Web-based Supplementary Materials for "A Modified Partial Likelihood Score Method for Cox Regression with Covariate Error Under the Internal Validation Design" by David M. Zucker, Xin Zhou, Xiaomei Liao, Yi Li, and Donna Spiegelman

Introduction, Notation, and Preliminaries

In this document we present the details of the asymptotic theory for the modified score estimator. At the end of the document we present tables with the simulation results for the common disease case that we referred to in the main paper. Denote the true value of $\boldsymbol{\theta}$ by $\boldsymbol{\theta}^*$. We assume that $\boldsymbol{\theta}$ lies in a (suitably large) compact set $\mathcal{Q}$ whose interior contains $\boldsymbol{\theta}^*$. Two additional assumptions will be introduced later, with clear signposting. Let τ denote the maximum follow-up time. All assertions below of uniform convergence of functions of $\boldsymbol{\theta}$ and/or t refer to $\boldsymbol{\theta} \in \mathcal{Q}$ and $t \in [0, \tau]$.

We use the notation used in the main paper. From now on, in the various S's defined in the main paper we will write $\boldsymbol{\theta}$ instead of $\boldsymbol{\beta}, \boldsymbol{\alpha}$. We now define some additional notation.

$$
\begin{aligned}
\mathbf{S}_{1e}(t, \boldsymbol{\theta}) &= \frac{1}{n}\sum_{j=1}^{n}(1-\omega_j)Y_j(t)\widehat{\mathbf{X}}_j(\boldsymbol{\alpha})\exp(\boldsymbol{\beta}^T\mathbf{X}_j) \\
\mathbf{S}_{2a}(t, \boldsymbol{\theta}) &= \frac{1}{n}\sum_{j=1}^{n}\omega_j Y_j(t)\mathbf{X}_j\mathbf{X}_j^T\exp(\boldsymbol{\beta}^T\mathbf{X}_j) \\
\mathbf{S}_{2b}(t, \boldsymbol{\theta}) &= \frac{1}{n}\sum_{j=1}^{n}(1-\omega_j)Y_j(t)\widehat{\mathbf{X}}(\boldsymbol{\alpha})\widehat{\mathbf{X}}(\boldsymbol{\alpha})^T\exp\{\boldsymbol{\beta}^T\widehat{\mathbf{X}}(\boldsymbol{\alpha})\} \\
\mathbf{S}_{2c}(t, \boldsymbol{\theta}) &= \frac{1}{n}\sum_{j=1}^{n}\omega_j Y_j(t)\widehat{\mathbf{X}}(\boldsymbol{\alpha})\widehat{\mathbf{X}}(\boldsymbol{\alpha})^T\exp\{\boldsymbol{\beta}^T\widehat{\mathbf{X}}(\boldsymbol{\alpha})\} \\
\mathbf{S}_{2d}(t, \boldsymbol{\theta}) &= \frac{1}{n}\sum_{j=1}^{n}\omega_j Y_j(t)\widehat{\mathbf{X}}(\boldsymbol{\alpha})\mathbf{X}^T\exp(\boldsymbol{\beta}^T\mathbf{X}) \\
\mathcal{E}_W(t, \boldsymbol{\theta}) &= E[Y(t)\exp\{\boldsymbol{\beta}^T\widehat{\mathbf{X}}(\boldsymbol{\alpha})\}] \\
\boldsymbol{\mathcal{E}}_{WW}(t, \boldsymbol{\theta}) &= E[Y(t)\widehat{\mathbf{X}}(\boldsymbol{\alpha})\exp\{\boldsymbol{\beta}^T\widehat{\mathbf{X}}(\boldsymbol{\alpha})\}] \\
\boldsymbol{\mathcal{E}}_{XXX}(t, \boldsymbol{\theta}) &= E[Y(t)\mathbf{X}\mathbf{X}^T\exp(\boldsymbol{\beta}^T\mathbf{X})] \\
\boldsymbol{\mathcal{E}}_{WWW}(t, \boldsymbol{\theta}) &= E[Y(t)\widehat{\mathbf{X}}(\boldsymbol{\alpha})\widehat{\mathbf{X}}(\boldsymbol{\alpha})^T\exp\{\boldsymbol{\beta}^T\widehat{\mathbf{X}}(\boldsymbol{\alpha})\}] \\
\boldsymbol{\mathcal{E}}_{WXX}(t, \boldsymbol{\theta}) &= E[Y(t)\widehat{\mathbf{X}}(\boldsymbol{\alpha})\mathbf{X}^T\exp\{\boldsymbol{\beta}^T\widehat{\mathbf{X}}(\boldsymbol{\alpha})\}]
\end{aligned}
$$

