APPLICATIONS OF THE POINT ESTIMATION METHOD FOR STOCHASTIC ROCK SLOPE ENGINEERING

Stanley M. Miller
Department of Civil Engineering, University of Idaho, Moscow, ID, USA

Jeffrey K. Whyatt & Edward L. McHugh
CDC-NIOSH Spokane Research Laboratory, Spokane, WA, USA

ABSTRACT: The point estimation method can be applied to the safety factor (SF) equation for any specified rock slope failure mode (such as plane shear, step path, or wedge) to obtain reliable estimates of the mean and standard deviation of the SF probability distribution. A gamma probability density function is recommended for modeling this probability distribution, because it allows only for positive values and is flexible enough to provide symmetrical shapes and right-skewed, exponential-type shapes for the SF distribution. The mean and standard deviation define this distribution, which then can be integrated numerically from 0 to 1 to obtain the probability of sliding, P_S (portion of the SF distribution where SF < 1.0). The overall probability of failure, P_F, for the potential slope failure mass is the joint probability that the rock discontinuities are long enough to allow kinematic failure (P_L) and that sliding occurs along the rock discontinuities (P_S); that is, $P_F = P_S P_L$. This method for estimating the probability of sliding is extremely efficient computationally, and thus, expedites slope stability simulation routines used by NIOSH software to stochastically describe rock slope behavior and assist the engineer in designing catch benches for large rock slopes. Enhanced bench design translates into increased operational efficiency and safer working conditions in open pit mines and quarries.

1. INTRODUCTION

Stochastic simulations of fractured rock masses can provide valuable information for the engineering design of rock slopes, particularly when the natural geologic discontinuities may form potential slope failure modes. An essential component of such engineering simulations is being able to compute the probability of sliding for a given potential failure mass once the geometry of that failure mass has been identified through spatial rock-fracture simulations. In cases where several thousand (or more) simulations of possible failure geometries are needed to provide a realistic representation of the rock slope, computational efficiency is essential for the repetitious protocol used to calculate the probability of sliding.

An example of applying this type of geotechnical approach to rock slope design was presented by Miller and others [1] in a paper focused on the design of catch benches for open pit mines and quarries. This issue is important to NIOSH (National Institute for Occupational Safety and Health) as part of its research mission to improve safety and health in the mining industry. Between 1995 and 2003 there were 42 reported fatalities due to slope failure accidents at surface mines in the United States, at least one and as many as eleven each year. Additional accident statistics collected by the Mine Safety and Health Administration (MSHA) have shown that loose material from slope and bench failures can pose significant safety hazards to miners. To address these concerns the NIOSH Spokane Research Laboratory has been developing and testing rock slope stability software over the past few years to provide advanced technical tools for analyzing bench stability. A key element of this software is a module used to compute the probability of sliding for a given viable failure geometry that has been simulated for the bench under study.

Several different methods can be used to compute the probability of sliding in a slope stability analysis, including Monte Carlo simulation [2], Taylor series expansion [2], Fourier analysis [3], and statistical point estimation [4]. Initial versions of the NIOSH bench stability software relied on the Fourier method [1] due to its numerical efficiency and its capability to provide a discretized, general output pdf (probability density function) for the factor of safety, rather than relying on a specific model for the pdf (e.g., a normal or lognormal pdf).

However, our recent experience with probabilistic studies of rock slope stability has indicated that the safety factor pdf tends to behave like a slightly right-skewed gamma pdf or a left-truncated normal pdf (truncated at zero, because the safety factor realistically cannot take on negative values). The right skew apparently is caused by a combination of the positive-only gamma shape of the input pdf for the shear strength (along the sliding plane) and the exponential-type shape of the input pdf for the fracture waviness. In this context, the waviness is measured on a scale of about 1-2 meters and is defined as the average dip of the fracture minus its minimum dip, as presented by Call and others [5]. Thus, assuming that the output pdf for the safety factor takes on the form of a gamma pdf, then the point estimation method clearly has considerable computational advantage even over the Fourier method (which relies heavily on mathematical manipulations of discretized pdf's [3]).

2. POINT ESTIMATION METHOD

When a random variable of interest can be expressed in an equation as the result of a mathematical operation of other random variables, then the point estimation method developed by Rosenblueth [6] provides a direct computational procedure to obtain moment estimates for that random variable. In particular, these statistical moments are the mean (i.e., the first moment about the origin) and the variance (i.e., the second moment about the mean). Geotechnical engineering applications of this method have been around for several decades, and recent publications [2, 4] have clearly presented such work. The particular shape of any pdf used for any input random variable is not critical to the analysis, because the pdf is represented by the mean and two hypothetical point masses located at plus and minus one standard deviation (s) from the mean (μ).

Consequently, required inputs for a probabilistic rock slope stability analysis are: 1) a defined performance function (i.e., safety factor equation), 2) estimated value for each input attribute if it is assumed to have negligible variability, and 3) estimated mean μ and standard deviation s of each input attribute treated as a random variable. Typical attributes for a rock slope failure mass with a defined geometry would include shear strength, rock mass unit weight, and fracture waviness. Calculation steps are presented below for the point estimation method using two random variables X_1 and X_2 in a performance function to obtain the mean and variance of F, the factor of safety.

