ELECTROMAGNETIC FIELD SOLUTIONS FOR INFINITE AND FINITE CABLES FOR CONDUCTING HALF-SPACE MODELS- BOTH FREQUENCY - AND TIME-DOMAIN*

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Introduction

The fields of an infinite line source in the presence of a conducting half-space have been examined by Wait and Spies (1971). In any real communication link using a current-carrying cable for the transmitting antenna, the cable is of finite length. Here we examine the fields of a finite length cable carrying a constant current in the presence of a homogeneous conducting half-space. The constant current assumption is valid at sufficiently low frequencies when an insulated cable is grounded at the end points (Wait, 1952; Sunde, 1968).

Here we examine both surface and buried cables since both downlink and uplink communication links are of interest. Also, we examine both frequency and time solutions since either CW or pulsed communications may be used. Finally, it is necessary to include both magnetic and electric field solutions since reception could be with either loops or dipoles.

Since the finite cable solution is more complicated than both the infinite line source and the short dipole solutions, we explore under what conditions a cable appears to be either infinitely long or very short. Some special cases, such as the low frequency limit, permit analytical solutions which exhibit the dependence on various parameters, such as cable length, quite clearly.

Frequency Domain Subsurface Fields

The geometry of a line source of length 2ℓ located on a conducting half-space with the observer in the half-space is shown in Figure 1. The fields

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of an incremental source can be derived from x and y components of a Hertz vector which are expressed in terms of Sommerfeld type integrals (Wait, 1961; Banos, 1966). If we assume that the frequency is sufficiently low that displacement currents can be neglected, then the Sommerfeld type integrals can be evaluated in closed form. The electric and magnetic field components then involve partial derivatives in x, y, and z. The details of the procedure are given by Hill and Wait (1973a).

To obtain the fields of the finite cable, the field components of the incremental source must be integrated over the cable length 2ℓ. Some of the field components can be integrated in closed form, but others must be integrated numerically. For an observer at a depth h, field components were computed as a function of ℓ. For an ℓ/h ratio of about 2 or greater, the results agree well with those of the infinite line source. For an ℓ/h ration less than 0.3, the results approximate those of a short dipole. In the intermediate range, only the finite cable formulation is valid. For the zero frequency limit, the expressions simplify, and all quantities can be expressed in closed form.

Transient Subsurface Fields

Transient waveforms are of interest in pulsed communications. The transient subsurface electric field of an infinite line source at the surface has been examined by Wait (1971). From Maxwell's equations, the time derivative of the magnetic field can be determined. This is the quantity of interest when reception is with a loop antenna. Results have been computed which illustrate pulse attenuation and dispersion as a function of observer position (Hill and Wait, 1973b).

The transient subsurface fields of a finite cable are more complicated, but the frequency domain results of the previous section are useful. The necessary inverse Fourier transform can be done in closed form for some of the field components, but a numerical inversion is required for others. Details of the procedures plus various numerical results are available (Hill and Wait, 1973c), and related transient problems have been treated by Wait (1960).

Surface Fields of a Buried Cable

In the uplink application where the transmitting antenna may be in a coal mine environment, the cable is not necessarily level. Even if it is level, the earth surface may not be. Consequently, the geometry with a tilted cable shown in Figure 2 was examined. Results simplify if the observer is on the surface (z = 0), and the necessary Sommerfeld type integrals can again be integrated in closed form if the quasi-static assumption is made. We are primarily interested in the magnetic field components since reception at the surface is normally with loop antennas.

Since the additional complication of an arbitrary cable angle α has been included, the source integration along the cable must in general be done numerically. The details of the formulation along with various numerical results are available (Hill, 1973). Again there is a wide range where the cable can be approximated by neither the short dipole nor the infinite line source.

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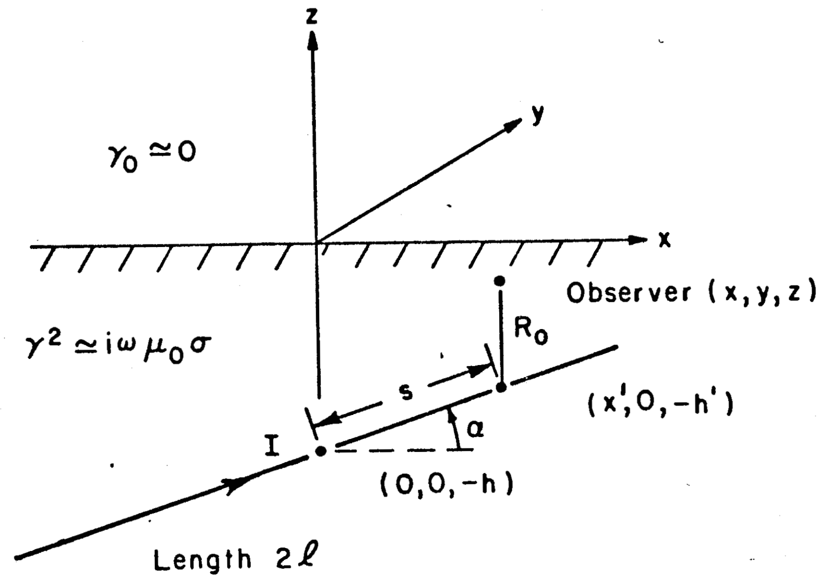

FIGURE 2. Tilted cable in a homogeneous half-space.

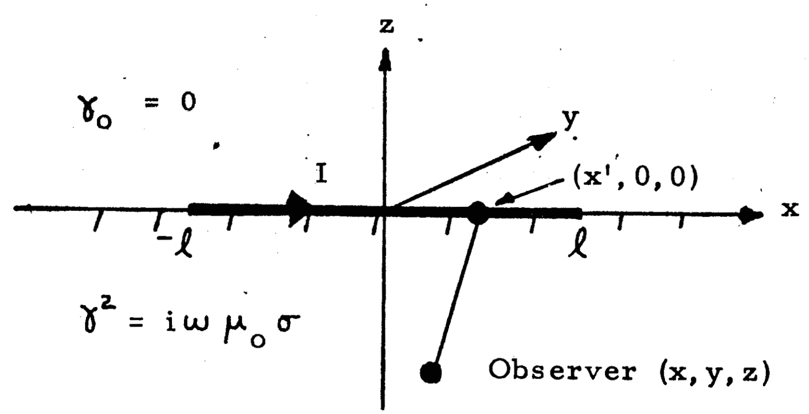

FIGURE 1. Finite length cable on a homogeneous half-space.