UNIVERSITY OF NEW SOUTH WALES COAL PILLAR STRENGTH DETERMINATIONS FOR AUSTRALIAN AND SOUTH AFRICAN MINING CONDITIONS

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ABSTRACT

A series of mine design accidents in the late 1980s resulted in a major research program at the University of New South Wales, Australia, aimed at developing pillar and mine design guidelines. A database of both failed and unfailed Australian underground coal mine pillar case studies was compiled. A procedure was developed to enable the effective width of rectangular pillars to be taken into account. The database was analyzed statistically using the maximum likelihood method, both independently and as a combined data set with the more extensive South African database. Probabilities of failure were correlated to factors of safety. It was found that there was less than a 4% variance in pillar design extraction ratios resulting from each of these approaches. There is a remarkable consistency between the design formulas developed from back-analysis of the two separate national pillar databases containing many different coal seams and geological environments.

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INTRODUCTION

In the 3-year period to 1992, 60 continuous miners were trapped by falls of strata for more than 7 hr in collieries in New South Wales, Australia. In the preceding 2 years, eight coal miners were killed in pillar extraction operations in New South Wales. In the New South Wales and Queensland coalfields, at least 15 extensive collapses of bord-and-pillar workings occurred unexpectedly in the 15-year period to 1992. Six of these collapses occurred in working panels; fortuitously, five occurred during shutdown periods and the sixth occurred while the continuous miner was being flitted to the surface for repairs.

One contributor to these events was the lack of a comprehensive pillar design procedure. Legislation in New South Wales at the time simply required coal pillars to have a minimum width of one-tenth depth or 10 m, whichever was greater. The influence of pillar height on strength received no recognition.

This set of circumstances led to funding by the New South Wales Joint Coal Board of a major research project on pillar design and behavior. The research was undertaken by the School of Mining Engineering at the University of New South Wales (UNSW). The primary objectives of the research were to improve the understanding of coal pillars and associated floor and roof strata behavior under various loading conditions and to incorporate these outcomes into the mine design knowledge base.

RESEARCH METHODOLOGY

The approach adopted to pillar design was based on that developed for square pillars by Salamon and Munro [1966, 1967]. However, the extensive use of rectangular and diamond-shaped pillars in Australia required more detailed consideration of the *effective width* of parallelepiped pillars and the effect of this width on pillar strength.

Firstly, an adequate Australian database of failed and unfailed pillar case histories was established. A relationship was then developed to factor in the influence of rectangular and diamond-shaped pillars, which comprised just over 50% of the database. This database was then subjected to rigorous statistical analysis using a range of techniques in order to quantify parameters associated with each of two generally accepted empirical formulas for describing pillar strength. This facilitated the establishment of correlation, for all strength expressions, between the probability that a formula would yield a successful design versus the respective design factor of safety.

The Australian database was also combined with the much larger and long-established South African database, and the analysis was repeated to determine if the two population bases could be considered as one. A close correlation was obtained, leading to an increased level of confidence in this methodology and to a number of more universal conclusions concerning pillar design.

EMPIRICAL COAL PILLAR STRENGTH ESTIMATIONS

The development of computer and numerical technologies in recent decades has facilitated, at least in principle, the analysis of stresses in pillars and their foundations, i.e., the roof and floor strata. Unfortunately, physical experimentation has not advanced equally rapidly. Hence, the understanding of the intrinsic constitutive laws controlling the behavior of yielding rocks is still unsatisfactory. More immediate problems include the significant discrepancies between the physical properties exhibited by rocks in situ and those measured in the laboratory by testing small specimens. These problems relate to the effects of size and shape on rock strength.

