FIELDS OF A MAGNETIC DIPOLE EXCITED BURIED CYLINDER

Allen Q. Howard, Jr.
U. S. Dept. of Commerce
Institute for Telecommunication Sciences
Office of Telecommunications
Boulder, Colorado 80302

Abstract

An approximate solution to the electromagnetic boundary value problem consisting of a horizontal cylindrical conductor buried in a lossy half-space and excited by an arbitrarily oriented magnetic dipole is found using an iterative perturbation technique in a double Fourier transform space. This model is used to gain insight into the anomalous fields due to strong scatterers such as pipes or tracks which would be in close proximity to an EM mine rescue operation. The novel three-dimensional aspect of the problem (i. e., the source) imposes the complexity that the current in the cylinder is not uniform. The field expressions are ideally suited to evaluation using FFT algorithm.

Introduction

The electromagnetic response of an inhomogeneous half-space continues to be a subject of high interest because of its direct application to EM prospecting techniques, mine rescue operations, and location of buried gas or water pipes. By in large, with the notable exceptions of D'Yakonov (1959), and Hill and Wait (1973), the theory applied to this class of problems has been restricted to two-dimensional time harmonic analyses. For example, the February '71 issue of Geophysics was devoted entirely to numerical solutions of this type. The comparatively simple sub-case consisting of an idealized buried cylindrical inhomogeneity has also received considerable attention (D'Yakonov (1959), Wait (1972), and Howard (1972)). We comment that D'Yakonov published no numerical results; the approximate iterated perturbation method due to Wait, which accounted for the interaction of the air-earth interface and the induced axial monopole current is readily evaluated. Numerical results based on Wait's method have been shown to be in complete agreement with the integral equation solution of Howard (1972). All three of these solutions are two-dimensional since the primary excitation in each case is taken to be a uniform line current parallel to the cylinder.

Herein, we consider a three-dimensional extension of these solutions. That the extension is non-trivial is attested by the effort involved in considering such problems in the absence of the air-earth interface (Wait, 1960). Thus, the uniform line source is replaced by a more realistic arbitrarily oriented magnetic dipole in the earth. This introduces several complications. The most serious are that we now need a two-dimensional transform representation and the problem is now intrinsically vector. The vector nature requires either the introduction of both electric and magnetic potentials (Wait (1958), Weaver (1970)) or the appropriate Green's function dyadic (Tai (1971)). For an arbitrary cylinder, the relevant two-dimensional Fourier integral dyadic can be used with a vector integral equation formalism. It is, under certain conditions, permissible and much simpler to by-pass the integral equation technique and use a perturbation analysis.

Thus, to keep the problem tractable, we will assume that the cylinder is perfectly conducting and electrically small so that only an axial surface current density has appreciable excitation. This allows us to perform a perturbation analysis similar to that of Hill (1970) and Wait (1972) to obtain an iterated approximant of the axial surface current. Note that in so doing, the features of an air-earth interface and an arbitrarily oriented magnetic dipole are not compromised. Hence, the problem remains three-dimensional; however, the simplifying assumptions divide the problem into three almost completely independent parts - each one of which is a well-defined boundary value problem.

As an overview, we give in section two a two-dimensional Fourier integral representation of the magnetic loop in a half-space. With an eye to the application of this "incident" field, we represent the resulting interface fields in terms of magnetic and electric vector potentials parallel to the cylinder axis which is introduced in the following step. In the third section, we obtain the zeroth order surface current on the cylinder which includes the "over and down" mode coupling. This is done for one spectral component of the incident field - i.e., an arbitrarily polarized plane wave constituent.

Then, in the fourth section, we obtain the interface dependent potentials for a given axial surface current. It then becomes a straightforward matter to iterate the correction to the surface current starting with the zeroth order current obtained in the previous section. The anomalous fields are, of course, written in terms of the potentials derived in section four.

II. Two-Dimensional Fourier Integral Incident Field

Here we consider the well-known problem of a magnetic dipole in a half-space. Our application, however, requires us to look at the problem afresh. The geometry is given in Figure 1.