1. Calculate the output value of F using the performance function evaluated with the values of mean-plus-one-s.d. for each of the two variables.

$$F_{++} = fn[(\mu_1 + s_1), (\mu_2 + s_2)] \quad (1a)$$

Repeat for other combinations, as follows:

$$F_{--} = fn[(\mu_1 - s_1), (\mu_2 - s_2)] \quad (1b)$$

$$F_{+-} = fn[(\mu_1 + s_1), (\mu_2 - s_2)] \quad (1c)$$

$$F_{-+} = fn[(\mu_1 - s_1), (\mu_2 + s_2)] \quad (1d)$$

2. Calculate the point-mass "weights" [6].

$$P_{++} = P_{--} = (1/4)(1 + \rho_{12}) \quad (2a)$$

$$P_{+-} = P_{-+} = (1/4)(1 - \rho_{12}) \quad (2b)$$

where ρ_{12} = correlation coefficient between input variables X_1 and X_2.

3. Calculate the expectation (mean, μ_F) of F [6].

$$E(F) = P_{++}F_{++} + P_{--}F_{--} + P_{+-}F_{+-} + P_{-+}F_{-+} \quad (3)$$

4. Calculate the variance (s_F^2) of F.

$$Var(F) = E(F^2) - [E(F)]^2 \quad (4)$$

where $E(F^2)$ is calculated using Eq. (3) with F^2 terms substituted for the F terms.

5. The standard deviation (s_F) of F then is calculated by taking the square root of s_F^2.

For three input variables X_1, X_2, X_3, there are eight calculations in Step 1, and the point-mass weights in Step 2 are given by [6]:

$$P_{+++} = P_{---} = (1/8)(1 + \rho_{12} + \rho_{23} + \rho_{31}) \quad (5a)$$

$$P_{+--} = P_{-++} = (1/8)(1 - \rho_{12} + \rho_{23} - \rho_{31}) \quad (5b)$$

$$P_{++-} = P_{--+} = (1/8)(1 + \rho_{12} - \rho_{23} - \rho_{31}) \quad (5c)$$

$$P_{+-+} = P_{-+-} = (1/8)(1 - \rho_{12} - \rho_{23} + \rho_{31}) \quad (5d)$$

Eq. (3) for *E(F)* is extended from a summation of four terms to a summation of eight terms for this case. The s_F value subsequently can be calculated using Eq. (4), after first using eight F^2 terms in the extended Eq. (3) to calculate $E(F^2)$.

3. PLANE SHEAR ANALYSIS

A probabilistic procedure based on the point estimation method can be applied to the two-dimensional slope stability analysis used for the plane shear failure mode (Fig.1). For a plane shear mass with a defined geometry (i.e., slope angle δ, dip of failure plane α, height of the failure mass H), the SF equation is assumed to contain two random variables, the shear strength and the waviness of the geologic discontinuity. Other input terms, such as the rock-mass density, which is used to calculate the weight of the potential failure mass, are treated as constants. The plane shear SF equation is:

$$F = \frac{L\tau + L\sigma_n \tan(r)}{W\sin(\alpha)} = \left(\frac{L}{W\sin(\alpha)}\right)\cdot \tau + \left(\frac{L\sigma_n}{W\sin(\alpha)}\right)\cdot \tan(r) \quad (6)$$

where: τ = shear strength, r = waviness, σ_n = effective normal stress, α = average dip of sliding surface, L = length of sliding surface, and W = weight of slide mass. The point estimation method can be applied to this expression if we have reliable estimates of the mean and standard deviation of each random variable.

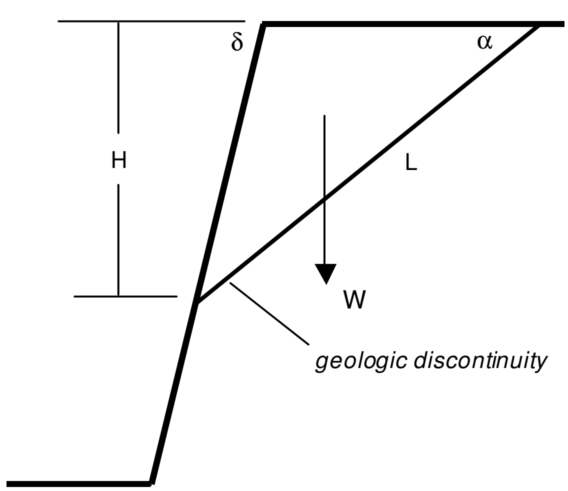


Fig. 1. Example of plane shear failure mode.

3.1. Mean and S.D. of Shear Strength

The mean of τ is obtained from the linear or power curve expression that relates the effective normal stress σ_n to the shear strength [7], as shown by the following options:

$$\text{Linear:} \quad \tau = c + \sigma_n \tan\phi \quad (7)$$

where: c = cohesion, ϕ = friction angle.

$$\text{Power:} \quad \tau = c + a\sigma_n^{\,b} \quad (8)$$

where: c = y-intercept of power curve, a,b = shape parameters for power curve. This model reverts to a linear model when b equals 1.

$$\text{JRC Power:} \quad \tau = \sigma_n \tan[\text{JRC}\cdot\log(\text{JCS}/\sigma_n) + \phi_b] \quad (9)$$

where: JRC= joint roughness coefficient, JCS= joint wall compressive strength, ϕ_b = base friction angle.

For a simple 2d rock-slope bench geometry (shown in Fig. 1), the effective normal stress is:

$$\sigma_n = \frac{W\cos(\alpha)}{L\cdot 1} - U \quad (10)$$

where: W = total weight of the failure mass, U = ground-water pore pressure on the sliding surface.

