

EMPIRICAL METHODS FOR COAL PILLAR DESIGN

By Christopher Mark, Ph.D.¹

ABSTRACT

Empirical methods involve the scientific interpretation of real-world experience. Many problems in ground control lend themselves to an empirical approach because the mines provide us with plenty of experience with full-scale rock structures. During the past 10 years, powerful design techniques have emerged from statistical analyses of large databases of real-world pillar successes and failures. These include the Analysis of Retreat Mining Pillar Stability (ARMPS), the Analysis of Longwall Pillar Stability (ALPS), the Mark-Bieniawski rectangular pillar strength formula, and guidelines for preventing massive pillar collapses. In the process, our practical understanding of pillar behavior has been greatly enriched.

¹Supervisory physical scientist, Pittsburgh Research Laboratory, National Institute for Occupational Safety and Health, Pittsburgh, PA.

INTRODUCTION

"Empirical" is defined by *Webster's Dictionary* [1988] as "relying upon or gained from experiment or observation." Until relatively recently, all pillar design methods used in the United States were empirical. The earliest, proposed by Bunting [1911], was based on case histories supplemented by laboratory testing. Later formulas followed the same basic pattern and were derived from laboratory tests (the Holland-Gaddy and Obert-Duvall formulas), large-scale in situ tests (the Bieniawski formula), or case histories (the Salamon-Munro formula).

Each of these "classic" pillar design formulas consisted of three steps:

- (1) Estimating the *pillar load* using tributary area theory;
- (2) Estimating the *pillar strength* using a pillar strength formula; and
- (3) Calculating the *pillar safety factor*.

In each case, the pillar strength was estimated as a function of two variables—the pillar's width-to-height (w/h) ratio and the coal seam strength. For many years, these classic formulas performed reasonably well for room-and-pillar mining under relatively shallow cover. Their key advantages were that they were closely linked to reality and were easy to use.

The greatest disadvantages of empirical formulas are that they cannot be easily extended beyond their original database, and they provide little direct insight into coal pillar mechanics. The growth of longwall mining exposed these shortcomings. Full extraction results in large abutment loads, which cannot be estimated by tributary area. More important is that longwall mining uses pillars that are much more "squat" (large w/h ratio) than those for which the classic formulas were developed. Testing such pillars in situ is prohibitively expensive, and laboratory tests of squat pillars are clearly inappropriate. Moreover, longwall mining raised some new issues even about the definition of what constitutes pillar "failure." The classic approach assumes that "pillars will fail when the applied load reaches the compressive strength of the pillars" and that "the load-bearing capacity of the pillar reduces to zero the moment the ultimate strength is exceeded" [Bieniawski 1992]. When large w/h longwall pillars "fail," however, their load-bearing capacity does not disappear. Rather, the gate roads become unserviceable.

During the 1970s, analytical methods began to emerge as an alternative to the classic formulas. Wilson [1972, 1983] of the British National Coal Board was the first to take a radically different approach to pillar design. He treated pillar design as a problem in mechanics, rather than one of curve-fitting to experimental or case history data. A pillar was analyzed as a complex structure with a nonuniform stress gradient, a buildup of confinement around a high-stress core, and progressive pillar failure. Although his mathematics were seriously limited [Mark

1987; Salamon 1992], Wilson's basic concepts are now broadly accepted.

The advent of powerful computer models gave a further boost to the analytical approach. The primary advantage of numerical models is that they can test assumptions about pillar behavior as affected by a variety of geometric and geologic variables. For example, independent studies reported by Gale [1992] and Su and Hasenfus [1997] concluded that for pillars whose $w/h > 6$, weak host rocks or partings have greater effects on pillar strength than the uniaxial compressive strength (UCS). Unfortunately, effective numerical modeling requires numerous assumptions about material properties, failure criteria, and post-failure mechanics.

In their insightful article, Starfield and Cundall [1988] introduced a classification of modeling problems (figure 1). One axis on the graph refers to the quality and/or quantity of the available data; the other measures the understanding of the fundamental mechanics of the problem to be solved. In many branches of mechanics, most problems fall into region 3, where there is both good understanding and reliable data. This is the region where numerical models can be built, validated, and used with conviction. Starfield and Cundall argued that problems in rock mechanics usually fall into the data-limited categories 2 or 4 and require a more experimental use of models.

