## Theory

A general analytical solution of the effect of an inclined contact separating two media of differing electrical properties on a normally incident plane electromagnetic field has been given (Geyer, 1972). These solutions for the electromagnetic response at any observation point $P(r, \varphi)$ are given in the form of inverse Lebedev-Kontorovich transforms. Two modes of excitation are considered: (1) case where current density flows across strike of the contact separating the media of differing electrical properties (only $E_r$, $E_\varphi$, and $H_x$ differ from zero) and (2) case where current density flows along strike (only $H_r$, $H_\varphi$, and $E_x$ differ from zero). The electromagnetic fields in the frequency range of interest and for the rock conductivities under consideration are quasi-static in nature so that to a first-order approximation the horizontal magnetic field at the surface of the earth is constant and continuous across the discrete discontinuity in resistivity for case (1) above, and so that the tangential derivative of the horizontal electric field at the surface of the earth is constant and continuous across the discrete discontinuity for case (2) above. Of course, it is well known that where large horizontal gradients in conductivity exist, there also exists large anomalous components of the vertical magnetic field (Weaver, 1963). The recognition of such anomalous components and their magnitudes due to the presence of lateral resistivity changes in the overburden becomes very important in any wavetilt type measurement which may be used to locate a miner.

When the lateral change in resistivity becomes quite large, the integral solutions (expressed as Lebedev-Kontorovich transforms) simplify considerably and allow ready numerical evaluation. Examples of the amplitude and phase responses of the radial electric, horizontal electric, and vertical magnetic field in the vicinity of the contrast in resistivity are shown in Figures 2, 3, and 4. Of particular significance is the fact that the anomalous vertical magnetic field is three orders of magnitude larger near the contact than far from the contact. This fact implies that where large resistivity contrasts do occur in the rocks overlying mine workings, that significant errors may be present in wavetilt measurements as used above for location criteria! Another observation insofar as the anomalous vertical magnetic field is concerned is the linear variation of phase as one proceeds away from strike in induction number (increased distance or frequency). For a pulse-excited communications system this phenomena would have a bearing on where the zero crossings would occur (i.e., 'pulse breadth') in the transient coupling between any given source-receiver configuration and at any given receiver site.

## Summary

Application of Lebedev-Kontorovich transforms together with the imposition of the quasi-static condition on the electromagnetic field at the air-earth interface have led to analytic integral solutions for the electric and magnetic fields inside two regions separated by a sloping interface where the normally incident fields were plane waves. In the derivation of the solutions the assumed boundary conditions were that the horizontal magnetic field (H polarization) and vertical derivative of the horizontal electric field (E polarization) are continuous and constant across the interface. Therefore the results are correct to $O(\omega\epsilon/\sigma_n)^{\frac{1}{2}}$. These solutions may be used to compute a correction term to the field in the air above the lateral resistivity contrast (and thus along the surface) which, in turn, might be used to evaluate field solutions on either side of the dipping contact to $O(\omega\epsilon/\sigma_n)$. This type of iteration may be continued indefinitely with each step improving the order of accuracy of the solutions in the quantity $(\omega\epsilon/\sigma_n)^{\frac{1}{2}}$.

The integral solutions obtained simplify considerably where large resistivity contrasts occur. By use of saddle-point techniques it is possible to evaluate the fields for $\pi/4 < \delta < 3\pi/4$ and at the same time to estimate the error involved in the asymptotic expansions in terms of the proximity to the interface separating the two regions of contrasting resistivity.

The numerical results indicate that one must exercise caution in wavetilt schemes for the location of a miner, especially in those regions where the presence of lateral resistivity contrasts in the rock overlying mine workings may affect the positioning of maxima and null phenomena in the surface magnetic fields produced by a buried loop antenna.

## References

Clemmow, P. C., 1953, Plane wave spectrum representation of electromagnetic fields.

Geyer, R. G., 1971, Research on the transmission of electromagnetic signals between mine workings and the surface: Quart. Tech. Rpt. for period Oct. 1, 1971 to Dec. 30, 1971 for U.S. Bureau of Mines, 106 p.