In addition, we define $\overline{N}(t) = n^{-1}\sum_{i=1}^{n} N_i(t)$ and $\mathcal{N}(t) = E\{N(t)\}$. We now introduce the following assumption:

Assumption 1: $\mathcal{E}_X(t, \boldsymbol{\theta})$ is bounded below over t and $\boldsymbol{\theta}$.

This assumption implies that there is a positive probability of reaching the maximum follow-up time τ with neither an event nor a censoring in the open interval $(0, \tau)$.

In view of the functional strong law of large numbers (Andersen and Gill, 1982, Appendix III), the following convergence relations hold (uniformly in t and $\boldsymbol{\theta}$) almost surely:

$$S_{0a}(t,\boldsymbol{\theta}) \to \pi\mathcal{E}_X(t,\boldsymbol{\theta}), \quad S_{0b}(t,\boldsymbol{\theta}) \to (1-\pi)\mathcal{E}_W(t,\boldsymbol{\theta}), \quad S_{0c}(t,\boldsymbol{\theta}) \to \pi\mathcal{E}_W(t,\boldsymbol{\theta}) \tag{1}$$

$$S_{0d}(t,\boldsymbol{\theta}) \to \mathcal{E}_X(t,\boldsymbol{\theta}), \quad \mathbf{S}_{1a}(t,\boldsymbol{\theta}) \to \pi\boldsymbol{\mathcal{E}}_{XX}(t,\boldsymbol{\theta}), \quad \mathbf{S}_{1b}(t,\boldsymbol{\theta}) \to (1-\pi)\boldsymbol{\mathcal{E}}_{WW}(t,\boldsymbol{\theta}) \tag{2}$$

$$\mathbf{S}_{1c}(t,\boldsymbol{\theta}) \to \pi\boldsymbol{\mathcal{E}}_{WW}(t,\boldsymbol{\theta}), \quad \mathbf{S}_{1d}(t,\boldsymbol{\theta}) \to \pi\boldsymbol{\mathcal{E}}_{WX}(t,\boldsymbol{\theta}), \quad \mathbf{S}_{1e}(t,\boldsymbol{\theta}) \to (1-\pi)\boldsymbol{\mathcal{E}}_{WX}(t,\boldsymbol{\theta}) \tag{3}$$

$$\mathbf{S}_{2a}(t,\boldsymbol{\theta}) \to \pi\boldsymbol{\mathcal{E}}_{XXX}(t,\boldsymbol{\theta}), \quad \mathbf{S}_{2b}(t,\boldsymbol{\theta}) \to (1-\pi)\boldsymbol{\mathcal{E}}_{WWW}(t,\boldsymbol{\theta}) \tag{4}$$

$$\mathbf{S}_{2c}(t,\boldsymbol{\theta}) \to \pi\boldsymbol{\mathcal{E}}_{WWW}(t,\boldsymbol{\theta}), \quad \mathbf{S}_{2d}(t,\boldsymbol{\theta}) \to \pi\boldsymbol{\mathcal{E}}_{WXX}(t,\boldsymbol{\theta}) \tag{5}$$

It follows that the following convergence relations hold (uniformly in t and $\boldsymbol{\theta}$) almost surely (note that $\phi\pi = 1 - \pi$):

$$S_0(t,\boldsymbol{\beta}) \to \boldsymbol{\mathcal{E}}_X(t,\boldsymbol{\theta}), \quad \mathbf{S}_1(t,\boldsymbol{\theta}) \to \mathbf{s}_1(t,\boldsymbol{\theta}) \tag{6}$$

Also, it is clear that $\widehat{\boldsymbol{\alpha}} \to \boldsymbol{\alpha}^*$ almost surely.