In the field of coal mine ground control, however, many problems may actually fall into Starfield and Cundall's region 1. Our understanding of the complex mechanical behavior and properties of rock masses may be limited, but the potential for data collection is huge. Hundreds of longwall and room-and-pillar panels are mined each year, and each one can be considered a full-scale test of a pillar design. As Parker [1974] noted: "Scattered around the world are millions and millions of

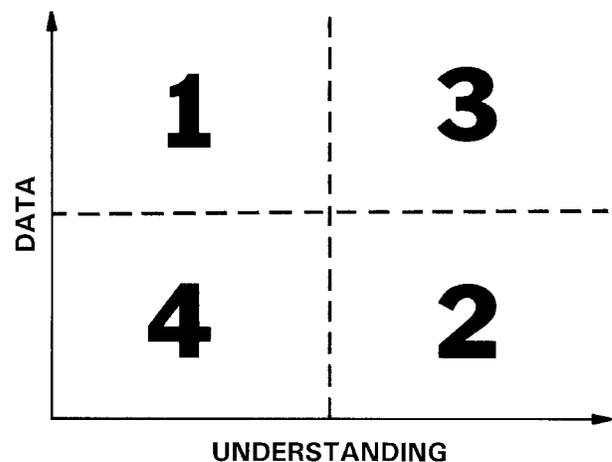


Figure 1.—Classification of modeling problems (after Starfield and Cundall [1988]).

pillars—the real thing—under all imaginable conditions; and tabulating their dimensions, the approximate loads, and whether they are stable or not would provide most useful guidelines for pillar design."

Actually, simply tabulating data does not necessarily lead to useful conclusions. Fortunately, today's data analysis techniques are far more powerful than those that were available to the pillar design pioneers. In the past 30 years, sciences like economics, sociology, psychology, anthropology, and epidemiology have all been transformed by quantitative data analysis using statistics [Encyclopedia Britannica 1989]. Sophisticated statistical packages enable researchers to efficiently comb large databases for significant relationships between the variables.

The empirical approach requires that the researcher begin with a clear hypothesis, often in the form of a simplified model

of the real world that abstracts and isolates the factors that are deemed to be important. It therefore requires, as Salamon [1989] indicated, "a reasonably clear understanding of the physical phenomenon in question." Without prudent simplification, the complexity of the problem will overwhelm the method's ability to discern relationships between the variables. However, a key advantage is that critical variables may be included, even if they are difficult to measure directly, through the use of "rating scales."

During the past 5 years, modern empirical techniques have been applied to a variety of problems in coal mine ground control. They have resulted in some very successful design techniques, as well as some new insights into pillar and rock mass behavior. This paper discusses some of them in more detail.

DESIGN OF LONGWALL GATE ENTRY SYSTEMS

In the 15 years after 1972, the number of U.S. longwall faces increased from 32 to 118 [Barczak 1992]. The new technology created a host of operational and safety problems, including the maintenance of stable travelways on the tailgate side. Researchers initially viewed gate entry ground control primarily as a pillar design issue. The clear correlation between larger pillars and improved conditions that had been established by trial and error at many mines supported this approach.

The most obvious difference between longwall pillars and traditional coal pillars is the abutment loading. The major contribution of the original Analysis of Longwall Pillar Stability (ALPS) was a formula for estimating the longwall pillar load based on numerous underground measurements [Mark 1990]. An evaluation of 100 case histories showed that 88% of the failed cases had stability factors <1.0; 76% of the successful cases had stability factors ≥ 1.0 [Mark 1992]. It was evident that ALPS had captured an essential element of the gate entry design problem.

On the other hand, there was a wide range of stability factors (approximately 0.5 to 1.2) in which both successful and unsuccessful designs occurred. Clearly, other variables in addition to the ALPS stability factor were influencing tailgate performance. A hypothesis was proposed stating that tailgate performance is determined by five factors:

- Pillar design and loading;
- Roof quality;
- Entry width;
- Primary support; and
- Supplemental support.

Attacking this extremely complex problem with traditional, deterministic rock mechanics using analytical or numerical models would have been extremely difficult. On the other hand, the problem was ideal for an empirical approach. The

empirical method could make full use of the wealth of full-scale case history data that had been collected. Moreover, it could focus directly on the variable of interest—tailgate performance.

