

# EXPERIENCE WITH THE BOUNDARY-ELEMENT METHOD OF NUMERICAL MODELING TO RESOLVE COMPLEX GROUND CONTROL PROBLEMS

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## ABSTRACT

The Mine Safety and Health Administration, Pittsburgh Safety and Health Technology Center, Roof Control Division, is routinely involved in the evaluation of ground conditions in underground coal mines. Assessing the stability of mined areas and the compatibility of mining plans with existing conditions is essential to ensuring a safe working environment for mine workers at a given site. Since 1985, the Roof Control Division has successfully used the boundary-element method of numerical modeling to aid in the resolution of complex ground control problems. This paper presents an overview of the modeling methodology and details of techniques currently used to generate coal seam, rock mass, and gob backfill input data. A summary of coal and rock properties used in numerous successful evaluations throughout the United States is included, and a set of deterioration indices that can aid in the quantification of in-mine ground conditions and verification of model accuracy is introduced. Finally, a case study is detailed that typifies the complexity of mining situations analyzed and illustrates various techniques that can be used to evaluate prospective design alternatives.

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## INTRODUCTION

Effective mine design has long been recognized as an essential element in establishing safe and productive mining operations. Numerous investigators have developed techniques to analyze pillar stability and maximize mining efficiency. The work of Holland and Gaddy [1964], Obert and Duvall [1967], and Bieniawski [1984], to name a few, served as a staple for mining engineers for many years. With the advent of longwall mining, new techniques were developed by Carr and Wilson [1982], Hsuing and Peng [1985], Choi and McCain [1980], and Mark [1990] to address design considerations for that technology. Most recently, the development of the Analysis of Retreat Mining Pillar Stability (ARMPS) methodology [Mark and Chase 1997] for the evaluation of retreat mining operations added an additional tool for engineers to design and evaluate full pillaring techniques.

Each of these methods can provide a reasonable estimate of pillar strength and stability under specific conditions and relatively simple mining geometries. In practice, however, situations often arise where areas of concern contain a number of pillar configurations with varying entry and crosscut widths, spacings, and orientations. Additional factors, such as non-uniform pillar lines, remanent stumps scattered throughout irregularly shaped gobs, and multiple-seam mining, can further complicate an analysis. In such instances, application of the previously mentioned empirical and analytical methods to accurately evaluate ground stability is difficult, if not totally impossible.

A primary function of the Roof Control Division, Pittsburgh Safety and Health Technology Center, is to provide technical assistance to the Mine Safety and Health Administration (MSHA) and the mining industry in the resolution of complex

roof control problems. In order to evaluate mining systems not easily treated by simplified empirical or analytical methods, boundary-element numerical modeling was initiated in 1984 and expanded in 1987 with acquisition of the BESOL system from Crouch Research, Inc., St. Paul, MN. The ability of the three-dimensional (3-D) boundary-element method to model large mine areas with complex geometries has enabled the Roof Control Division to successfully simulate conditions and identify potential solutions to ground control problems in mines throughout the United States. The technique has been applied to a variety of mining scenarios, including longwall and room-and-pillar operations using both conventional and yield pillar configurations. The influence of vertical and horizontal stress has been modeled to simulate underground conditions ranging from deteriorating roof and persistent falls to areas of squeezing ground and complete pillar failure.

In the process of developing numerical models for the various mining operations analyzed during the last 10 years, a systematic simulation methodology has evolved. Techniques to estimate the necessary coal, rock, and gob backfill properties have been established, and a deterioration index was developed to quantify in-mine roof, floor, and pillar behavior to assist in calibrating model parameters and evaluating potential mine design alternatives. This paper presents a brief description of the BESOL system, an overview of the simulation process used, and details of methods used to construct models and estimate rock mechanics parameters. A discussion of the deterioration index system and details of a case study typifying an actual mine simulation and techniques used to evaluate conditions and proposed mining options is also included.

## BESOL SYSTEM DESCRIPTION

BESOL is a system of computer programs for solving rock mechanics problems based on the boundary-element displacement discontinuity method of analysis. The 3-D MS221 version (yielding and multiple-seam capability) was acquired from Crouch Research, Inc., and has been used by the Roof Control Division to evaluate complex mining systems since 1987. The BESOL system is complete with graphic pre- and postprocessors that greatly simplify model construction and output data interpretation.

Figure 1 presents a generalized BESOL boundary-element model that illustrates a tabular seam or ore body surrounded by

a homogenous, isotropic, linearly elastic rock mass. Input data include elastic rock mass properties and rock strength criteria, seam properties, and backfill or artificial support characteristics. A definition of the seam plane(s), detailed geometry of the excavation, mining depth, seam height, and a complete 3-D in situ stress state of the model are also required. Output capabilities include stress, strain, and displacement calculations within user-selected areas (both on and off the seam plane), failure index (Mohr-Coulomb or Hoek-Brown roof and floor safety factors) calculations at variable locations in the rock mass, and energy release estimates in yielding areas.

BESOL was selected by the Roof Control Division because it offered a number of features considered essential in simulating complex mining situations. These include:

- 3-D capability
- Large fine-mesh grid (180 by 270 elements)
- Yielding seam option (user-defined)
- Multiple-seam capability
- Backfill and artificial support materials

Other features that made the package attractive were:

- PC-based operation
- Off-seam stress/strain capability
- Failure index calculation (Mohr-Coulomb/Hoek-Brown)
- Graphic pre- and postprocessors
- Multiform hard-copy output capability

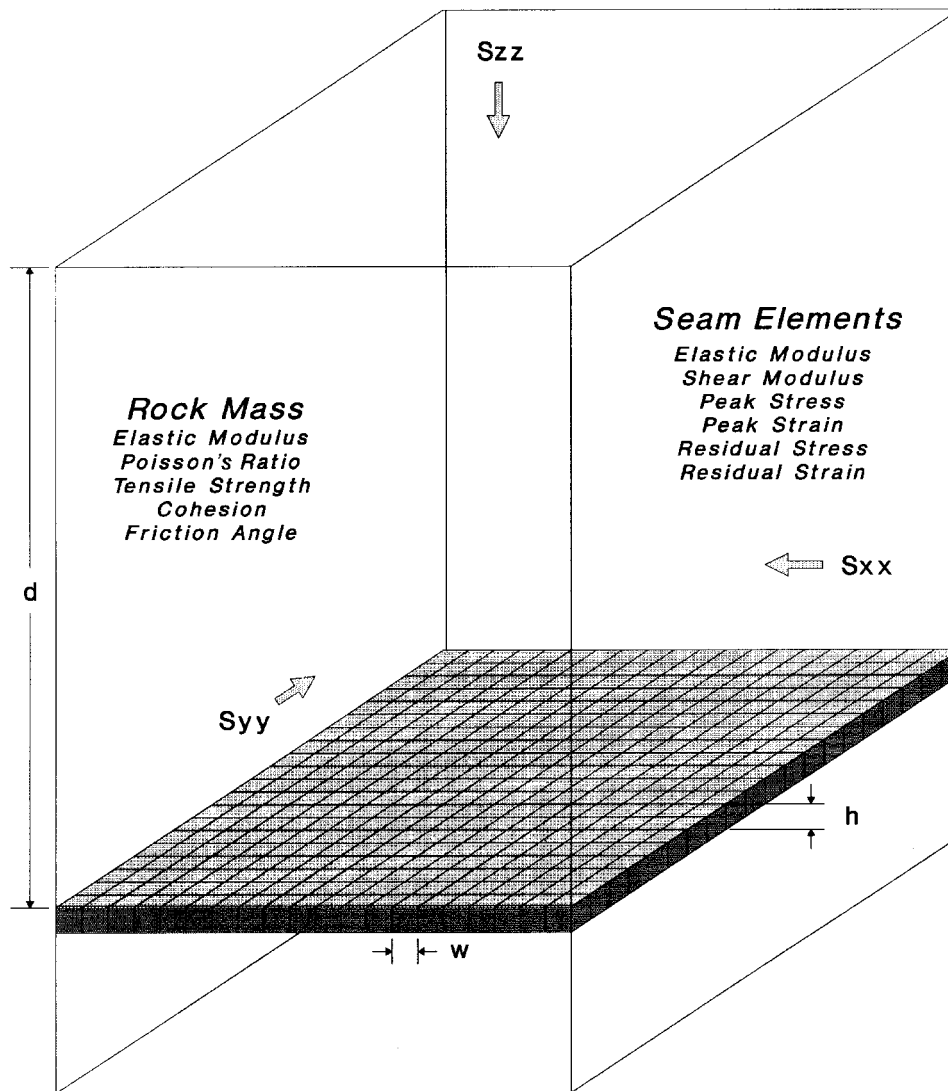


Figure 1.—Generalized BESOL boundary-element model.

## SIMULATION PROCESS

Figure 2 presents an eight-step process used by the Roof Control Division during the simulation of underground mining systems. Although it is specifically directed to numerical modeling applications, it can also be used in conjunction with empirical or analytical methods.

1. *Observe Underground Areas:* This is an essential first step in solving ground control problems regardless of the methodology employed. Mine conditions should be categorized

in a number of areas where differing pillar sizes, panel configurations, and overburden levels are found. The deterioration index system, which will be discussed later in this paper, can aid in the description of in-mine ground conditions.

2. *Estimate Model Parameters:* Coal, rock, and gob properties must be established consistent with the requirements of a particular numerical method. Ideally, these properties will be based on coal and rock tests of the specific mine site. In the absence of these data, published properties of adjacent or same seam mines can be used. When no site-related data are available, general coal and mine roof rock properties can be used. Regardless of the source of data, it cannot be over-emphasized that they represent only a *first estimate* of mine roof and rock properties that must be validated.

3. *Model Observed Areas:* The third step of the process involves modeling each of the areas observed underground. The properties estimated above are tested under various geometric and overburden conditions to determine their usability. Successfully modeling many areas under a variety of different conditions increases confidence in the properties used.

4. *Verify Model Accuracy:* This is the most critical step in the entire simulation process. Each of the areas modeled must be closely examined to ensure that the results correlate with observed conditions. If reasonable correlations cannot be made, the model must be recalibrated (material properties adjusted) and the process repeated. It should be noted that relating the output of numerical models (stress, convergence, etc.) to observed conditions (pillar sloughing and roof or floor deterioration) is often difficult given the complexities of the underground environment. The use of regression techniques to define actual conditions as a function of model output parameters (using the deterioration index rating system) can simplify that task.

5. *Establish Threshold Limits:* Once the accuracy of the model is verified, threshold limits delineating acceptable and unacceptable mining conditions must be established in order to evaluate the effectiveness of proposed design alternatives. Stress or convergence levels corresponding to deteriorating ground conditions can be identified. Other factors such as the extent of pillar yielding or predicted pillar, roof, and floor conditions from a more comprehensive regression analysis can also be used.

6. *Model New Configurations:* Having established an effective model and a means of evaluating the results of analyses, new mining techniques can be simulated. Generally, several alternatives are modeled under the conditions expected at the mine location where the design will be implemented.