Many investigators have proposed simple empirical formulas to describe the strength of coal pillars. The most common feature of most of these empirical relationships is that they define strength ostensively only in terms of the linear dimensions of the pillars and a multiplying constant, representing the strength of the unit volume of coal. Investigators over the years have proposed formulas that belong to one of two types. One type defines pillar strength simply as a linear function of the width-to-height (w/h) ratio:

$$\mathsf{F}_{s1} = K_1 \left[r + (1 - r)\,\frac{w}{h} \right], \tag{1}$$

where K_1 is the compressive strength of a cube and r is a dimensionless constant. The quantities of w and h are the width and height of the pillar, respectively.

If the notation

$$R = w/h \tag{2}$$

is introduced, then equation 1 becomes

$$F_{s1} = K_1[r + (1 - r)R]. \tag{3}$$

According to this formula, geometrically similar pillars have the same strength regardless of their actual dimensions.

A second commonly used pillar strength formula takes the form

$$F_{s2} = K_2 \left(\frac{w}{w_0} \right)^{\alpha} \left(\frac{h}{h_0} \right)^{\beta}, \tag{4}$$

which is expressed in a dimensionally correct form. α and β are dimensionless parameters; w and h are the linear dimensions of the pillar. Multiplier K_2 is the strength of a reference body of coal of height h_0 and a square cross section with side length w_0.

In most instances, the reference body is taken to be cube of unit volume for convenience's sake, in which case h_0 and w_0 are both unity and can be omitted from the formula. Expressions belonging to this family are referred to as *power law* strength formulas. In contrast to formulas of the form of equation 1, these formulas are also volume-sensitive.

EFFECTIVE WIDTH OF PARALLELEPIPED PILLARS

The development of statistically based pillar design formulas rests minimally upon the premise that a fairly large and tolerably reliable database of unfailed and failed pillar panels can be compiled. Salamon et al. [1996] have identified a number of strict criteria that must be satisfied before a case can be included in the database. One of these that must be appreciated when applying the outcomes of this pillar design research is that these outcomes apply only to competent roof and floor environments, i.e., the database relates only to failures of the coal pillar element of the pillar system, not to the roof or floor elements.

Against this background, an Australian database of 19 failed and 16 unfailed cases was assembled. Rectangular pillars comprised eight of the failed and nine of the unfailed cases. Diamond-shaped pillars comprised one failed case. In order to preserve in these circumstances the availability of the strength formulas derived for square pillars, many researchers have proposed the introduction of an *effective* width.

One of the most basic approaches is to define the effective width, w_e, as

$$w_e = \sqrt{w_1 w_2}\ , \tag{5}$$

where w_1 = minimum pillar width (measured along roadway)

and w_2 = maximum pillar width (measured along roadway).

In situations where w_2 is not extremely different to w_1, this approach has merit. However, when $w_2 \gg w_1$, the equation produces an unrealistic effective pillar width (table 1).

Table 1.—Application of various effective pillar width formulas
(Width and height in meters)

w_1	w_2	h	$\sqrt{w_1 w_2}$	$4A_p/C_p$	w1
100	100	3	100.0	100	100
80	100	3	89.4	88.9	88.9
50	100	3	70.7	66.7	66.7
30	100	3	54.7	46.2	46.2
20	100	3	44.7	33.3	33.3
15	100	3	38.7	26.1	21.7
10	100	3	31.6	18.2	10.7
1	100	3	10.0	2.0	1

The most promising recommendation has come from Wagner [1974, 1980], who, making use of the concept of hydraulic radius, suggested that the effective width be defined as

$$w_e = 4\,\frac{A_p}{C_p}, \tag{6}$$

where A_p and C_p are the cross-sectional area and the circumference of the pillar, respectively.

Application of equation 6 produces effective pillar width similar to that of equation 5 when w_1 is greater than about $0.5w_2$ (table 1). At moderate to low values of w_1 ($0.4w_2 \leq w_1 \leq 0.2w_2$), equation 6 predicts a smaller effective width, which is more sensible from a mechanistic viewpoint. However, at very low values of w_1 ($w_1 < 0.2w_2$), the equation is still considered to overestimate the effective pillar width. This is because when a pillar is narrow, failure is likely to occur across the narrow dimension before sufficient confinement is generated in the longitudinal direction to be of benefit.

