NEW STRENGTH FORMULA FOR COAL PILLARS IN SOUTH AFRICA

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ABSTRACT

For the last 3 decades, coal pillars in the Republic of South Africa have been designed using the well-known strength formula of Salamon and Munro that was empirically derived after the Coalbrook disaster. The database was recently updated with the addition of failures that occurred after the initial analysis and the omission of failures that occurred in a known anomalous area. An alternative method of analysis was used to refine the constants in the formula. The outcome was a new formula that shows that the larger width-to-height ratio coal pillars are significantly stronger than previously believed, even though the material itself is represented by a reduced constant in the new formula. The formula predicts lower strength for the smaller pillars, explaining the failure of small pillars that were previously believed to have had high safety factors. Application of the new formula will result in improved coal reserve utilization for deeper workings and enhanced stability of shallow workings.

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INTRODUCTION

The Coalbrook disaster in January 1960, in which more than 400 men lost their lives when the mine's pillars collapsed, led to a concerted research effort that eventually resulted in the creation of two formulas for the prediction of coal pillar strength: the power formula of Salamon and Munro [1967] and the linear equation of Bieniawski [1968]. The Bieniawski formula was based on in situ tests of large coal specimens; the Salamon-Munro formula, on a statistical analysis of failed and stable pillar cases. The South African mining industry adopted the Salamon-Munro formula, even though the differences between the two formulas were not significant for the range of pillar sizes that were mined at the time.

It is characteristic of the Salamon-Munro formula that the strength increases at a lower rate as the width-to-height (w/h) ratios of the pillars increase. Later, this was rectified by the so-called squat pillar formula refined by Madden [1991]. This formula is valid for w/h ratios >5 and is characterized by an accelerating strength increase with increasing w/h ratios.

An intriguing aspect of the Salamon-Munro formula is the relatively high value of the constant in the formula that represents the strength of the coal material—7.2 MPa. This compares with the 4.3 MPa used in the Bieniawski formula. The question has always been why the statistical back-analysis yielded a higher value than the direct underground tests. An attempt by van der Merwe [1993] to explain the significantly higher rate of pillar collapse in the Vaal Basin yielded a constant for that area of 4.5 MPa, more similar to Bieniawski than to Salamon and Munro, but not directly comparable because it was valid for a defined geological district only.

In the process of analyzing coal pillar failures for other purposes, an alternative method of analysis was used that resulted in a formula that is 12.5% more effective in distinguishing between failed and stable pillars in the database. This paper describes the method of analysis and the results obtained.

REQUIREMENTS OF A SAFETY FACTOR FORMULA

A safety factor formula should satisfy two main requirements: (1) it should successfully distinguish between failed and stable pillars and (2) it should provide the means whereby relative stability can be judged. The third requirement, simplicity, has become less important with the widespread use of computers, but is still desirable.

These fundamental requirements are conceptually illustrated in figure 1. Figure 1*A* shows the frequency distributions of safety factors of the populations of failed and stable pillars, respectively. The area of overlap between the populations can be seen as a measure of the success of the formula; the perfect formula will result in complete separation of the two populations. Figure 1*B* is a normalized cumulative frequency distribution of the safety factors of the failed cases plotted against safety factors. At a safety factor of 1.0, one-half of the pillars should have failed, or the midpoint of the distribution of failed pillars should coincide with a safety factor of 1.0.