7. *Evaluate New Configurations:* The various alternatives can be evaluated relative to the threshold limits established. For instance, if specific stress and convergence values were found to correspond to deteriorating ground conditions, an alternative

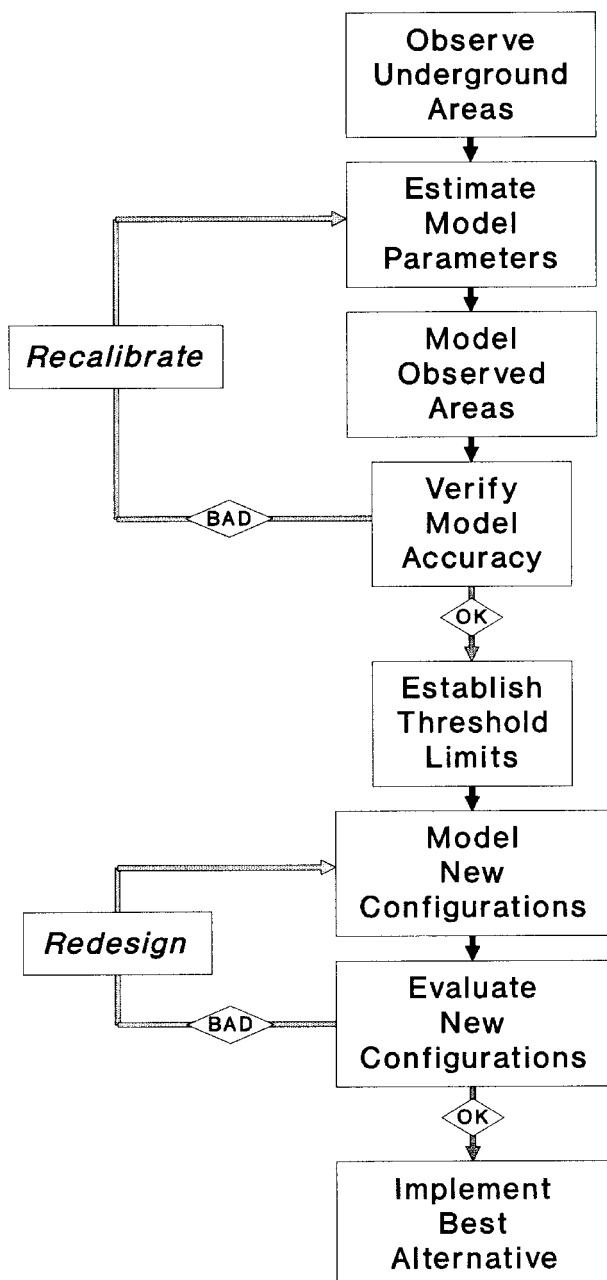


Figure 2.—Simulation process.

that produces levels lower than those values would be desired. However, if none of the configurations evaluated meet the threshold requirement for stable conditions, new alternatives must be developed and analyzed.

8. *Implement Best Alternative:* Once the best alternative is identified (either meeting the threshold criteria or providing the most favorable conditions), it can be *cautiously* implemented.

The level of confidence in achieving a successful design is directly proportional to the breadth of the evaluation and the degree of correlation noted in the model verification process. In any event, conditions should be closely monitored as the design is implemented; any deviations from the expected behavior warrants recalibration of the model.

### MINING GEOMETRY AND INITIAL STRESS

An essential element in the simulation process is creating a model grid that duplicates the in-mine geometry. The seam must be broken into elements of a size that allows the entry, crosscut, and pillar dimensions to be accurately reproduced. Seam elements must be small enough to model details of the mine geometry and produce discernable differences in performance, yet large enough to allow broad areas of the mine to be included in the simulation.

Generally, setting the element size at 1/2 the entry width (figure 3) has provided acceptable results in most coal mining

applications. A 10-ft element width (for a 20-ft-wide entry/crosscut configuration) enables a large area (1,800 by 2,700 ft) to be modeled, yet provides the stress and convergence detail needed to effectively evaluate conditions. Both larger (one-entry width) and smaller (1/4-entry width) element sizes have been used out of necessity in specific applications, but are limited in application to scenarios where detail (large elements) or influence area (small elements) are not critical.

A number of other geometric guidelines have been identified that can aid in creating an effective boundary-element model:

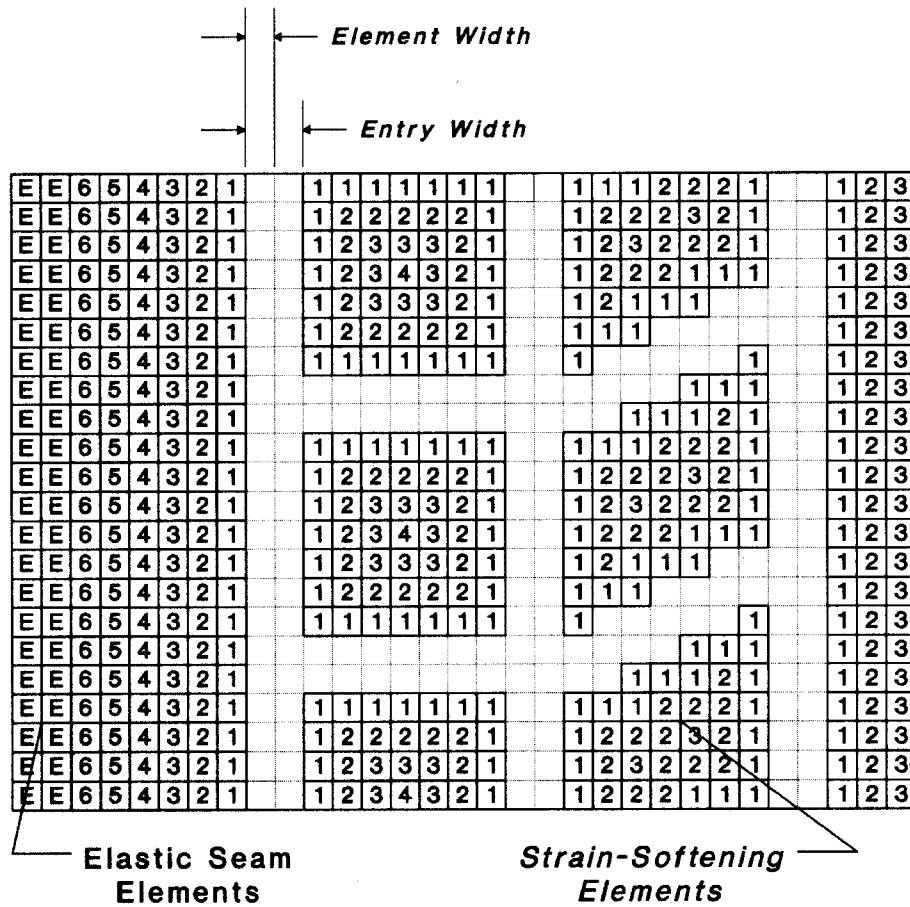


Figure 3.—Model elements and strain-softening locations.

- To the extent possible, locate model boundaries over solid coal or known stable areas to reduce the likelihood of erroneous loading conditions (resulting from the exclusion of transferred stress from adjacent yielded areas in the zone of interest).
- Orient the model such that the primary areas of interest are positioned away from the boundaries to minimize end effects.
- Known or potential yielding pillars should not contain linear-elastic elements that could erroneously affect the stress transfer to adjacent areas.
- Known or potential yielding pillars should contain an odd number of elements across the minimum dimension to ensure accurate pillar strength and peak core stress calculations.
- Care should be taken when entries or crosscuts are not oriented at 90° angles (see figure 3) to ensure that the effective widths and percent extraction match the actual mine geometry.

Initial stress conditions on the rock mass, in the absence of known high horizontal stress fields, have generally been as follows:

$S_{zz}$  (vertical) ' 1.1 psi per foot of depth

$S_{xx}$  (x-horizontal) ' 50% of the vertical stress

$S_{yy}$  (y-horizontal) ' 50% of the vertical stress

These values have resulted in effective simulations of in-mine conditions in the vast majority of cases modeled, even when the influence of horizontal stress was suspected. High horizontal stress was rarely found to actually control mine conditions, and high horizontal stress values are only used when clear evidence of their existence and magnitude is available.

### ROCK PROPERTIES

The rock mass properties needed for boundary-element models are minimal because the assumption of a linearly elastic material is inherent. The BESOL system requires only estimates of the modulus of elasticity and Poisson's ratio of the rock mass. Initially, it may seem that treating a complex rock structure in such a simplistic manner is not appropriate. However, considering that seam stresses are generated through massive main roof loading (generally remaining in elastic compression), it is not unreasonable to expect that an effective representation of pillar loading (the crux of a boundary-element model) would result.

The Roof Control Division uses a weighted-average technique to calculate the rock mass modulus of elasticity. As many borehole logs as possible located over areas to be modeled are examined, and the percentages of the various rock types (e.g., shale, sandstone, coal) in each core are identified (table 1). These values are averaged, multiplied by the modulus of elasticity of each rock type to calculate composite portions,

then summed to estimate the rock mass modulus of elasticity. Ideally, individual strata moduli are established by site-specific tests. If those data are not available, then published data for local mine roof strata or typical rock properties must be used. It should be noted that published data for particular rock types vary widely, and some judgment is needed in selecting appropriate values. The specific rock moduli listed in table 1 have been used successfully in a number of instances when on-site data were not available.

A similar weighted-average process is recommended for the calculation of Poisson's ratio. Again, the use of site-specific data would be ideal, but estimates based on published data are generally used. Poisson's ratios ranging from 0.20 to 0.25 have been acceptable in the analyses made to date.

The properties used to define the rock mass can have a significant effect on the accuracy of a simulation. Overestimating the rock modulus results in lower pillar stresses within a panel or mined area (gob) and higher loads over the

**Table 1.—Composite rock modulus calculation**

Rock type	Percent in borehole					Rock modulus, psi	Composite portion, psi
	Hole No. 1	Hole No. 2	Hole No. 3	Hole No. 4	Average		
Dirt . . . . .	10.84	8.07	11.51	15.64	11.52	50,000	5,750
Coal . . . . .	1.52	1.60	1.34	0.96	1.36	473,000	6,409
Shale . . . . .	51.15	26.86	21.79	48.22	37.01	900,000	333,090
Slate . . . . .	1.18	0.78	2.54	0	1.13	1,250,000	14,125
Sandstone . . . . .	22.28	28.63	23.70	26.31	25.23	2,200,000	555,060
Limestone . . . . .	0	0	0	0	0	3,200,000	0
Sandy shale . . . . .	11.47	31.70	36.01	7.78	21.74	1,500,000	326,100
Fireclay . . . . .	1.57	2.35	3.11	1.08	2.03	900,000	18,270
<b>TOTAL . . . . .</b>							<b>1,258,804</b>

$E$  ' 1,260,000 psi

adjacent abutments due to the enhanced bridging action (less

deformation) of the rock strata. Conversely, underestimating

the rock modulus leads to higher panel pillar stress or gob loading in mined areas and lower stresses on the adjacent abutments.

As noted previously, the BESOL system contains a failure index (safety factor) calculation to evaluate the rock strength/stress ratios using either a Mohr-Coulomb or Hoek-Brown failure criterion. Essentially, the state of stress of a point in the rock mass is calculated in terms of 3-D principal stresses, and the "available strength" of the rock (as influenced by confinement) is compared to the existing stress level. To date, only the Mohr-Coulomb technique has been used, which requires input of cohesion, friction angle, and tensile strength of the rock (roof or floor) material. Because the analysis of the rock structure is completely elastic, exact properties (although desirable) are not required. The failure index analysis is treated

in a relative manner (higher failure indices indicate a more stable condition), and the following parameters have provided reasonable results:

Tensile strength - 1,000 psi  
Cohesion - 800 psi  
Friction angle - 25°

The failure index has been successfully used to indicate high stress locations and the effect of mining changes to relieve those stresses. Although they can be calculated anywhere in the rock mass, failure index calculations made at the immediate roof or floor lines have been most useful. Coupling them with stress and convergence data provides a more complete picture of mine stability that can be correlated to observed or expected conditions.

## COAL PROPERTIES

Establishing representative coal properties for a boundary-element analysis is the most critical step in model formulation. The need for yielding seam capability is clear to accurately simulate the complex underground environment where localized coal failure results in the redistribution and concentration of stress in adjacent areas. The strain-softening approach [Crouch and Fairhurst 1973] has been identified as a reasonable method of describing coal seam behavior. Although that concept has been widely discussed, little specific information is available concerning the actual construction of a strain-softening model.

The Roof Control Division has established a technique to make a *first approximation* of the stress and strain values needed to describe the strain-softening characteristics of a specific coal seam. As generalized in figure 4, peak and residual (postpeak) stress and strain levels are required for seam elements located at various distances from a mined area. BESOL allows up to six user-defined elements (each characterized by three stress-strain values), and model elements located farther away from a free face are treated as linearly elastic (figure 3).

Peak coal strength values are estimated at the center of each of the six yielding seam elements by the following equation:

$$S_p(i) = S_1 (0.78 + 1.74 x/h), \quad (1)$$

where  $S_p(i)$  = peak strength of element (i), psi,

$S_1$  = in situ coal strength, psi,

$x$  = distance from element (i) center to free face, ft,

and  $h$  = seam height, ft.

