THE PERTURBATION OF ALTERNATING ELECTROMAGNETIC FIELDS BY THREE-DIMENSIONAL BODIES

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Abstract

The perturbation of alternating electromagnetic fields by three-dimensional structures is considered. The general model is that of a semi-infinite conducting half-space which consists of regions of different conductivities. A numerical method is used to obtain the solution for the equations and boundary conditions. The effects on the fields at the surface of the half-space due to the conductivity discontinuities are shown by three-dimensional amplitude and phase plots.

General Method

During the last year or so a numerical method has been developed for studying three-dimensional electromagnetic induction problems. The conductive structure of interest is defined over a three-dimensional mesh of grid points and a solution for the equations and boundary conditions is obtained by a finite iterative technique. By means of this method, the effects of buried anomalies have been studied (Jones and Pascoe, 1972) as well as islands and coastlines (Lines and Jones, 1973a, b) and cubic conductors (Jones, 1973) for uniform inducing fields.

The general method is described in detail by Lines (1972). If we neglect displacement currents as is normal for the slowly varying fields we wish to consider, and assume a time variation exp($i\omega t$), we obtain from Maxwell's equations (in e.m.u.):

$$\nabla \times \nabla \times \vec{E} = -4\pi\sigma i\omega \vec{E} \tag{1}$$

or

$$\nabla^2 \vec{E} - \nabla(\nabla \cdot \vec{E}) = i\eta^2 \vec{E} \tag{2}$$

where $\eta^2 = 4\pi\sigma\omega$

Equation (2) must be solved in all regions. Also, the usual boundary conditions must be satisfied. This equation may be written as three scalar equations in Cartesian coordinates:

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \frac{\partial}{\partial x}\left[\frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right] = i\eta^2 E_x \tag{3}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} - \frac{\partial}{\partial y}\left[\frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z}\right] = i\eta^2 E_y \tag{4}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} - \frac{\partial}{\partial z}\left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}\right] = i\eta^2 E_z \tag{5}$$

In the method these equations are written in finite difference form and are solved simultaneously for E_x, E_y, and E_z at each point of the mesh by the Gauss-Seidel iteration technique.

The external boundaries of the mesh are set by assuming that the conducting half-space is uniform or uniformly layered there. That is, the external boundaries are placed far enough from any conductivity discontinuities so that the perturbations of the fields at the boundaries by such discontinuities will be negligible. Within the mesh the usual boundary conditions are employed. However, when discontinuities in conductivity are encountered, the mixed partial derivatives which arise from the $\nabla(\nabla \cdot \vec{E})$ term in (2) (and which insure the proper coupling of the three field components) are evaluated by central differences in the same manner as for the other derivatives. The component of $\vec{E}$ normal to an abrupt boundary between conductivity regions will be discontinuous and such a discontinuity cannot be represented by a point value at the boundary. The use of a double mesh to accomodate all possible occurences of double valued functions for the arbitrary models we wish to consider is prohibitive in computer time and cost. Therefore, at discontinuities the average of the values of the $\vec{E}$ component normal to the boundary on either side of the discontinuity is used to represent the value at the boundary. When this is done, and the boundary conditions applied to $\vec{E}$, η^2 in the difference equations representing (3), (4), and (5) is replaced by $\overline{\eta^2}$, the average of η^2 for all the regions surrounding the point being considered. This implies that the conductivity discontinuity is represented by a transition zone from one conductivity region to the other. This is a reasonable approximation for geophysical situations. If the boundary is a transition zone from one conductivity region to the other, then $\vec{J}$ ($= \sigma \vec{E}$) must be continuous, and if σ varies continuously through the transition, $\vec{E}$ will vary as we have assumed.

After the electric field components are approximated by the Gauss-Seidel iterative technique, the magnetic field components are calculated by taking the curl of the electric field, i.e.,

$$\vec{H} = -\nabla \times \vec{E} / i\omega \tag{6}$$

A model of geophysical interest is shown in Fig. 1. This is an anomaly embedded in a half-space. This anomaly is just below the surface and $\sigma_1 = 10\,\sigma_2$. The source field is such that $\hat{E}$ is polarized in the x-direction. In Fig. 2 are shown three-dimensional plots of the amplitudes and phases of the electric field components over the surface of the conducting region. The magnetic field components are shown in Fig. 3. From these figures it is clear that the uniform fields are considerably perturbed by the presence of the buried anomaly.