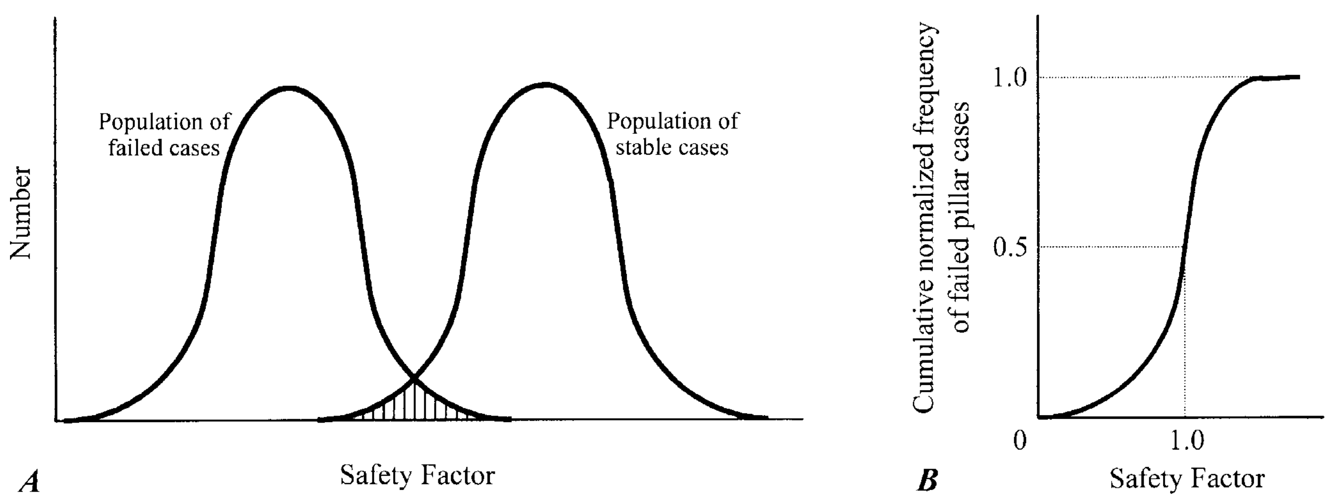

Figure 1.—Concept of the measure of success of a safety factor formula. *A*, The overlap area between the failed and stable cases should be a minimum. *B*, At a safety factor of 1.0, one-half of the pillars should have failed.

EXISTING FORMULAS IN SOUTH AFRICA

The safety factor is a ratio between pillar strength and pillar load. In its simplest form, the load is assumed to be the weight of the rock column overlying the pillar and the road around the pillar, i.e., the tributary area theory is normally used. This is widely held to be a conservative, and thus safe, assumption. However, it has at least one complication when this load is used to derive a safety factor empirically: if the load used to determine pillar strength is greater than the actual load, then the strength derived will also be greater than the actual pillar strength. If an alternative method is then used later to calculate pillar load, such as numerical modeling, and the strength is not modified, then the calculated safety factor will be greater than the real safety factor.

For purposes of this paper, the tributary area loading theory is used, and the restriction must then be added that the derived strength is only valid for situations where the tributary area load is used. This is not a unique restriction; even if not explicitly stated, it is also valid for any other empirical safety factor formula for which the tributary area loading assumption was used, such as the Salamon-Munro formula.

It then remains to determine a satisfactory formula for the calculation of pillar strength. The strength of a pillar is a function of the pillar dimensions, namely, width and height for a square pillar, and a constant that is related to the strength of the pillar material. According to Salamon and Munro [1967], the strength is

$$\sigma = k w^{\alpha} h^{\beta}, \tag{1}$$

where h = pillar height,

w = pillar width,

and k = constant related to material strength.

The parameters k, α, and β are interdependent. Salamon and Munro [1967] used the established greatest likelihood method to determine their values simultaneously and found:

$k = 7.2$ MPa,

$\alpha = 0.46$,

and $\beta = -0.66$.

The linear formula of Bieniawski [1968] is

$$\sigma = 4.3(0.64 + 0.36\ w/h). \tag{2}$$

With the addition of new data on failures after 1966 to the Salamon and Munro database, Madden and Hardman [1992] found:

$k = 5.24$ MPa,

$\alpha = 0.63$,

and $\beta = -0.78$.

These new values, however, did not result in sufficiently significant changes to safety factors to warrant changing the old formula, and they were not used by the industry. Note, however, the increases in values of α and β and reduction of k.