The standard deviation of τ is estimated using the standard error (s_e) obtained from a least-squares regression analysis of data obtained by laboratory direct shear tests of natural fractures [7]. For a linear regression model, we can estimate the standard deviation of τ at a given value of effective normal stress σ_n by the expression [8, p. 371]:

$$s_\tau = s_e \sqrt{\frac{1}{n} + \frac{(\sigma_n - m_\tau)^2}{SSQ - n\cdot m_\tau^{\,2}}} \quad (11)$$

where: s_e = standard error of the regression; n = number of data pairs used in the regression; m_τ = sample mean of the n τ values; and

$$SSQ = \sum_n \tau_i^{\,2} \qquad \text{(sum of the squared } \tau \text{ values)}$$

For a power-curve regression model, Eq. (11) also should give reasonable results if the s_e value from that power-curve model is used. Example results from laboratory direct-shear tests of natural rock joints for actual rock-slope design projects are summarized in Table 1. Stress units for the reported shear strengths are given in tonnes per square meter (tsm). As seen by the s_e values for the three rock types, the regression errors are small for the latite porphyry (a “tight” regression fit) and quite high for the granodiorite (a “noisy” regression fit). These examples in Table 1 were selected to show typical s_e values for shear strength regression models.

Estimated standard deviations of τ reported at the bottom of Table 1 were computed using Eq. (11). The maximum normal stress acting on potential sliding surfaces in a 25-m high bench is about 20 tsm; this justifies the 2, 10, and 18 tsm σ_n values in the table. The representative s_τ results shown in Table 1 provide general guidance for reasonable and acceptable values to be used in stochastic rock slope studies, especially when direct-shear test results are not available. If the engineer anticipates little variability in discontinuity shear strength for the critical failure mode in a mine bench, then a reasonable value for s_τ is 0.1 tsm (0.98 kPa). If a lot of variability in shear strength is expected, then a value up to 0.6 tsm (5.88 kPa) should be assumed. Our experience has indicated that in most cases reasonable “default” values are 0.3 to 0.4 tsm for s_τ.

Table 1. Shear strength estimates for power-curve regressions.

Regression Terms	Latite Porphyry	Quartz Monzonite	Granodiorite
a	1.3802	1.5412	0.8099
b	0.8377	0.7381	0.8468
c	0.0	0.0	0.0
s_e (tsm)	0.1823	0.3953	0.5718
Est. τ @ 2, 10, 18 tsm	2.47 9.50 15.54	2.57 8.43 13.01	1.46 5.69 9.36
SD of τ @ 2, 10, 18 tsm	0.118 0.105 0.202	0.260 0.213 0.382	0.383 0.294 0.482

3.2. *Mean and S.D. of Waviness*

The mean and standard deviation of tan(r) are computed by applying a random variable transformation from waviness r to tan(r), then numerically integrating the probability density function of the new transformed variable. Assuming the waviness can be modeled as an exponentially distributed random variable, the pdf of the new random variable V (i.e., v = tan(r)) is given by [3]:

$$f_V(v) = \begin{cases} \dfrac{e^{[-\arctan(v)/\mu_r]}}{\mu_r(1+v^2)} & if \quad v \geq 0 \\ 0 & if \quad v < 0 \end{cases} \tag{12}$$

where: μ_r = mean waviness (in degrees).

Thus, for any given mean value of the fracture waviness, the pdf of V is defined and can be integrated numerically to obtain the mean and variance of V (in other words, the mean and variance of tan(r)). These procedures are outlined below.

Mean of V:

$$\mu_v = E[V] = \int_0^\infty v \cdot f_V(v) \cdot dv \tag{13}$$

Variance of V:

$$Var[V] = E[V^2] - \mu_v^2 \tag{14}$$

where: $E[V^2] = \int_0^\infty v^2 \cdot f_V(v) \cdot dv$

Standard deviation of V:

$$s_v = \sqrt{Var[V]} \tag{15}$$

Numerical integrations needed to evaluate Eqs. (13) and (14) have been completed for waviness values from 1^o to 12^o, and the results are listed in Table 2. Note that values for 0^o waviness are not needed, because the expression in Eq. (6) reduces to only one random variable for $r = 0^o$.

Therefore, once a specified (simulated) mean waviness value is known for a given plane shear failure, the corresponding pdf of V (which has an exponential-type shape) is defined by Eq. (12). Waviness values should only be reported and used as integer, whole numbers. Then, the appropriate values for the mean and standard deviation of V can be obtained from Table 2.

Table 2. Calculated means and standard deviations of random variable V; note that v = tan(r).

Waviness (deg.)	Mean of V	S.D. of V
1	0.017464	0.014337
2	0.034992	0.035167
3	0.052654	0.053264
4	0.070523	0.072055
5	0.088691	0.091989

6	0.107277	0.114034
7	0.126444	0.140049
8	0.146394	0.172409
9	0.167338	0.212872
10	0.189444	0.261799
11	0.212803	0.318279
12	0.237413	0.380713

3.3. *Example Calculations*

To illustrate the point estimation procedures for computing the probability of sliding for a 2-d plane shear failure mode, a simple bench geometry is considered with the following terms (see Fig. 1):

Bench face angle: $\delta = 65^o$
Height of failure mass at face: H = 4.0 m
Dip of sliding surface: $\alpha = 32^o$
Waviness of sliding surface: $r = 3^o$
Unit weight of rock mass: $\gamma = 2.6$ tcm (25.5 kN/m^3)
Calculated length of sliding surface: L = 7.55 m
Calculated weight of slide mass: W = 23.588 tonne
Calculated effective normal stress: $\sigma_n = 2.650$ tsm
Shear strength parameters:
a = 0.6512 b = 0.988 c = 0.0
Calculated mean τ: $\mu_\tau = 0.6512\sigma_n^{0.988} = 1.706$ tsm
Standard deviation of τ: $s_\tau = 0.3$ tsm
For 3^o waviness, $\mu_v = 0.052654$ and $s_v = 0.053264$