This leads to the concept that rectangular and irregular pillars need to be of a critical minimum width before benefit is gained from confinement generated in the longitudinal direction. This benefit can be expected to ramp up to a plateau level as the minimum width increases. Furthermore, it is reasonable to expect that this minimum critical width will be a function of mining height, increasing with increasing mining height.

The need to nominate a minimum critical pillar width has been incorporated into the analysis by modifying equation 6 on the basis that almost all pillars can be regarded as parallelepipeds, i.e., their bases are parallelograms (figure 1). Pillars therefore have side lengths w_1 and w_2 ($w_1 \leq w_2$) and an internal angle $\theta \leq 90°$. Equation 6 then becomes

$$w_{eo} = \zeta_o w, \tag{7}$$

where w is the minimum width of the pillar, i.e.,

$$w = w_1 \sin \theta \tag{8}$$

and the dimensionless factor ζ_o is defined by

$$\zeta_o = \frac{2w_2}{w_1 + w_2}. \tag{9}$$

The range of this factor is $1 \leq \zeta_o < 2$, which is encountered as the aspect ratio moves from unity toward infinity. Experience indicates that much before the complete failure of a pillar, its edges are already yielding. Thus, if the w/h ratio in one direction of a rectangular pillar is low, one of the principal stresses confining its core will remain small, and this stress, together with the maximum stress, will control failure.

Hence, the extra confinement that may arise from the aspect ratio will have little or no effect. It is suggested that such apprehension may be catered for by postulating that the effective width is the minimum width, i.e., $w_e = w$ as long as $R < R_l$, and it becomes $w_e = w_{eo}$ when $R > R_u$.

In the intermediate range, i.e., when $R_l \leq R \leq R_u$, the effective width changes smoothly in accordance with

$$w_e = w\,\zeta_o^{\frac{R-R_l}{R_u-R_l}} = w\zeta. \tag{10}$$

Here, the choice of the limiting w/h ratios is open to judgment. It appears reasonable, however, to use the following values:

$$R_l = 3 \qquad R_u = 6 \tag{11}$$

Table 1 and figure 2 show the effects of the various approaches when applied to calculating the effective width of a 100-m-long, 3-m-high rectangular pillar.

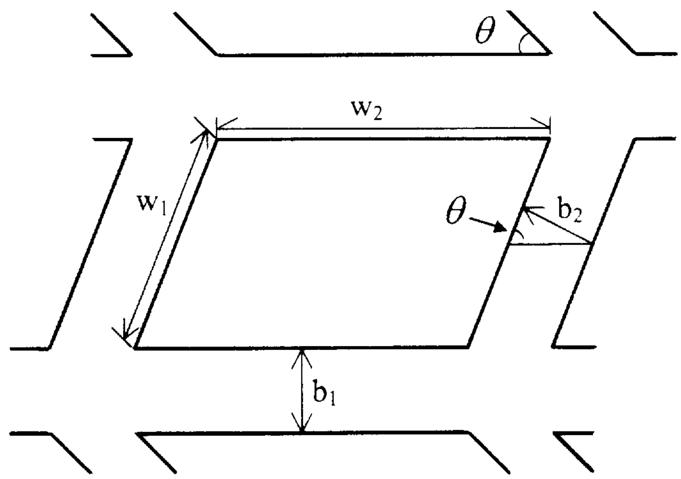

Figure 1.—Definition of mining variables associated with a parallelepiped pillar.