Equation 1 was based on the derivations of Mark and

Iannacchione [1992] for estimating the stress gradient in the yield zone of several empirical pillar design formulas and represents an average of the Bieniawski and Obert-Duvall methods. The in situ coal strength is usually based on uniaxial compression tests of samples acquired from the mine, although published data have also been used when site-specific data were not available. Strength reduction factors of 1/5 for 2-in cubes and 1/4 for 3-in cubes have been used to estimate in situ strength from test data and have generally provided acceptable results. Figure 5 presents a summary of peak strengths measured (with borehole pressure cells) at various depths into coal pillars at three mines where pillar yielding was evident. Data are shown as a ratio of the measured peak stress to that estimated by equation 1; the majority fall within 10% of the predicted stress level. Because the seam is considered to behave elastically until peak stress is reached, the total strain at that level is simply

$$e_p(i) = S_p(i)/E, \quad (2)$$

where  $e_p(i)$  = strain at peak strength of element (i), in/in,

$S_p(i)$  = peak strength of element (i), psi,

and  $E$  = coal seam modulus of elasticity, psi.

Residual (postpeak) seam stress and strain values are approximated by the following relationship:

$$S_{R1}(i) = (0.1385 + \ln(x) + 0.413) (S_p(i)) \quad (3)$$

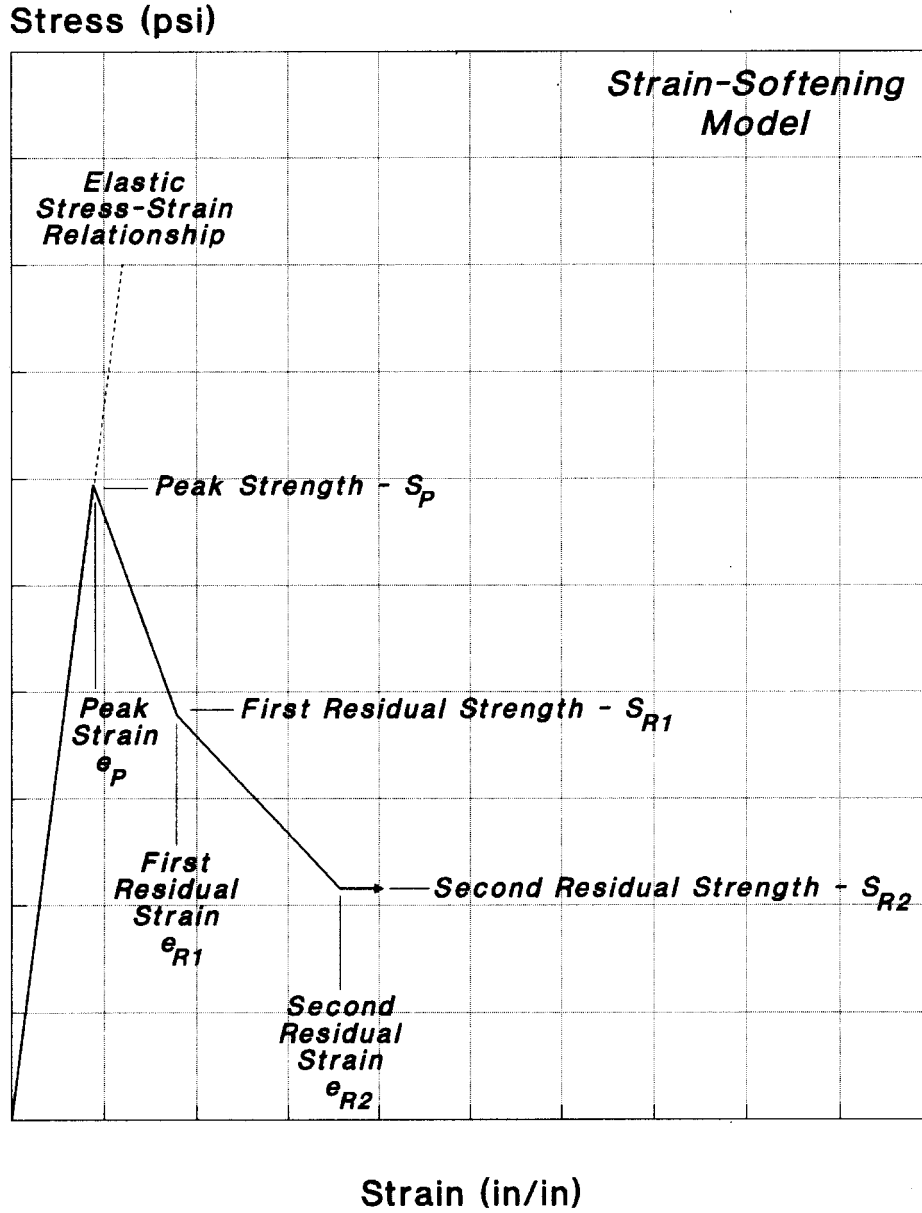


Figure 4.—General strain-softening element characteristics.

$$e_{R1}(i) = 2 ( e_p(i) ) \tag{4}$$

$$SR2(i) = (0.2254 ( \ln (x) ) ( S_p(i) ) \tag{5}$$

$$e_{R2}(i) = 4 ( e_p(i) ) \tag{6}$$

$e_{R2}(i)$  ' strain of element (i) at second residual stress level, in/in,

and  $x$  ' distance from element (i) center to free face, ft.

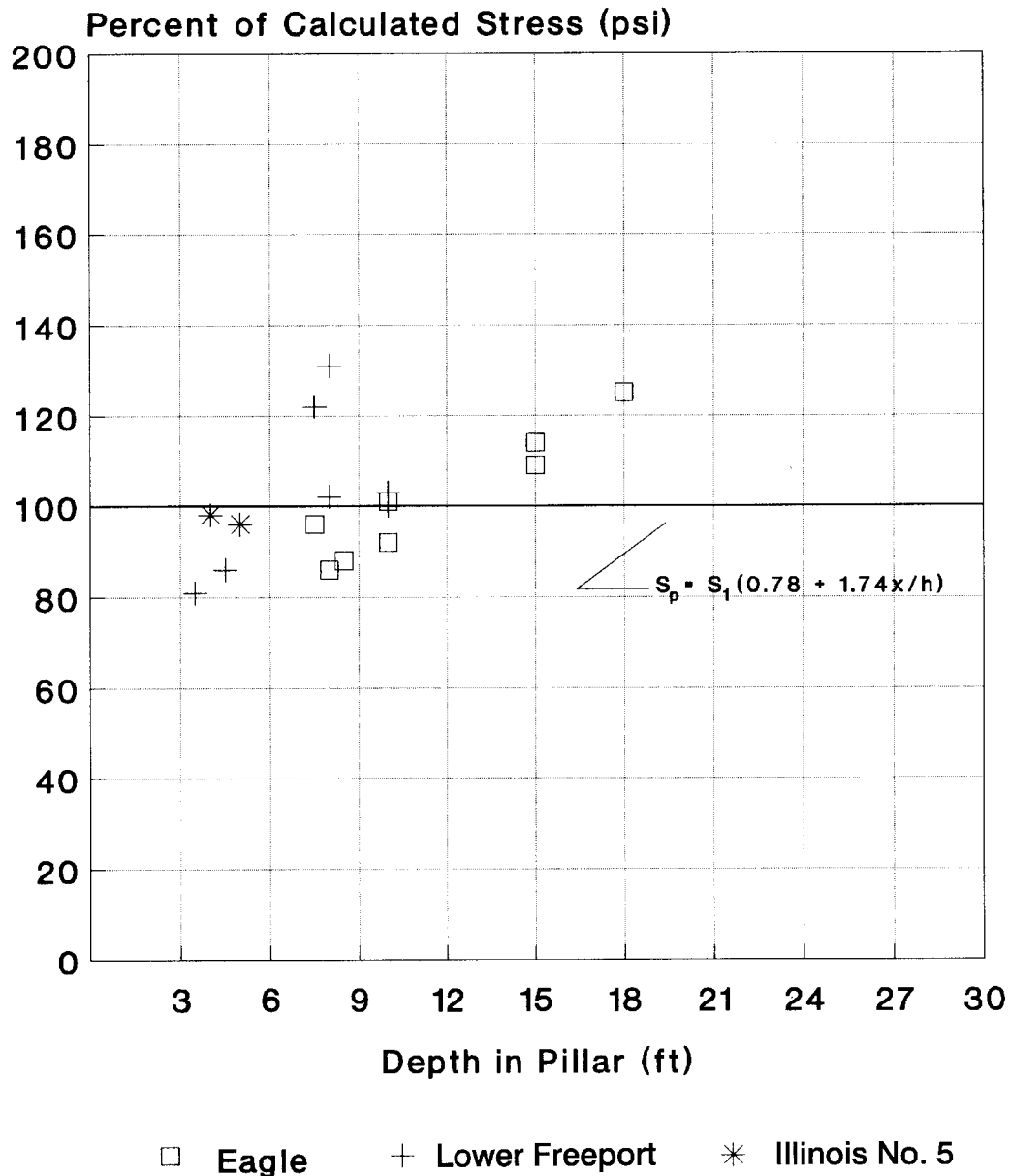
where  $S_{R1}(i)$  ' first residual stress level of element (i), psi,

$e_{R1}(i)$  ' strain of element (i) at first residual stress level, in/in,

$S_{R2}(i)$  ' second residual stress level of element (i), psi,

These relationships were patterned after the load/deflection response of coal samples under uniaxial testing, yield pillar stress and entry convergence measurements made at one mine site, and the assumption that at increasing depth into the pillar core a higher residual strength would be maintained.



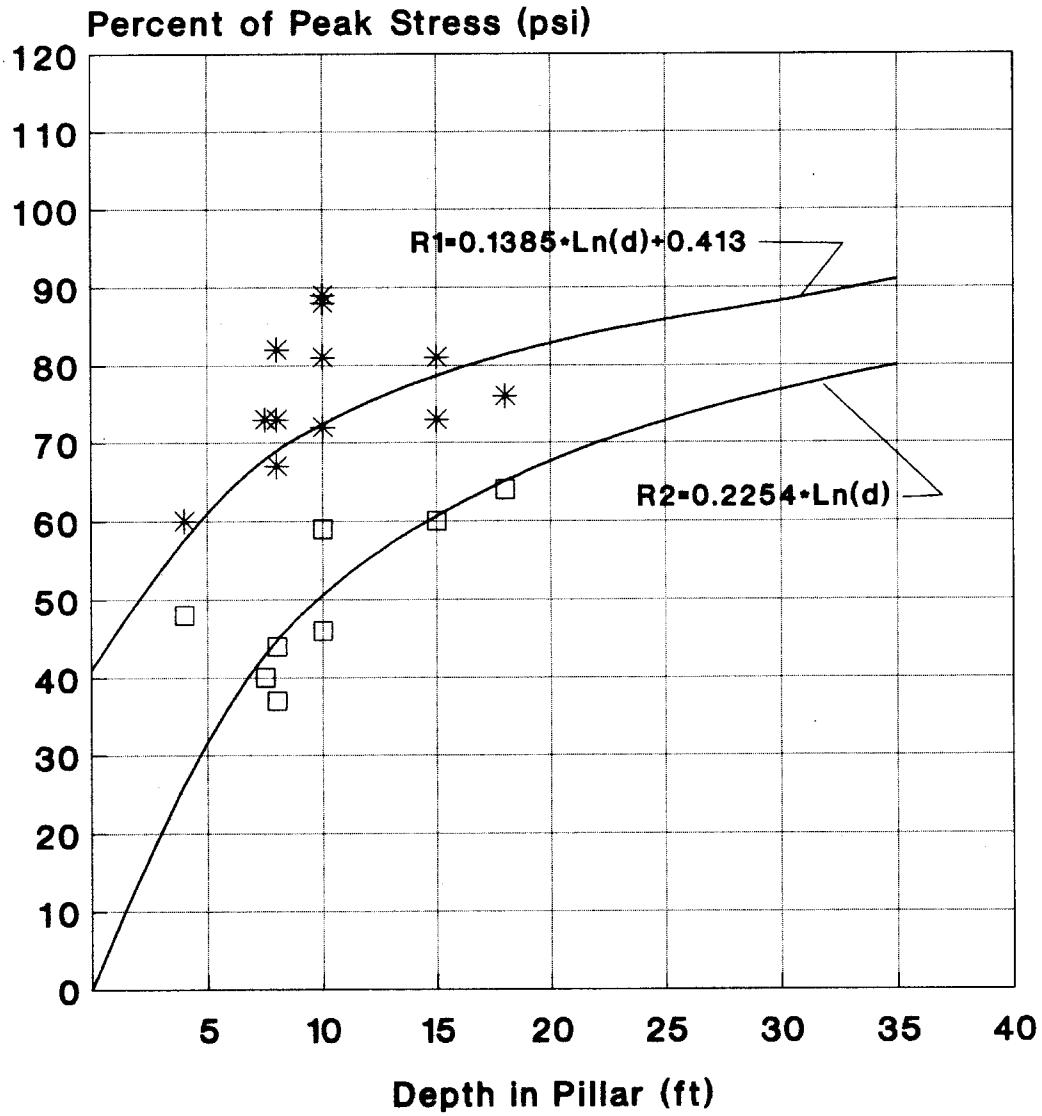


**Figure 5.—Measured versus calculated peak coal strength.**

Figure 6 presents a summary of residual stress levels measured at various depths at four mines where pillar yielding was monitored. The data are illustrated as a percentage of measured peak stress and compared to levels predicted by the above equations. The R1 levels represent the initial drop in stress once the peak has been reached; the R2 values indicate the final magnitude after substantial convergence. Both are difficult to identify because deformation plays a significant role in the unloading process; however, figure 6 represents a best estimate of those stress levels for the pillars monitored.