Recall the counting process representation of the score function $\mathbf{U}_{MS}(\boldsymbol{\theta})$:

$$\mathbf{U}_{MS}(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n}\int_0^\tau \{\omega_i\mathbf{X}_i + (1-\omega_i)\widehat{\mathbf{X}}_i(\boldsymbol{\alpha})\}dN_i(t) - \int_0^\tau \frac{\mathbf{S}_1(t,\boldsymbol{\theta})}{S_0(t,\boldsymbol{\theta})}d\overline{N}(t) \tag{7}$$

We can write

$$\mathbf{U}_{MS}(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n}\int_0^\tau \{\omega_i\mathbf{X}_i + (1-\omega_i)\widehat{\mathbf{X}}_i(\boldsymbol{\alpha})\}dN_i(t) - \int_0^\tau \frac{\mathbf{s}_1(t,\boldsymbol{\theta})}{\mathcal{E}_X(t,\boldsymbol{\theta})}d\overline{N}(t) + R_n(\boldsymbol{\theta})$$

with

$$R_n(\boldsymbol{\theta}) = \int_0^\tau \left\{\frac{\mathbf{S}_1(t,\boldsymbol{\theta})}{S_0(t,\boldsymbol{\theta})} - \frac{\mathbf{s}_1(t,\boldsymbol{\theta})}{\mathcal{E}_X(t,\boldsymbol{\theta})}\right\} d\overline{N}(t)$$

Given the uniform convergence of $S_0(t,\boldsymbol{\theta})$ and $\mathbf{S}_1(t,\boldsymbol{\theta})$, the assumption that $\mathcal{E}_X(t,\boldsymbol{\theta})$ is bounded below, and the fact that $\overline{N}(\tau) \leq 1$, we obtain the result that $R_n(\boldsymbol{\theta}) \to 0$ uniformly in $\boldsymbol{\theta}$.

Recall now the definition $dM_i(t) = dN_i(t) - Y_i(t)\exp\{\boldsymbol{\beta}^{*T}\mathbf{X}_i(t)\}\lambda_0(t)dt$. According to counting process theory (Gill 1984), $M_i(t)$ is a mean-zero martingale process w.r.t. the history defined by $\mathcal{F}_t = \sigma(\mathbf{X}_i; Y_i(s),\ N_i(s), s \in [0,t], i = 1, \ldots, n)$. Given the noninformative measurement error assumption and the fact that ω_i is independent of all the other basic random variables associated with individual i, $M_i(t)$ is also a martingale w.r.t. the history defined by $\mathcal{G}_t = \sigma(\mathbf{X}_i, \mathbf{W}_i, \omega_i, Y_i(s), N_i(s), s \in [0,t], i = 1, \ldots, n)$.

Consistency

We can write

$$\frac{1}{n}\sum_{i=1}^{n}\int_0^{\tau} \omega_i \mathbf{X}_i \, dN_i(t) = \frac{1}{n}\sum_{i=1}^{n}\int_0^{\tau} \omega_i \mathbf{X}_i \, dM_i(t) + \int_0^{\tau} \mathbf{S}_{1a}(\boldsymbol{\beta}^*, \boldsymbol{\alpha})\lambda_0(t)\, dt$$

and

$$\frac{1}{n}\sum_{i=1}^{n}\int_0^{\tau} (1-\omega_i)\widehat{\mathbf{X}}_i(\boldsymbol{\alpha}) \, dN_i(t) = \frac{1}{n}\sum_{i=1}^{n}\int_0^{\tau} (1-\omega_i)\widehat{\mathbf{X}}_i(\boldsymbol{\alpha}) \, dM_i(t) + \int_0^{\tau} \mathbf{S}_{1e}(\boldsymbol{\beta}^*, \boldsymbol{\alpha})\lambda_0(t)\, dt$$

Similarly,

$$\int_0^{\tau} \frac{\mathbf{s}_1(t,\boldsymbol{\theta})}{\mathcal{E}_X(t,\boldsymbol{\theta})}\, d\overline{N}(t) = \frac{1}{n}\sum_{i=1}^{n}\int_0^{\tau} \frac{\mathbf{s}_1(t,\boldsymbol{\theta})}{\mathcal{E}_X(t,\boldsymbol{\theta})}\, dM_i(t) + \int_0^{\tau} \frac{\mathbf{s}_1(t,\boldsymbol{\theta})}{\mathcal{E}_X(t,\boldsymbol{\theta})}\, S_{0d}(t,\boldsymbol{\theta}^*)\lambda_0(t) dt$$