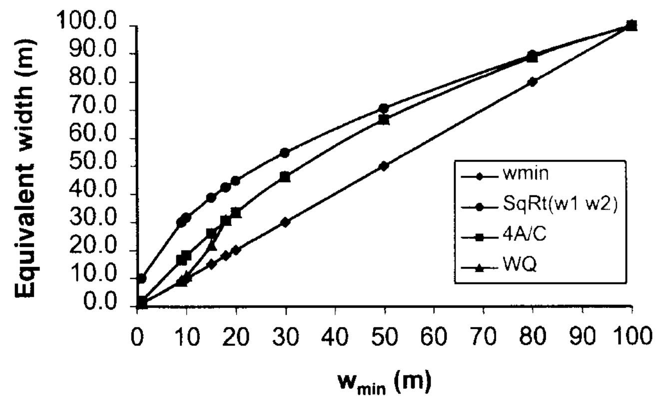

Figure 2.—Comparison of the various proposals for calculating the effective width of a 100-m-long, 3-m-high rectangular pillar.

Using the concept of effective width, the power law in equation 4 can be rewritten for pillars with a general parallelepiped shape:

$$F_{s2} = K_2 w^{\alpha} h^{\beta} \zeta^{\alpha} \tag{12}$$

An alternative form of this formula expresses the strength as the function of the pillar volume V and the w/h ratio R:

$$F_{s2} = K_2 V^{a} R^{b} \zeta^{\alpha}, \tag{13}$$

where the volume refers to a dummy square pillar of width w and height h, and the w/h ratio is calculated from the minimum pillar width:

$$V = w^2 h \qquad R = \frac{w_1 \sin \theta}{h} = \frac{w}{h} \tag{14}$$

The new constants a and b can be defined in terms of constants α and β:

$$a = \frac{1}{3}(\alpha + \beta) \qquad b = \frac{1}{3}(\alpha - 2\beta) \tag{15}$$

Experience has shown that the original power law formula (equation 4) tends to underestimate the strength of squat pillars, i.e., pillars with a w/h ratio in excess of about 5. To cater for this problem, Salamon and Wagner [1985] suggested an extension of equation 4 into the range of higher w/h ratios. This extension, after adaptation to pillars of parallelepiped shape, is

$$F_{s2} = K_2 V^{a} R_o^{b} \Theta^{\alpha} \left\{ \frac{b}{\varepsilon} \left[\left(\frac{R}{R_o} \right)^{\varepsilon} - 1 \right] + 1 \right\}, \tag{16}$$

which is valid if $R > R_o$ and where Θ is defined in equation 10. This particular form was chosen to ensure that there is a smooth transition between this and equation 13 at $R = R_o$ [Salamon and Wagner 1985]. Here, R_o and ε are appropriately chosen constants. The expression is often referred to as the *squat pillar strength* formula. Since its inception, it has been applied widely in the Republic of South Africa using the following pair of constants:

$$R_o = 5 \qquad \varepsilon = 2.5 \tag{17}$$

In critical situations, the judgment exercised in deriving the effective pillar width relationship may be regarded as too speculative. This concern can be addressed by either choosing an elevated design factor of safety to account for this level of uncertainty or reverting to the use of the minimum pillar width in pillar strength calculations.

Another aspect to the use of rectangular pillars is the calculation of pillar load. In calculating the tributary load, the true dimensions need to be employed. Thus, the pillar load assumes the following form:

$$q_m = \gamma H \delta \frac{(w + b_1)(w_2 + b_2 / \sin \theta)}{w w_2} \tag{18}$$

In this relationship, δ is a modifier. It is unity in all cases where the pillar burden is the conventional tributary load. If, however, due to secondary extraction the pillar load is believed to differ from this value, the load can be adjusted by applying this factor. Moreover, to remain consistent with earlier calculations, γ is taken to be: $\gamma = 1.1$ psi/ft $= 24.8827$ kN/m^3 $= 24.8827$ kPa/m.