References

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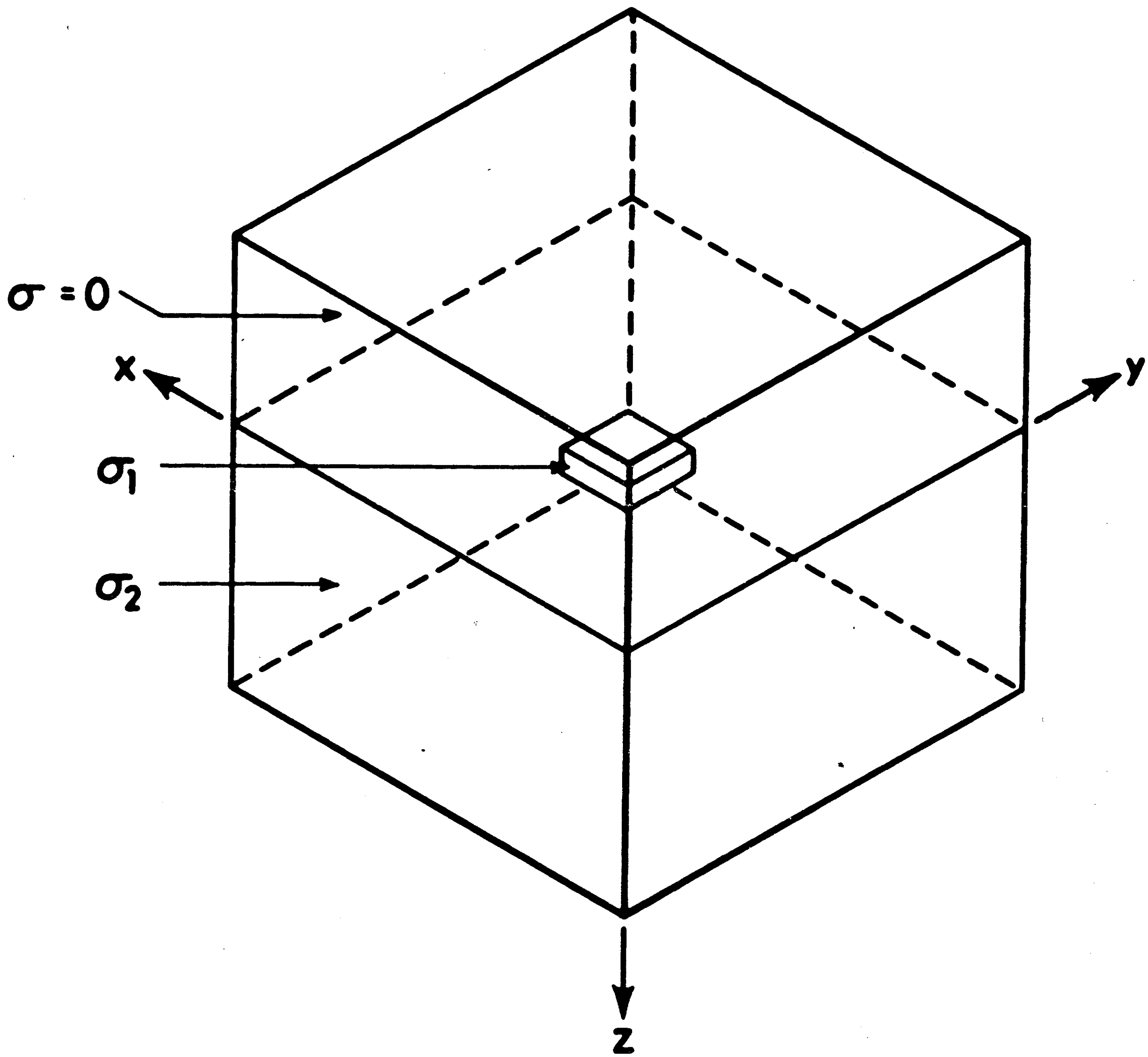

