrations in the unfilled upper part of the silo were measured with a handheld portable gas detector. One end of a 100-foot section of 3/8-ID flexible plastic tubing marked at 5-foot intervals was attached to the gas detector; the other end was lowered into the silo for sampling. Occasionally, samples were taken in vacuum bottles, which were later analyzed using a gas chromatograph. Methane concentrations measured using the gas detector and chromatograph agreed within ± 0.1 pct.

Figure 4 shows typical curves for the methane concentration versus the distance from the top of the silo. The curves are based on concentration measurements at 10-foot intervals from the top near the center axis of the silo down to the coal surface. If the end of the tube actually touched the coal surface, concentrations of a few percent were often measured; therefore, to avoid measurement inconsistencies, the tubing end was always lifted a few inches from the coal surface. As seen from figure 4, there are no significant concentration changes throughout the unfilled portion of the silo. However, all of the open-top and most of the closed-top silos investigated do display a slight decrease in methane toward the top of the silo owing to better circulation.

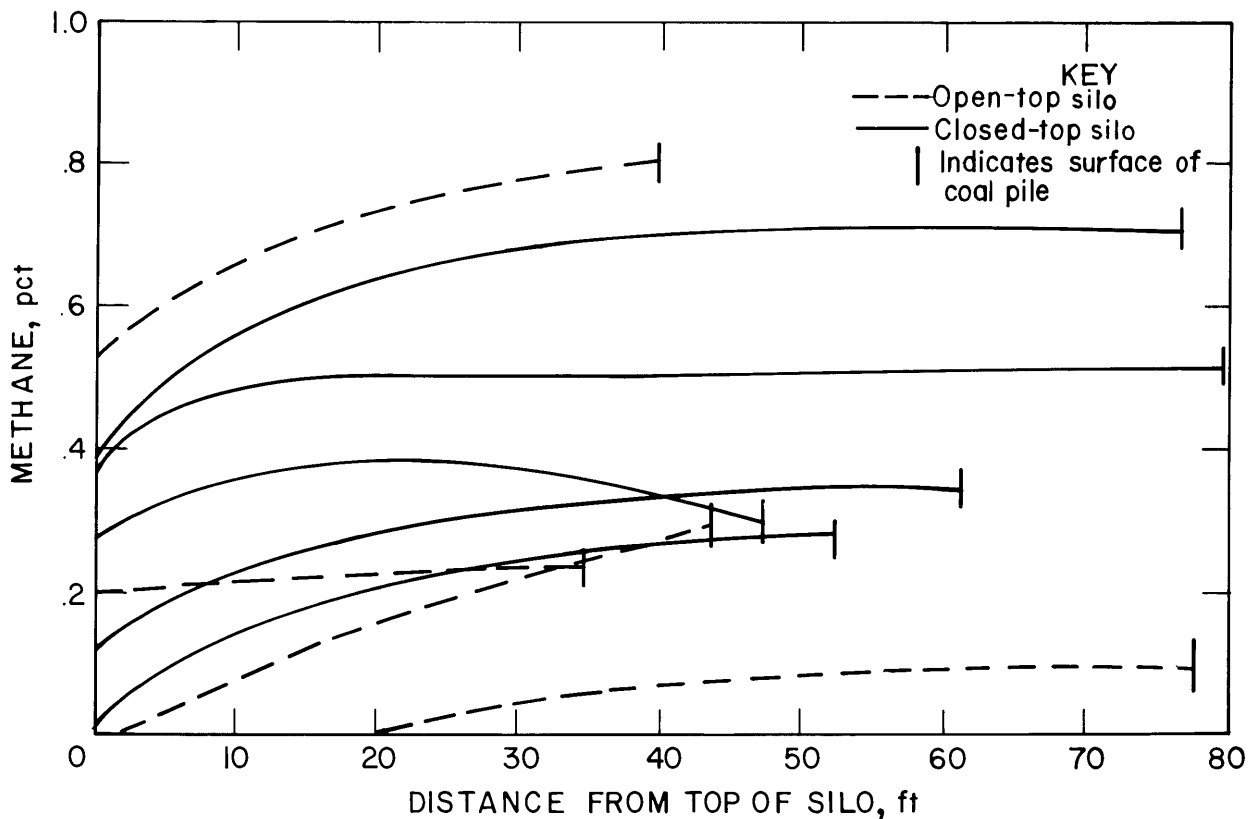


FIGURE 4. - Methane concentration gradient in coal silo.

Figure 5 is a plot of the average methane concentration in the silo above the coal pile versus the gassiness of the coal as determined from the random grab samples for each silo. Since the coal is usually stored for approximately 24 hours, this time period was used to determine the gassiness of the coal from the emission curves. The average methane concentration above the coal was estimated from the methane concentration midway between the top of the silo and the coal surface. For the silos shown in figure 4, the deviation from the average⁵ and midway concentration is less than 10 pct.