Geyer, R. G., 1972, The effect of a dipping contact on the behavior of the electromagnetic field: Geophysics, v. 37, n. 2, p. 337-350.

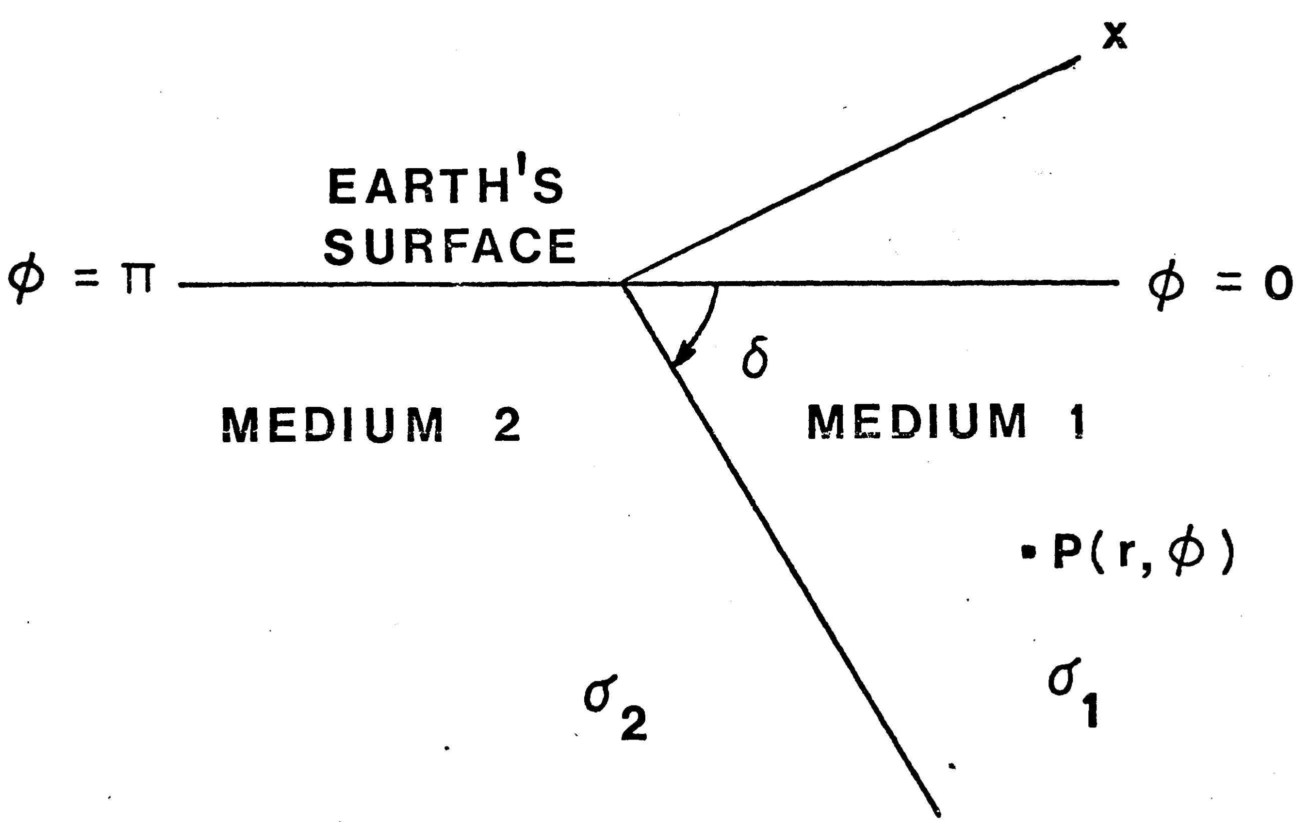


Fig. 1. Geometry of model where discrete contrasts in electrical properties occur.

