CLASSIFICATION-BASED PROBABILISTIC DESIGN OF GROUND SUPPORT

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ABSTRACT

Probabilistic design is gaining wider acceptance in the rock engineering community since it allows more rigorous determination of risk relating to ground fall or excavation instability. Risk analysis can be conducted by various means, but the basis is formed by either objectively or subjectively determining probability of occurrence of an event. In the case of rock engineering, this event is either instability or excavation failure. Rock mass classification systems provide objective analysis of data collected on a typically subjective basis that also relate closely to excavation stability. A probabilistic analysis technique is presented that uses statistical distributions of rock mass and material properties, ground support fixture specifications, stress conditions, opening geometry, and ground support installation quality to more rigorously determine probability of failure for an underground opening and the subsequent risk to personnel.

INTRODUCTION

The philosophy governing much ground support practice in underground excavation relies on the concept of constructing a support arch by harnessing the frictional and cohesive strength inherent in the rock mass through reinforcement. Rock masses, in some cases, are self-supporting and need few, if any, additional elements to mobilize their strength. However, in instances where the rock mass requires added elements to be stable, three essential components must be present in order to effectively construct an arch. The first two components are rock reinforcement and surface support. Presence of competent abutments for the support arch to stand upon is the third vital component.

Field data collection systems are useful tools to gauge the effectiveness of these three components in creating a stable support arch. These systems typically are intimately tied to empirical design methods, design graphs, and deterministic approaches, as are the principles of the geomechanical concepts put forth in this paper. However, with the technique described herein, both objective and subjective probabilistic methods are suggested to derive a basis for design, as well as an overall picture of system integrity and risk analysis.

In the course of rock mass data collection, reduction, and design, varying degrees of uncertainty exist concerning all input parameters. Ignorance or simply the unknowability of specific values for often critical design factors makes a probabilistic approach to rock mass classification and ground support design a useful tool. Empirical and deterministic design approaches do not incorporate uncertainty into the process aside from ad hoc methods or simply by pure overdesign. Probabilistic methods also allow the production of more objective end products from input variables that are frequently quite subjective. Objective products resulting from the design process make their contribution in a risk, financial, or other decision-making analysis more rigorous. If little or no geotechnical data exist, this process can be also be used to conduct "what if" or sensitivity analysis for specified components of a feasibility study.

BASIC PROBABILITY CONCEPTS

Some basic probability concepts as they apply to this particular problem are outlined below.

Cause and Effect

Human interaction is a world of complex cause and effect. Causes to effects that we pursue are often effects of lesser-order causes. A cause of an effect is an effect of one or more identifiable underlying causes. Human shortcomings, in principle, constitute the lowest order of causes because humans ultimately hold the initiative to all action.

Cause: underlying factor that leads to a particular event
Effect: outcome resulting from a particular event

Independent and Dependent Causes

Two separate types of causes are considered in this paper. Independent causes take place separately from other causes and are represented by a logical "OR" statement. Dependent causes require other factors in order to occur and are represented by a logical "AND" statement. Probabilities of occurrence of independent causes are added with each other to exclude joint occurrences, and those for dependent causes are multiplied together.

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Thresholds for Probability

International thresholds for probability of loss are given after Cole [1993] in Table 1 in terms of total losses of life, property, and money. Total loss of life denotes fatality, whereas property denotes fixed assets and money, and business ventures. Thresholds are selected in terms of voluntary and involuntary exposure of the affected aspects to the hazards. They apply to the overall probability of loss, which includes the probabilities of failure and exposure.

The thresholds are expressed in terms of lifetime frequencies, which are defined as the probable unit number of times that a hazard would occur during the life of the person affected. A natural lifetime is on average 70 years and a working lifetime is 50 years, which corresponds to 250×8 working hours per annum or $250 \times 8 \times 50 =$ 100,000 total working hours. Expressed as a percentage, the lifetime probability of loss is equal to one-tenth of the fatality accident rate. The fatality accident rate is equal to the number of deaths from 1,000 people who are involved in a hazardous activity for their entire lives. The upper limit of lifetime probability of loss is equal to 7,000%, which by definition is the product of 70 as the average lifetime in years and 100% as the probability of occurrence of an event that will certainly occur in every year. The probabilities of failure that may be determined for engineering systems represent lifetime frequencies because they represent the unit number of times that the systems may fail in the conceivable future.