UNSW INITIAL DESIGN FORMULAS

In 1992, following a number of serious incidents related to the lack of restriction on pillar height, the Chief Inspector of Coal Mines in New South Wales required operators to obtain approval to mine at heights exceeding 4 m. To address the need for a pillar design methodology, the UNSW research team undertook in 1995 a preliminary analysis of its database [Hocking et al. 1995]. At the time, the database comprised 14 collapsed cases and 16 stable cases that satisfied the selection criteria. The database was analyzed statistically using the full maximum likelihood method. Galvin and Hebblewhite [1995] subsequently published the following pillar design formulas, which find current application in Australia:

$$F_{s2} = 7.4 \frac{w^{0.46}}{h^{0.66}} \quad \text{(MPa)} \tag{19a}$$

and its squat pillar version ($R > 5$):

$$F_{s2} = \frac{19.24}{w^{0.133} h^{0.067}} \left\{ 0.237 \left[\left(\frac{w}{5h} \right)^{2.5} - 1 \right] + 1 \right\} \quad \text{(MPa)} \tag{19b}$$

A conservative approach was adopted, and the minimum pillar width was proposed as the effective width. It follows, therefore, that $\Theta = 1$ in these expressions. There was little difference in the pillar strength obtained by allowing all parameters to float in the statistical analysis as opposed to allowing only the K values to float and fixing the other parameters to be the same as those used for many years in the Republic of South Africa. To avoid confusion and to facilitate the introduction of the formulas, therefore, only those formulas derived by allowing the K values to float were presented to operators. The formula for strength based on the linear relationship took the following form:

$$F_{s1} = 5.36(0.64 + 0.36R) \quad \text{(MPa)} \tag{20}$$

UNSW REFINED (RECTANGULAR) FORMULAS

In 1996, a more comprehensive statistical analysis of the expanded Australian database was completed that incorporated the effective width of rectangular pillars as defined earlier [Salamon et al. 1996]. Statistical methods included least squares, limited maximum likelihood, and full maximum likelihood. Both power law models and linear law models were evaluated, and all parameters were allowed to float. In all instances, the power law model gave better correlations.

The following strength formulas were found to best describe the observed behavior of pillars in New South Wales and Queensland:

$$F_{s2} = 8.60 \frac{(w\Theta)^{0.51}}{h^{0.84}} \quad \text{(MPa)} \qquad (21a)$$

The corresponding expression for squat pillars is given by

$$F_{s2} = \frac{27.63\,\Theta^{0.51}}{w^{0.220}\,h^{0.110}} \left\{ 0.290 \left[\left(\frac{w}{5h} \right)^{2.5} - 1 \right] + 1 \right\} \quad \text{(MPa)} \qquad (21b)$$

In these expressions, $w = w_1 \sin \theta$, and the effective width factor Θ is as defined in equation 10.

The relationship between pillar strength and pillar load produced by these equations for each point in the database is shown in figure 3. Design factors of safety associated with the probability of achieving a stable design are shown in table 2.

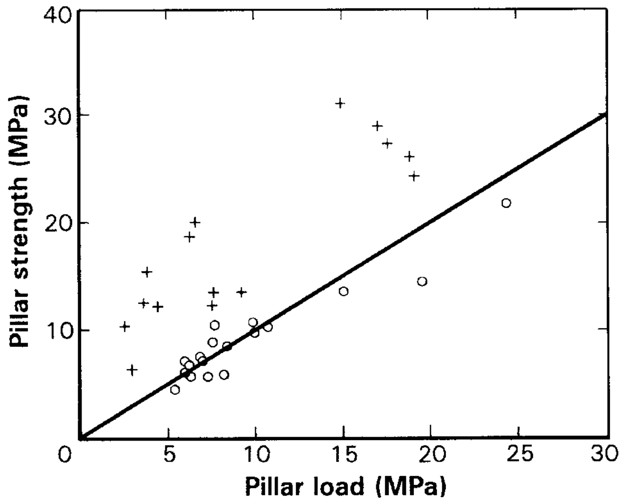

Figure 3.—Pillar strength and pillar load relationship for both the failed (o) and unfailed (+) Australian cases.