According to Madden [1991], the squat pillar formula, valid only for pillars with a w/h > 5, is

$$\sigma = k \frac{R_0^{\,b}}{V^{\,a}} \left\{ \frac{b}{\gamma} \left[\left(\frac{R}{R_0} \right)^{\gamma} - 1 \right] + 1 \right\}, \tag{3}$$

where R = pillar w/h ratio,

R_0 = pillar w/h ratio at which formula begins to be valid = 5.0,

and V = pillar volume.

Substituting $k = 7.2$ MPa, $a = 0.0667$, $b = 0.5933$, $R_0 = 5.0$, and $\gamma = 2.5$ results in a somewhat simplified form of the formula that is sometimes used:

$$\sigma = \frac{0.0786}{V^{0.0667}} \{R^{2.5} + 181.6\} \tag{4}$$

For quick calculations, equation 4 can be approximated with negligible error by

$$\sigma = 0.0786 \frac{w^{2.366}}{h^{2.5667}} + 9\,. \tag{5}$$

ALTERNATIVE METHOD OF ANALYSIS

Although α, β, and k are interdependent, they can be separated for purposes of analysis. It was found that changing α and β affected the overlap area of the populations of failed and stable pillars. Modifying k does not affect this relationship; it causes an equal shift toward higher or lower safety factors in both populations. Therefore, α and β can be modified independently to minimize the overlap area between the two populations; once that is done, k can be adjusted to shift the midpoint of the population of failed pillars to a safety factor of 1.0.

DETERMINATION OF α AND β

The data bank for failed pillars for the analysis described here was that quoted by Madden and Hardman [1992], which was the original Salamon and Munro data. The post-1966 failures were added to the data, and the three Vaal Basin failures were removed because the Vaal Basin should be treated as a separate group (see van der Merwe [1993]). (Note that a subsequent back-analysis indicated that the changes to the data bank did not meaningfully affect the outcome.)

For the first round of analysis, α and β were both varied between 0.3 and 1.2 with increments of 0.1. Safety factors were calculated for each case of failed and stable pillars. For each of the 100 sets of results, the area of overlap between the populations of failed and stable pillar populations was calculated. A standard procedure was used for this, taken from Harr [1987]. This involved the simplifying assumption that the distributions were both normal, but because it was only used for comparative purposes, the assumption is valid. Using the same procedure, the overlap area for the Salamon-Munro formula was also calculated. This was used as the basis from which an improvement factor was calculated for each of the new data sets.

The safety factor, S, was

$$S = \frac{\text{Strength}}{\text{Load}}. \tag{6}$$

The tributary area theory was used to calculate the load:

$$\text{Load} = \frac{\rho g H (w + B)^2}{w^2}, \tag{7}$$

where H = mining depth,

w = pillar width,

and B = bord width.

Then, the strength was varied, as follows:

$$\text{Strength} = 7.2 \frac{w^{\alpha}}{h^{\beta}}, \tag{8}$$

where w = pillar width,

h = pillar height,

α = 0.3 to 1.2 with 0.1 increments,

and β = 0.3 to 1.2 with 0.1 increments.

Equations 6 through 9 were applied to each of the cases of failed and stable populations, thus creating 100 sets of populations of safety factors of failed and stable cases. For each set, a comparative improvement factor was calculated. The first step was to calculate "f" for each of the 100 sets:

$$f = \frac{M_s - M_f}{\sqrt{S_s^2 + S_f^2}}, \tag{9}$$

where M_s = mean safety factor of the population of stable pillars,

M_f = mean safety factor of the population of failed pillars,

S_s = standard deviation of the safety factors of the stable pillars,

and S_f = standard deviation of the safety factors of the failed pillars.

Then,

$$R = 0.5 - \frac{1}{f} (2\pi)^{-0.5} \exp\left(\frac{-f^2}{2}\right) \tag{10}$$

and the overlap area between the two populations is

$$A = 0.5 - R. \tag{11}$$

Finally, the improvement factor, I, for each set is

$$I = \frac{A_s - A_n}{A_s}, \tag{12}$$

where A_s $=$ overlap area with the original Salamon-Munro formula,

and A_n $=$ overlap area with the new formula.