Hence,

$$\begin{aligned}\mathbf{U}_{MS}(\boldsymbol{\theta}) \;&=\; \int_0^{\tau} \mathbf{S}_{1a}(\boldsymbol{\beta}^*, \boldsymbol{\alpha})\lambda_0(t)\, dt + \int_0^{\tau} \mathbf{S}_{1e}(\boldsymbol{\beta}^*, \boldsymbol{\alpha})\lambda_0(t)\, dt - \int_0^{\tau} \frac{\mathbf{s}_1(t,\boldsymbol{\theta})}{\mathcal{E}_X(t,\boldsymbol{\theta})}\, S_{0d}(t,\boldsymbol{\theta}^*)\lambda_0(t) dt \\ &\quad + R_n(\boldsymbol{\theta}) + \mathcal{M}(\boldsymbol{\theta})\end{aligned}$$

where

$$\mathcal{M}(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n}\int_0^{\tau} \left\{ \omega_i \mathbf{X}_i + (1-\omega_i)\mathbf{W}_i - \frac{\mathbf{s}_1(t,\boldsymbol{\theta})}{\mathcal{E}_X(t,\boldsymbol{\theta})} \right\} dM_i(t)$$

Now, $\mathcal{M}(\boldsymbol{\theta})$ is the sample mean of i.i.d. mean-zero r.v.'s, and therefore converges a.s. to 0, and since $\mathcal{Q}$ is compact and $\mathcal{E}_X(t,\boldsymbol{\theta})$ and $\mathbf{s}_1(t,\boldsymbol{\theta})$ are continuous in $\boldsymbol{\theta}$ (uniformly over t) the convergence is uniform in $\boldsymbol{\theta}$. Using this result, along with the convergence results (1)-(5) and the fact that $\widehat{\boldsymbol{\alpha}} \to \boldsymbol{\alpha}^*$ a.s., we find that $\mathbf{U}_{MS}(\boldsymbol{\beta}, \widehat{\boldsymbol{\alpha}})$ converges uniformly almost surely to

$$\mathbf{u}(\boldsymbol{\beta}) = \int_0^\tau \left\{ \mathbf{s}_1(t, \boldsymbol{\theta}^*) - \frac{\mathbf{s}_1(t, \boldsymbol{\beta}, \boldsymbol{\alpha}^*)}{\mathcal{E}_X(t, \boldsymbol{\beta}, \boldsymbol{\alpha}^*)} \, \mathcal{E}_X(t, \boldsymbol{\theta}^*) \right\} \lambda_0(t) \, dt.$$

We have that -1 times the matrix of derivatives of $\mathbf{U}_{MS}(\boldsymbol{\theta})$ with respect to $\boldsymbol{\beta}$ is given by

$$\mathbf{D}_{\beta\beta}(\boldsymbol{\theta}) = \int_0^\tau \left[\frac{\dot{\mathbf{S}}_1(t, \boldsymbol{\theta})}{S_0(t, \boldsymbol{\theta})} - \left\{ \frac{\mathbf{S}_1(t, \boldsymbol{\theta})}{S_0(t, \boldsymbol{\theta})} \right\} \left\{ \frac{\dot{\mathbf{S}}_0(t, \boldsymbol{\theta})}{S_0(t, \boldsymbol{\theta})} \right\}^T \right] d\overline{N}(t) \tag{8}$$

where

$$\dot{\mathbf{S}}_0(t, \boldsymbol{\theta}) = \mathbf{S}_{1a}(t, \boldsymbol{\theta}) + \mathbf{S}_{1b}(t, \boldsymbol{\theta}) \left\{ \frac{S_{0a}(t, \boldsymbol{\theta})}{S_{0c}(t, \boldsymbol{\theta})} \right\} + S_{0b}(t, \boldsymbol{\theta}) \left[\frac{\mathbf{S}_{1a}(t, \boldsymbol{\theta})}{S_{0c}(t, \boldsymbol{\theta})} - \left\{ \frac{S_{0a}(t, \boldsymbol{\theta})}{S_{0c}(t, \boldsymbol{\theta})} \right\} \left\{ \frac{\mathbf{S}_{1c}(t, \boldsymbol{\theta})}{S_{0c}(t, \boldsymbol{\theta})} \right\} \right]$$