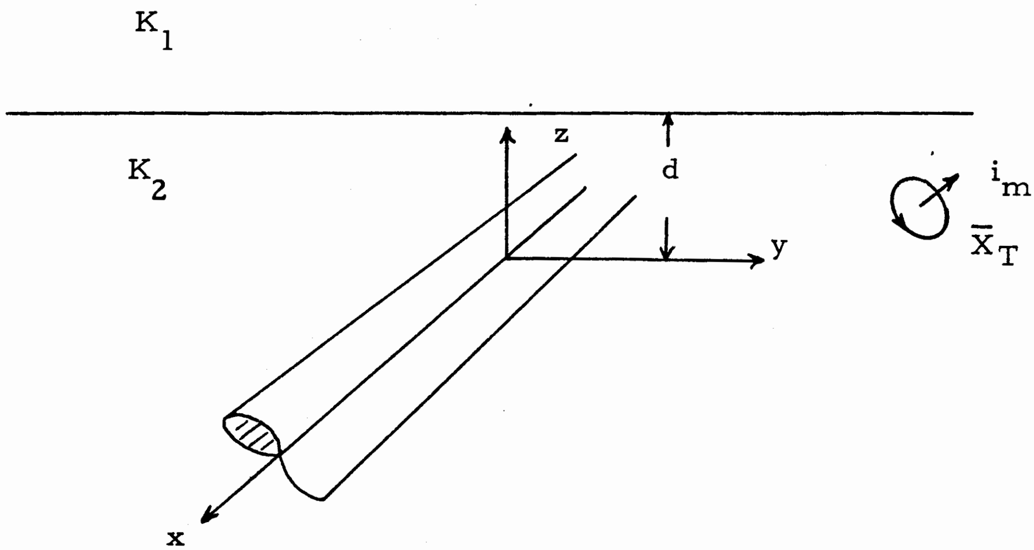

Figure 1. Scattering geometry.

Our solution is not standard in that for numerical reasons, it is preferable to use Cartesian coordinates, and the potentials are conveniently chosen parallel to the cylinder axis. We require a solution then to

$$\nabla \times \bar{H} = -i\omega \epsilon_2' \bar{E} \quad , \quad \epsilon_2' = \epsilon_2 + i\sigma_2/\omega$$

$$\nabla \times \bar{E} = i\omega\mu_0 (\bar{H} + \bar{M}) \tag{2.1}$$

by way of the potentials $\bar{F}$ and $\bar{A}$, i.e.,

$$\bar{H} = 1/\mu_0 \; \nabla \times \bar{A} + \frac{1}{\mu_0 K_2} \nabla \times \nabla \times \bar{F}$$

$$\bar{E} = \frac{i\omega}{K_2}(\nabla \times \bar{F}) + \frac{1}{K_2} \nabla \times \nabla \times \bar{A} \tag{2.2}$$

For an elementary magnetic dipole, $\overline{M}$ is given by

$$\overline{M} = \overline{m}\,\delta(\overline{x} - \overline{x}_T), \quad \overline{m} = I_o dA\overline{i}_m \tag{2.3}$$

Now for a homogeneous space $\overline{A} = 0$ and

$$\overline{F} = \overline{F}_o \equiv k_2 \mu_o \overline{m} \frac{e^{ik_2 R}}{4\pi R}, \quad R = |\overline{x} - \overline{x}_T| \tag{2.4}$$

This, in the representation required, is also written

$$\overline{F}_o = \frac{ik_2 \mu_o \overline{m}}{8\pi^2} \int_{-\infty}^{\infty} \frac{d^2K}{K_z^-} e^{i\overline{K}\cdot(\overline{x} - \overline{x}_T)}, \quad K_z = (K_2^2 - K_x^2 - K_y^2)^{1/2}$$

$$Im(K_z) \geq 0$$

$$z - z_T \geq 0 \tag{2.5}$$

where $d^2K = dK_x dK_y$

To account for the interface at $z = d$, we define

$$\overline{F} = \begin{cases} \overline{F}_o + \overline{F}^- & z \leq d \\ \overline{F}^+ & z > d \end{cases} \qquad \overline{A} = \begin{cases} \overline{A}^- & z \leq d \\ \overline{A}^+ & z > d \end{cases} \tag{2.6}$$

where

$$\overline{F}^{\pm} = \overline{i}_x \int d^2K\, f^{\pm}(K^{\pm})\, e^{iK^{\pm}\cdot\overline{x}}$$

$$K^{\pm} = (K_x, K_y, \pm K_z^{\pm}) \tag{2.7}$$

The pertinent solution is

$$a^- = \frac{i e^{2iK_z^- d}}{K_2} \Big\{ \left[K_z^- K_x^2 - K_2^+ K_\rho^+\right]\left[K_2^2 f_o^{(2)} - K_y \overline{K}\cdot\overline{f}_o\right] + K_x K_y K_2^2 \left[K_z^- f_o^{(1)} - K_x f_o^{(3)}\right] + K_x K_y \left[(K_2^2 f_o^{(1)} - K_x \overline{K}\cdot\overline{f}_o)(K_z^+ + K_z^-)\right]\Big\} \Big/ \left[K_y^2 K_2^2 + K_z(K_x^2 K_z^- - K_z^+ K_\rho^2)\right] \tag{2.8}$$

where $\overline{f}_o = \dfrac{iK_2 \mu_o \overline{m}}{8\pi^2 K_z^-} e^{-i\overline{K}^-\cdot\overline{x}_T}$