It was then possible to construct contours of the improvement factors for variations of α and β (figure 2). Figure 2 shows that the greatest improvement was for α between 0.7 and 0.8 and for β between 0.75 and 0.85. Fine tuning was then done by repeating the procedure with increments of 0.01 for α from 0.7 to 0.8 and for β between 0.75 and 0.85. The resulting contours are shown in figure 3.

On the basis of the contours of improvement factors in figure 3, it was concluded that for $\alpha = 0.81$ and $\beta = 0.76$, the improvement in efficiency of the formula to distinguish between failed and stable pillar cases is 12.5%.

DETERMINATION OF "k"

The last step was to determine k for the new exponents of α and β. This was done by adjusting k so that the midpoint of the population of failed pillars coincided with a safety factor of 1.0. It was found that a value of $k = 4.0$ MPa satisfied this condition; this is shown in figure 4.

FINAL NEW FORMULA

The full new formula for pillar strength in the Republic of South Africa is then as follows:

$$\text{Strength} = 4\,\frac{w^{0.81}}{h^{0.76}} \tag{8}$$

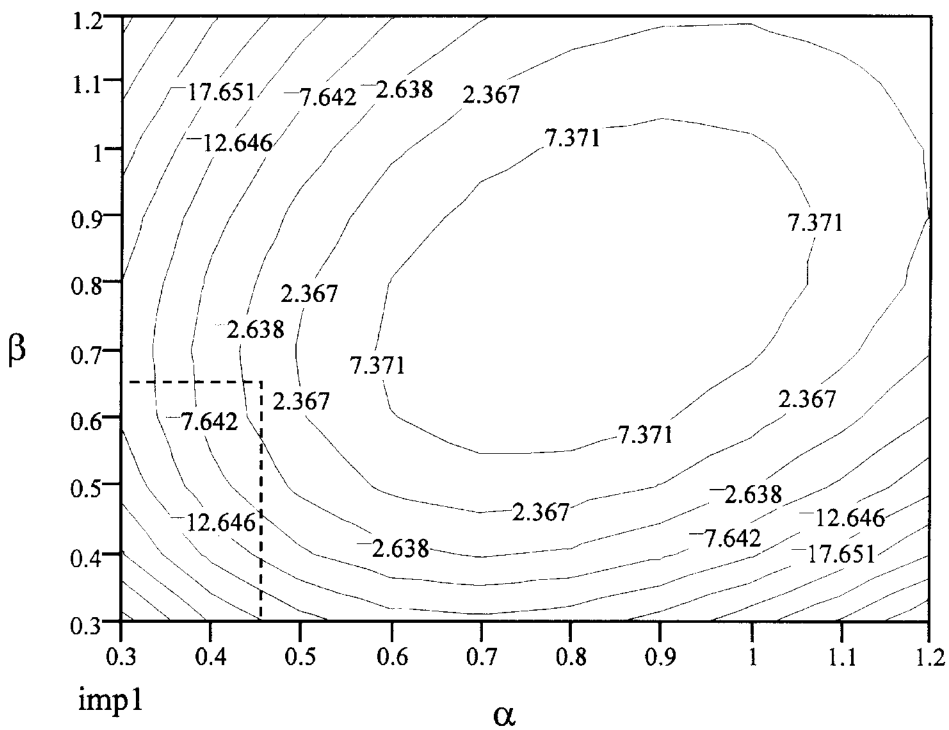

Figure 2.–Contour plot of percentage improvement in efficiency of formula to separate failed and stable pillar cases for variations of α between 0.3 and 1.2 and for β between 0.3 and 1.2. The Salamon and Munro [1967] combination is shown by the dotted lines.