Figure 7 illustrates a family of six curves representing a strain-softening model with an element size of 10 ft, a seam height of 2.8 ft, an elastic modulus of 500,000 psi, and an in situ coal strength of 967 psi. Curve No. 1 represents the behavior of free-face or pillar perimeter elements; the remaining curves represent the stress-strain relationship of elements located successively deeper into the pillar core.

The BESOL system also requires estimates of the seam shear modulus (G) and similar shear stress-strain characteristics for the six yieldable elements described above. These geotechnical



\* Actual R1 Data      □ Actual R2 Data

Figure 6.—Measured versus calculated residual strength.

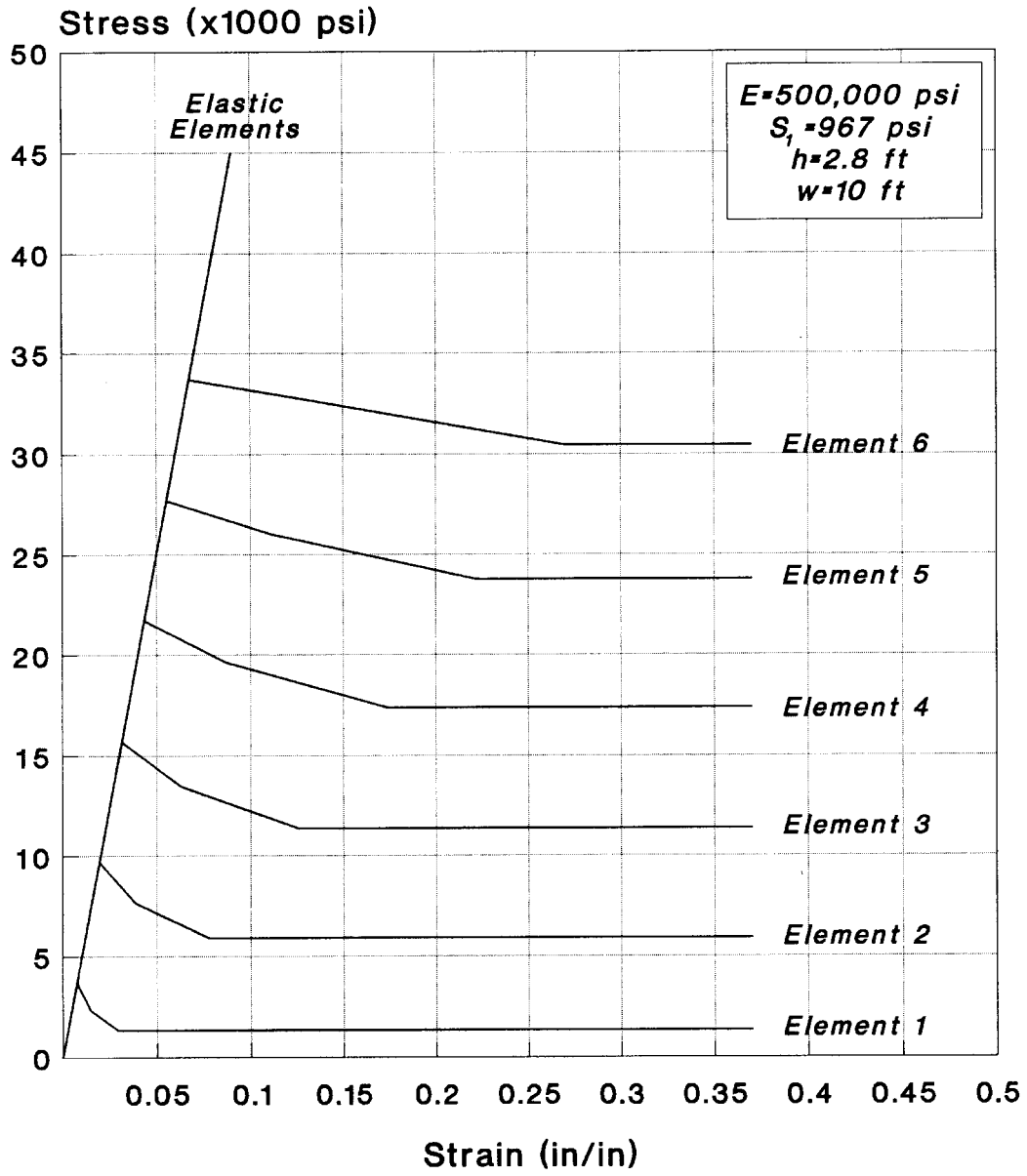


Figure 7.—Typical strain-softening seam behavior.

data are rarely available, and estimates (using the previously described procedure) based on a shear modulus equal to 1/2 to 1/3 of the elastic modulus and shear strengths of 1/2 to 1/3 of the strain softening values have been used.

It must again be emphasized that, although the methodology described above has been successfully used to estimate coal strain-softening properties, the properties generated are only a *first approximation* that must be verified for accuracy. Although in situ measurements have generally validated properties assigned to near-excavation locations, peak and residual stress levels deeper than 20 ft into a pillar or solid coal (where yielding rarely occurs) are largely unverified. Further, the procedure has been applied only to a limited number of coal seams, none of which experienced bump problems. The application of this technique to bump coal is not recommended because the strength increase due to confinement would likely exceed that predicted by the peak stress equations.

The suitability of assigned coal properties can be assessed by comparing the simulation output to observed pillar conditions. Test models should include underground areas (varying depths and pillar sizes) where definite observed pillar behavior can be isolated. For instance, if a model with 8-ft-wide elements predicts corner yielding, significant sloughing and crushing for

a length of 8 ft from the pillar corner should be obvious. A similar condition would be expected along the sides of pillars if perimeter yielding were projected. In general, more observed pillar deterioration than that projected by the model suggests that the coal strength has been overestimated; less sloughing than predicted indicates that it has been underestimated. There are occasions, however, where the element size itself can contribute to erroneous interpretations. A model using 10-ft elements may indicate elevated stress at the pillar corners, but no yielding. Underground observations of 4-ft crushed zones at the pillar corners may suggest that the model coal strength has been overestimated. Remodeling the area using 4-ft elements (with corresponding recalculation of element properties) may in fact result in the prediction of corner yielding that would match the in-mine conditions.

When constructing calibration models to verify coal strength, it is essential that:

- The element size selected is appropriate to illustrate phenomena (yielding) observed underground; and
- Element properties are recalculated when element sizes are changed; smaller elements have lower strength values than larger ones because of their proximity to the free face.

## GOB PROPERTIES

When numerical models contain large mined areas, such as longwall or pillar line gobs, some mechanism must be employed to simulate caving and stress relief associated with those areas. Without it, the full weight of the overburden would be transferred to adjacent areas and result in a significant overestimation of abutment loads. The stress relief process is complex and comprises caving, bulking, and subsequent compaction of the gob material. Although a number of investigators, including Pappas and Mark [1993], have evaluated the behavior of gob material, little published data exist regarding the simulation of caving in 3-D boundary-element numerical models.

The BESOL system provides a fill material that has been used to absorb a portion of the gob loads and provides a measure of stress relief associated with caving. The stress-strain relationship for the fill material is based on the work of M. D. G. Salamon and is of the form [Crouch Research, Inc. 1988]:

$$F_n = a (e_n / (b + e_n)), \quad (7)$$

where  $F_n$  = normal stress on the fill element,

$e_n$  = normal strain of the fill element,

$b$  = limiting value of normal strain (total compaction),

and  $a$  = stress to compress fill 1/2 of  $b$ .

For a first approximation, values for the necessary constants have been estimated as:

$$\begin{aligned} a &= 100 \text{ psi} \\ b &= 0.50 \text{ in/in} \end{aligned}$$

Figure 8 illustrates the relatively soft stress-strain response of backfill using these parameters. That material was tested in a number of general scenarios; resultant abutment loads were compared with those predicted by the inverse square decay function used by Mark [1990] in the Analysis of Longwall Pillar Stability (ALPS) methodology. As typified by figure 9, a reasonable agreement in resultant abutment stress distributions was found. The peak stress of the BESOL model exceeds that of the inverse square decay function; the average stress over the first 150 ft of the abutment (usually the zone of concern) is nearly identical. It appears that the use of a relatively soft backfill compensates for the tendency of boundary-element models to distribute abutment loads over a wide area and results in a reasonable approximation of near-gob stresses. Fill material of this type has been placed in gob areas during the BESOL simulation of nine mines (starting 20-30 ft from solid coal to allow an area of hanging roof) that have been successfully evaluated.

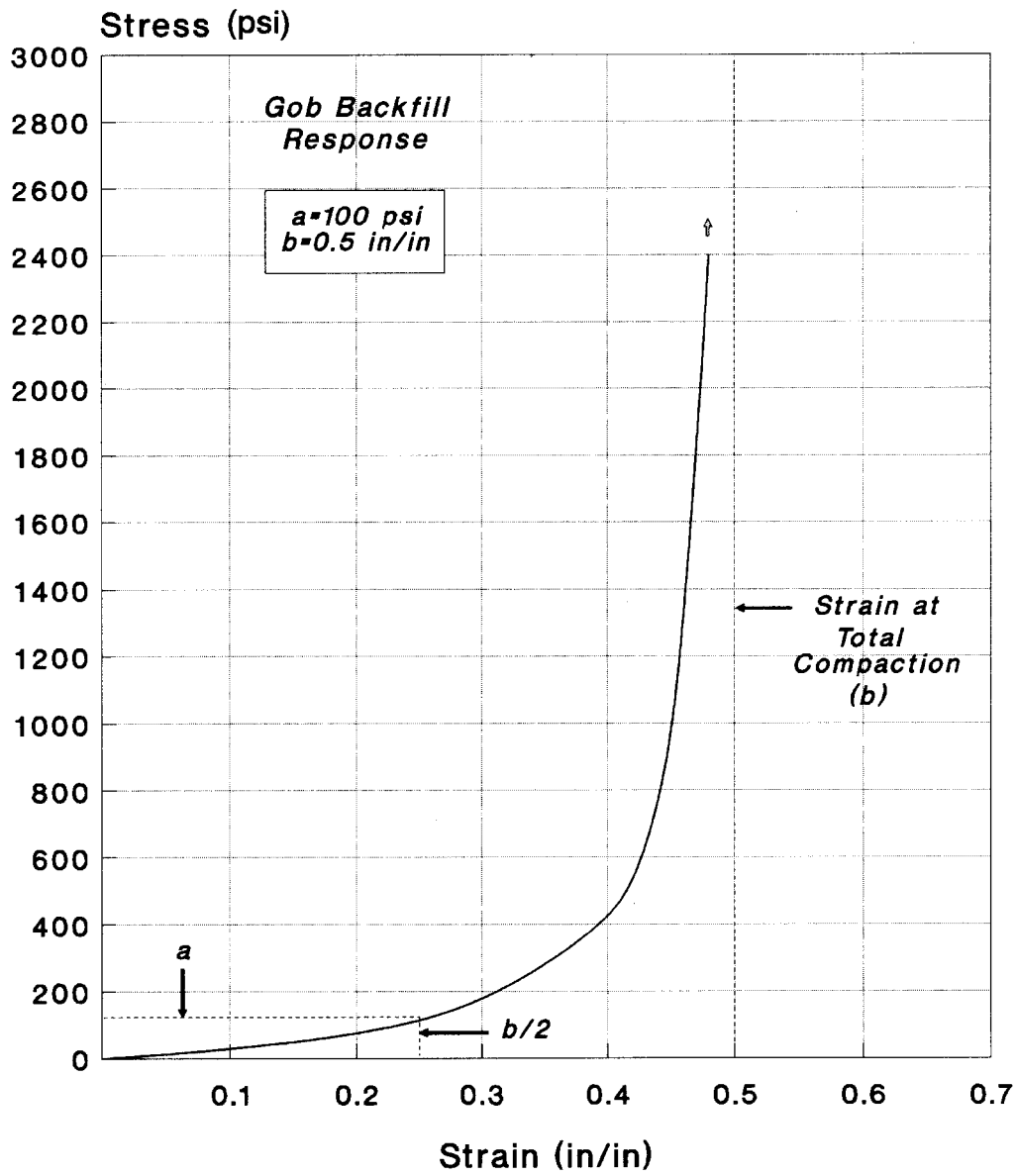


Figure 8.—BESOL strain-hardening backfill behavior.

