1 Checking identifiability of the process parameters

Since the marginal distribution of $\mathbf{y}$ is a multivariate normal, we only need check the identifiability of the parameters in the first two moments, i.e., the mean vector and the covariance matrix of $\mathbf{y}$. In particular, we consider the parameters in $\mathbf{\Sigma}_{\mathbf{y}}(\boldsymbol{\theta}_2, \boldsymbol{\theta}_3)$. The less obvious situation arises when $\mathbf{A}$ has structure "LT". Then,

$$\mathsf{VAR}\left\{\mathbf{y}\left(t_i\right)\right\} = \begin{bmatrix} a_1^2 & a_1 a_3 \\ a_1 a_3 & a_2^2 + a_3^2 \end{bmatrix} + \begin{bmatrix} \tau_1 & \tau_{12} \\ \tau_{12} & \tau_2 \end{bmatrix}$$

$$\mathsf{COV}\left\{\mathbf{y}\left(t_i\right), \mathbf{y}\left(t_j\right)\right\} = \begin{bmatrix} a_1^2\rho_1(\varphi_1; t_i, t_j) & a_1 a_3 \rho_1(\varphi_1; t_i, t_j) \\ a_1 a_3 \rho_1(\varphi_1; t_i, t_j) & a_2^2\rho_2(\varphi_2; t_i, t_j) + a_3^2 \rho_1(\varphi_1; t_i, t_j) \end{bmatrix}$$

where $i, j \in \{1, \cdots, n\}$ and $i \neq j$. Switching a_1^2 and τ_1 will still render the same $\mathsf{VAR}\left\{\mathbf{y}\left(t_i\right)\right\}$ but it will affect $\mathsf{COV}\left\{\mathbf{y}\left(t_i\right), \mathbf{y}\left(t_j\right)\right\}$, which ensures identifiability. The consequence is similar when we switch $a_1 a_3$ and τ_{12}, or $a_2^2 + a_3^2$ and τ_2. Therefore, we can say that the a's and τ's are identifiable. Likewise, if we switch a_k^2 and $\rho_k(\varphi_k; t_i, t_j)$ in $\mathsf{COV}\left\{\mathbf{y}\left(t_i\right), \mathbf{y}\left(t_j\right)\right\}$, it impacts $\mathsf{VAR}\left\{\mathbf{y}\left(t_i\right)\right\}$, where $k = \{1, 2\}$. Therefore, the a's and ρ's are also identifiable.

We point out the need to impose a restriction on the domain of a_1 and a_2. In particular, notice that $(a_1 = x, a_3 = y)$ and $(a_1 = -x, a_3 = -y)$, where $x > 0$ and $y > 0$, returns the same $\mathsf{VAR}\left\{\mathbf{y}\left(t_i\right)\right\}$ and $\mathsf{COV}\left\{\mathbf{y}\left(t_i\right), \mathbf{y}\left(t_j\right)\right\}$. Likewise, when $a_2 = z$ and $a_2 = -z$, where $z \neq 0$. Therefore, to ensure identifiability of these parameters, we restrict a_1 and a_2 to be greater than 0.