The first constant in Eq. (6) is computed as:

$$C_1 = \frac{L}{W\sin(\alpha)} = 0.6039$$

The second constant in Eq. (6) is computed as:

$$C_2 = \frac{L\sigma_n}{W\sin(\alpha)} = 1.6003$$

Point estimate calculations from Eq. (1):

$$F_{++} = 0.6039(1.706 + 0.3) + 1.6003(0.052654 + 0.053264) = 1.3807$$

$$F_{--} = 0.6039(1.706 - 0.3) + 1.6003(0.052654 - 0.053264) = 0.8479$$

$$F_{+-} = 0.6039(1.706 + 0.3) + 1.6003(0.052654 - 0.053264) = 1.2102$$

$$F_{-+} = 0.6039(1.706 - 0.3) + 1.6003(0.052654 + 0.053264) = 1.0184$$

Because the shear strength τ often is based on small-scale laboratory test specimens (~15 cm) and the waviness is measured over 1-2 m in the field, it is reasonable to assume these two attributes are not dependent on each other. Thus, the correlation coefficient between the two is zero. The statistical point-mass weights then are calculated from Eq. (2):

$$P_{++} = P_{--} = P_{+-} = P_{-+} = (1/4)(1 + 0) = 0.25$$

Calculate the mean safety factor from Eq. (3):

$$\mu_F = 0.25(1.3807 + 0.8479 + 1.2102 + 1.0184)$$
$$\mu_F = 1.114$$

Then, calculate the variance of the safety factor from Eq. (4):

$$s_F^2 = 0.25(1.3807^2 + 0.8479^2 + 1.2102^2 + 1.0184^2) - 1.114^2 = 0.0401$$

Thus, the standard deviation of the safety factor is:

$$s_F = 0.2002$$

A gamma pdf [8, p.161-163] for the safety factor then can be defined and numerically integrated from 0 to 1 to estimate the probability of sliding for this plane shear analysis. The u and z shape parameters of the gamma pdf are computed directly from the mean and variance.

$$u := \frac{1.114^2}{0.0401} \qquad z := \frac{0.0401}{1.114}$$

$$P_s := \frac{1}{\left(z^u \cdot \Gamma(u)\right)} \int_0^1 e^{-\left(\frac{x}{z}\right)} \cdot x^{u-1}\, dx$$

Probability of sliding: $P_s = 0.298$

The safety factor gamma pdf is shown in Figure 2. It is worthy to note that even though the mean safety factor is 1.114 (indicating stability), this plane shear actually has a 0.30 probability of sliding, which results directly from the variability of input values.

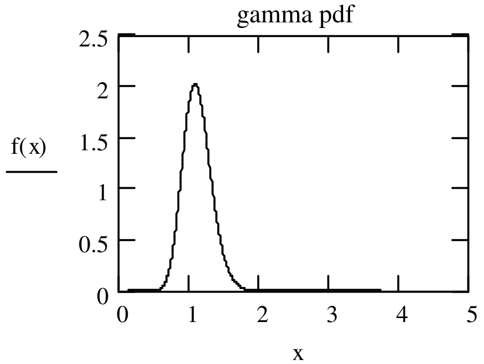


Fig. 2. Safety factor pdf for plane shear example based on point estimation method; mean = 1.114, s.d.= 0.200, P_S = 0.30.

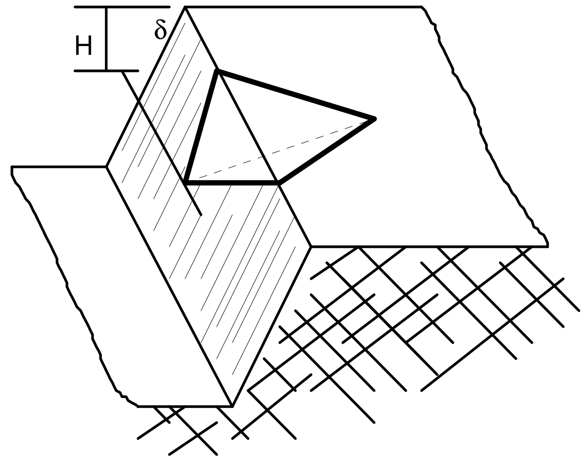


Fig. 3. Example of 3-d wedge failure mode.

4. WEDGE ANALYSIS

The probability of sliding also can be estimated for a three-dimensional, tetrahedral wedge that may form in the rock slope as a result of two intersecting planar discontinuities (Fig. 3). The safety factor equation for this type of wedge can be expressed as:

$$F = \frac{A_L \tau_L + A_L \sigma_L \tan(r_L)}{D_f} + \frac{A_R \tau_R A_R \sigma_R \tan(r_R)}{D_f}$$

$$F = \left(\frac{A_L}{D_f}\right)\tau_L + \left(\frac{A_R}{D_f}\right)\tau_R + \left(\frac{A_L}{D_f}\right)\sigma_L \tan(r_L) + \left(\frac{A_R}{D_f}\right)\sigma_R \tan(r_R) \qquad (16)$$

where: A_L = sliding area of left side of wedge, A_R = sliding area of right side of wedge, D_f = driving force of the wedge, τ_L = shear strength along left side of wedge, τ_R = shear strength along right side of wedge, σ_L = effective normal stress on left side of wedge, σ_R = effective normal stress on right side of wedge, r_L = waviness of left side of wedge, and r_R = waviness of right side of wedge.

