1 Prior settings for simulation study in Section 5.1

Table 1 shows the prior distributions for the physical parameters. Physical considerations and expert judgment usually lead to reasonably informative priors. Indeed, the generation and ventilation rates are customarily assigned more informative priors based upon their plausible ranges. One way to determine the hyperparameters in the prior distribution for

Table 1: Prior distributions for parameters in the analysis of the simulation study.

Model	β	Q	G
PBBM, BNLR	U$(0, 14.5)$	U$(12, 18)$	U$(73.5, 136.5)$

β is to write β as the product of the random airspeed (RA) at the boundary of the near field and one half of the free surface area (SA) of the near field, i.e., $\beta = \frac{1}{2}SA \times RA$. The advantage of doing this is that SA is usually available and an estimate of RA can be obtained with a non-directional anemometer, thus giving some prior information about β.

Matters are somewhat more delicate with process parameters. Unlike the physical parameters, the process parameters cannot be gleaned from physical considerations. In particular, the ϕ_i's and ν_i's are usually weakly identifiable from the data and will require weakly informative priors. We choose such priors based upon mechanistic considerations. For example, the ν_i's control the smoothness of the latent process. Allowing excessive smoothness for this process will not only impair inference but also cause numerical instabililities in the fitting algorithm. Therefore, we assume that $\nu_i \sim$ DU$(9, 0.5, 2.5)$, where $X \sim$ DU(k, a, b) denotes a discrete uniform distribution such that $P(X = a + sb) = 1/k$, for $s = 0, 1, \cdots, k-1$.

The ϕ_i's control the strength of temporal correlations in the two fields. We use the *practical range* as a basis for assigning priors. The practical range is informally defined as the time separation at which the correlation has dropped close to 0, say 0.05. We assume each $\phi_i \sim$ DU$(10, 1, 20)$, where the hyperparameters were chosen such that the prior mean for the practical range is about half of the maximum absolute time separation.

Subsequent inference is much more robust to the prior assumptions on the entries in $\mathbf{A}$. The prior distributions for the different structural specifications of $\mathbf{A}$ are shown in Table 2. Because the diagonal entries in $\mathbf{A}$ must be positive, so they are assigned log-normal distributions, while the off-diagonal entry is modeled with a normal distribution. Fairly vague, but proper, priors on $\mathbf{A}$ seem to render robust inference.

Table 2: Prior distributions for the unknown entries in $\mathbf{A}$

Structure	a_1	a_2	a_3
V, D	LN$(-5.7, 2.1)$	LN$(-4.3, 2.1)$	–
LT	LN$(-5.7, 2.1)$	LN$(-4.7, 2.1)$	N$(0.1, 1)$

For the simulation studies, we have 100 simulated datasets. Assigning a different set of priors for each study is infeasible. To maintain consistency across the different generated datasets we choose hyperparameters such that the prior mean for $\mathbf{A}\mathbf{A}^{\mathrm{T}}$ is roughly the estimated variance of $\mathbf{y}(t)$ (averaged over the 100 simulated datasets) and the prior coefficient of variation for each parameter in $\mathbf{A}$ is roughly 10.

Lastly, we adopt an inverse-Wishart (IW) distribution for $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}(\boldsymbol{\theta}_3)$. Subsequent inference is robust with respect to these parameters. We assume $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}(\boldsymbol{\theta}_3) \sim IW(\mathbf{S}, r)$, where $r = 5$ is the degrees of freedom and the inverse scale matrix is $\mathbf{S} = \begin{bmatrix} 0.002 & 0.007 \\ 0.007 & 0.038 \end{bmatrix}$. Since the expectation of $\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}(\boldsymbol{\theta}_3)$ equals $\dfrac{\mathbf{S}}{r-2-1}$, $\mathbf{S}$ was chosen from rough preliminary estimates of the residual variance and covariance.

2 Prior settings for analysis of misaligned experimental data in Section 5.2

Table 3 presents the prior distribution adopted in each model. The • indicates the set of parameters in each model. Recall from Section 3 that Q and G are known for this experiment. Inferential interest focuses upon estimation of β and the subsequent estimation of the bivariate distribution for the concentrations in the two fields.

Table 3: Prior Distributions for model parameters for the analysis of the workplace data.

Parameter	Prior Distribution	Model		
		BNLR	D	LT
β	$\mathrm{U}(0, 13)$	•	•	•
a_1	$\mathrm{LN}(-4.5, 2.1)$	–	•	•
a_2	$\mathrm{LN}(-3.4, 2.1)$	–	•	–
	$\mathrm{LN}(-5.0, 2.1)$	–	–	•
a_3	$\mathrm{N}(0.3, 3)$	–	–	•
ϕ_1, ϕ_2	$\mathrm{DU}(19, 1, 5.5)$	–	•	•
ν_1, ν_2	$\mathrm{DU}(33, 0.5, 2.5)$	–	•	•
$\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}(\boldsymbol{\theta}_3)$	$\mathrm{IW}\left(5, \begin{bmatrix} 0.0226 & 0.0715 \\ 0.0715 & 0.2359 \end{bmatrix}\right)$	•	•	•