Cole [1993] determined the threshold probabilities of losses in Table 1. They are generally much more stringent than those recommended in the literature prior to 1987, but are substantially in agreement with recommendations of various authors since. The "Acceptable and Tolerable Risk Criteria" given in Appendix H of the *Landslide Risk Management Concepts and Guidelines* published by the Australian Geomechanics Society [2000] correspond accurately with these thresholds.

Assignment of Probabilities

Probabilities of failure can be derived from randomly sampled distributions of input variables to generate a distribution of end product capacity versus demand values. They can also be subjectively assigned through experience, engineering judgment, and use of the eight-point scale shown in Table 2.

Objective Assignment of Probability

The likely occurrence of an event may be objectively determined by the following process. First, random sampling of the distributions for the governing input parameters produces a population of rock mass values that are translated into ground support demand. Next, a second group of values of ground support capacity is generated in the same manner. Values from both of these distributions are then randomly sampled to generate a distribution of capacity versus demand. The probability of failure is then the area under the probability density function left of the value where capacity equals demand. Distributions can be derived from statistical analysis of the input data or from predetermined functions if the statistical parameters are not well established. Figures 1–3 are normal, triangular, and uniform probability density functions commonly used for these purposes.

Table 1.—Acceptable lifetime probabilities of total losses [Cole 1993]

Degree of risk	Attitude to reliability		Probability (%) of total loss of		
	Voluntary	Involuntary	Life	Property	Money
Very risky..........	Very concerned	Totally unacceptable	70 (–) (deep-sea diving or rock climbing)	700 (–) (volcano or avalanche)	7,000 (–) (gambling)
Risky.................	Concerned	Not acceptable	7 (1.60) (deep-sea diving or rock climbing)	70 (–) (volcano or avalanche)	700 (–) (gambling)
Some risk..........	Circumspect	Very concerned	0.7 (2.50) (car, airplane, or home accident)	7 (1.60) (undermining or earthquake)	70 (–) (small business failure)
Slight chance	Of little concern	Concerned	0.07 (3.22) (car, airplane, or home accident)	0.7 (2.50) (undermining or earthquake)	7 (1.60) (small business failure)
Unlikely	Of no concern	Circumspect	0.007 (3.82) (public transport accident)	0.07 (3.22) (flooding)	0.7 (2.50) (company failure)
Very unlikely	Of no concern	Of little concern	0.0007 (4.35) (fatality in public place)	0.007 (3.82) (failure of foundation on soil)	0.07 (3.22) (failure of banks or building societies)
Practically impossible.........	Of no concern	Of no concern	0.00007 (4.83) (failure of nuclear powerplant)	0.0007 (4.35) (failure of foundation on rock)	0.007 (3.82) (collapse of National Savings)

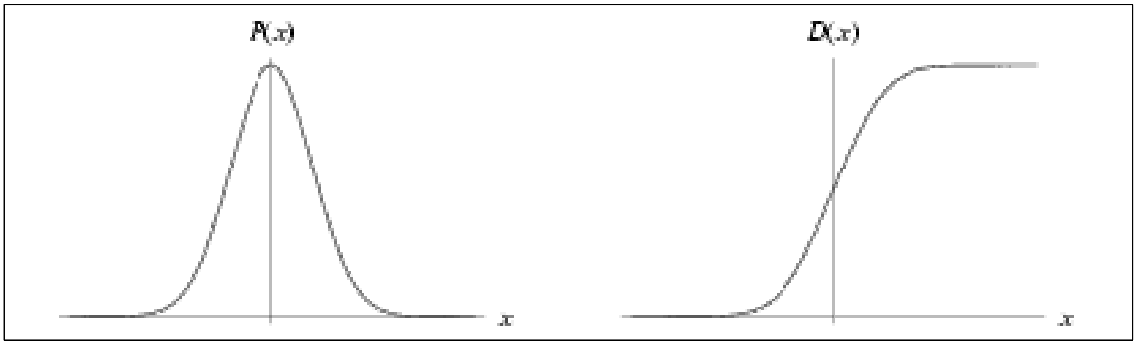

Figure 1.—Normal probability density function and cumulative distribution function [MathWorld 2007].