Prior to this investigation, it was thought that there might be a correlation between the coal gassiness and the average methane concentration above the coal surface. Higher methane concentrations might be observed for gassier coals. However, figure 5 shows no obvious correlation, probably because the gassier coals were stored in open-top silos where better air circulation reduces the methane concentration.

An example of this improved air circulation is the Beatrice silo at Keen Mountain, Va. (fig. 5). In 1970, this Beatrice silo, which was

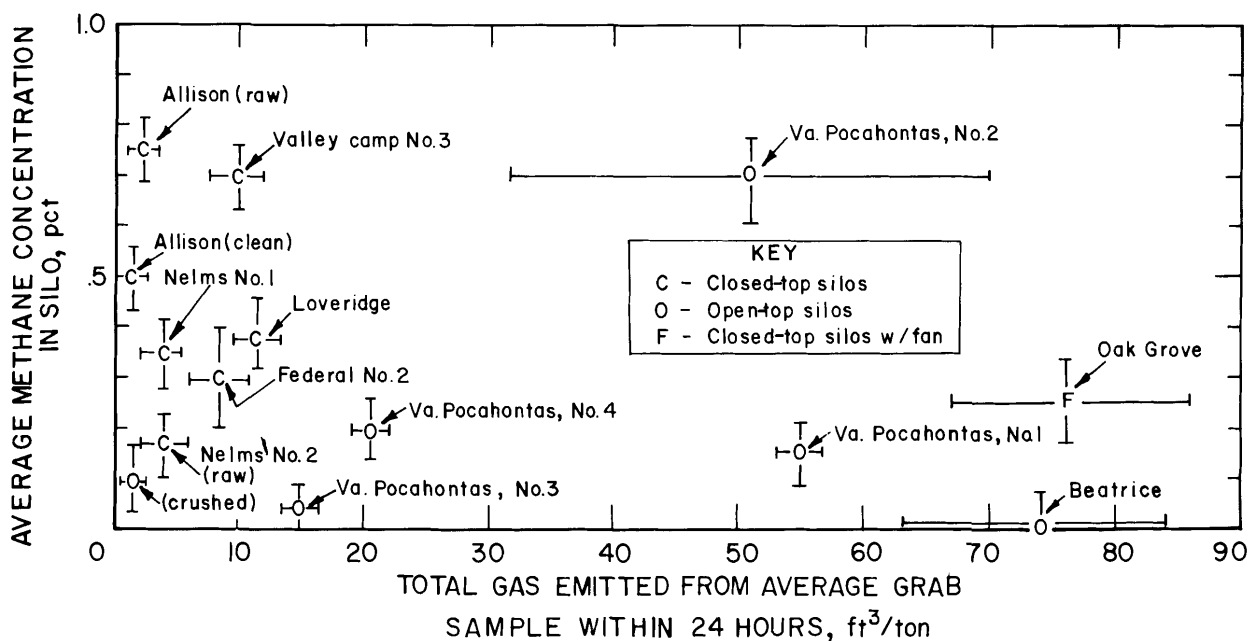


FIGURE 5. - Average methane concentration in silo versus gas emitted within 24 hours from the corresponding grab sample.

⁵The average methane concentration was determined from the methane concentrations taken from the top of the silo down to the coal surface at 10-foot intervals.

originally constructed as a closed-top silo, exploded. Subsequently, it was reconstructed as an open-top silo, and current tests indicated essentially no methane above the coal.

METHANE CONCENTRATION WITHIN THE STORED PILE

To investigate the methane concentration in the coal pile, a section of polyethylene tubing was extended halfway down into an empty closed-top silo, where it was buried by coal as the silo was filled over an 8-hour period. About 2 hours after the silo was filled, a methane concentration of about 14 pct was measured in the coal pile. This is similar to what one would estimate from a methane calculation assuming no ventilation.

Assuming no ventilation into the coal pile, the percent of methane concentration, C , which is initially zero, is governed by the following relation:

$$C = (1 - e^{-Q_g t / V}) \times 100, \quad (2)$$

where Q_g , V , and t are the gas emission rate, free space in the coal pile, and time, respectively. For the silo studied, V is approximately 50,000⁶ cu ft, and the average methane emission during the first 10 hours was about 20 cu ft/min. The average methane emission was determined from the random grab samples taken from this silo and using equation A-13, given in the appendix. Equation 2 predicts an approximate methane concentration of 21 pct after 10 hours.

The fair agreement between the calculated and measured concentration in the coal pile indicates that there was little ventilation throughout the pile. The methane concentration above the coal pile was less than 0.5 pct. It appears that although the methane concentration above the coal is low, large concentrations in the coal pile may exist.