The shear strengths on either side of the wedge are treated as random variables, and all other inputs are considered as constants. Thus, the first two terms in Eq. (16) are used in the point estimation method, with the sum of the last two terms being added later to the estimated mean of the safety factor. That is, the last two terms in Eq. (16) effectively comprise a waviness constant that increases the mean safety factor but does not affect its variance.

4.1 Mean and S.D. of Shear Strength

Shear strengths for the left and right sides of the wedge are handled in a similar fashion to that presented in Section 3.1. In some cases (e.g., for a symmetrical wedge), the mean shear strength will be the same for both the left and right sides of the wedge, but in the general case the two shear strengths do not have to be the same. This is due to the weight of the wedge being shared between the two sliding surfaces. Thus, the effective normal stress applied to each side of the wedge can vary significantly, depending on the steepness of the wedge plunge, the dihedral angle of the wedge, and other geometric factors.

Also, application of the same type of shear strength model to both sides of the wedge is not required for this stability analysis. For example, the engineer can specify a power-curve regression model for the left side and a linear model for the right side. Once the effective normal stress is computed for each sliding plane of the wedge, the corresponding mean and standard deviation of τ for each side are obtained independently.

4.2. Example Calculations

To illustrate the procedures for a 3-d wedge, the following example analysis is provided for a simple bench geometry:

Bench face angle: $\delta = 65^{o}$
Bench face dip direction: $D_{dir} = 160^{o}$
Height of failure mass at face: H = 4.2 m
Dip direction and dip of left plane: 106^{o}, 51^{o}
Dip direction and dip of right plane: 219^{o}, 52^{o}
Effective waviness of left plane: 3.3^{o}
Effective waviness of right plane: 4.8^{o}

(Note: Effective waviness is the apparent angle of waviness observed in the vertical plane in the direction of sliding rather than in the direction of down-dip, so it is always less than the true waviness angle.)

Shear strength parameters:
Left: a = 0.5543 b = 0.997 c = 0.0
Right: a = 0.5317 b = 0.988 c = 0.0
Unit weight of rock mass: γ = 2.6 tcm (25.5 kN/m^3)

Calculated length of wedge intersection: L = 7.23 m
Calculated volume of slide mass: V = 14.37 m^3
Calculated weight of slide mass: W = 37.36 tonne
Calculated bearing and plunge of the wedge intersection: 162°, 36°
Calculated driving force: D_f = 21.73 tonne
Calculated area of left side: A_L = 12.92 m^2
Calculated area of right side: A_R = 12.17 m^2
Calculated effective normal stresses:
σ_L = 1.565 tsm σ_R = 1.652 tsm
Calculated mean τ:
Left: $\mu_{\tau L} = 0.5543\sigma_L^{0.997} = 0.866$ tsm
Right: $\mu_{\tau R} = 0.5317\sigma_R^{0.988} = 0.910$ tsm
Standard deviation of τ:
$s_{\tau L}$ = 0.3 tsm $s_{\tau R}$ = 0.5 tsm

The first two constants in Eq. (16) are computed:

$A_L/D_f = 0.5945$ $A_R/D_f = 0.5600$

The waviness constant is computed:

$$0.5945(1.565)\tan(3.3^\circ) + 0.5600(1.652)\tan(4.8^\circ) = 0.132$$

Point estimate calculations:

$$F_{++} = 0.5945(0.866 + 0.3) + 0.5600(0.910 + 0.5) = 1.4827$$

$$F_{--} = 0.5945(0.866 - 0.3) + 0.5600(0.910 - 0.5) = 0.5660$$

$$F_{+-} = 0.5945(0.866 + 0.3) + 0.5600(0.910 - 0.5) = 0.9227$$

$$F_{-+} = 0.5945(0.866 - 0.3) + 0.5600(0.910 + 0.5) = 1.1261$$

Calculate the mean safety factor from Eq. (3):

$$\mu_F = 0.25(1.4827 + 0.5660 + 0.9227 + 1.1261) = 1.024$$

Then, calculate the variance of the safety factor from Eq. (4):

$$s_F^2 = 0.25(1.4827^2 + 0.5660^2 + 0.9227^2 + 1.1261^2) - 1.024^2 = 0.111$$

Thus, the standard deviation of the safety factor is:

$$s_F = 0.333$$

A gamma pdf for the safety factor distribution that results from these stochastic procedures then can be defined and numerically integrated from 0 to 1 to estimate the probability of sliding for this 3-d wedge analysis. The u and z shape parameters of the gamma pdf are computed directly from the mean and variance.

$$u := \frac{1.024^2}{0.111} \qquad z := \frac{0.111}{1.024}$$

$$P_S := \frac{1}{\left(z^u \cdot \Gamma(u)\right)} \int_0^1 e^{-\left(\frac{x}{z}\right)} \cdot x^{u-1}\, dx$$

Probability of sliding: $P_s = 0.514$

The safety factor gamma pdf is shown in Figure 4. It is worthy to note that even though the mean safety factor is 1.024 (indicating stability), this 3-d wedge actually has a 0.51 probability of sliding, which results directly from the variability of input values.