It quickly became clear that roof quality was the key. Studies conducted as early as the 1960s had concluded that "whether or not the stress [from an extracted longwall panel] will influence a roadway depends more on the strength of the rocks which surround the roadway itself than on the width of the intervening pillar" [Carr and Wilson 1982]. Yet the variety and complexity of geologic environments had defied effective measurement.

The Coal Mine Roof Rating (CMRR) overcame this obstacle by providing a quantitative measure of the structural competence of coal mine roof [Molinda and Mark 1994; Mark and Molinda 1996]. The CMRR applies many of the principles of Bieniawski's Rock Mass Rating (RMR), with the following significant differences:

- The CMRR focuses on the characteristics of bedding planes, slickensides, and other discontinuities that determine the structural competence of sedimentary coal measure rocks.
- It is applicable to all U.S. coalfields and allows a meaningful comparison of structural competence, even where lithologies are quite different.
- It treats the bolted interval as a single structure while considering the contributions of the different lithologic units that may be present within it.

The CMRR weighs the importance of the geotechnical factors that determine roof competence and combines these values into a single rating on a scale from 0 to 100.

Data on tailgate performance were collected from approximately 55% of all U.S. longwall mines; these mines were selected to represent a geographic and geologic cross section of the U.S. longwall experience. A total of 64 case histories were

classified as "satisfactory" or "unsatisfactory" based on the conditions in the tailgate [Mark et al. 1994]. Each case history was described by the ALPS stability factor (SF), entry width, and primary support rating, as well as the CMRR.

Multivariate statistical analysis showed that when the roof is strong, smaller pillars can safely be used. For example, when the CMRR is 75, an ALPS SF of 0.7 is adequate. When the CMRR drops to 35, the ALPS SF must be increased to 1.3. Significant correlations were also found between the CMRR and both entry width and the level of primary support [Mark et al. 1994]. A simple design equation related the required ALPS SF to the CMRR:

$$\text{ALPS SF} = 1.76 - 0.014 \text{ CMRR} \quad (1)$$

THE ALPS database was recently revisited, with several new variables added. These include:

Rectangular pillar strength formula: All of the SFs were recalculated with the Mark-Bieniawski formula (see the section below on "Interactions With Numerical Models") substituted for the original Bieniawski formula. The new result is designated as the ALPS (R) SF.

Uniaxial compressive strength: Nearly 4,000 laboratory tests were compiled from the literature into the Database of Uniaxial Coal Strength (DUCS) [Mark and Barton 1996]. From these data, typical seam strength values were obtained for 60 U.S. coalbeds.

Width-to-height (w/h) ratio: The w/h of the largest pillar in the gate entry system was included as an independent variable to check if the pillar strength formula could be improved.

Depth of cover (H): H was included as an independent variable primarily to check the loading formulation.

The entry width and the primary support were included as before.

The statistical analysis showed that the ALPS (R) SF and the CMRR still correctly predicted 85% of the outcome, including

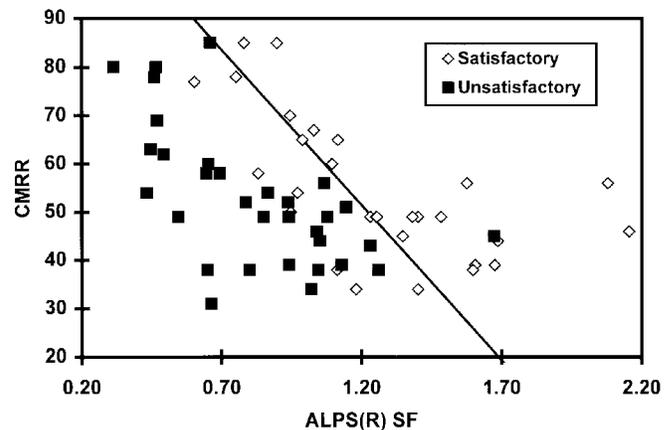


Figure 2.—U.S. longwall case histories showing the modified design equation for ALPS (R) with the Mark-Bieniawski pillar strength formula.