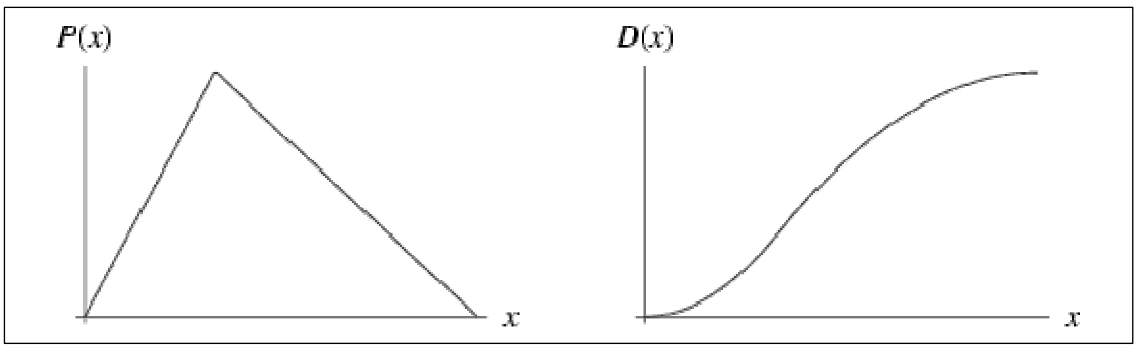

Figure 2.—Triangular probability density function and cumulative distribution function [MathWorld 2007].

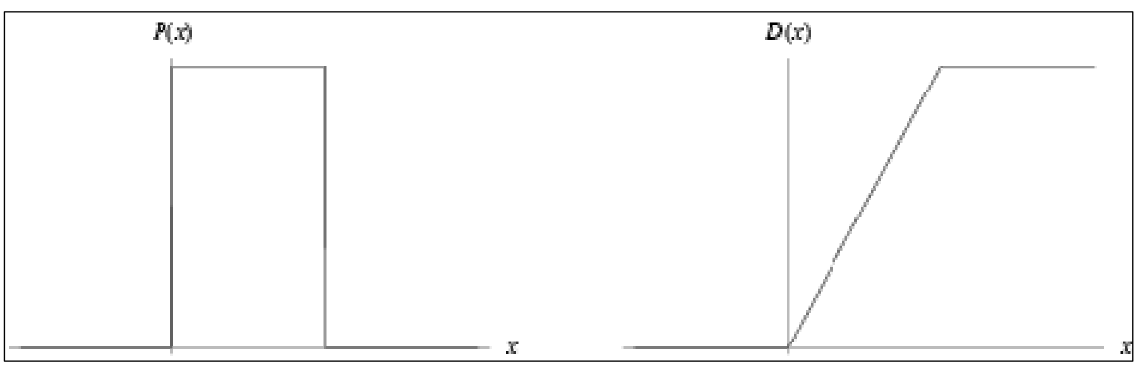

Figure 3.—Uniform probability density function and cumulative distribution function [MathWorld 2007].

SUBJECTIVE ASSIGNMENT OF PROBABILITY

The eight-point scale described below was developed for evaluating operational safety in South African coal mines. It can consistently evaluate operational safety, system integrity, economic viability, process reliability, and environmental protection and rehabilitation in a wide range of engineering systems [Kirsten 1999]. The six-point scale Risk Assessment Table published by the Institution of Civil Engineers and the Faculty and Institute of Actuaries [1998] in the United Kingdom is identical in concept. The six-point scale in Appendix G of the *Landslide Risk Management Concepts and Guidelines* published by the Australian Geomechanics Society [2000] is almost identical to Table 2 in both qualitative and quantitative levels of probability.

Table 2.—Classes for probability of occurrence

Qualitative evaluation	Quantitative evaluation	
Certain	Every time	1.0
Very high	1 in 10	10^{-1}
High	1 in 100	10^{-2}
Moderate	1 in 1,000	10^{-3}
Low	1 in 10,000	10^{-4}
Very low	1 in 100,000	10^{-5}
Extremely low	1 in 1 million	10^{-6}
Practically zero	1 in 10 million	10^{-7}

GEOMECHANICAL DATA

Rock mass parameters are closely tied and most often differ with variations in rock type, which usually also defines the spatial relationships of the parameters and the excavation. The system used in this paper for assessing rock mass quality is the Q-system developed by Barton et al. [1974]. The methods described in this paper would work equally well for virtually any rock mass classification system.