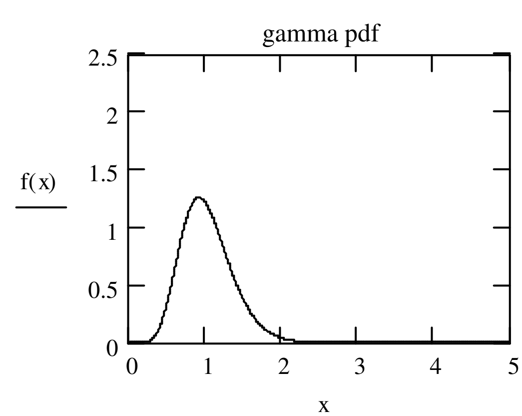


Fig. 4. Safety factor pdf for 3-d wedge example based on point estimation method; mean = 1.024, s.d.= 0.333, P_S = 0.51.

5. STEP-PATH ANALYSIS

Calculating the probability of sliding for step-path failure modes (see Fig. 5) requires the application of three variables in the safety factor equation. The safety factor equation used here is based on the work of Jaeger [9] and subsequent adaptations of the step-path analysis to rock slope engineering by others [10, 11]:

$$F = \frac{L\tau + L\sigma_n \tan(r) + TL}{W\sin(\alpha)}$$

$$= \left(\frac{L}{W\sin(\alpha)}\right)\cdot\tau + \left(\frac{L\sigma_n}{W\sin(\alpha)}\right)\cdot\tan(r) + \left(\frac{L}{W\sin(\alpha)}\right)\cdot T \quad (17)$$

where: L = effective sliding length of the potential step-path failure; r = waviness of the master joint set; α = average dip of the master joint set; W = weight of the potential failure mass; and T is the effective tensile strength (i.e., resisting force) due to intact rock bridges, given by:

$$T = F_{ir}(h_t)(T_o)/L \quad (18)$$

where: F_{ir} = fraction of intact rock contained in tensile rock bridges along the step path; h_t = perpendicular height of the potential step-path failure mass; and T_o = estimated tensile strength of the intact rock. For a given step-path geometry, all the terms in Eq. (18) are constants except for T_o, the tensile strength, which typically is estimated by a set of Brazilian disk tension tests. If this set consists of at least four specimens, then a mean and standard deviation can be computed for T_o, which then are used to describe this input variable in the safety factor computations. Thus, the third term in Eq. (17) can be rewritten as:

$$\left(\frac{I}{W\sin(\alpha)}\right)\cdot T_o \quad \text{where } I = F_{ir}(h_t).$$

The following intermediate calculations are useful for helping to define the final constants to be used in the point estimation method.

Effective length of sliding (side AC of triangle ABC in Fig. 5; obtained by applying the law of sines):

$$L = \frac{H\sin(\alpha_c - \beta)}{\sin(\beta)\sin(\alpha_c - \alpha)} \quad (19)$$

where: α_c = average dip of cross joint set; β = overall average angle of the step path.

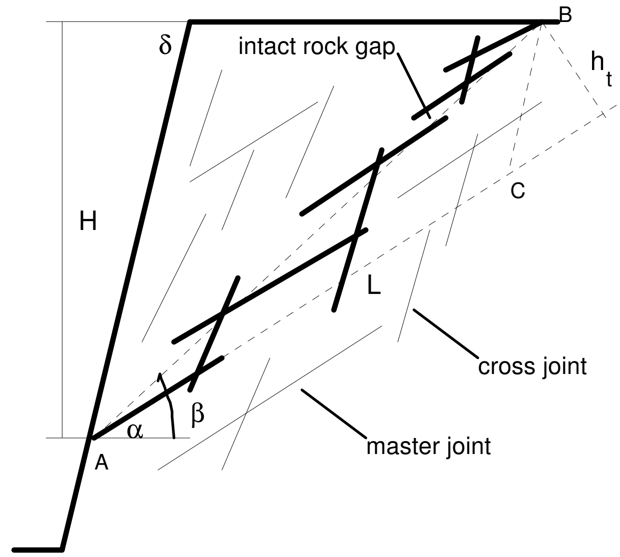


Fig. 5. Example of step-path failure mode.

Weight of the potential failure mass:

$$W = \frac{\gamma H^2 \sin(\delta - \beta)}{2\sin(\delta)\sin(\beta)} \quad (20)$$

where: γ = unit weight of the rock mass.

Perpendicular height of the potential failure mass:

$$h_t = \frac{H}{\sin(\beta)}\sin(\beta - \alpha) \quad (21)$$

5.1. Practical Considerations

Simulated step paths are generated by combining realistic fractures from both the master joint set and the cross joint set using the method presented in [10]. A path is “continuous” if it continues uninterrupted from the daylight point on the slope face all the way to the top of the bench. In this case, the F_{ir} value is zero, meaning that the third term in Eq. (17) can be ignored. However, in many cases a continuous step path does not form (e.g., a cross joint length is not long enough to span the spacing between adjacent master joints), and a new path is initiated from the gap, leaving an intact rock “bridge” that must fail in tension in order for the entire failure mass to slide. The fraction of intact rock is estimated as the summed length of all such rock bridges divided by the value of h_t. Our experience has shown that when this value exceeds about 0.08 the probability of sliding is nil. Thus, in computer routines the P_s is assigned a very small value (say, 0.000001) when $F_{ir} > 0.08$, and the point estimation calculations are not needed.

If the master joint set is very planar with a waviness angle of zero, then the second term in Eq. (17) can be ignored, meaning that only two input variables are analyzed. However, for the general case of step-path analysis, all three variables are used in the point estimation method as described earlier in Section 2. These three variables are:

τ, shear strength along the master joint set;

tan(r), tangent of the waviness angle for the master joint set;

T_o, tensile strength of the intact rock

Their respective constants (multipliers) are:

$L/(W\sin\alpha)$; (22a)

$L\sigma_n/(W\sin\alpha)$; (22b)

$I/(W\sin\alpha)$. (22c)

It is worthy to note that engineering units of force (actually, force per unit width for the 2-d analysis) are preserved in the numerators for all three of the terms in the safety factor equation (tonne/m). Such units cancel the same units in the denominators (i.e., weight per unit width, or tonne/m) to provide a unitless value for the safety factor. This was also the case for the 2-d plane shear analysis presented previously in Section 3.