94% of the failures. None of the other new variables would be included even at the 50% confidence level (a 90% confidence level would be required for a covariate to be considered statistically significant). Figure 2 shows the distribution of the case histories and the revised design equation

$$\text{ALPS (R) SF} = 2.0 - 0.016 \text{ CMRR} \quad (2)$$

Since 1987, ALPS has become the most widely used pillar design method in the United States. The ALPS-CMRR method directly addresses gate entry performance and makes U.S. longwall experience available to mine planners in a practical form. ALPS reduces a multitude of variables (e.g., depth of cover, pillar widths, seam height, entry width, roof quality) into a single, meaningful design parameter—the stability factor. ALPS has been accepted because it is easy to use, its essential concepts are easy to grasp, and it has been thoroughly verified with case histories. Most importantly, ALPS gives reasonable answers that make sense in terms of experience. Tailgate blockages are far less common today than 10 years ago; ALPS can surely claim some of the credit.

PILLAR DESIGN FOR ROOM-AND-PILLAR MINING

Room-and-pillar mining still accounts for nearly 50% of the underground coal mined in the United States (even after excluding longwall development). Most room-and-pillar mines operate under relatively shallow depth, often working small, irregular deposits. Approximately 20% of room-and-pillar coal is won during pillar recovery operations [Mark et al. 1997b].

Room-and-pillar mines still suffer from large-scale pillar failures, including sudden collapses and the more common "squeezes." The classical empirical pillar strength formulas were developed precisely to prevent these types of failures, but they have never been entirely satisfactory. First, they did not consider the abutment loads that occur during pillar recovery

operations. Second, laboratory testing to determine coal strength has remained controversial despite the fact that textbooks have considered it an integral part of pillar design for 30 years. Third, because the empirical formulas were developed from tests on relatively slender specimens, their applicability to squat pillars has been open to question. Finally, attempts to verify the formulas' accuracy with U.S. case histories have been incomplete and conspicuously lacking in examples of pillar failure [Holland 1962; Bieniawski 1984].

An intensive research effort to develop an improved design method culminated in the Analysis of Retreat Mining Pillar

Stability (ARMPS). ARMPS employs many of the same basic constructs as ALPS, adapted to more complex and varied retreat mining geometries [Mark and Chase 1997]. The abutment load formulas were adapted to three dimensions to account for the presence of barrier pillars and previously extracted panels. Because the pillars used in retreat mining are often rectangular, the Mark-Bieniawski pillar strength formula was developed to estimate pillar strength. Features such as varied entry spacings, angled crosscuts, and slab cuts in the barrier can all be modeled.

To verify ARMPS, more than 200 retreat mining case histories were obtained from field visits throughout the United States. The case histories come from 10 States and cover an extensive range of geologic conditions, roof rock caveability, extraction methods, depths of cover, and pillar geometries. Ground conditions were characterized in each case as satisfactory or unsatisfactory. Where possible, data were also collected to assess the CMRR. Site-specific data on coal strength were not generally available for individual case histories, but DUCS again provided estimates of UCS for most coalbeds. Finally, the depth of cover and the w/h were also included as independent variables in the analysis. Details on the individual case histories have been presented elsewhere [Mark and Chase 1997].

When the entire data set was evaluated, it was found that 77% of the outcomes could be correctly predicted simply by setting the ARMPS SF to 1.46. Including either the depth or the w/h increased the correlation coefficient, r^2 , slightly without improving the accuracy (figure 3). The depth and the w/h ratio were strongly correlated with each other within the data set.

The accuracy improved when the data set was divided into two parts. One group included only cases where cover was shallow ($H < 200$ m (650 ft)) and where the pillars were not squat ($w/h < 8$). For this group, when the ARMPS SF = 1.5, 83% of the outcomes were correctly predicted. However, for the deep cover/squat pillar group, only 58% of the cases were correctly predicted at ARMPS SF = 0.93. No other variables could be included in either group at the 90% confidence level. It seems clear that ARMPS works quite well at shallow depth and moderate w/h ratios, but that other factors must be considered when squat pillars are used at greater depths.