III. Zeroth Order Surface Current

We now consider the interaction of a plane-wave constituent of the previous section with the cylinder surface current. Basically, we require a transverse vector cylindrical wave expansion of a plane wave constituent of the "incident" field.

$$\bar{\mathcal{E}}^{\circ} e^{i \cdot \bar{x}} = \int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} dh \sum_{n=0}^{\infty} (a_n \bar{M}_{en\lambda}^{(1)} + b_n \bar{N}_{en\lambda}^{(1)}) \qquad (3.1)$$

$$\bar{M}_{en\lambda}^{(j)} = \nabla \times (\psi_{en\lambda}^{(j)}(h)\, \hat{i}_x), \quad \bar{N}_{en\lambda}^{(j)}(h) = \frac{\nabla \times \bar{M}_{en\lambda}^{(j)}}{(\lambda^2 + h^2)^{1/2}}$$

$$\psi_{en\lambda}^{(j)}(h) = Z_n^{(j)}(\lambda\rho) \cos n\varphi\, e^{ihx}, \quad Z_n^{(1)}(\cdot) = J_n(\cdot), \; Z_n^{(3)}(\cdot) = H_n^{(1)}(\cdot)$$

The M and N functions are orthogonal; it is easy to solve (3.1) for a_n and b_n. Now again we match boundary fields, this time at the cylinder surface. Thus let

$$\bar{\mathcal{E}}_s(\bar{x}) = \sum_{0}^{\infty} (c_n \bar{M}_{enk_\rho}^{(3)}(K_x) + d_n \bar{N}_{enk_\rho}^{(3)}(K_x)) \qquad (3.2)$$

and one finds

$$c_n = -(J_n^{1'}(\alpha)/H_n^{(1)'}(\alpha))a_n, \quad d_n = -(J_n(\alpha)/H_n^{(1)}(\alpha))\, b_n \qquad (3.3)$$

$$= K_\rho a, \quad K_\rho = (k_2^2 - K_x^2)^{\frac{1}{2}}$$

The surface current in amperes/meter on a perfectly conducting body is proportional to the total magnetic intensity H, i.e.,

$$\hat{n} \times \bar{H} = \bar{K}\Big|_{\rho = a}, \quad \hat{n} = \hat{i}_\rho \qquad (3.4)$$

This condition yields the zeroth order transformed axial surface current density

$$K_x^{\circ}(K_x, K_y) = \frac{-2[\bar{K} \times (\bar{K} \times \bar{\mathcal{E}}^{\circ})] \cdot \hat{i}_x}{K_\rho^2 \pi \mu_0 a H_0^{(1)}(K_\rho a)} \qquad (3.5)$$

where
$$\bar{\mathcal{E}}^{\circ} = -\frac{\omega}{K_2}\left[\bar{K} \times (\bar{f}^- + \bar{f}_0) + \frac{i}{K_2} \bar{K} \times (\bar{K} \times \bar{a})\right] \qquad (3.6)$$

Hence (3.5) becomes

$$K_x^{o}(K_x, K_y) = \frac{2i\{a^- - ik_z \bar{K}\cdot(\hat{i}_x \times \bar{f}_o)\}}{\pi a \mu_o H_o^{(1)}(K_\rho a)\, K_\rho^2}$$

$$= K_{x2}^{o}(K_x, K_y) + K_{x1}^{o}(K_x, K_y) \quad (3.7)$$

Define

$$K_{x1}^{o}(K_x) = \int_{-\infty}^{\infty} K_{x1}^{o}(K_x, K_y)\, dK_y$$

$$= \frac{K_z^2\, e^{-iK_x x_T} H_o^{(1)}(K_\rho \rho_T)(m_y|z_T| + m_z(y_T))}{4\pi^2 K_\rho a\, H_o^{(1)}(K_\rho a)\, \rho_T}, \quad \rho_T = (y_T^2 + z_T^2)^{1/2} \quad (3.8)$$

Note that K_{x1} is the interface independent zeroth order axial surface current transform. The interface term $K_{x2}(K_x)$ is unfortunately a numerical integral.