As noted earlier, when investigating the air space above the coal pile, if the sampling-tube end actually touched the coal pile, methane measurements of a few percent were obtained. Also, at some of the silos visited, the workers noted that similar methane concentrations are detected below the silo in the vicinity of the feeders when coal is being unloaded from the silo. Similar problems have been reported during stockpile operations (7), further indicating that there can be high methane concentrations within the coal pile.

Large methane concentrations in the coal pile might generate an explosive atmosphere below the silo during reclaiming operations if proper ventilation is not maintained. At the Valley Camp No. 3 mine in West Virginia, two 10,000-cu ft/min exhaust fans force air into the vicinity of the feeders, reducing the methane concentration below 1 pct during reclaiming operations. The fans operate simultaneously with the conveyor belt that unloads coal out of the silo.

⁶The free space in the coal pile was determined by calculating the empty volume of the silo minus the volume of coal stored using 1.3 g/cu cm as the density.

Although this investigation was designed to study the possibility of hazardous methane concentrations in and above the coal pile, it appears now that a followup study of the silo reclaiming area is also warranted.

CONCLUSIONS

1. Even for stored gassy coals, open-top silos appear to provide sufficient ventilation above the coal to keep the methane concentration far below 1 pct. If degradation of the coal is a concern, a slanted metal roof of the sort used at the Nelms No. 2 mine can provide protection while maintaining good ventilation.

2. High methane concentrations can exist in the coal pile. It is possible that this gas is released during reclaiming operations, resulting in high concentrations below the silo if this area is not ventilated properly. However, to substantiate this, further research is required.

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⁷Title enclosed in parentheses is translation from the language in which the item was originally published.

APPENDIX.--THEORETICAL CALCULATION OF THE GAS
EMISSION FROM A COAL SILO

To determine the total gas emitted from a silo, one should take into account the filling time of the silo. This can be considered by dividing the silo into discrete vertical intervals, n , of equal size and summing the gas released from the coal in each interval. The total amount of coal stored, F , can be expressed as the sum of the coal stored in each interval; namely,

$$F = f_1 + f_2 + \dots + f_i + \dots + f_n, \quad (\text{A-1})$$

where f_i is the amount of coal stored in the i^{th} interval. Assuming a constant silo filling rate, the total time to fill the silo, T_f , is

$$T_f = n\Delta t, \quad (\text{A-2})$$

where Δt is the time to fill an interval; that is,

$$\Delta t = f_i/b, \quad (\text{A-3})$$

where b is the filling rate.

Now, if $Q_1(t)$ equals the total gas emitted from the i^{th} interval at time, t , then the total gas emitted, $Q_T(t)$, for $t > T_f$ is

$$Q_T(t) = Q_1(t) + Q_2(t - \Delta t) + Q_3(t - 2\Delta t) + \dots + Q_n(t - [n-1]\Delta t). \quad (\text{A-4})$$

According to Müller (4), $Q_1(t)$ can be expressed as

$$Q_1(t) = af_1 t^x, \quad (\text{A-5})$$

where a and x are constants depending upon the coal stored,

$$Q_T(t) = af_1 [t^x + (t-\Delta t)^x + \dots + (t - [n-1]\Delta t)^x]. \quad (\text{A-6})$$

The gas emission rate from the silo, $E(t)$, for $t \geq T_f$ can then be written as

$$E(t) = \lim_{\Delta t \rightarrow 0} \frac{Q_T(t+\Delta t) - Q_T(t)}{\Delta t}. \quad (\text{A-7})$$

Using equations A-6, A-3, and A-2 and taking the limit

$$E(t) = \lim_{\Delta t \rightarrow 0} a \frac{f_1}{\Delta t} [(t+\Delta t)^x - (t - [n-1]\Delta t)^x]; \quad (\text{A-8})$$

$$E(t) = \lim_{\Delta t \rightarrow 0} ab [(t+\Delta t)^x - (t-T_f+\Delta t)^x]; \quad (\text{A-9})$$

$$E(t) = ab [t^x - (t-T_f)^x], \text{ for } t \geq T_f. \quad (\text{A-10})$$

Similarly for $t \leq T_f$ by summing the gas emitted for only the filled intervals, the total gas emission rate can be expressed as

$$E(t) = abt^x, \text{ for } t \leq T_f. \quad (\text{A-11})$$

The total gas emitted can be determined from the emission rates by integration; that is,

$$Q_T = \frac{ab}{x+1} t^{x+1}, \text{ for } t \leq T_f; \quad (\text{A-12})$$

and

$$Q_T = \frac{ab}{x+1} [t^{x+1} - (t-T_f)^{x+1}], \text{ for } t \geq T_f. \quad (\text{A-13})$$

Flügge (2) developed an equation similar to A-12 for the specialized case $x = 0.5$ using a double integration technique.