The analysis also found that using laboratory UCS tests did not improve the accuracy of ARMPS at all. This finding confirms the results of a previously published study [Mark and

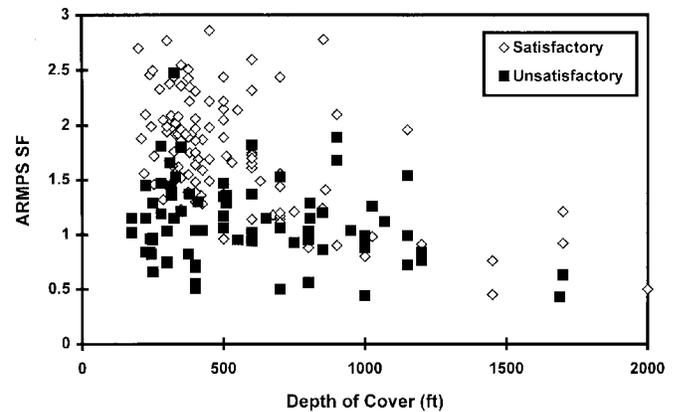


Figure 3.—U.S. room-and-pillar case histories.

Barton 1996], which showed that ARMPS was *more* reliable when the in situ coal strength was always assumed to be 6.2 MPa (900 psi). It also showed that the "size effect" varies dramatically from seam to seam depending on the coal cleat structure.

Studies in the Republic of South Africa and Australia have also found that a uniform coal strength worked reasonably well in pillar design formulas [Salamon 1991; Galvin and Hebblewhite 1995]. It has already been noted that ARMPS is significantly less reliable for squat pillars. It seems likely that while the strength of the intact coal (which is what is measured in a laboratory test) is not related to pillar strength, large-scale geologic features like bedding planes, clay bands, rock partings, and roof and floor rock may determine the strength of squat pillars. Such features influence the amount of confinement that can be generated within the pillar and therefore the load-bearing capacity of the pillar core. Similar conclusions have been reached by researchers using numerical models [Su and Hasenfus 1997; Gale 1992].

Although the CMRR was not found to be significant in the overall data set, one local study indicated that caveability may affect pillar design. More than 50 case histories were collected at a mining complex in southern West Virginia. Analysis showed that satisfactory conditions were more likely to be encountered under shale roof (figure 4) than under massive sandstone roof (figure 5). The implication is that better caving occurs with shale, resulting in lower pillar loads.

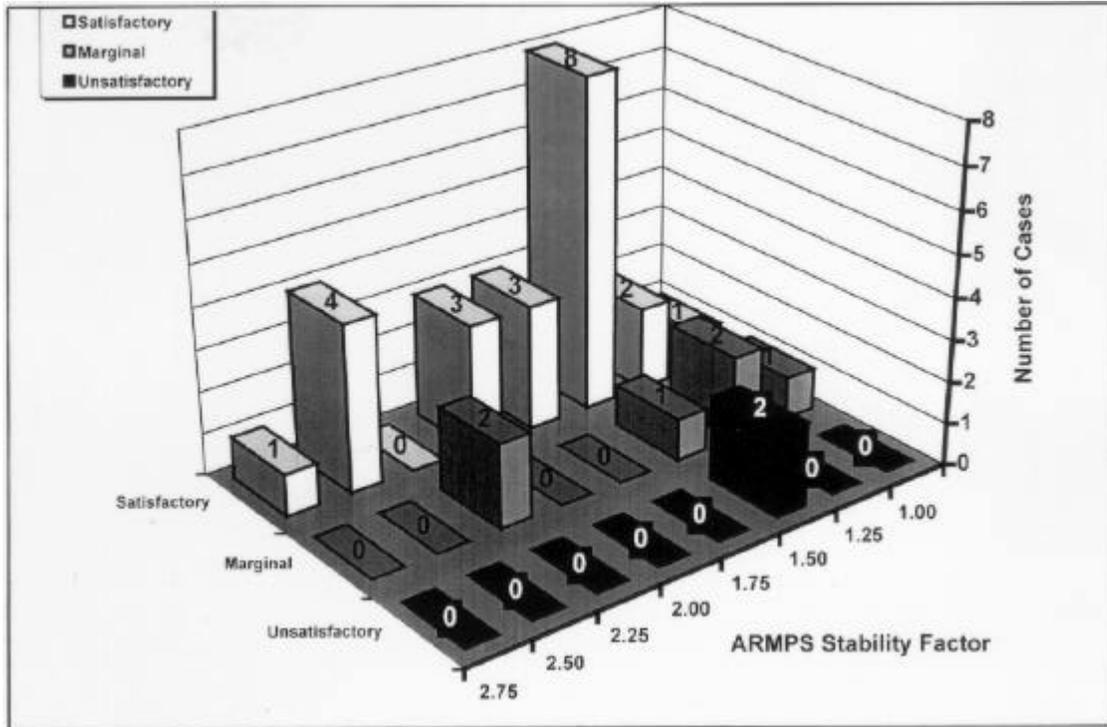


Figure 4.—Pillar performance under different roof geologies at a mining complex in West Virginia—shale roof.