Fig. 1. The general three-dimensional model
(from Jones and Pascoe, 1972)

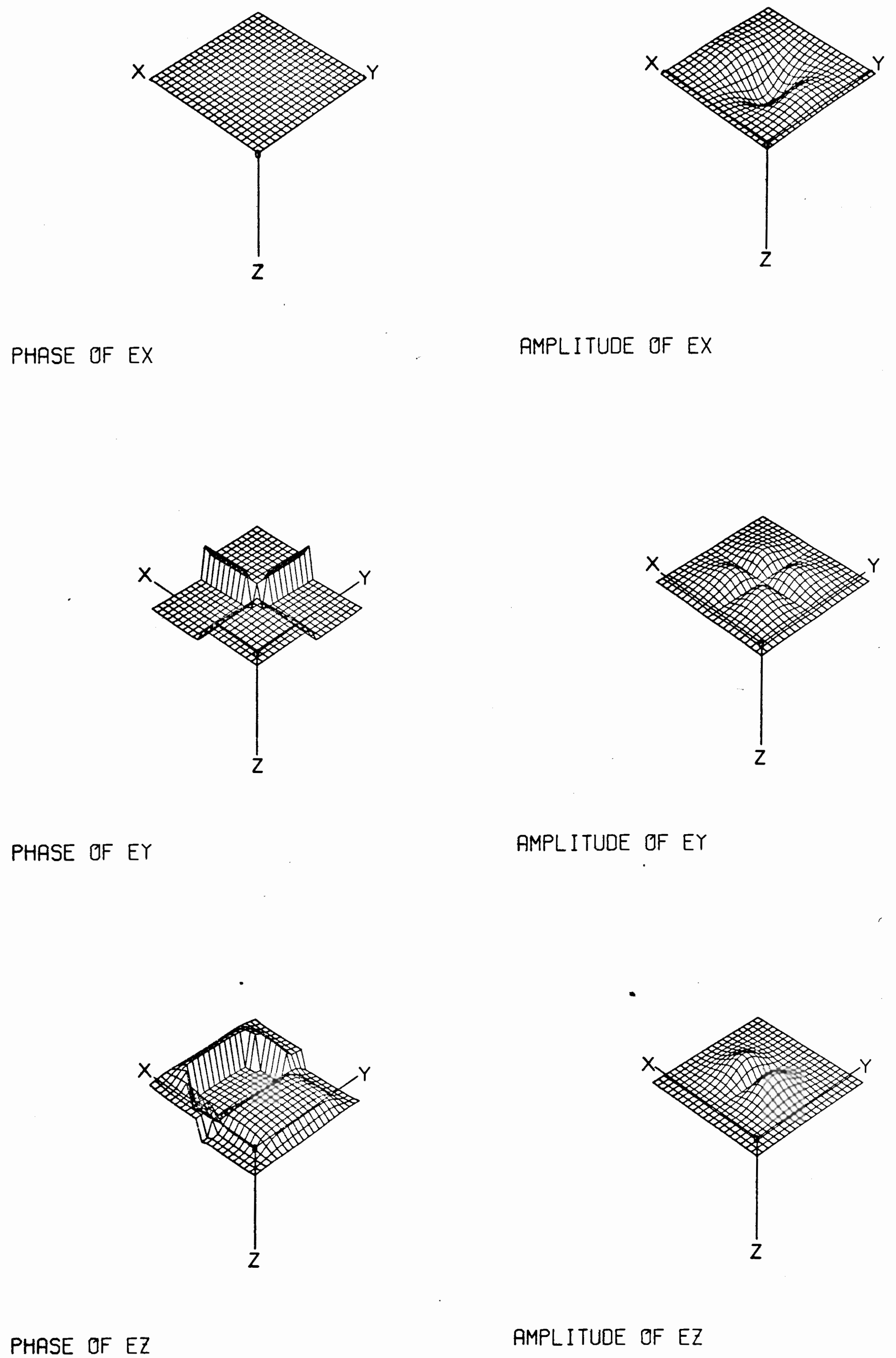

Fig. 2. Electric field component amplitudes and phases for the model as in Fig. 1, for $\sigma_1 > \sigma_2$. (From Jones and Pascoe, 1972).

