STATE OF KNOWLEDGE OF ANALYTICAL TECHNIQUES FOR THRU-THE-EARTH ELECTROMAGNETIC WAVE PROBLEMS RELEVANT TO MINE RESCUE

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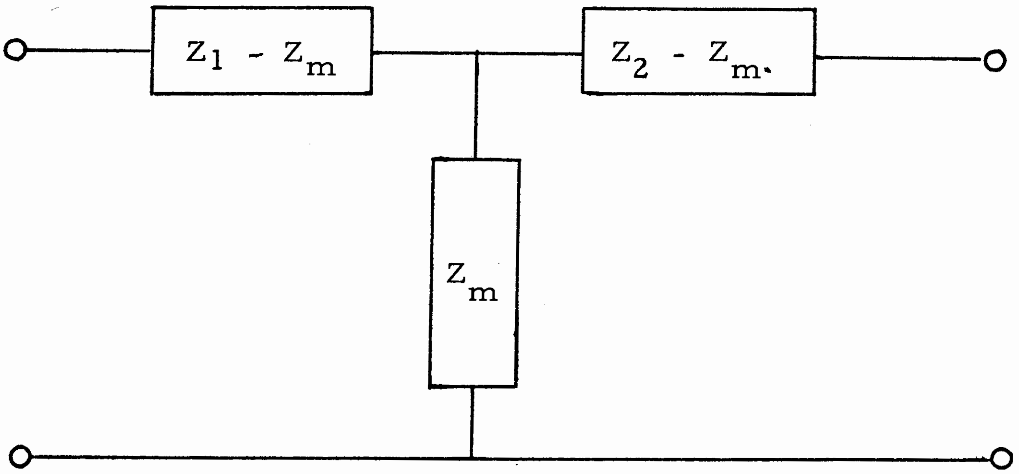
It is the purpose of this paper to review theoretical model concepts in through-the-earth transmission of electromagnetic waves. While the subject is very broad in its defined scope, we will be conscious of the specific applications to mine rescue and emergency telecommunications in coal mine environments.

The plan of the paper is to outline previous theoretical efforts without attempting to attach names or organizations to the specific developments. However, we include a guide to the literature as an appendix. Without pretending to be entirely impartial we include a critique of the limitations of the available analytical models.

As I see it the central problem is to calculate the steady state mutual impedance of a four-terminal network that is the black box characterizing the through-the-earth path. Problems of secondary importance include the determination of the input impedances of the two pairs of terminals and the associated transient responses.



The simple equivalent circuit given tells us a lot about the problem that we are seeking a solution for. The mutual impedance Z_m describes the propagation phenomena involved in the transmission from input (source) to the output (receiver). Z_1 and Z_2, on the other hand, are the input impedances. Clearly the latter are needed in a total systems study since we have only a finite amount of power available at the source and the receiver must be matched in some sense to the output terminals of the receiving antenna.

Most of the past literature deals with the calculation or merely the derivation of the mutual impedance Z_m for some highly idealized geometrical configuration. For example, the source for an ungrounded loop is a magnetic dipole that is assigned a fixed magnetic moment. In the most extreme case this magnetic dipole is an infinitesimally small loop (of area dA) with a total circulating current of I amps. The moment is IdA! Now we can still use this concept for a multi turn loop if I is the total azimuthal current and provided the uniform current assumption is valid. Also, of course, we recognize that the finite size of the loop needs to be accounted for in some cases.

In some of the earliest work the fields of this dipole were calculated as if the conduction currents in the earth were negligible. Of course, this is a useful beginning. The next refinement was to correct these magneto-static fields by assuming that we need only multiply them by $\exp(-s/\delta)$ where s was the transmission range and δ was the skin-depth in the earth for the operating frequency being adopted. With the benefit of hindsight we can criticize this correction as being grossly pessimistic for many cases of interest to mine rescue or to communication to a receiving station above or below the source loop.

By using the correct electrodynamic forms of the magnetic dipole fields in an homogeneous conductive medium of infinite extent, we can obtain a better estimate of the relevant value of Z_m. For example, if we are transmitting between two small coaxial loops, we can easily show that

$$Z_m = Z_o(1 + \Gamma s)e^{-\Gamma s}$$

where $\Gamma = (1 + i)\delta^{-1}$ and Z_o is the static or D.C. coupling limit.

Now usually the transmission takes place from the earth's surface to a buried receiving terminal. Or the converse situation may exist if we are dealing with up-link communication. In both cases the air-earth interface must be considered. Here we can immediately call attention to the classical formulation of Arnold Sommerfeld that dates back to 1909 in its earliest version. This was used as the basis of much analysis in later years but the interest was mainly for the case where the range exceeded a free-space wavelength. Here we are interested in the near zone where the significant distances are small compared with the wavelength in the air but such distances (i.e., depth or offset) may be comparable with the wavelength in the earth. This is quasi-statics in the vernacular of the current workers. To obtain field estimates we now have to "do" some integrations. Fortunately it was found that identities in Bessel function theory, recognized by mathematicians of the late 19th century were ripe for the picking. Thus some closed form field expressions were obtained and published in the 1950's for a fairly broad class of such problems. Numerical results required the manipulation of modified Bessel functions whose arguments were complex. Fortunately for cases of interest, the phase angles were near $\pi/4$ radians or 45° so that the tabulated Thomson's functions could be used. These are sometimes called the Kelvin functions and denoted ber, bei, kei, etc. (Note William Thomson became Lord Kelvin who was also noted for his work on laying the first Trans-Atlantic cable.)