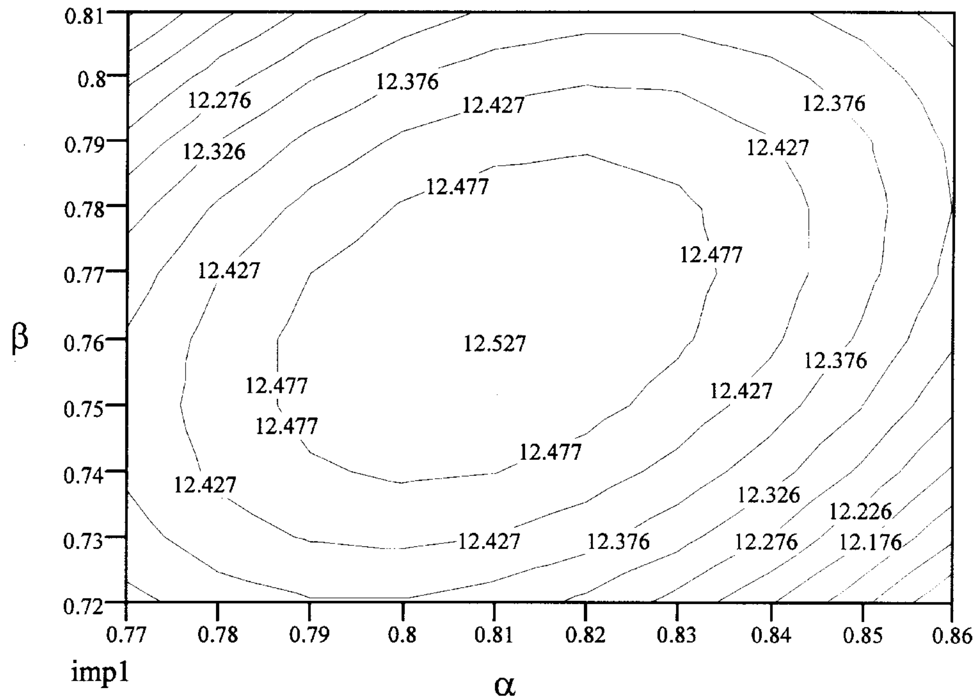

Figure 3.—Contour plot of percentage improvement in efficiency of formula to separate failed and stable pillar cases for variations of " between 0.77 and 0.86 and for $ between 0.72 and 0.81.

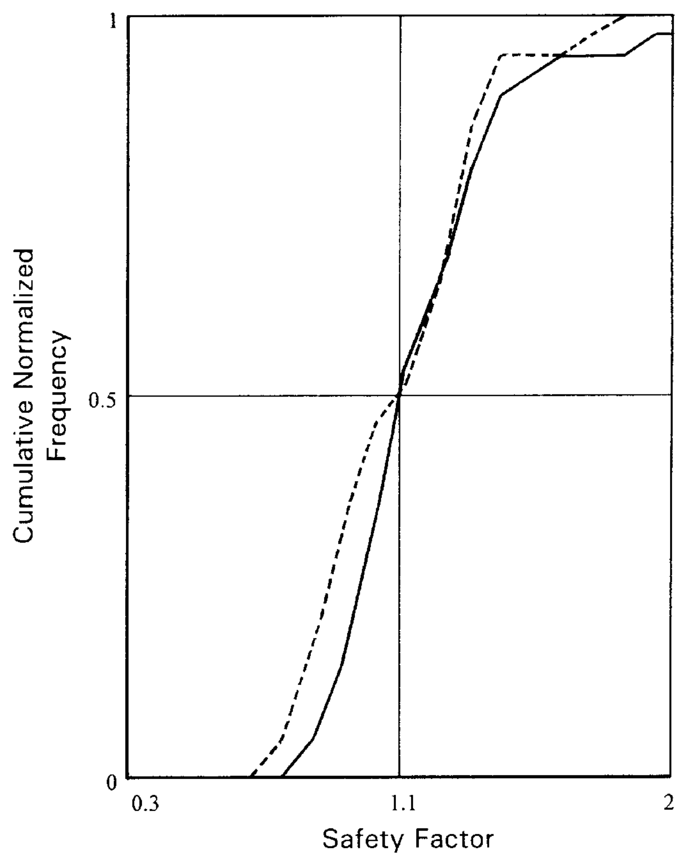

Figure 4.—Plot of cumulative normalized frequency against safety factors calculated with the Salamon-Munro formula (solid line) and the new formula (broken line). For the new formula, k = 4 MPa, " = 0.81, and $ = 0.76.