5.2. *Example Calculations*

To demonstrate the point estimation procedures for analyzing a 2-d step-path failure mode, a simple bench geometry is considered with the following terms (refer to Fig. 5):

Bench face angle: $\delta = 65^o$
Height of failure mass at face: H = 6.1 m
Average dip of master joint set: $\alpha = 33^o$
Waviness of master joint set: $r = 4^o$
Average dip of cross joint set: $\alpha_c = 79^o$
Unit weight of rock mass: $\gamma = 2.6$ tcm (25.5 kN/m^3)
Mean tensile strength of intact rock: 270 tsm
S.D. of tensile strength of intact rock: 28 tsm

Step-path simulation provides a failure path with one intact rock gap:

Overall step-path angle: $\beta = 51^o$
Total span of intact rock gap(s): I = 0.035 m
Calculated effective length of step path: L = 5.12 m
Calculated weight of slide mass: W = 16.62 tonne
Calculated effective normal stress: $\sigma_n = 2.722$ tsm
Shear strength parameters:
a = 0.5549 b = 0.988 c = 0.0
Calculated mean τ: $\mu_\tau = 0.5549\sigma_n^{0.988} = 1.492$ tsm
Standard deviation of τ: $s_\tau = 0.4$ tsm
For 4^o waviness, $\mu_v = 0.070523$ and $s_v = 0.072055$

The first constant from Eq. (22a) is:

$$C_1 = \frac{L}{W\sin(\alpha)} = 0.5656$$

The second constant from Eq. (22b) is:

$$C_2 = \frac{L\sigma_n}{W\sin(\alpha)} = 1.5396$$

The third constant from Eq. (22c) is:

$$C_2 = \frac{I}{W\sin(\alpha)} = 0.0038$$

Point estimate calculations adapted from Eq. (1):

$$F_{+++} = 0.5656(1.492 + 0.4) + 1.5396(0.070523 + 0.072055) + 0.0038(270 + 28) = 2.4220$$

$$F_{---} = 0.5656(1.492 - 0.4) + 1.5396(0.070523 - 0.072055) + 0.0038(270 - 28) = 1.5349$$

$$F_{++-} = 0.5656(1.492 + 0.4) + 1.5396(0.070523 + 0.072055) + 0.0038(270 - 28) = 2.2092$$

$$F_{+--} = 0.5656(1.492 + 0.4) + 1.5396(0.070523 - 0.072055) + 0.0038(270 - 28) = 1.9873$$

$$F_{-++} = 0.5656(1.492 - 0.4) + 1.5396(0.070523 + 0.072055) + 0.0038(270 + 28) = 1.9694$$

$$F_{--+} = 0.5656(1.492 - 0.4) + 1.5396(0.070523 - 0.072055) + 0.0038(270 + 28) = 1.7477$$

$$F_{+-+} = 0.5656(1.492 + 0.4) + 1.5396(0.070523 - 0.072055) + 0.0038(270 + 28) = 2.2002$$

$$F_{-+-} = 0.5656(1.492 - 0.4) + 1.5396(0.070523 + 0.072055) + 0.0038(270 - 28) = 1.7567$$

Assuming independence between each pair of the three variables, the correlation coefficients are zero. The statistical point-mass weights then are given by Eq. (5): $(1/8)(1 + 0) = 0.125$.

Calculate the mean safety factor from an expanded form of Eq. (3):

$$\mu_F = 0.125(2.422 + 1.5349 + 2.2092 + 1.9873 + 1.9694 + 1.7477 + 2.2002 + 1.7567) = 1.978$$

Then, calculate the variance of the safety factor by applying Eq. (4):

$$s_F^2 = 0.125(2.422^2 + 1.5349^2 + 2.2092^2 + 1.9873^2 + 1.9694^2 + 1.7477^2 + 2.2002^2 + 1.7567^2) - 1.978^2$$

$$= 0.0765$$

Thus, the standard deviation of the safety factor is:

$$s_F = 0.2766$$

A gamma pdf for the safety factor distribution that results from these stochastic procedures then can be defined and numerically integrated from 0 to 1 to estimate the probability of sliding for this step-path analysis. The u and z shape parameters of the gamma pdf are computed directly from the mean and variance.

$$u := \frac{1.978^2}{0.0765} \qquad z := \frac{0.0765}{1.978}$$

$$P_S := \frac{1}{\left(z^u \cdot \Gamma(u)\right)} \int_0^1 e^{-\left(\frac{x}{z}\right)} \cdot x^{u-1}\, dx$$

Probability of sliding: $P_s = 0.000074$

The safety factor gamma pdf is shown in Figure 6. The calculated probability of sliding is small here, primarily due to the relatively high mean value of the safety factor.