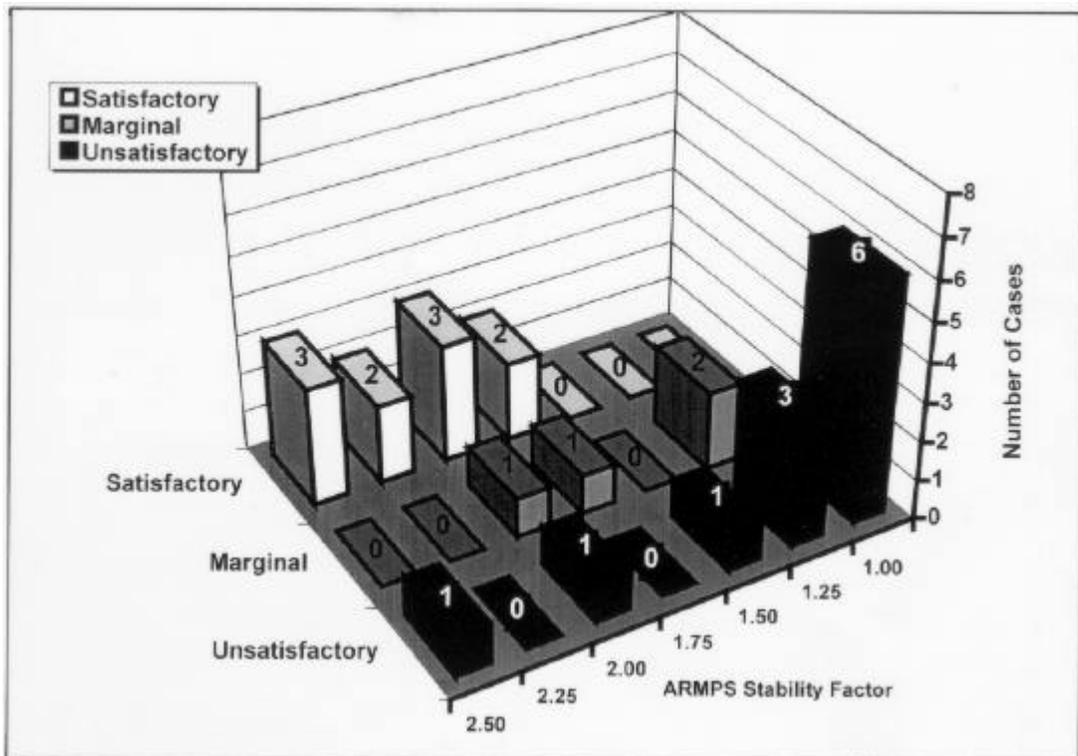


Figure 5.—Pillar performance under different roof geologies at a mining complex in West Virginia—sandstone roof.

MASSIVE PILLAR COLLAPSES

Most of the pillar failures included in the ARMPS database are "squeezes" in which the section converged over hours, days, or even weeks. There are also 15 massive pillar collapses that form an important subset [Mark et al. 1997a]. Massive pillar collapses occur when undersized pillars fail and rapidly shed their load to adjacent pillars, which in turn fail. The consequences of such chain-reaction failures typically include a powerful, destructive, and hazardous airblast.

Data collected at 12 massive collapse sites revealed that the ARMPS SF was <1.5 in every case and <1.2 in 81% of the cases (figure 6). What really distinguished the sudden collapses from the slow squeezes, however, was the pillar's w/h ratio. Every massive pillar collapse involved *slender* pillars whose w/h was <3. The overburden also included strong, bridging strata in every case.

In this instance, the empirical analysis led to a hypothesis about the mechanism of the failure. Laboratory tests have shown that slender coal specimens typically have little residual strength, which means that they shed almost their entire load when they fail. As the specimens become more squat, their residual strength increases, reducing the potential for a rapid domino-type failure. The mechanism of massive collapses was replicated in a numerical model [Zipf and Mark 1997], providing further support for the hypothesis.

