THE ELECTROMAGNETIC RESPONSE OF A BURIED SPHERE FOR BURIED DIPOLE EXCITATION*

D. A. Hill and J. R. Wait
U. S. Department of Commerce
Office of Telecommunications
Institute for Telecommunication Sciences
Boulder, Colorado 80302

Introduction

The feasibility of locating a buried vertical magnetic dipole source (horizontal loop) from surface measurements of the vertical and horizontal magnetic field components has been investigated by Wait (1971). For sufficiently low frequencies, the magnetic fields have a simple static-like behavior, and a single observation of the ratio and relative phases of the vertical and horizontal field components is sufficient for location when the earth is homogeneous. However, when inhomogenieties are present, the surface fields will be modified, and source location may be more difficult.

In order to obtain a quantitative idea of the surface field modifications, we consider a spherical conducting zone as a perturbation to the homogeneous half-space. A rigorous solution for the buried sphere problem has been formulated by D'Yakanov (1959). Unfortunately, his solution is restricted to azimuthally symmetric excitation, and even then the solution is not in a convenient computational form. However, if the sphere is electrically small and is located at a sufficient distance from both the dipole source and the interface, the scattered fields can be identified as the secondary fields of induced dipole moments. The latter are equal to the product of the incident fields and the polarizabilities of the sphere. The details of the approach are given by Hill and Wait (1973).

Wait (1968) has used this induced dipole moment approach for scattering by a small sphere above a conducting half-space. The method has the advantage that it is easily generalized to scatterers of other shapes for which both the electric and magnetic polarizabilities are known, such as spheroids (Van de Hulst, 1957). This concept has

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also been considered by Ward (1967) in the context of electromagnetic detection of massive sulfide ore bodies from airborne platforms.

Induced Dipole Moments

The geometry of the situation is shown in Figure 1. The buried sphere has radius a, conductivity σ_s, permeability μ_s, and permittivity ϵ_s.

Although the distances involved are much less than a free-space wavelength, they are not necessarily small compared with a wavelength in the lower half-space. Consequently, the induced electric dipole moment must be considered as well as the induced magnetic dipole moment. The unperturbed magnetic field has both horizontal and vertical components at the sphere center, H_x^o and H_y^o, and the unperturbed electric field has only a horizontal component, E_y^o. The components can be obtained from Wait (1951; 1971), Banos (1966), or Ward (1967). The induced magnetic dipole moments, m_x and m_y, are given by the product of the magnetic polarizability and the unperturbed magnetic field (Wait, 1960; 1968). The induced electric dipole moment, p_y, is given by the product of the electric polarizability and the unperturbed electric dipole moment (Van de Hulst, 1957; Wait, 1960).

In order that higher order multipoles are not important, it is necessary that the sphere radius is small compared to both the wavelength in the lower half-space (Stratton, 1941) and the geometric mean of the source and observer distances (Wait, 1960). The sphere radius must also be small compared to the depth so that interactions between the sphere and the interface are not important. Actually, interface interaction terms have been computed for a buried cylinder (Wait, 1972), but the interaction terms are more complicated for a sphere because of coupling between the electric and magnetic dipole modes (Hill, 1970). In the static limit, the interaction has shown to be unimportant for a sphere-interface separation of at least two sphere radii (Hill and Wait, 1972).

Surface Fields

The total magnetic field at the surface is the sum of the vertical magnetic dipole source field and the reradiated fields of the dipole moments induced in the sphere. The fields of buried vertical and horizontal magnetic dipoles are given by Wait (1971; 1972), and the fields of a buried horizontal electric dipole are given by Wait (1961) and Banos (1966).

The actual calculations involve the evaluation of numerous Sommerfeld type integrals which must be done numerically. However, all of the integrals can be evaluated in closed form in the zero-frequency limit. If these limiting forms are subtracted, the convergence of the remaining integrals is significantly improved. These static limits may even be of direct interest if the frequency is sufficiently low. In the static limit the contribution from the electric dipole moment goes to zero, but the contribution from the magnetic dipole moment goes to zero only for a non-magnetic ($\mu_s = \mu_o$) sphere (Ward, 1959).

Discussion and Conclusions

A computer program was written to calculate the magnitude and phase of the ratio of the vertical and horizontal magnetic field components at the earth's surface. These are the measurable quantities which Wait (1971) has suggested for location of the vertical magnetic dipole source. For spheres of radius less than .2 times the source depth, the change in surface fields are found to be insignificant. For a sphere of radius .3 times the source depth located at half the source depth, some noticeable changes in the surface fields begin to occur. However, resultant errors in source location should still be small.

Larger errors can be expected when the sphere is either larger or closer to the source or interface, but the simplified theory presented here is not valid under such conditions. The study of such cases is a worthwhile extension.

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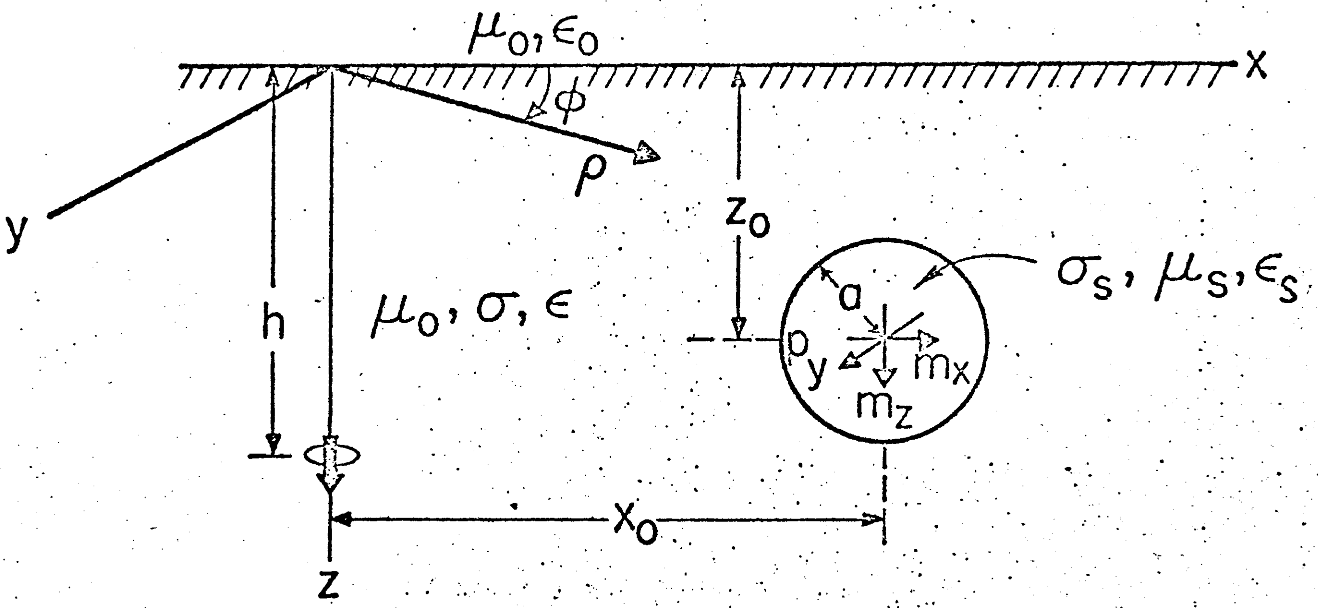

FIGURE 1. Geometry for source loop and buried sphere with induced dipole moments.