Table 2.—Probability of failure versus factor of safety

Probability of failure	Factor of safety
8 in 10	0.87
5 in 10	1.00
1 in 10	1.22
5 in 100	1.30
2 in 100	1.38
1 in 100	1.44
1 in 1,000	1.63
1 in 10,000	1.79
1 in 100,000	1.95
1 in 1,000,000	2.11

REANALYSIS OF THE SOUTH AFRICAN DATABASE

The original extensive South African coal pillar database used by Salamon and Munro in 1966 has since been updated and supplemented by Madden and Hardman [1992]. This combined South African database comprises 44 failed and 98 unfailed cases. It has also been reanalyzed using the same statistical techniques used for the Australian database. Two failed cases were later omitted from the data set [Salamon et al. 1996].

This analysis has produced the following strength formulas:

$$F_{s2} = 6.88 \frac{(w\Theta)^{0.42}}{h^{0.60}} \quad \text{(MPa)} \qquad (22a)$$

The corresponding expression for squat pillars ($R > 5$) is given by

$$F_{s2} = \frac{16.36\,\Theta^{0.42}}{w^{0.116}\,h^{0.058}} \left\{ 0.215 \left[\left(\frac{w}{5h} \right)^{2.5} - 1 \right] + 1 \right\} \quad \text{(MPa)} \qquad (22b)$$

The linear version of the strength estimator is simply

$$F_{s1} = 5.60(0.69 + 0.31R) \quad \text{(MPa)} \qquad (23)$$

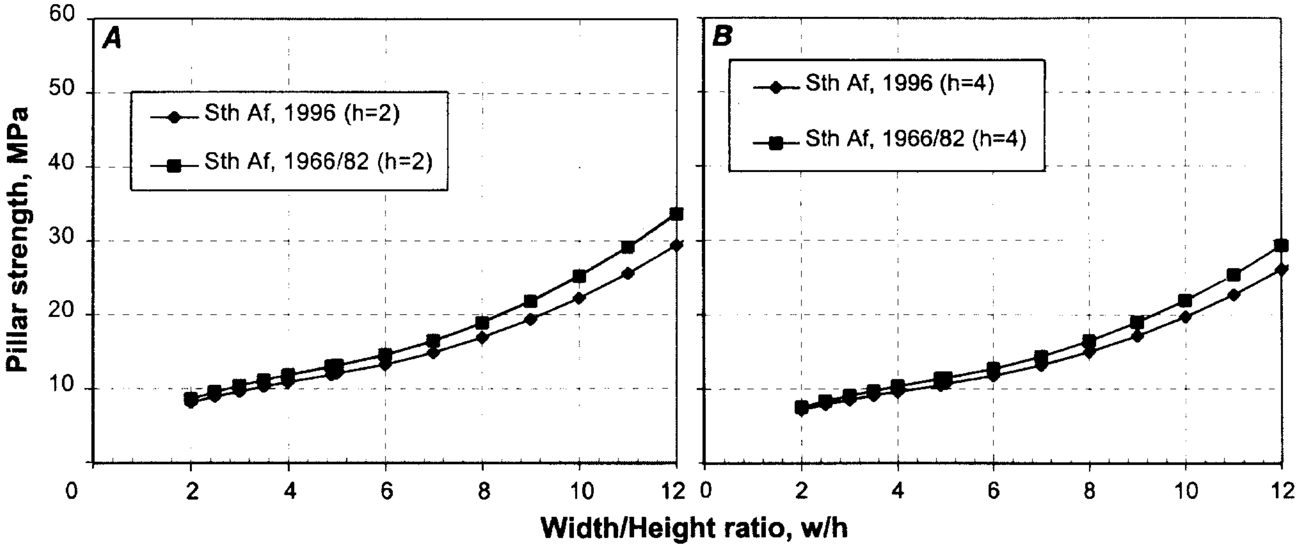

Figure 4.—Comparison between South African power formulas, 1966/82 and 1996. ***A*****, $h = 2$ m;** ***B*****, $h = 4$ m.**

Figure 4 shows the comparison between the pillar strength produced by equations 22a and 22b and that predicted by the original Salamon and Munro formula and its modified squat pillar form. In the case of a mining height of 2 m, the figure shows that for a given pillar strength, pillars designed with the updated formulas may need to be about 2 m wider. For a bord width of 6 m at a w/h ratio of 10, this results in about 3% less resource recovery. For similar circumstances in a 4-m mining height, the increase in pillar size is on the order of 3.2 m.