Rock mass quality, Q, varies on a logarithmic scale from 0.001 to 1,000 and is determined by Equation 1:

$$Q = (RQD/J_n) \times (J_r/J_a) \times (J_w/SRF) \qquad (1)$$

RQD/J_n is an estimate of block size. J_r/J_a generally represents the strength of the discontinuities demarcating the blocks. J_w/SRF is a measure of the active stress present in the rock mass [Barton et al. 1974].

All of the inputs for the equation can be defined as random variables and, as such, may belong to populations easily defined by statistical distributions. Five of the six parameters needed to define rock mass quality (Q) listed below can be sampled from their own distribution. The last, stress reduction factor (SRF), can be either chosen from a distribution of SRF or calculated from the UCS and depth, each of which can also be picked from a specific distribution [Kirsten 1988]. Any one of these random variables can also be held to a constant value. The rock mass input parameters are listed below.

- RQD is a measure of the degree of fracturing in the rock mass;
- J_n represents the total number of discrete joint sets in the rock mass;
- J_r is a measure of friction and, to a somewhat lesser degree, cohesion of a discontinuity;
- J_a represents the amount of both cohesion and friction of a discontinuity;
- J_w is the amount of water inflow affecting the rock mass; and
- SRF quantifies the effect of the excavation on the rock mass.

Other geomechanical parameters that can be selected out of specified distributions are—

- UCS;
- Angle of internal friction;
- Unit weight;
- Maximum principal primitive stress;
- Minimum principal primitive stress;
- Maximum principal primitive stress direction;
- Depth below surface;
- Span variation;
- Excavation support ratio; and
- Geologic structure variations.

All of these rock mass and geomechanical properties combine with excavation size and geometry to place a demand of some magnitude on the ground support system. The designed ground support system must reinforce and confine the rock mass to the point that the effects of this demand are counteracted in order to provide excavation stability.

SUPPORT COMPONENTS

Variation in ground support elements is typically less pronounced than rock mass properties because they are produced by a relatively well controlled manufacturing process. However, the installation process, design layout, and excavation profile often vary widely. For a specified rock-reinforcing fixture, any of the following support component parameters can be sampled randomly from an appropriate distribution:

- Fixture length;
- Angle from normal;
- Material properties;
- Fixture specifications;
- Hole diameter;
- In-plane spacing;
- Out-of-plane spacing;
- Plate properties; and
- Properties of the fixture rock interface(s).

Statistical distributions can represent the shotcrete properties listed below:

- Thickness;
- Span-thickness ratio;
- Compressive strength;
- Shear strength; and
- Reinforcement properties.

Surface support elements also conform to this process. These are:

- Area per meter;
- Caternary rise;
- Strand tensile strength;
- Mesh anchor shear area;
- Weld strength; and
- Strand spacing.

When the distribution of rock mass and geomechanical demand has been generated, it is compared to a distribution of ground support capacity. This comparison results in a distribution of capacity versus demand. The probability of failure determined from the capacity versus demand distribution is the area under the best-fit curve left of the point where capacity equals demand. This represents probability of structural failure (p_{sfw}), and the process of deriving the probability is applied to each wall of the excavation in turn. Probability of failure for each wall will then be combined with the probabilities for the other walls by a logical "OR" statement since failure can occur in any wall independent of the others. From this point, the other aspects determining total probability failure and overall threat may now be applied.

Failure of ground support subsystems such as installation quality increase the overall probability of failure, but due to the difficulty in objective measurement, the probabilities of failure are best subjectively applied. These probabilities of the factors listed below are subjectively assessed and applied to each wall separately. The subjective application of the probabilities should be derived from the eight-point scale in Table 2. In general terms, the subsystems are described by the following list:

- Fixtures installed per manufacturer's specifications and standard industry practice
- Proper anchorage for the specified fixture
- Rock bolt plates tight against the rock face
- Angle of the installed fixture as close to normal to the bearing surface as possible
- Significant structures or weak contacts crossed and locked together
- Systematic and regular support installation
- Adherence to specified design or ground support standard
- Installed support adequate for the ground type
- Rock not excessively damaged due to blasting
- Blast holes drilled on line and not out into the walls or up into the back
- The walls have relatively smooth profile
- No excessive loading of ground support elements
- Surface support elements secured tightly against the rock surface