With the ready availability of high speed computers, the use of intricate closed form field formulas is giving way to direct numerical integration of the Sommerfeld integrals. This is OK provided the programmer has some limiting checks or if he can refer back to some of the earlier work where the more elegant closed-form expressions in terms of special functions are used. Also, as has been shown quite recently, in the treatment of a finite-length source elements the special function representations for the dipole source is a convenient starting point. Otherwise double numerical integration is needed!

As we have indicated above, the air-earth interface problem is treated by regarding the overburden as a homogeneous half-space. A rather straightforward extension is the stratified half-space wherein the intermediate interfaces are plane and parallel to the air-earth interface. Further extensions involve electric dipoles rather than magnetic dipoles. These are appropriate when dealing with grounded electrodes connected by insulated cables. No new basic difficulties are encountered here.

In the class of problems discussed above, all three of the impedance elements Z_m, Z_1, and Z_2 are determinable. Thus, in principle, a complete determination or prediction of the system performance is possible. This includes down-link and up-link communication efficiency <u>and</u> estimates of source location accuracy for the models assumed.

Traditionally, the source current is assumed to vary harmonically in time. For dealing with transient excitation or in estimating signalling rate we need to understand the time-domain behavior of the system. Formally this is obtained by Fourier or Laplace transform inversion of the preceding transfer functions. In general, this is not a trivial task. However, some rather interesting closed-form results can be obtained if the original spatial wavenumber integration can be deferred until after the Laplace transforms are inverted.

Now that the homogeneous and stratified half-space models have been exhausted it is appropriate to consider more realistic situations. For example, the earth's surface is not always flat. One approach to allow for this situation was to adapt cylindrical and spherical boundaries. In such cases, the local radius of curvature was chosen to (more-or-less) match the local topography. Results of calculations from such models indicated that transmission ranges were not markedly affected but that source location errors could be significant. An even simpler method conceptually was to retain the uniform half-space model but to allow the plane interface to be tilted at such an angle that the local slope of the terrain was matched. Further analytical work on such models is justified since much insight can be gained without excessive numerical effort. Unfortunately, formally exact solutions for such models are strictly limited. One class of boundary that could be treated for line source excitation is a parabolic or hyperbolic interface. The three dimensional version involving dipoles in the presence of a paraboliodal interface is extremely intricate but it may be worth doing also.

One must clearly recognize that the overburden is not a homogeneous slab whether it be bounded by plane or uniformly curved interfaces. Thus, the influence of omni-present inhomogeneities needs to be understood. An extreme case is when metallic tracks and pipes are in the vicinity of the normal transmission path. To treat such configurations, two dimensional models have been used. An example is the line source excited buried cylinder of infinite length. This is not a trivial problem if one considers the interactions between the buried cylindrical inhomogeneity and the air-earth interface. Two specific analytical techniques have been used to get numerical results for this problem. One is essentially a perturbation procedure that involves successive reflections between the cylinder and the plane interface while the other is a sophisticated integral equation procedure. The results between those methods agree in a common region of validity. The extension to three dimensional versions of these problems is nontrivial. Some progress has been made, however, in using perturbation procedures. An example is the buried sphere in the presence of a surface based dipole source. Some work has also been done on cylinders of finite length where certain assumptions were made about the nature of the axial induced current flow at the ends of the cylinder. Among other things this analysis showed that predictions based on infinite cylinders may be quite misleading.

Within the scope of these analytical techniques, we encompass both active and passive location concepts. In the active method, of course, the unknown source is energized by the to-be-rescued party. On the other hand, in the passive method, the target to be located is typically a loop of wire or a similar configuration. Various geometries, for such problems, have been considered in both the frequency and the time domain.

The foregoing account of theoretical analyses of electromagnetic induction problems is, by no means, claimed to be exhaustive. The selection has been based mainly on work that the author is familiar with.

APPENDIX

A rather brief "guide" to the literature is as follows:

E. D. Sunde - Earth Conduction Effects in Transmission Systems - Dover Publications, New York, 1968 - (Comprehensive review of much of the early work in the 1930-1940 period carried out at Bell Telephone Labs. - includes many useful formulations for mutual impedances of grounded circuits relevant to coupling between power and communication circuits).

D. B. Large, L. Ball, and A. J. Farstad - Radio Transmission from Underground Coal Mines - Theory and measurement trans. IEEE Comm. 21, 194-10204, 1973 (good recent account of Westinghouse's investigations).

R. E. Collin and F. J. Zucker (editors) - Antenna Theory Part II - McGraw-Hill, 1969 - (Chap. 24 by J. R. Wait gives a basic account of the theory for sources in conducting media - also an extensive bibliography is included).

L. L. Vanyan - Electromagnetic Depth Soundings (edited and translated by G. V. Keller) - Plenum Press, 1967 - (An authorative review of Russian work on electromagnetic induction in the earth).

Also see recent volumes of the Journal Radio Science, IEEE Transactions on Antennas and Propagation, Journal of Applied Physics, and Geophysics.