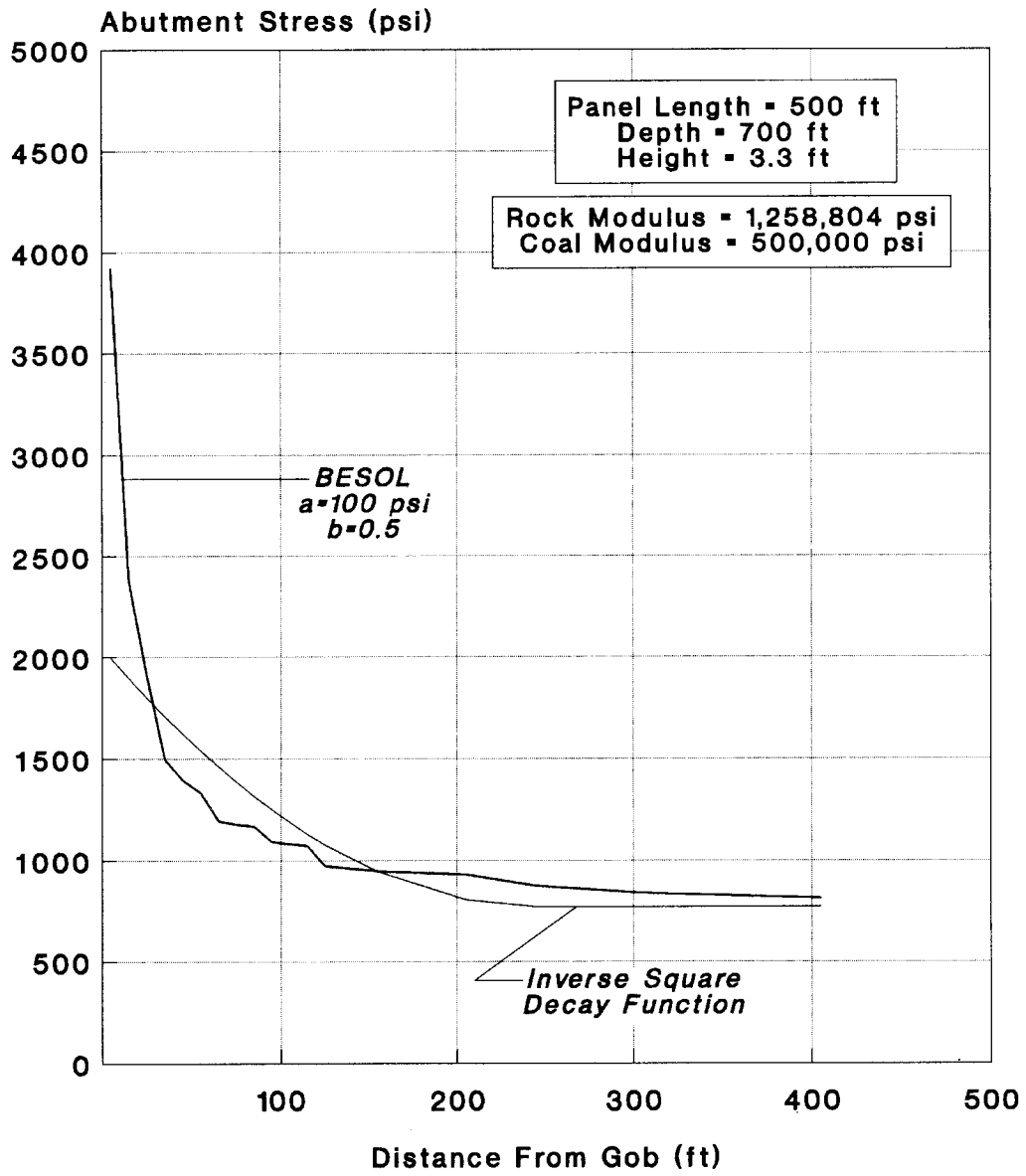


Figure 9.—BESOL versus inverse square decay function.

As with the other material properties discussed in this paper, the suitability of gob backfill based on the above or any other parameters must be verified. Obviously, the use of backfill that is too stiff will result in excessive gob loading and reduced abutment loads. Conversely, a gob material that is too soft will generate excessive abutment loads and low-gob stress. The

modulus of elasticity of the rock mass and other geometric parameters (depth, panel width, etc.) can have a significant impact on backfill loading and must be considered. Examining backfill stress in gob areas can indicate the amount of relief simulated by the model and can be compared to known or anticipated cave heights associated with those areas.

## SUMMARY OF PROPERTIES

In the process of simulating ground conditions at mines throughout the United States (12 coal seams in 5 States), a host of coal and rock properties have been generated. Table 2 summarizes the in situ coal strength, coal modulus of elasticity,

and rock moduli of elasticity used in 18 successful evaluations. The mining depth of each simulation is also shown in the table. The data are presented for reference purposes and illustrate the variation in properties that can be expected at different sites.

**Table 2.—Successfully applied coal and rock properties**

State and coal seam	Mining depth, ft	In situ coal strength, psi	Coal modulus of elasticity, psi	Rock modulus of elasticity, psi
PA:				
Lower Freeport . . . . .	420	<sup>1</sup> 462	<sup>1</sup> 550,000	<sup>2</sup> 1,000,000
Upper Freeport . . . . .	700	<sup>1</sup> 405	<sup>1</sup> 200,000	<sup>1</sup> 590,000
Upper Freeport . . . . .	360	<sup>1</sup> 775	<sup>1</sup> 200,000	<sup>1</sup> 740,000
Pittsburgh . . . . .	950	<sup>2</sup> 790	<sup>2</sup> 350,000	<sup>2</sup> 2,100,000
Pittsburgh . . . . .	650	<sup>2</sup> 900	<sup>2</sup> 500,000	<sup>2</sup> 3,280,000
Pittsburgh . . . . .	575	<sup>2</sup> 790	<sup>2</sup> 350,000	<sup>2</sup> 2,140,000
Lower Kittanning . . . . .	375	<sup>2</sup> 679	<sup>2</sup> 300,000	<sup>2</sup> 1,850,000
WV:				
Cedar Grove . . . . .	900	<sup>1</sup> 705	<sup>2</sup> 500,000	<sup>2</sup> 1,800,000
Dorothy . . . . .	150	<sup>1</sup> 290	<sup>1</sup> 121,000	<sup>2</sup> 910,000
Eagle . . . . .	950	<sup>1</sup> 712	<sup>1</sup> 490,000	<sup>1</sup> 880,000
Eagle . . . . .	850	<sup>1</sup> 850	<sup>1</sup> 500,000	<sup>1</sup> 810,000
Lower Lewiston . . . . .	260	<sup>1</sup> 583	<sup>1</sup> 200,000	<sup>2</sup> 2,400,000
Sewell . . . . .	470	<sup>1</sup> 312	<sup>1</sup> 250,000	<sup>2</sup> 1,400,000
KY:				
Elkhorn No. 3 . . . . .	420	<sup>1</sup> 951	<sup>1</sup> 548,000	<sup>2</sup> 1,750,000
Hazard No. 4 . . . . .	900	<sup>1</sup> 967	<sup>2</sup> 500,000	<sup>2</sup> 1,260,000
Hazard No. 4 . . . . .	950	<sup>2</sup> 967	<sup>2</sup> 500,000	<sup>2</sup> 1,260,000
IL:				
Illinois No. 5 . . . . .	700	<sup>2</sup> 620	<sup>2</sup> 330,000	<sup>2</sup> 1,000,000
AL:				
Blue Creek . . . . .	1,200	<sup>2</sup> 750	<sup>2</sup> 580,000	<sup>2</sup> 1,440,000

<sup>1</sup>Based on site-specific tests.

<sup>2</sup>Estimated from published data provided by the mine or found in literature reviews.

## DETERIORATION INDICES AND ANALYSIS

As mentioned previously, the most critical phase of the simulation process is verifying the accuracy of a model through correlation with actual underground conditions. To aid in that exercise, a set of deterioration indices was established to quantify pillar, roof, and floor behavior. Observed sites are assigned a numerical rating on a scale of 0 to 5 (0 is the best condition; 5 is the most severe) in each of the three categories. The deterioration index levels are reasonably well defined to minimize subjectivity of observations and promote consistency in ratings from site to site.

The pillar deterioration index (PDI) establishes observable sloughing levels that can be directly related to numerical model projections. A rating of 1 indicates corner crushing for a distance equal to one element width (usually 1/2-entry width) in the boundary-element model. A rating of 2 indicates some perimeter sloughing, but to a depth of less than one element width. This corresponding model would indicate yielding of some, but not all, of the perimeter seam elements. At the 2.5 level, sloughing is severe enough to cause concern over the stability of the area. A PDI of 3.5 represents a situation where

sloughing caused widening of the entry to a point that supplemental support (cribs or posts) was required to narrow the roadway. A corresponding model would indicate yielding of all perimeter elements and elevated pillar core stresses. PDIs of 4 and 5 represent progressively more severe conditions. A model response equivalent to a level 4 would indicate deeper pillar yielding and core stresses approaching the maximum capacity; a level of 5 indicates total pillar yielding and elevated convergence.

*Pillar deterioration index (PDI)*

0	Virtually no sloughing
1.0	Corner sloughing
2.0	Light perimeter sloughing
2.5	Onset of pillar stability concerns
3.0	Significant perimeter sloughing
3.5	Supplemental support required
4.0	Severe perimeter sloughing
5.0	Complete pillar failure

The roof deterioration index (RDI) defines a rating scale to quantify the condition of the roof strata in observed areas. Unlike the PDI, however, roof deterioration cannot be directly correlated to model output. The levels were established to correspond to progressively more significant observable phenomena ranging from roof flaking or sloughing (level 1) to widespread and massive roof falls (level 5). The severity of each feature can be identified within a one-point band. For instance, areas with only a hint of roof cutters would be rated at 1.6; those containing many severe cutters (a situation causing roof stability concerns) would receive a 2.5 rating. A roof deterioration index of 3.5 corresponds to conditions where supplemental support was required to maintain stability.

*Roof deterioration index (RDI)*

0	Virtually no deterioration
1.0	Flaking or spalling
2.0	Cutter roof
2.5	Onset of roof stability concerns
3.0	Broken roof
3.5	Supplemental support required
4.0	Significant roof falls
5.0	Widespread and massive roof falls

The floor deterioration index (FDI) provides a measure of mine floor stability relative to fracturing and the level of heave experienced. Like the RDI, this index cannot be directly

correlated to the model output, and the established levels represent progressively more serious floor conditions. An FDI of 2.5 represents the occurrence of heave that causes concern over floor stability; a level of 3.5 indicates a condition that impedes passage and requires grading to maintain an active travelway.