DISCUSSION

Each of these subjectively assigned probabilities of subsystem failure can now be combined with the probability of structural failure for a given wall with a logical "OR" statement, giving a probability of overall structural failure (p_{osfw}) calculated by:

$$P_{osfw} = 1 - [(1 - p_{sfw}) * (1 - p_{ss1}) * (1 - p_{ss2}) * (1 - p_{ssn+1})] \quad (2)$$

where p_{osfw} = P(overall structural failure wall);
p_{sfw} = P(probability of structural failure wall);
p_{ss1} = P(probability of subsystem failure 1);
p_{ss2} = P(probability of subsystem failure 2);
and p_{ssn+1} = P(probability of subsystem failure n).

Three additional aspects needed in determining total probability of failure and overall threat to personal injury must now be considered. The first of these is probability of ejection freedom. This concept is based on the degree of confinement or restraint against spontaneous block ejection resulting from a gravitational or seismic acceleration provided to the rock mass as the level of ground support increases. As the number of ground support elements multiply, the probability that a block of rock can spontaneously be ejected, taking a worker by surprise, decreases. The probability of surprise or ejection freedom with no support installed is 100%, or certain. As the support quantity increases, the probability of ejection freedom decreases by an order of magnitude as shown in Table 3.

Table 3.—Probability of ejection freedom

Support level	Probability of ejection freedom	
No support…………………………	Every time	1.0
Light support……………………	1 in 10	10^{-1}
Moderate support………………	1 in 100	10^{-2}
Heavy support………………….	1 in 1,000	10^{-3}
Very heavy support…………….	1 in 10,000	10^{-4}
Extremely heavy support……...	1 in 100,000	10^{-5}

The next facet of the overall threat to consider is the probability of personnel appearance. This probability can be calculated as the percentage of the entire work shift that personnel spend exposed to potential excavation instability. It can also be referenced from Table 4.

Table 4.—Probability of personnel appearance

Personnel appearance	Probability
Continuous…………………….......	1.0
Very regular……………………..	0.3
Regular…………………………	0.03
Occasional…………………….......	0.003
Very occasional……………......	0.0003
Rare…………………………..........	0.00003

The final variable needed to complete this calculation is the probability of personnel coincidence. Coincidence is essentially calculated by a 0.5-m width of a person divided by the total length of excavation that exposes that person to a rock fall hazard. Thus, for one person in 50 m of tunnel, the personnel coincidence is 0.5/50, or 0.1.

In order to now determine total probability of failure, the probability of overall structural failure for each wall is "OR" gated to the other walls to obtain the probability of failure for the entire excavation.

$$P_{osfe} = 1 - [(1 - p_{osfw1}) * (1 - p_{osfw2}) * (1 - p_{osfw3})] \quad (3)$$

where p_{osfe} = P(overall structural failure excavation);
p_{osfw1} = P(overall structural failure wall 1);
p_{osfw2} = P(overall structural failure wall 2);
and p_{osfw3} = P(overall structural failure wall 3).

In order to calculate the overall threat of injury for the entire excavation, the overall probability of structural failure for the excavation is "AND" gated with the probability of ejection freedom, probability of personnel appearance, and probability of personnel coincidence, as shown in Equation 4:

$$T = p_{osfe} * p_{ejec} * p_{app} * p_{coin} \quad (4)$$

where T = overall threat of injury;
p_{osfe} = P(overall structural failure excavation);
p_{ejec} = P(ejection freedom);
p_{app} = P(personnel appearance);
and p_{coin} = P(personnel coincidence).

When the overall threat is calculated, it can be compared to the thresholds shown in Table 1. If the overall threat is below an acceptable level, the design can stand. If it is not, a number of methods can be employed to reduce the threat. Rock mass variable distributions should be checked for plausibility or appropriate application and assumptions regarding input variables recalibrated. The design can be reconsidered and altered to increase ground support capacity, thereby reducing the possibility of structural failure. Subsystem shortfalls found to contribute significantly to the probability of structural failure can be remediated. The probability of ejection freedom decreases with an increase in support quantity, and the probability of personnel appearance and coincidence can be reduced by limiting access to the area.