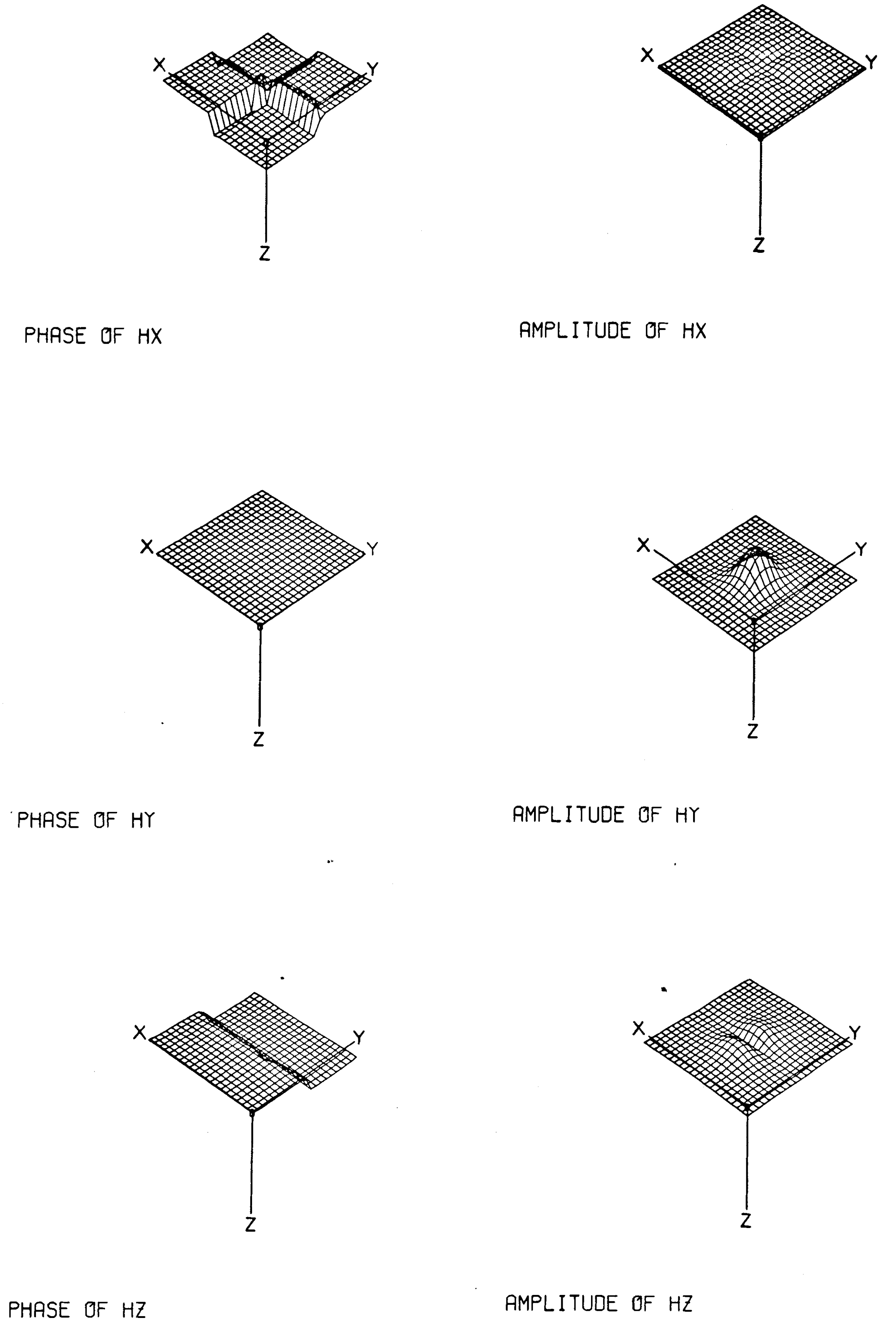

Fig. 3. Magnetic field component amplitudes and phases for the model as in Fig. 1. for $\sigma_1 > \sigma_2$. (From Jones and Pascoe, 1972).