*Floor deterioration index (FDI)*

0	Virtually no deterioration
1.0	Sporadic cracks
2.0	Consistent localized cracks
2.5	Onset of floor stability concerns
3.0	Widespread cracks and obvious heave
3.5	Travel impeded; grading required
4.0	Significant floor displacement
5.0	Complete entry closure

The deterioration indices have been effectively used to describe in-mine ground conditions and to correlate BESOL output data to those observations. While simulation output such as stress and convergence can often be directly related to in-mine conditions, many instances arise where the combined influence of a number of factors affects ground behavior. To better establish those relationships and provide an effective means of evaluating potential design alternatives, a multiple linear regression can be used to relate model output to observed (deterioration index) conditions.

Table 3 presents a partial listing of BESOL output (stress, convergence, and failure index (FI) at the immediate roof line) and deterioration indices for a number of areas modeled and observed during an actual mine analysis. Other BESOL output (i.e., horizontal stress or displacement) could be included if applicable to a particular situation, but the three parameters listed are those routinely used. After model and observation data for all of the evaluated areas are compiled, multiple linear regression analyses are performed to define each deterioration index as a function of model output. In the sample instance in table 3, the various deterioration indices were related to maximum stress, maximum convergence, and minimum failure index at the roof line, and the resultant regression equations and correlation coefficients are listed.

Once the model accuracy is verified by comparing predicted to observed pillar yielding, examining the regression correlation coefficients, and using the regression equations to back-calculate deterioration indices for the observed (modeled) areas, design alternatives can be modeled and expected conditions predicted. Table 4 contains projected deterioration indices at a critical pillar line location for various pillar sizes and depths of



cover as predicted by BESOL output and the verified regression equations. The difference in expected conditions with each design alternative is clear.

The deterioration index/regression equation technique has proved to be a viable method of verifying numerical model accuracy and evaluating the potential of design alternatives *provided that* relatively consistent mining conditions exist. When changing roof, pillar, or floor strengths are encountered, the usability of the regression technique may be greatly reduced. Further, the relationships established are based on strata reaction at a particular mine, and only those observed (which are limited by current mine design and environment) can be included in the database. This is a particular concern when the use of yield

pillars as an alternative configuration is considered, but no complete pillar yielding is evident at the mine.

The Roof Control Division is currently exploring the use of normalizing parameters in the regression analysis to alleviate these difficulties. Factors such as in situ coal strength and seam height (for the PDI), a roof rock rating such as the Coal Mine Roof Rating (CMRR) [Molinda and Mark 1993] for the RDI, and a floor characterization number (for the FDI) are being evaluated to determine their usefulness in the regression analysis to buffer the variations found within a given mine and also between mines. If successful, the resultant technique could enhance individual mine analyses and allow the experience of many mines to be used.

**Table 3.—Partial BESOL/deterioration index listing and regression equations**

Location and entry	BESOL output			Deterioration indices					
	Maximum stress, psi	Maximum convergence, ft	Minimum failure index (FI)	Observed			Back-calculated		
				PDI	RDI	FDI	PDI	RDI	FDI
Face area:									
1 .....	4,000	0.113	1.04	1.5	1.5	0.0	1.5	1.2	0.2
2 .....	6,800	0.195	1.09	2.0	1.8	0.3	2.5	2.4	1.2
3 .....	8,100	0.251	0.96	3.5	3.0	1.0	3.0	2.9	1.7
4 .....	8,800	0.289	0.89	4.0	4.2	2.5	3.3	3.3	2.0
5 .....	8,800	0.307	0.87	4.0	3.5	4.0	3.3	3.4	2.1
1 crosscut outby:									
1 .....	3,100	0.083	1.11	1.2	1.5	0.0	1.1	0.9	0.0
2 .....	5,400	0.161	1.16	1.5	1.5	0.0	2.0	1.9	0.8
3 .....	7,000	0.207	1.11	2.0	2.0	0.5	2.6	2.5	1.3
4 .....	7,500	0.230	1.02	3.0	3.0	1.0	2.8	2.7	1.5
5 .....	7,500	0.223	0.94	3.0	3.0	3.0	2.7	2.6	1.4
3 crosscuts outby:									
1 .....	2,710	0.063	1.25	1.0	0.5	0.0	1.0	0.7	0.0
2 .....	3,900	0.089	0.93	1.5	0.8	0.0	1.3	1.1	0.0
3 .....	6,000	0.150	1.16	1.5	1.5	0.2	2.2	2.0	0.9
4 .....	7,000	0.182	1.13	2.0	2.0	1.0	2.5	2.4	1.2
5 .....	7,300	0.204	1.21	3.0	2.5	2.0	2.7	2.6	1.4
3-Right:									
2 .....	2,240	0.059	1.53	1.0	0.5	0.0	1.0	0.7	0.0
4 .....	2,560	0.070	1.41	1.4	1.0	0.0	1.1	0.8	0.0
5 .....	2,820	0.072	1.45	1.5	1.4	0.1	1.2	0.9	0.0
1-Right:									
2 .....	1,530	0.040	2.13	1.0	0.2	0.0	1.0	0.6	0.0
4 .....	1,700	0.047	1.91	1.0	1.0	0.0	1.0	0.6	0.0
5 .....	1,780	0.047	2.00	1.0	1.0	0.0	1.0	0.7	0.0
PDI ' 0.000268 ( STR % 3.259622 ( CONV % 0.379665 ( FI & 0.383740				$r^2$ ' 0.79					
RDI ' 0.000263 ( STR % 4.603502 ( CONV % 0.309200 ( FI & 0.643870				$r^2$ ' 0.80					
FDI ' 0.000170 ( STR % 6.094244 ( CONV % 0.600442 ( FI & 1.82412				$r^2$ ' 0.60					

**Table 4.—Full pillaring BESOL output and predicted deterioration index**

Pillar size (ft), depth, and location	Maximum stress, psi	Maximum convergence, ft	PDI	RDI	FDI
50 by 50 (900-ft depth):					
1 .....	<sup>1</sup> 8,300	<sup>1</sup> 0.291	<sup>2</sup> 3.0	<sup>2</sup> 3.1	<sup>3</sup> 1.7
2 .....	<sup>1</sup> 8,200	<sup>2</sup> 0.247	<sup>2</sup> 3.1	<sup>2</sup> 3.1	<sup>3</sup> 1.9
3 .....	<sup>3</sup> 5,900	<sup>2</sup> 0.185	<sup>3</sup> 2.1	<sup>3</sup> 2.0	<sup>3</sup> 0.8
4 .....	<sup>3</sup> 5,600	<sup>3</sup> 0.161	<sup>3</sup> 2.2	<sup>3</sup> 2.0	<sup>3</sup> 1.0
40 by 40 (900-ft depth):					
1 .....	<sup>1</sup> 9,690	<sup>1</sup> 0.385	<sup>1</sup> 3.8	<sup>1</sup> 4.0	<sup>2</sup> 2.7
2 .....	<sup>1</sup> 9,690	<sup>1</sup> 0.343	<sup>1</sup> 3.8	<sup>1</sup> 3.9	<sup>2</sup> 2.6
3 .....	<sup>1</sup> 8,700	<sup>2</sup> 0.245	<sup>2</sup> 3.0	<sup>2</sup> 3.0	<sup>3</sup> 1.6
4 .....	<sup>1</sup> 8,300	<sup>2</sup> 0.230	<sup>2</sup> 3.1	<sup>2</sup> 3.0	<sup>3</sup> 1.7
40 by 40 (800-ft depth):					
1 .....	<sup>1</sup> 9,690	<sup>1</sup> 0.305	<sup>1</sup> 3.5	<sup>1</sup> 3.6	<sup>3</sup> 2.2
2 .....	<sup>1</sup> 9,690	<sup>1</sup> 0.269	<sup>1</sup> 3.6	<sup>1</sup> 3.5	<sup>3</sup> 2.2
3 .....	<sup>2</sup> 6,800	<sup>2</sup> 0.198	<sup>3</sup> 2.4	<sup>3</sup> 2.3	<sup>3</sup> 1.0
4 .....	<sup>2</sup> 6,600	<sup>2</sup> 0.182	<sup>2</sup> 2.5	<sup>3</sup> 2.4	<sup>3</sup> 1.3
40 by 40 (600-ft depth):					
1 .....	<sup>2</sup> 7,300	<sup>2</sup> 0.204	<sup>2</sup> 2.6	<sup>2</sup> 2.5	<sup>3</sup> 1.2
2 .....	<sup>2</sup> 7,150	<sup>3</sup> 0.171	<sup>2</sup> 2.7	<sup>2</sup> 2.5	<sup>3</sup> 1.4
3 .....	<sup>3</sup> 3,500	<sup>3</sup> 0.095	<sup>3</sup> 1.2	<sup>3</sup> 1.0	<sup>3</sup> 0.0
4 .....	<sup>3</sup> 3,400	<sup>3</sup> 0.087	<sup>3</sup> 1.3	<sup>3</sup> 1.0	<sup>3</sup> 0.1
40 by 30 (400-ft depth):					
1 .....	<sup>3</sup> 4,400	<sup>3</sup> 0.116	<sup>3</sup> 1.5	<sup>3</sup> 1.3	<sup>3</sup> 0.2
2 .....	<sup>3</sup> 4,200	<sup>3</sup> 0.098	<sup>3</sup> 1.4	<sup>3</sup> 1.2	<sup>3</sup> 0.1
3 .....	<sup>3</sup> 2,660	<sup>3</sup> 0.063	<sup>3</sup> 1.0	<sup>3</sup> 0.7	<sup>3</sup> 0.0
4 .....	<sup>3</sup> 2,320	<sup>3</sup> 0.060	<sup>3</sup> 1.1	<sup>3</sup> 0.8	<sup>3</sup> 0.0

<sup>1</sup>Severe conditions.<sup>2</sup>Borderline conditions.<sup>3</sup>Desirable mining conditions.

## CASE STUDY

An investigation was conducted at a coal mine in eastern Kentucky to determine the cause of a roof fall and deteriorating ground conditions that were encountered on a full pillaring section. The mine is located in the Hazard No. 4 Seam and has a mining height of 32-40 in. Figure 10 presents an illustration of the 1-Left Mains in the vicinity of the roof fall. These mains were developed as a five-entry system on 50- by 60-ft centers with 20-ft-wide entries and crosscuts. Panels were driven to the right and retreated as the mains were advanced (13 panels total). Following development of the mains (and panels) to the property boundary, retreating of those pillars was initiated. As figure 10 illustrates, a roof fall occurred one crosscut outby the pillar line as the 18th row of blocks was being extracted. Cover at the face was about 800 ft, but ranged from 480 ft near the mouth of the section (about 2,400 ft outby) to over 950 ft several hundred feet inby and to the right of the fall. The immediate roof strata were composed of a 15-ft-thick laminated shale and were overlain by a 20-ft-thick sandstone layer. Roof support was provided by 4-ft-long fully grouted bolts installed in a 4- by 4-ft pattern throughout the mains.