Another methodology that has been employed is to conduct a survey of numerous excavations at a project such as a large mining operation and apply this process to each excavation. When the overall threat for an appropriate number of workings have been calculated, the distribution of threats can be plotted, as shown in Figure 4.

An appropriate design level of threat for work in underground excavations is less than 10^{-4}, or 1 chance in 10,000. As a basis for comparison, this probability of occurrence is equivalent to acceptable risk of injury on a public transport system [Kirsten 1999]. Values above this threshold indicate a need to promptly address conditions, while values below this indicate acceptable level of threat or risk. Probabilities that are several orders of magnitude below the threshold indicate less than optimal ground support economy. Figure 4 shows an example distribution of threat from 42 hypothetical cases.

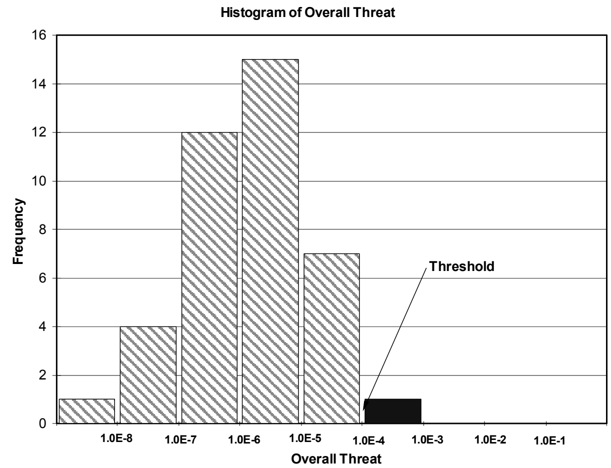

Figure 4.—Example histogram of threat.

As previously stated, when overall threat exceeds the threshold, access by personnel to the area can be limited until ground support remediation has been completed. It is advisable to install remedial ground support only from under supported ground that has an overall threat below a threshold acceptable to operations management. Prohibiting access to an area effectively decreases exposure so that the threat is reduced to below the acceptable level of threat. A reasonable goal is to not let more than 5% of headings exceed an acceptable threshold of 10^{-4} at any instant of time.

SUMMARY

Typical rock mass classification design systems involve data collection, data reduction, and then plotting the reduced data on empirical design curves. From this step, empirical or deterministic criteria are applied and a final design proposed. This type of process is quite adequate for many rock engineering problems. Some, however, lend themselves to probabilistic analysis due to the inhomogeneous nature of the rock mass and inherent uncertainty of its characterization.

Rock mass data collection, reduction, and design involve varying degrees of uncertainty due to the variability of all input parameters. Parameters collected during the course of rock mass classification and excavation design are random variables and, as such, belong to populations naturally expressed by statistical distributions.

Unknowns regarding specific values needed in ground support allow a probabilistic design approach to provide inputs that can be used with a known degree of confidence. That degree of confidence may be low or high, but it is known and was systematically derived. This process can also produce more objective end products from frequently subjective input variables. Objective products resulting from the design process make their contribution in a risk, financial, or other decision-making analysis more rigorous. If little or no geotechnical data exist, this process can be also be used to conduct "what if" or sensitivity analysis for specified components of a feasibility study.

Probability of loss thresholds are specified for total loss of life, property, and money. Thresholds are chosen with regard to voluntary and involuntary exposure to hazards and expressed in terms of lifetime frequencies. Probabilities of failure determined for engineering systems represent lifetime frequencies, since they correspond to the unit number of times the systems could fail in a potential lifetime.

After generation of a rock mass and geomechanical demand distribution, it is compared to a ground support capacity distribution that gives rise to a capacity versus demand distribution from which the probability of structural failure is calculated. Subjectively derived probabilities of failure for ground support subsystems are added to the probability of structural failure to give a probability of overall structural failure. The probability of ejection freedom, personnel appearance, and personnel coincidence are multiplied with the overall probability of structural failure to give the overall threat of injury.

When the overall threat of injury is above an acceptable threshold, several approaches can be taken to lessen the threat, from recalibrating input variables to increasing the quantity of ground support and remediating ground support subsystem shortfalls. Increasing support quantity decreases the probability of ejection freedom, and limiting access to the area lowers the probability of personnel appearance and coincidence.

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