Three alternative strategies were proposed to prevent massive pillar collapses:

- *Prevention:* With the prevention approach, the panel pillars are designed so that collapse is highly unlikely. This can be accomplished by increasing either the SF of the pillars or their w/h ratio.
- *Containment:* In this approach, high extraction is practiced within individual compartments that are separated by

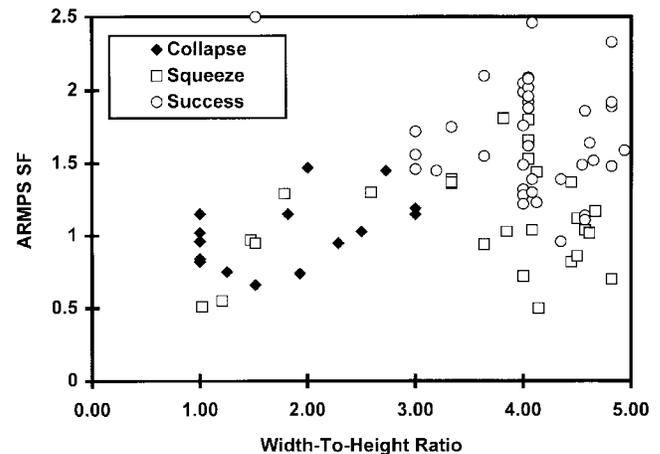


Figure 6.—A portion of the room-and-pillar case history database showing examples of pillar collapse.

barriers. The small pillars may collapse within a compartment, but because the compartment size is limited, the consequences are not great. The barriers may be true barrier pillars, or they may be rows of development pillars that are not split on retreat. The containment approach has been likened to the use of compartments on a submarine.

- *High extraction:* By removing enough coal during retreat mining, failure of the overburden may be induced, which would remove the airblast hazard.

The empirical analysis, using case histories, has allowed the first two of these approaches to be quantified in terms of the w/h ratio and the ARMPS SF. The guidelines are now being implemented in southern West Virginia, where the majority of these events have occurred.

INTERACTIONS WITH NUMERICAL MODELS

A number of important links have developed between empirical methods and numerical models. Because they were obtained from real-world data, empirical models are a good starting point for material property input to models. For example, Mark [1990] analyzed numerous field measurements of abutment stress and determined that the stress decay over the ribside could be approximated as an inverse square function. Karabin and Evanto [1999] adjusted the gob parameters in the BESOL boundary-element model to obtain a reasonable fit to the inverse square function. Similarly, Heasley and Salamon [1996a,b] used the same stress decay function to calibrate the LAMODEL program.

Empirical formulas have also helped provide coal properties for some models. Although empirical formulas do not explicitly consider the effect of internal pillar mechanics, it is apparent that they imply a nonuniform stress distribution because of the w/h effect. A derivation of the implied stress gradients was published by Mark and Iannacchione [1992]. For example, the Bieniawski formula

$$S_p = S_1 (0.64 - 0.36 w/h) \quad (3)$$

implies a stress gradient within the pillar at ultimate load of

$$S_v = S_1 (0.64 + 2.16 x/h), \quad (4)$$

where S_p = pillar strength,

S_1 = in situ coal strength,

S_v = vertical pillar stress,

and x = distance from pillar rib.

The stress gradient defines the vertical stress within the pillar at maximum load as a function of the distance from the nearest rib.

These empirical stress gradients have been widely used to estimate coal properties for use in boundary-element models that use strain-softening pillar elements. In the models, the peak stress increases the further the element is from the rib. The empirical stress gradients help ensure that the initial strength estimates are reasonable.

The same empirical stress gradient was used to extend a classic pillar strength formula to rectangular pillars. The original Bieniawski formula was derived for square pillars and underestimates the strength of rectangular pillars that contain proportionately more core area. By integrating equation 4 over

the load-bearing area of a rectangular pillar, the Mark-Bieniawski pillar strength formula is obtained:

$$S_p = S_1 (0.64 + 0.54 w/h + 0.18 (w^2/Lh)), \quad (5)$$

where L = pillar length.

The approach is illustrated in figure 7 and described in more detail by Mark and Chase [1997].