COMBINED AUSTRALIAN AND SOUTH AFRICAN DATABASES

A further step in the research program was to combine the South African and Australian databases and to analyze them as a combined population, then compare and contrast them with the two independent data populations for each country. This combined database comprised 177 cases of pillar systems, including 61 collapsed cases. This produced the following formulas:

$$F_{s2} = 6.88\,\frac{\sqrt{w1}}{h^{0.7}} \quad \text{(MPa)} \qquad (24a)$$

For R > 5, the squat version of this expression takes the following form:

$$F_{s2} = \frac{19.05\sqrt{1}}{w^{0.133}\,h^{0.066}}\left\{0.253\left[\left(\frac{w}{5h}\right)^{2.5} - 1\right] + 1\right\} \quad \text{(MPa)} \qquad (24b)$$

The corresponding linear formula is simply

$$F_{s1} = 5.41(0.63 + 0.37R) \quad \text{(MPa)} \qquad (25)$$

Figure 5 shows failed and unfailed cases in the load plane. The figure illustrates a fairly good discrimination between the two sets of points. Only one unfailed point occurs on the wrong side of the $s = 1$ line, and the median failed cases is 1.039.

Figure 6 shows a comparison between pillar strengths using power law estimators derived from the Australian, South African, and combined Australian-South African databases. The closeness of the predictions is remarkable considering the geographical separation of the Australian and South African coalfields.

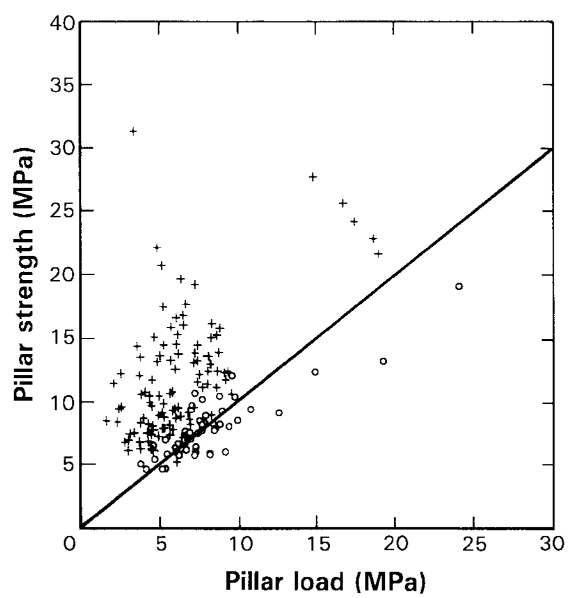

Figure 5.—The failed (o) and unfailed (+) cases in a pillar strength versus pillar load plot using the combined Australian-South African database.