COMPARISON OF THE DIFFERENT FORMULAS

Again using the accepted Salamon-Munro formula as a basis, the formulas of Bieniawski [1968] and Madden and Hardman [1992] were also compared for relative changes in the overlap area of failed and stable pillar populations. The method used was the one described in the previous section. The relevant strength formulas were used in turn for the calculation of safety factors, and the overlap areas were calculated and compared with the original Salamon-Munro formula. The results are summarized below.

The table shows that the Bieniawski [1968] formula was only slightly less efficient than the Salamon-Munro formula; Madden and Hardman [1992] was slightly more efficient, although the decision not to implement the latter was probably correct because the improvement is small. The formula derived in this paper, referred to in the table above as the "new formula," is, however, 12.5% more efficient, which is considered significant.

Strength formula	*Improvement factor, %*
Bieniawski [1968]	*&1.5*
Madden and Hardman [1992] . .	*% 2.3*
New formula	*%12.5*

DISCUSSION AND IMPLICATIONS FOR THE INDUSTRY

The new formula yields higher values of safety factors for most pillars than either of the formulas proposed previously for South African coals. The exceptions are the small pillars, such as those typically found at shallow depth. The new formula is more successful in explaining the "anomalous" pillar collapses of small pillars at shallow depth.

Figure 5 compares pillar strengths obtained with the various formulas for different w/h ratios of the pillars. Note that due to the different exponents of width and height, the relationships are ambiguous (except for the linear formula of Bieniawski [1968] and the Mark-Bieniawski formula described by Mark and Chase [1997]). For purposes of this comparison, the pillar heights were fixed at 3 m and the widths adjusted to obtain the different ratios.

An important feature of the comparison is the close correlation between the Mark-Bieniawski formula and the new formula. They were derived independently using different databases in different countries. Both predict stronger pillars for the same dimensions as the other formulas. The new formula only deviates meaningfully from Mark-Bieniawski in the lower range of the w/h ratio, where it predicts weaker pillars. This is in accordance with observations where the failure of small pillars was previously regarded as anomalous.

The major implication for the coal mining industry is that higher coal extraction can be obtained without sacrificing stability. In effect, this is nothing more than a correction of the overdesign that has been implemented over the past decades. Figure 6 shows examples of the benefits with regard to the percentage extraction. The greater the depth and the higher the required safety factor, the greater the benefit.

As the new formula deals with underground pillar stability, it is inherently linked to the safety of underground mine personnel. In particular, it will enhance the stability of shallow workings, which has hitherto been a shortcoming of the Salamon-Munro formula. For deeper workings and for cases where surface structures are undermined, the new formula will enable mines to extract more coal without sacrificing stability.