Other sections of this paper have indicated areas where numerical models and empirical methods have reached similar conclusions about important aspects of pillar mechanics. In light of these insights, old concepts of pillar "failure" have given way to a new paradigm that identifies three broad categories of pillar behavior:

- *Slender pillars* ($w/h < 3$), which have little residual strength and are prone to massive collapse when used over a large area;
- *Intermediate pillars* ($4 < w/h < 8$), where "squeezes" are the dominant failure mode in room-and-pillar mining and where empirical pillar strength formulas seem to be reasonably accurate; and
- *Squat pillars* ($w/h > 10$), which can carry very large loads and are strain-hardening, and which are dominated by entry failure (roof, rib, and floor) and by coal bumps.

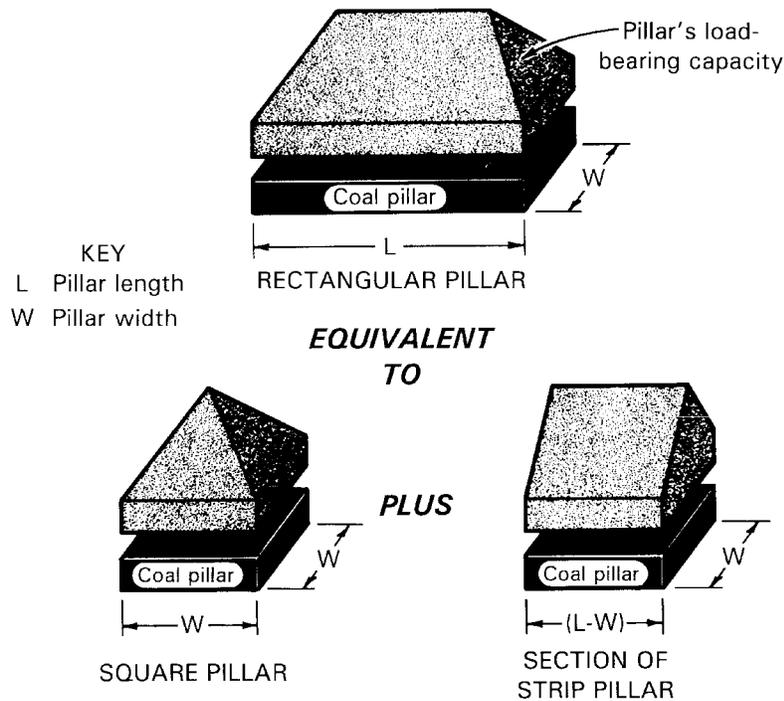


Figure 7.—Conceptual depiction of the Mark-Bieniawski pillar strength formula.

CONCLUSIONS

Empirical methods rely on the scientific interpretation of actual mining experience. Because they are so firmly linked to reality, they are particularly well suited to practical problems like pillar design. Empirical methods like ALPS and ARMPS have met the mining community's need for reliable design techniques that can be used and understood by the nonspecialist.

Successful empirical research has three central elements:

- A hypothesis or model that simplifies the real world, yet incorporates its most significant features;
- A large database of case histories, developed using consistent and thorough in-mine data collection techniques; and
- Quantitative analysis using appropriate statistical techniques.

Empirical techniques are not, of course, the only tool in the ground control specialist's kit. Indeed, one of the most satisfying developments in recent years is the synergy that has developed between empirical techniques and numerical modeling. The two approaches seem to have converged on a number of important conclusions, including:

- Laboratory testing of small coal samples, particularly UCS tests, are not useful for predicting pillar strength;
- The strength becomes more difficult to predict as the pillar becomes more squat;
- The w/h ratio is important for predicting not only the pillar strength, but also the mode of failure; and
- Many ground control problems must be considered from the standpoint of entry stability, where pillar behavior is just one component.

Certainly, more work remains before the age-old questions of pillar design are finally solved. In particular, much remains to be learned about the mechanics of squat pillars and roof-pillar-floor interactions. Currently, there is no accepted way to determine the frictional characteristics of the contacts, bedding planes, and partings that are so crucial to pillar strength. It is similarly difficult to characterize the bearing capacity of the floor. Simple, meaningful field techniques for estimating these properties will be necessary for further progress with either numerical or empirical techniques. Indeed, the cross-pollination between the numerical and empirical methods that has characterized the recent past can be expected to bear further fruit in the future.

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