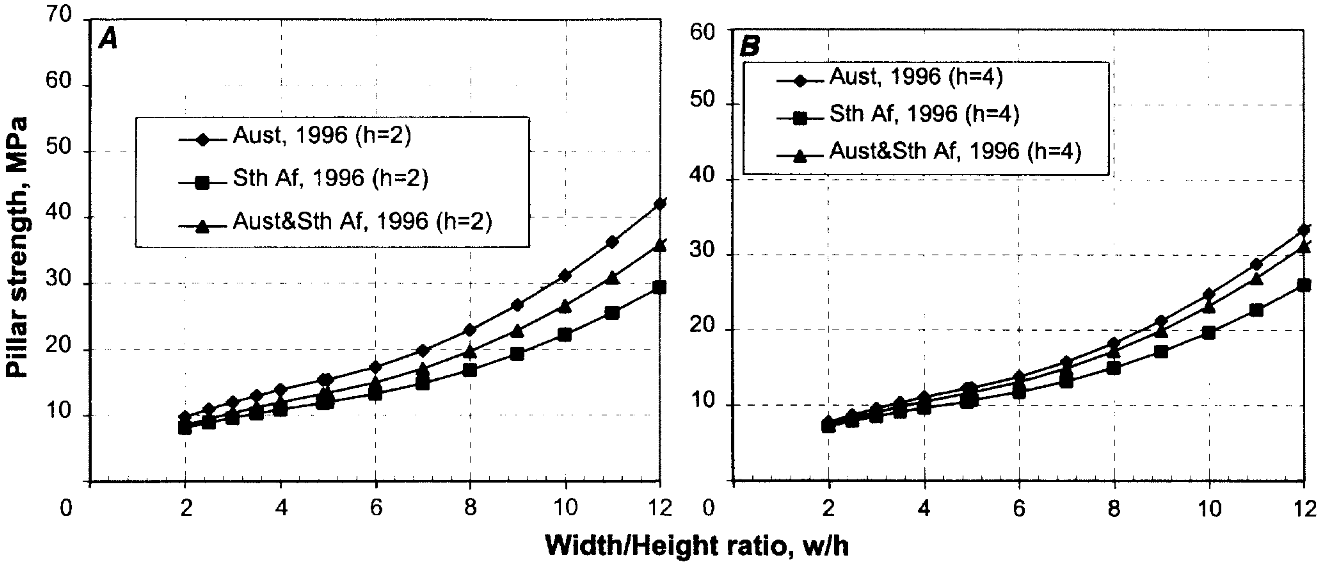

Figure 6.—Comparison between power law strength formulas derived for the Australian, South African, and combined databases. *A*, h ' 2 m; *B*, h ' 4 m.

CONCLUSIONS

The statistical analysis of the Australian database indicates that the method proposed for calculating the effective width of parallelepiped pillars produced sensible outcomes. However, it must be remembered that, although of sufficient size to be statistically significant, the parallelepiped database is small. The method should therefore be used with caution.

In order to enhance confidence in the pillar design procedure, including the use of the effective pillar width method, additional research was undertaken. It was noted that the formula derived from the initial Australian database closely resembled the original Salamon-Munro expression. This somewhat surprising resemblance prompted further research and enlargement of the database. The larger database yielded pillar strengths that again were similar to those obtained from the initial UNSW research and by Salamon and Munro. The combination of the Australian and South African databases reinforced the original impression, namely, that the underlying pillar strengths in these countries resembled each other closely.

The outcome of the investigation lends support to the view expressed by Mark and Barton [1996]. They suggested that strength values obtained in the laboratory cannot be utilized in a meaningful way in pillar design and that the variation in the strength of pillars of the same size can be disregarded in many instances. Mark and Barton [1996] emphasize that they do not claim that the in situ strength of all U.S. coal is the same. Their study merely showed that a uniform strength is a better approximation than one based on laboratory testing. Although the UNSW research conclusions are encouraging, complacency is not justified. The formulas are based on competent roof and floor conditions. Significantly different pillar strengths may be associated with abnormal strata behavior mechanisms. Because pillars with w/h ratios greater than 10 have not been tested to destruction, it must also be recognized that neither linear nor power law formulas have been validated at w/h ratios greater than about 8.

It cannot be overemphasized that, because the design formulas have been developed on a probabilistic basis, they need to be reviewed periodically as the database expands and the understanding of pillar mechanics advances. A fundamental rule of empirical research is that the results should be used within the range of data used in their derivation. Extrapolation with empirical formulas is always fraught with danger.

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