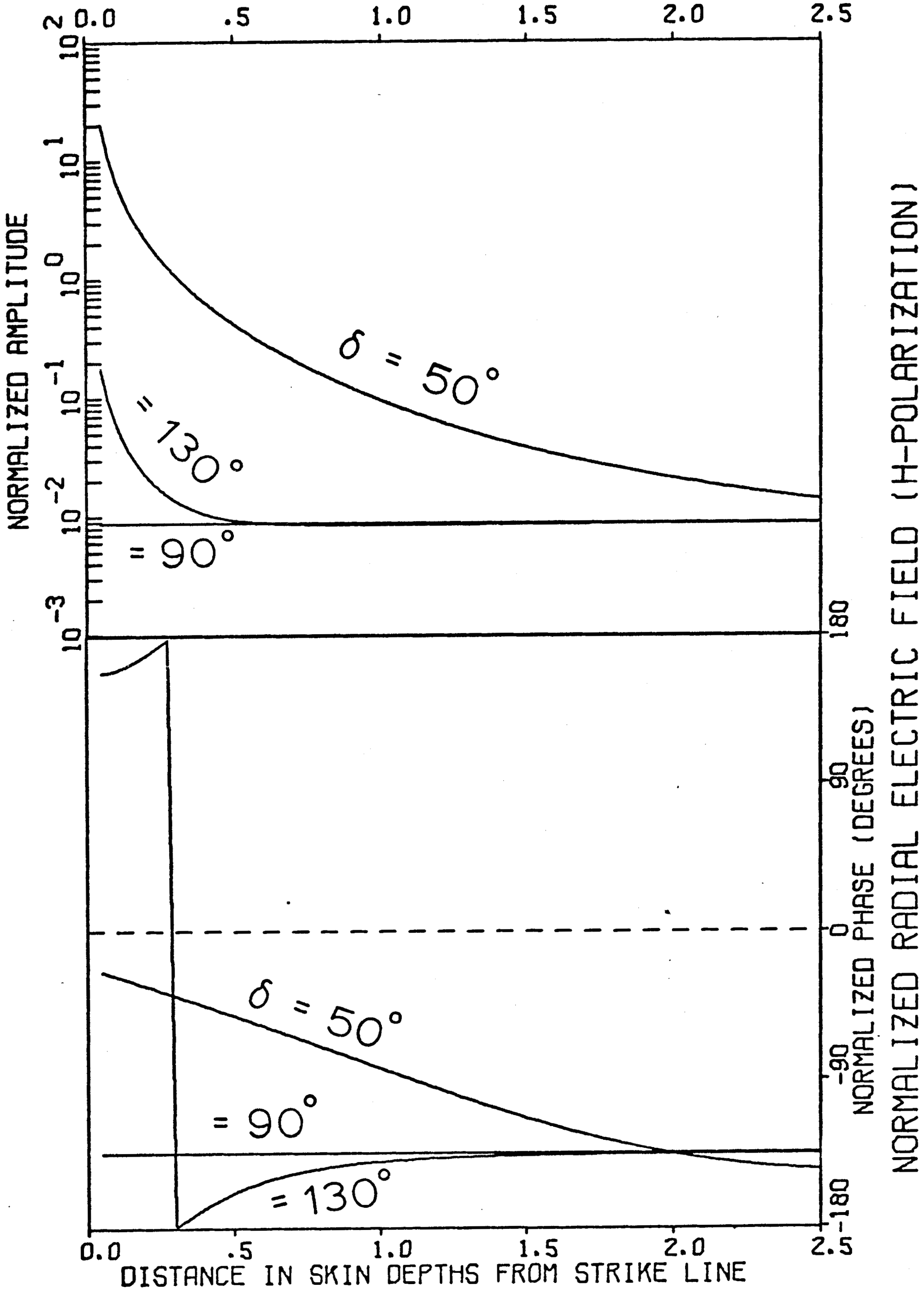


Fig. 2. Normalized radial electric field response over poorly conducting region for several dip angles of the interface (H-polarization).