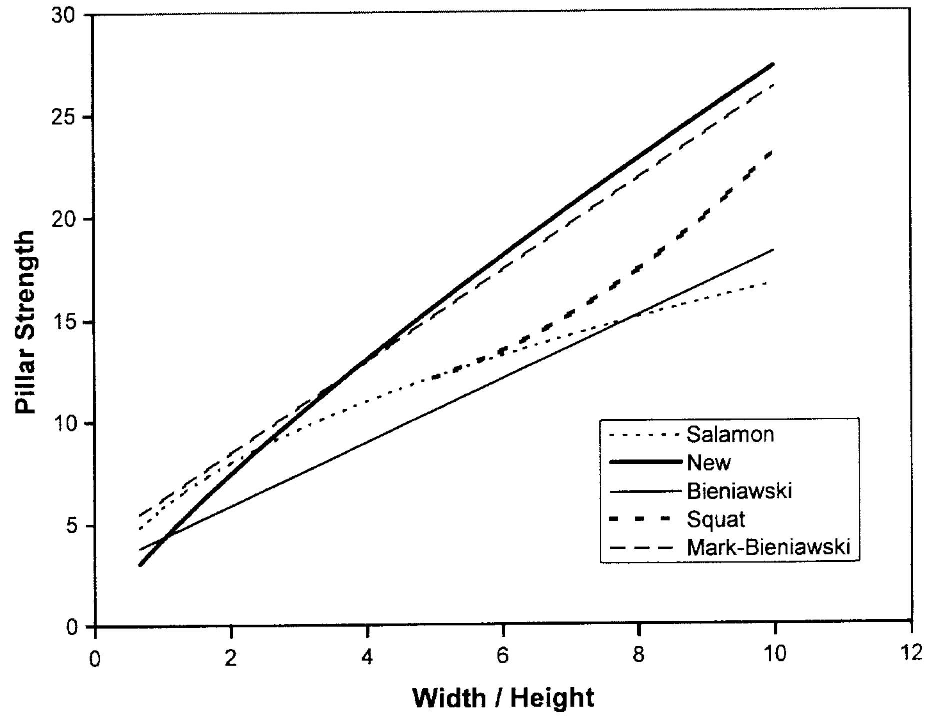

Figure 5.—Comparison of the strength increase with increasing width to height of pillars. The new formula results in higher strength values for most of the pillar sizes. This comparison is included for demonstration purposes only, because the relationship between width to height and pillar strength is ambiguous for all cases where the exponents of width and height are not equal. Note the similarity between the new formula and the Mark-Bieniawski formula.

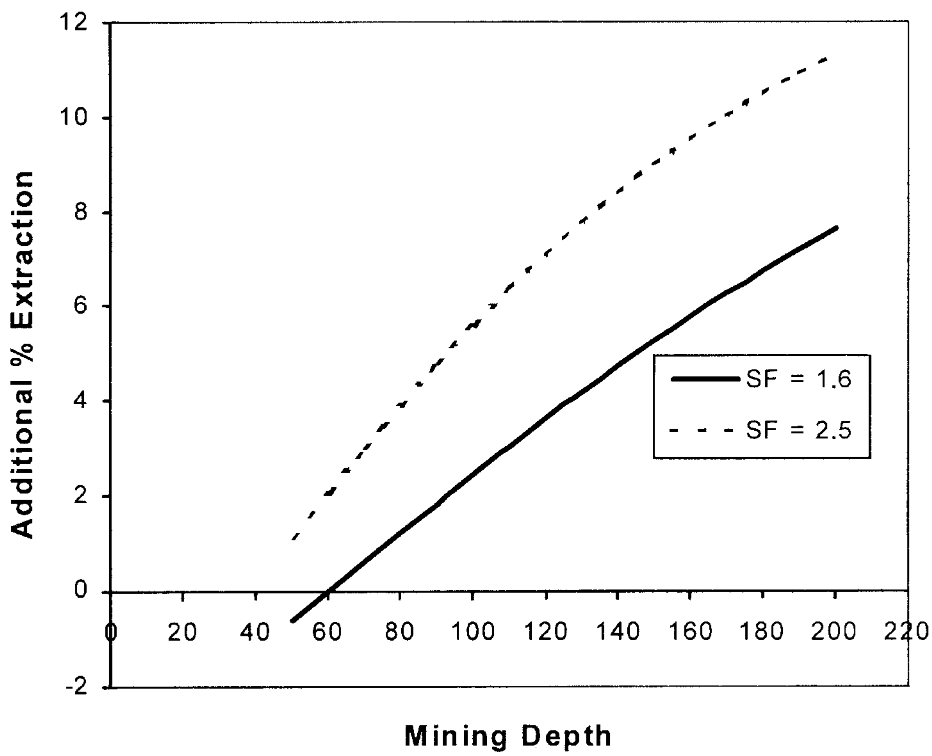

Figure 6.—Illustration of the benefit obtained by using the new formula. As the safety factors and depth of mining increase, more extraction can be obtained without sacrificing stability. For purposes of this comparison, the mining height was 3 m and the road width was 6.6 m.

ACKNOWLEDGMENTS

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