IV. Iterated Current and Anomalous Fields

We now temporarily assume the current is known; i.e.,

$$\bar{J}(\bar{x}') = \hat{i}_x K_x(x')\, \delta(\rho' - a) \quad (4.1)$$

The excitation potential for this section is thus

$$\bar{A}^o = \mu_o \int_R G_o(\bar{x}, \bar{x}')\, \bar{J}(\bar{x}')\, d^3x' \quad (4.2)$$

Now, substituting (4.1), and G_o as given by (2.5) into (4.2), and carrying out the spatial integrations gives

$$\bar{A}^o = \int e^{i\bar{K}\cdot\bar{x}}\, \bar{a}^o(\bar{K})\, d^2K, \quad \bar{a}^o = \hat{i}_x \frac{i\mu_o a}{2K_z^-} K^o(K_x) J_o(K_\rho a) \quad (4.3)$$

Again, we now introduce potentials $A^\pm$, $F^\pm$, match fields across the interface and obtain in the quasi-static approximation

$$a^-(\bar{K}) = Q(\bar{K}) a^o(-), \quad Q(\bar{K}) = e^{2iK_z d} \frac{[f(\bar{K}) - K_z^2 K_y^2]}{[f(\bar{K}) + K_z^2 K_y^2]}$$

$$f(\bar{K}) = K_z[(K_z^+ - K_z^-)K_x^2 - K_z^2 K_z^+] \quad (4.4)$$

Thus, the air-earth interface to first order modifies the potential amplitude a^o; it becomes $(1 + Q)\, a^o$. In fact, the iteration is equivalent to a geometric series which formally can be summed to give as an infinite iterant $a^o/(1 - Q)$. Thus, the corrected potential $\bar{A}$ is given by

$$\bar{A} = \int d^2K \, \frac{a^o}{1 - Q} \left\{ e^{i\bar{K}\uparrow\cdot\bar{x}} + Q\, e^{i\bar{K}\downarrow\cdot\bar{x}} \right\} \tag{4.5}$$

and the surface current is, from (3.4),

$$K_x(x) = 1/\mu_0 \, \partial/\partial\rho \, \langle A \rangle \Big|_{\rho = a}, \quad \langle A \rangle = \frac{1}{2\pi} \int_0^{2\pi} A(\bar{x})\, d\varphi \tag{4.6}$$

It turns out that since Q is exponentially small for large K_y, the averaging integral and the K_y integral do not commute; hence, we first substract out the asymptotic limit to the integral (4.5) and add it on in integrated form giving

$$K_x(K_x) = i\alpha J_o(\alpha)\, K_x^o\, (K_x) \left[J_1(\alpha)\, P(K_x, d) + \pi/2\, H_1^{(1)}(\alpha) \right] \tag{4.7}$$

where

$$\alpha = K_\rho a, \quad P(K_x, d) = \int_{-\infty}^{\infty} dK_y \, \frac{Q}{K_z^-\,(1-Q)}$$

An important limiting case of (4.8) is when $d \to \infty$, then $P \to 0$; it can be shown that the remaining term is then identical to the exact expression in the absence of the interface. Using the key result (4.7), the anomalous magnetic field H_a in the lower half-space is given in terms of the potentials A^o, A^-, and F^-.

$$\bar{H}_a(\bar{x}) = \bar{H}_a^o(\bar{x}) + \bar{H}_a^i \tag{4.8}$$

where H^o depends on the interface only through the current (it corresponds to A^o with iterated current (4.7)). In particular

$$H_{ax}^o = 0, \quad H_{ay}^o = -z/\rho \;\; h^o, \quad H_{az}^o = y/\rho \;\; h^o \tag{4.9}$$

where

$$h^o = ia \int_0^{\infty} \cos(K_x x) K_x(K_x)\, J_o(K_\rho\, a)\, H_1^{(1)}(K_\rho\, \rho)\, K_\rho\, dK_x \tag{4.10}$$

The H_a^i term is given by A^-, F^- through (2.2). For completeness these potential coefficients are determined to be

$$a^- = \frac{i\mu_o a}{2\, K_z^-} \, K_x(K_x)\, J_o(K_\rho a)\, Q(K)$$

$$f^{-} = - \frac{\mu_0 K_z a K_x K_y Q(K) K_x(K_x) J_0(K_\rho a)}{f(\bar{K}) + K_y^2 K_z^2} \tag{4.11}$$

This completes the formal solution to our problem.

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