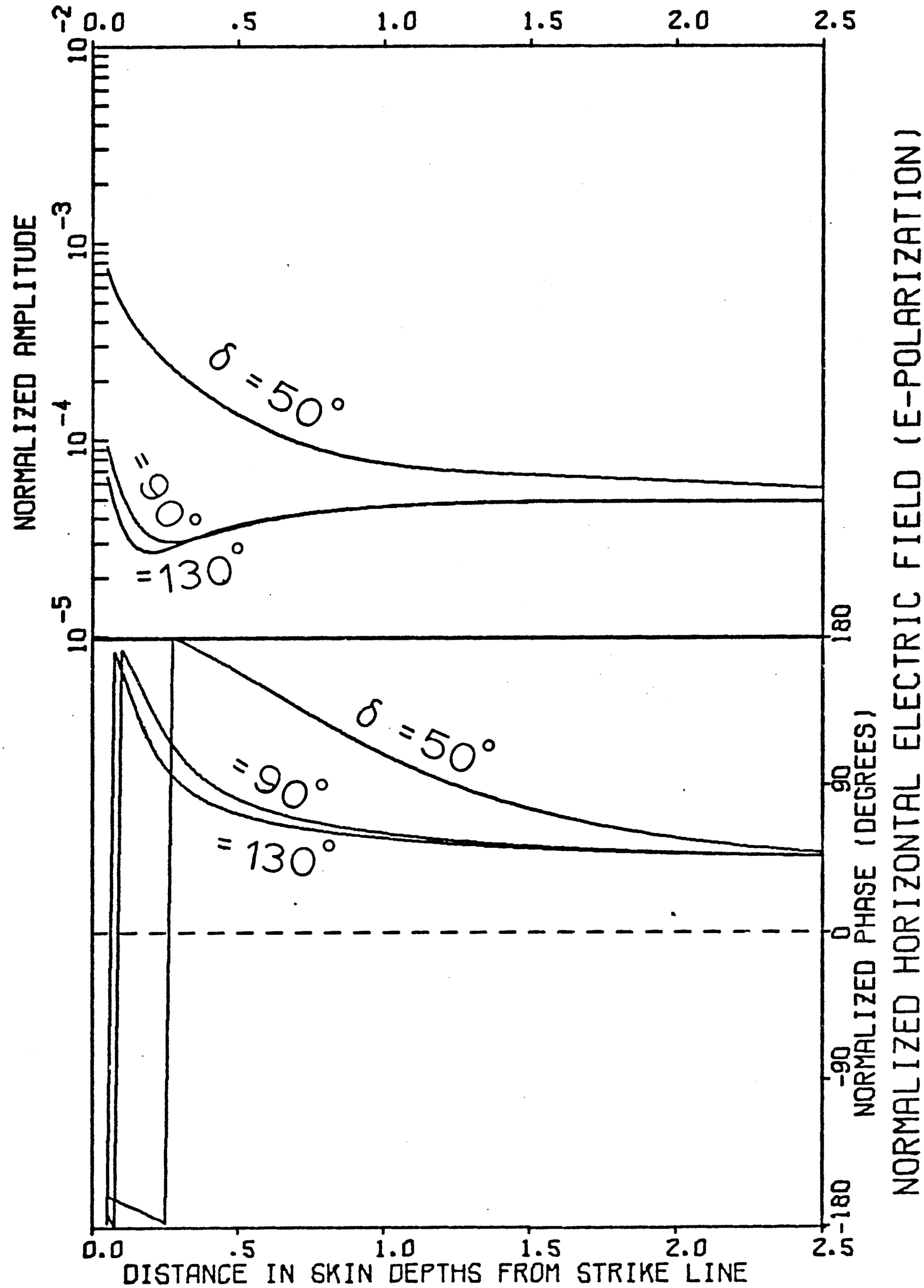


Fig. 3. Normalized horizontal electric field response over poorly conducting region for several dip angles of the interface (E-polarization).