Observations were made throughout the 1-Left Mains to characterize ground conditions under various depths of cover and degrees of gob influence. Significant deterioration (heavy pillar sloughing, cutters, and broken roof zones) was noted in the face area; conditions were most severe in the immediate vicinity of the roof fall. Outby the face, conditions gradually improved, although the right side of the mains consistently showed heavier deterioration than the left side. The most significant conditions noted in the outby area corresponded to zones of heavier cover, suggesting that overburden depth and the adjacent gob areas contributed to the deteriorating conditions. Detailed deterioration index ratings were made throughout the observed areas to quantify the roof, floor, and pillar behavior. The data presented in table 3 represent a partial listing of these ratings in a number of entry locations (crosscut conditions were also quantified and used in the analysis). Higher PDI, RDI, and FDI levels correspond to more severe deterioration, which were observed in the face area and along the right side of the mains. Cover at the face was about 800 ft and about 650 ft and 480 ft over the 3-Right and 1-Right outby

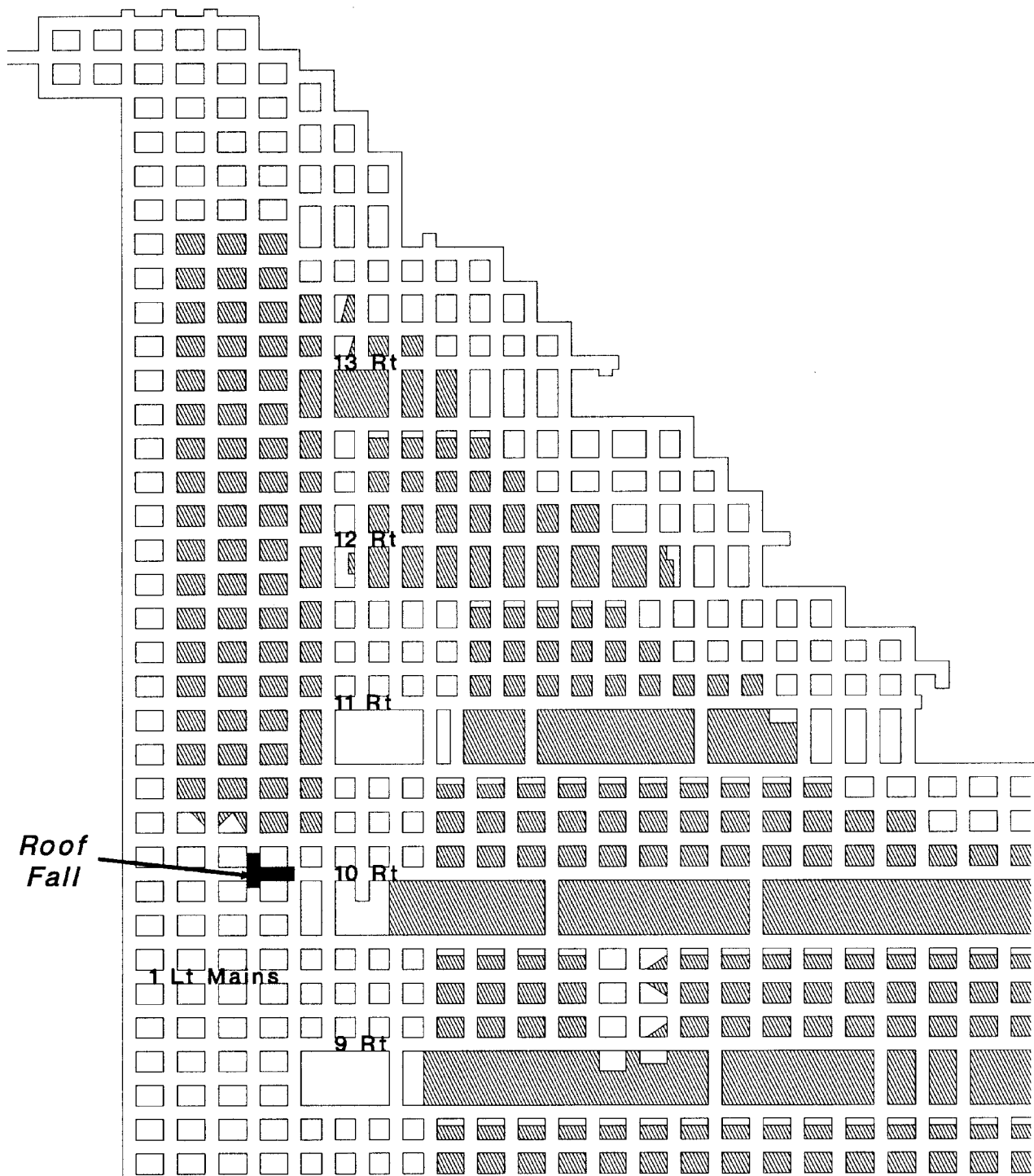
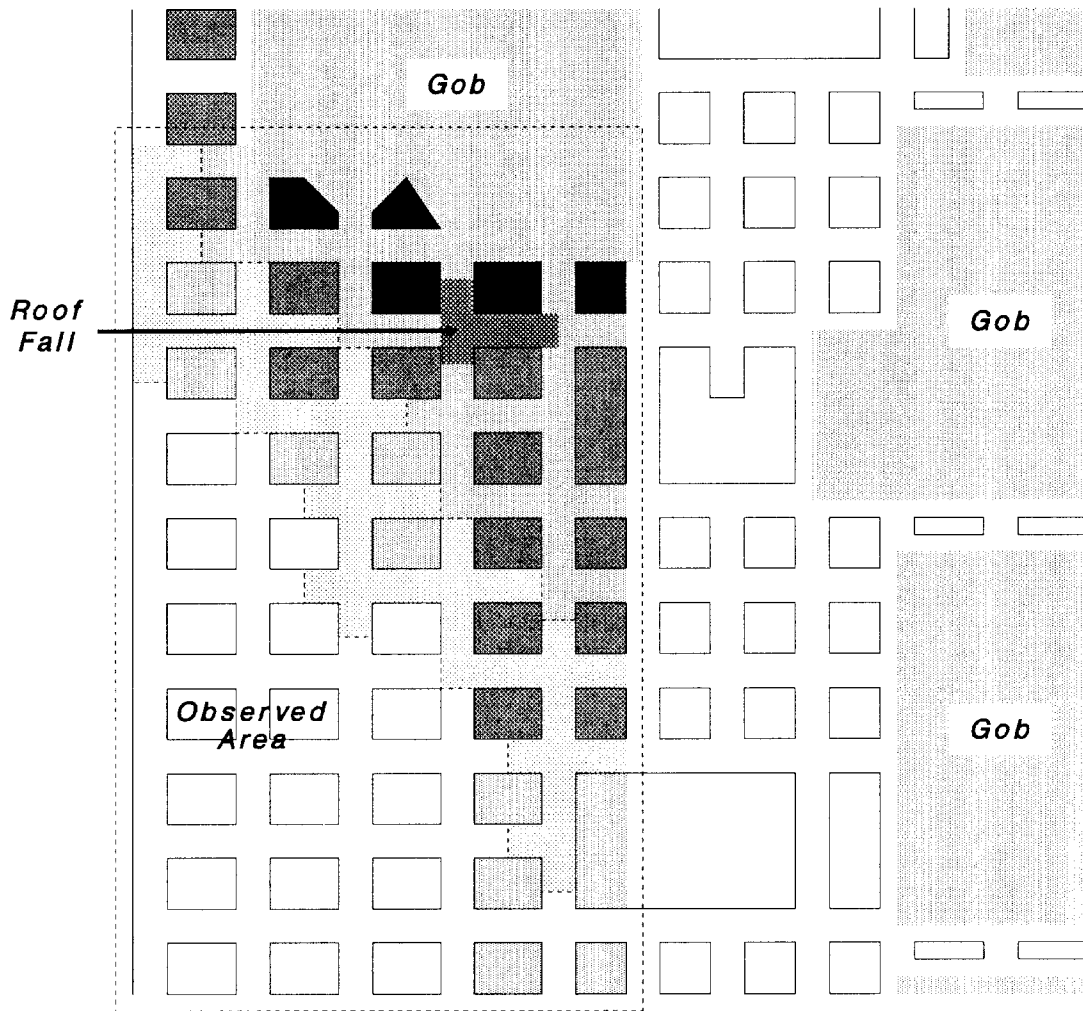


Figure 10.—Case study: partial mine map of pillaring section - roof fall area.

areas, respectively, where conditions were much improved. Figure 11 presents a composite deterioration index drawing of conditions observed at and just outby the face, illustrating the concentration of deterioration in the vicinity of the roof fall and along the right side of the section.

A series of three BESOL models was subsequently created to simulate conditions in the areas observed during the underground investigation. The first model (covering the area shown in figure 10) was used to simulate mining at the time of the roof fall and also at inby and proposed outby face positions



Pillar Deterioration	
	Severe (3.5-4.5)
	Significant (2.5-3.5)
	Moderate (1.5-2.5)
	Light (<1.5)

Roof/Floor Deterioration	
	Severe (2.75-4.0)
	Moderate (1.5-2.75)
	Light (<1.5)

Figure 11.—Case study: observations on pillaring section - roof fall area.

where cover was approximately 800 ft. Additional models were constructed of the outby areas (3-Right (650-ft cover) and 1-Right (480-ft cover)) to provide model verification under significantly differing conditions. Vertical stress applied to the models equaled 1.1 psi per foot of depth, and a horizontal stress of 1/2 the vertical stress was assumed in both the x and y directions. The element size used in the simulations was 10 ft, or 1/2 the 20-ft-entry width.

A composite rock modulus of 1,260,000 psi was based on data obtained from four boreholes in the vicinity, as shown in table 1. The individual rock moduli were estimated from published data for the specific strata contained in each borehole. A Poisson's ratio of 0.21 and the default Mohr-Coulomb properties (cohesion = 800 psi, friction angle = 25°, and tensile strength = 1,000 psi) were used because no site-specific data were available.

Coal properties were based on an in situ strength of 967 psi (site-specific coal strength data were provided by the mine); the peak and residual strength levels were calculated as outlined previously in this paper. A seam height of 2.8 ft was used, and a coal modulus of elasticity of 500,000 psi was assumed. The stress-strain curves of figure 7 represent the strain-softening model used in the analysis. Shear stress-strain properties were based on a shear modulus of 200,000 psi (0.4E).

Gob caving was simulated using the Salomon backfill discussed earlier with the constants  $a = 100$  psi and  $b = 0.50$ . The comparison of abutment loading between BESOL and the inverse square decay function of figure 9 was based on the rock mechanics parameters used in this simulation.

Maximum pillar stress, maximum roof/floor convergence, and minimum failure index values were determined from the 3 models for 37 locations (entries and crosscuts) corresponding to the observed areas. The stress and convergence data compiled indicate the highest levels found in or adjacent to the 37 locations; the failure index values represent the lowest levels detected at the roof line in each area. A portion of these data (entry locations) is listed in table 3. A series of multiple linear regression analyses was made to relate the deterioration indices observed to the BESOL data and resulted in the equations also listed in table 3. The R-squared values for the PDI (0.79) and the RDI (0.80) were very good, but marginal for the FDI (0.60). It should be noted that the characterization of floor conditions was not a primary concern during the investigation, but sketchy data acquired were used to illustrate the process. The BESOL output was then inputted into the regression equations to predict (back-calculate) deterioration indices for the observed locations; these values describing entry conditions are also listed in table 3. Most of the predicted PDI and RDI levels match the observed data fairly well, and the trend of higher deterioration indices in areas of more severe conditions was evident, even with the FDI.

Figure 12 presents a composite of maximum pillar stress and convergence levels predicted by the BESOL model of the roof fall site. Note the correlation of BESOL stress and convergence

with the degree of deterioration observed underground. The zone of high convergence (>0.25 ft) and stress (>9,500 psi) encompasses the area of deteriorating conditions at the pillar line, including the roof fall. Lower stress and convergence levels also correspond to zones of lesser deterioration, and the more severe conditions predicted on the right side of the mains (indicating the influence of the adjacent gob) also match the conditions observed underground. These correlations, coupled with the good fit of the regression analysis (deterioration indices), confirmed the accuracy of the model (and properties used) to simulate conditions at the mine. Confidence was further enhanced by an evaluation of the BESOL model with a face position several crosscuts inby the roof fall. The results showed significantly lower stress and convergence levels in the face area that correlated to the better mining conditions actually encountered.

It was concluded that the roof fall (and deteriorating conditions) resulted from a combination of stresses from the active and adjacent gobs overriding the pillar line (yielding) and focusing outby the face. The small pillar size employed (30 by 40 ft) on the mains, the lack of protection provided by the combination of chain and barrier pillars from the adjacent gob, and the depth of cover (>800 ft) contributed to the problems encountered.