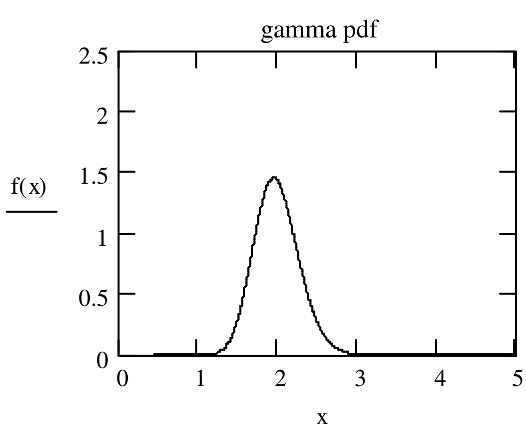


Fig. 6. Safety factor pdf for step-path example based on point estimation method; mean = 1.978, s.d.= 0.277, P_S = 0.00007.

For this same step-path geometry with no intact rock gaps (i.e., I = 0.0), we can neglect the contribution from tensile strength and use only the first two terms in Eq. (17). This situation leads to a calculated mean safety factor of 0.952 and s.d. of 0.2537, which yield a value of 0.60 for the probability of sliding. This simple example clearly shows the significant increase in stability provided by small intact rock bridges along a step-path failure surface.

The driving force for this 2-d step path example is $W\sin(\alpha)$ = 9.05 tonne. The expected contribution to stability from the small intact rock bridge is found by: (0.035 m)(1 m)(270 tsm) = 9.45 tonne, which is the tensile force that can be carried by the intact rock bridge without rupturing. Thus, we see why the mean safety factor can increase from 0.952 to 1.978 when the effect of the intact rock bridge is included in the stability analysis. If observed field conditions do not indicate solid, intact zones of rock substance (e.g., rock masses that are highly fractured or that contain weathered or altered zones) that can form such rock bridges, then the engineer should consider reducing the input value for the mean tensile strength. This is especially important when the tensile strength has been estimated from small, intact specimens used in laboratory testing.

6. DISCUSSION

The point estimation method provides a computationally efficient method to calculate the mean and standard deviation of the safety factor for common rock slope failure modes. The shape of the safety factor pdf must be specified by the engineer in order to estimate the probability of sliding by numerically integrating the area under this pdf from zero to one. This pdf shape will closely resemble that of the pdf chosen for the shear strength. It is prudent to use a pdf model for shear strength that only takes on positive values, such as the gamma pdf. We recommend this choice, which rationally leads to an assumption of a gamma pdf for the safety factor as well. For this model, the shape of the safety factor distribution appears fairly bell-shaped for small values of the standard deviation, but becomes more right-skewed as the variance increases (Fig. 7).

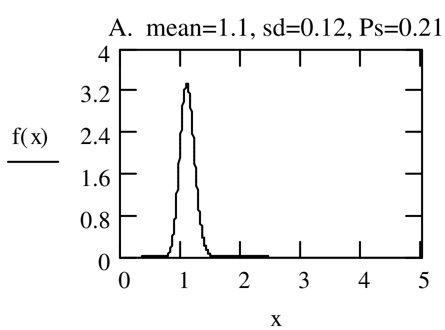
For the plane-shear and wedge failure modes, the probability that the failure surface is long enough to extend all the way to the top of the bench must be considered in the final calculation to obtain the probability of failure (note: this probability of sufficient length is not needed for the step-path mode, which presumes a continuous failure path including any small intact rock bridges present). The length required for a continuous plane shear is shown by L in Fig. 1. The length required for a wedge failure is measured along the intersection line of the wedge, and both planes must have trace lengths at least as long as this intersection for the wedge to be viable.

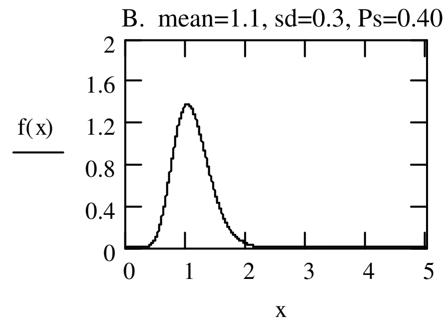
A reasonable and convenient model for the length distribution in a fracture set is the exponential pdf [1, 5], which is a one-parameter pdf defined by its mean. The cumulative distribution function (cdf; denoted by F(x)) of an exponential variable can be used directly to obtain the probability that length in a fracture set will take on a value at least as long as that required for the failure mode to be viable:

$$P_L = 1 - F(x) = 1 - (1 - e^{-x/m}) = e^{-x/m} \qquad (23)$$

where: m = mean length of the fracture set. Thus, for the plane shear mode, the probability of failure is: $P_F = P_S P_L$, and for the wedge mode it is given by: $P_F = P_S(P_{L1}P_{L2})$, which reflects the joint probability of sufficient length for the two planar discontinuities that form the wedge. There are accepted methods for estimating the mean trace length of a fracture set based on field data obtained by scan-line or window mapping [12]. Such procedures provide the mean length to be used in Eq. (23) to obtain the needed P_L values.

These methods for estimating the probability of failure have been programmed into several software





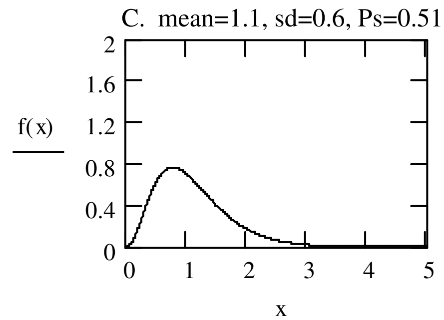


Fig. 7. Comparison of safety factor gamma pdf's for different standard deviation values; the mean SF in all figures is 1.1.

components used in NIOSH bench simulation software [1], which can be used to help engineers stochastically describe rock slope behavior. The tools also can be applied to other aspects of rock slope engineering whenever estimates of the probability of failure are desired.

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