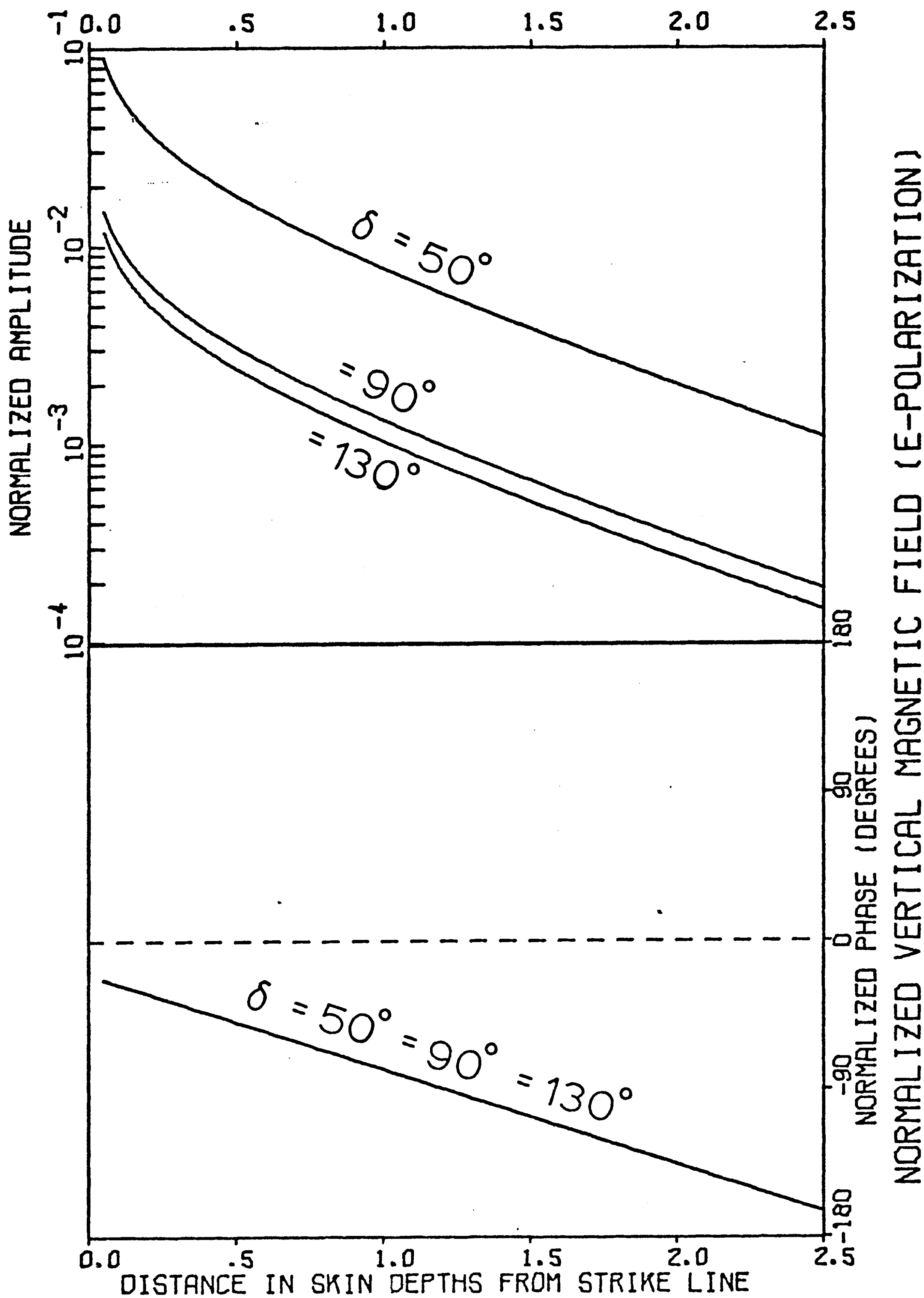


Fig. 4. Normalized anomalous vertical magnetic field over poorly conducting region for several dip angles (E-polarization).