A series of additional models was created to evaluate the performance of various pillar sizes at different mining depths that would be encountered. Figure 13 illustrates the pillaring plan to be implemented using a 200-ft barrier between adjacent panels that would be roomed and retreated along with the panel being extracted. Stresses and convergences were examined at four entry locations near the face (during retreat of the second panel), as illustrated in figure 14. Threshold levels delineating expected conditions (from the 1-Left models) were established as follows:

*Severe conditions:*

Stress > 8,000 psi; convergence > 0.25 ft  
PDI > 3.5; RDI > 3.5; FDI > 3.5

*Borderline conditions:*

Stress = 6,500 to 8,000 psi; convergence = 0.18 to 0.25 ft  
PDI = 2.5 to 3.4; RDI = 2.5 to 3.4; FDI = 2.5 to 3.4

*Desirable mining conditions:*

Stress < 6,500 psi; convergence < 0.18 ft  
PDI < 2.5; RDI < 2.5; FDI < 2.5

It was predetermined that good (desirable) mining conditions should exist at locations 3 and 4 since no supplemental supports (posts) would be installed in those areas. Borderline conditions could be tolerated at locations 1 and 2 (posts are set in this area),

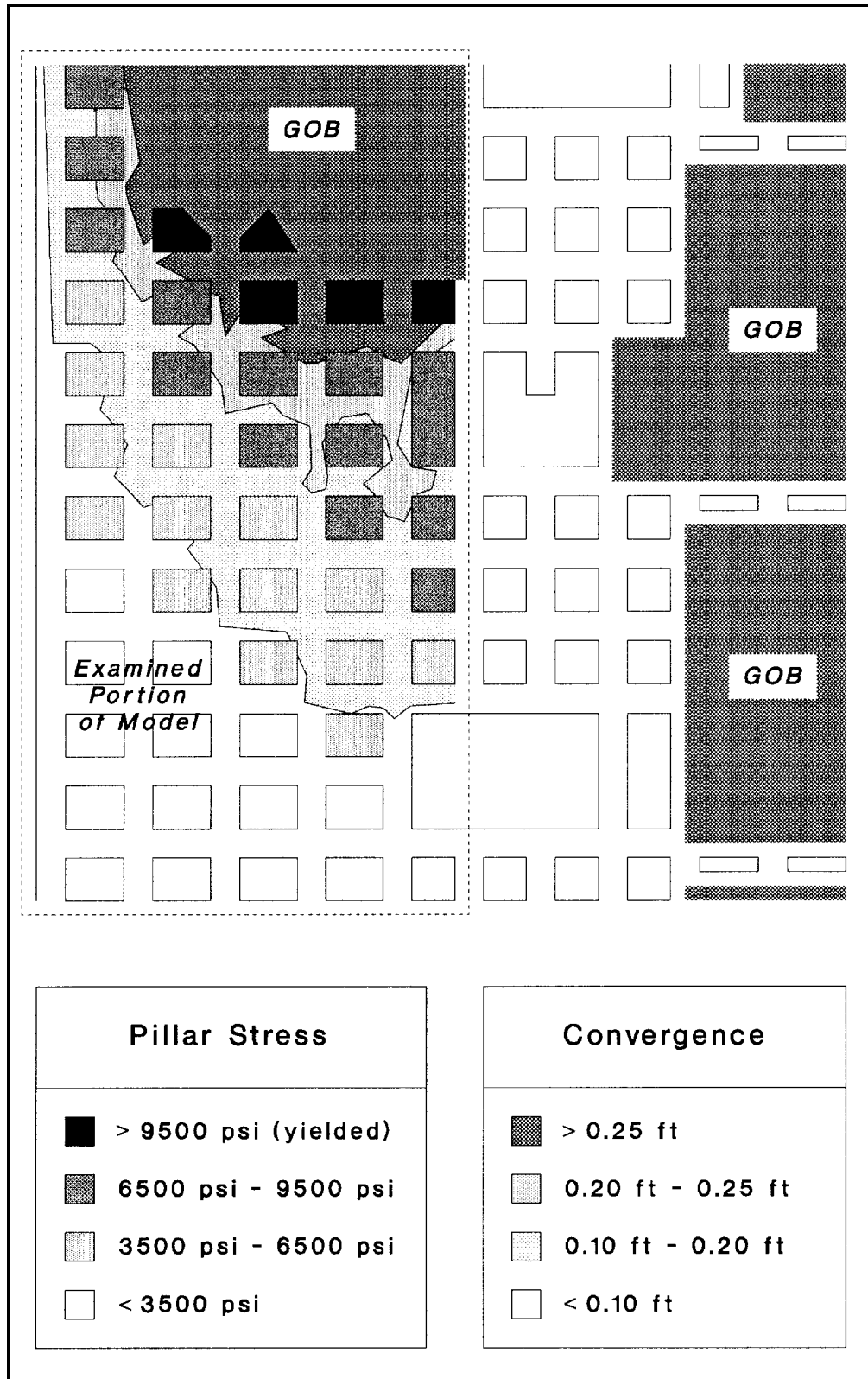


Figure 12.—Case study: BESOL output pillaring section - roof fall area.

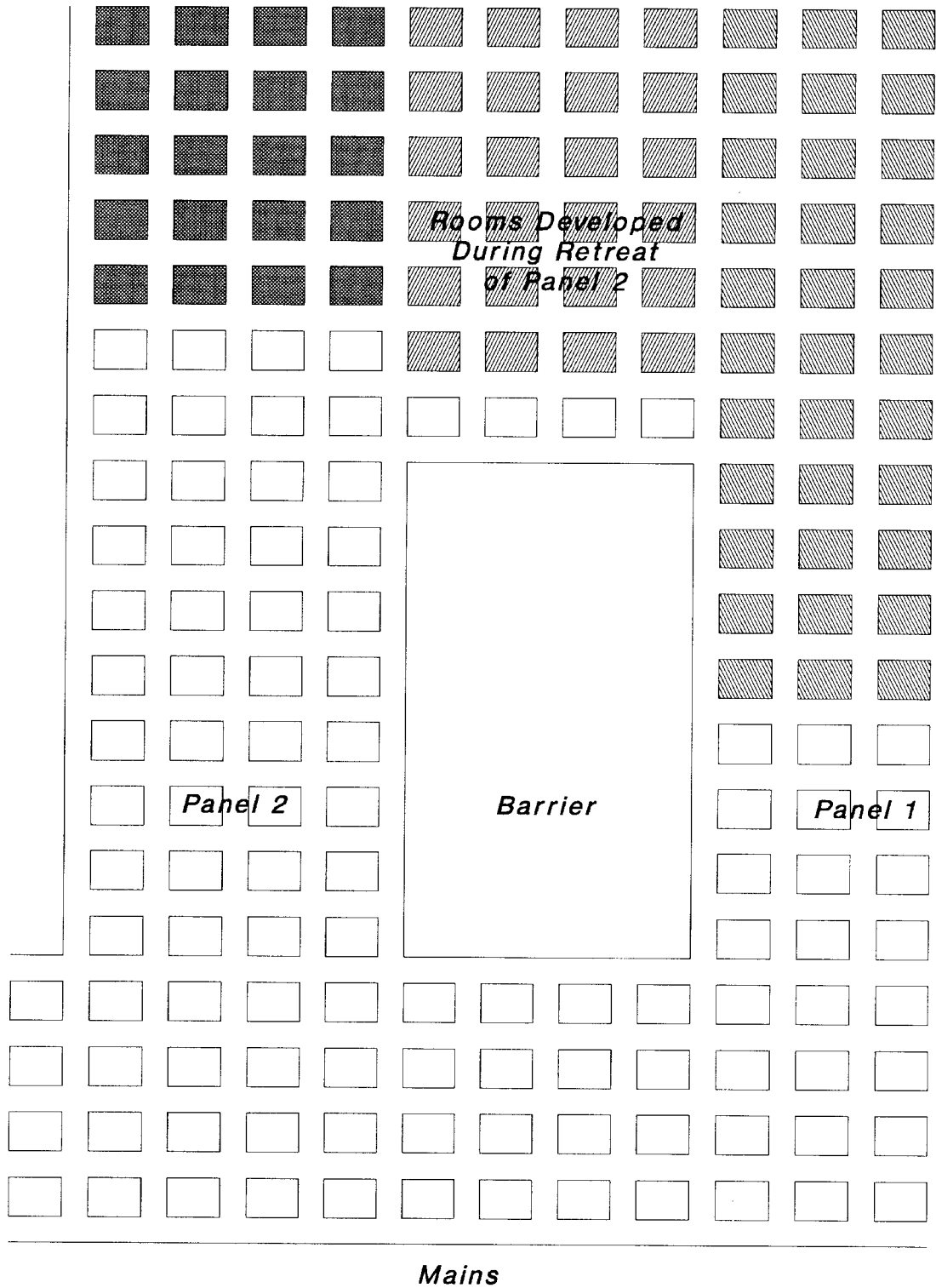


Figure 13.—Case study: full pillaring plan.

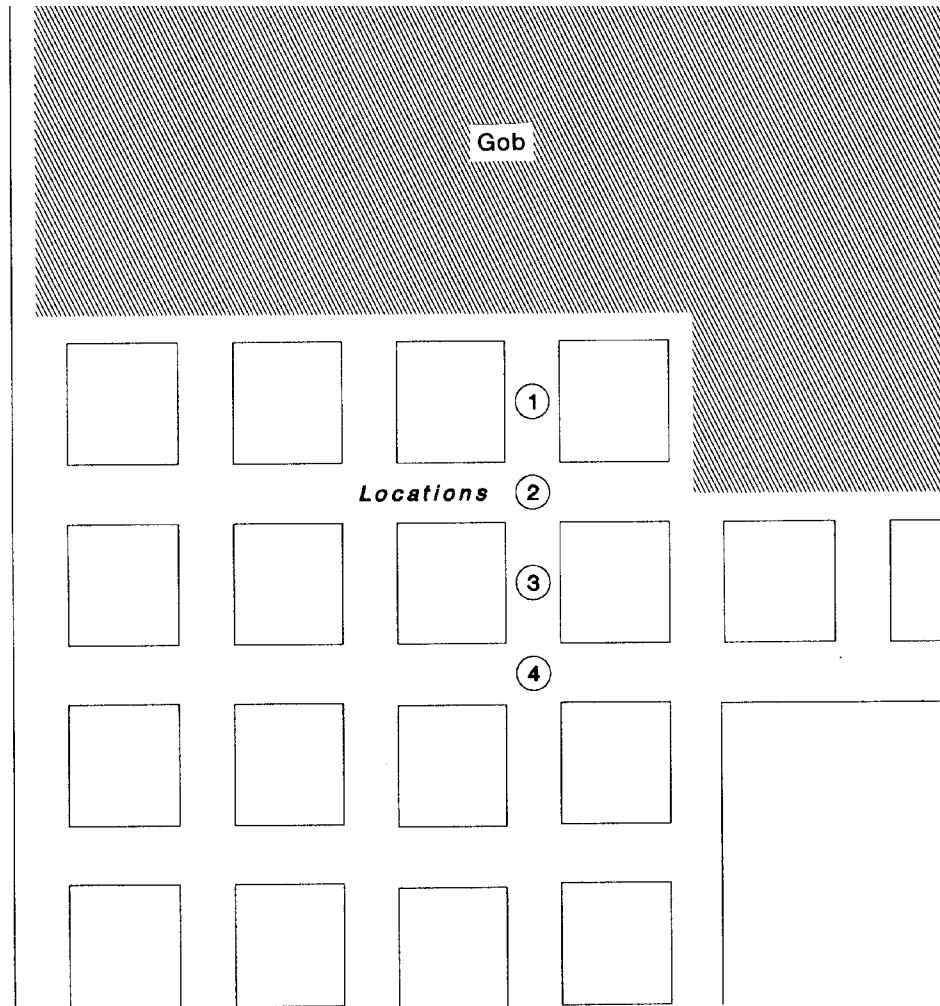


Figure 14.—Case study: full pillaring analysis locations.

but the occurrence of severe conditions should be avoided or at least limited to location 1.

Table 4 presents the BESOL and predicted deterioration index data for each of the four locations for a number of scenarios. The analysis indicated that the use of 40- by 30-ft

pillars would result in good conditions through a depth of 400 ft and that 40- by 40-ft pillars would be effective up to 600 ft of cover. Pillars 50- by 50-ft in size would be needed for deeper cover areas, although severe conditions could be possible at locations 1 and 2 as the depth approaches 900 ft.

## CONCLUSION

Boundary-element modeling has proven to be an effective tool for mining engineers to resolve complex ground control problems. The techniques set forth in this paper describing coal, rock, and gob behavior have been effectively used to evaluate a variety of mining scenarios. Although they are supported by a number of in situ measurements and have resulted in near duplication of underground conditions in many instances, they provide only a first estimate of parameters that must be validated. Successful numerical simulation requires a substantial

effort, including the observation of conditions in many areas and the often repetitive process of calibrating model parameters. The use of techniques such as the deterioration index/regression method has greatly facilitated the linking observed and simulated mine conditions. It cannot be overemphasized, however, that in order to be of any value, a numerical model *must* be validated and provide a realistic representation of the underground environment for which it is applied.



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