$$\begin{aligned} \dot{\mathbf{S}}_1(t, \boldsymbol{\theta}) = {} & \mathbf{S}_{2a}(t, \boldsymbol{\theta}) + \mathbf{S}_{2b}(t, \boldsymbol{\theta}) + \left\{ \frac{S_{0b}(t, \boldsymbol{\theta})}{S_{0c}(t, \boldsymbol{\theta})} \right\} (\mathbf{S}_{2d}(t, \boldsymbol{\theta}) - \mathbf{S}_{2c}(t, \boldsymbol{\theta})) \\ & + (\mathbf{S}_{1d}(t, \boldsymbol{\theta}) - \mathbf{S}_{1c}(t, \boldsymbol{\theta})) \left[\frac{\mathbf{S}_{1b}(t, \boldsymbol{\theta})}{S_{0c}(t, \boldsymbol{\theta})} - \left\{ \frac{\mathbf{S}_{0b}(t, \boldsymbol{\theta})}{S_{0c}(t, \boldsymbol{\theta})} \right\} \left\{ \frac{\mathbf{S}_{1c}(t, \boldsymbol{\theta})}{S_{0c}(t, \boldsymbol{\theta})} \right\} \right]^T \end{aligned}$$

The limiting value of $\mathbf{D}_{\beta\beta}(\boldsymbol{\theta})$ is given by

$$\begin{aligned} \mathbf{d}_{\beta\beta}(\boldsymbol{\theta}) = {} & \pi \int_0^\tau \left[\frac{\boldsymbol{\mathcal{E}}_{XXX}(t, \boldsymbol{\theta})}{\mathcal{E}_X(t, \boldsymbol{\theta})} - \left\{ \frac{\boldsymbol{\mathcal{E}}_{XX}(t, \boldsymbol{\theta})}{\mathcal{E}_X(t, \boldsymbol{\theta})} \right\} \left\{ \frac{\boldsymbol{\mathcal{E}}_{XX}(t, \boldsymbol{\theta})}{\mathcal{E}_X(t, \boldsymbol{\theta})} \right\}^T \right] \mathcal{E}_X(t, \boldsymbol{\theta}^*) \lambda_0(t) dt \\ & + (1 - \pi) \int_0^\tau \left[\frac{\boldsymbol{\mathcal{E}}_{WXX}(t, \boldsymbol{\theta})}{\mathcal{E}_X(t, \boldsymbol{\theta})} - \left\{ \frac{\boldsymbol{\mathcal{E}}_{WX}(t, \boldsymbol{\theta})}{\mathcal{E}_X(t, \boldsymbol{\theta})} \right\} \left\{ \frac{\boldsymbol{\mathcal{E}}_{XX}(t, \boldsymbol{\theta})}{\mathcal{E}_X(t, \boldsymbol{\theta})} \right\}^T \right] \mathcal{E}_X(t, \boldsymbol{\theta}^*) \lambda_0(t) dt \end{aligned}$$

which is -1 times the matrix of derivatives of $\mathbf{u}(\boldsymbol{\theta})$ with respect to $\boldsymbol{\beta}$. The first term in the above expression is a positive definite matrix for all $\boldsymbol{\theta}$.

At this point, we introduce an additional assumption:

Assumption 2: $\mathbf{d}_{\beta\beta}(\boldsymbol{\theta}^*)$ is nonsingular.

By inspection of $\mathbf{u}(\boldsymbol{\theta})$, we see that $\mathbf{u}(\boldsymbol{\beta}^*, \boldsymbol{\alpha}^*) = \mathbf{0}$. Given this fact, the assumed nonsingularity of $\mathbf{d}_{\beta\beta}(\boldsymbol{\theta}^*)$, the convergence of $\widehat{\boldsymbol{\alpha}}$ to $\boldsymbol{\alpha}$, and the uniform convergence of $\mathbf{U}_{MS}(\boldsymbol{\theta})$ to $\mathbf{u}(\boldsymbol{\theta})$ and $\mathbf{D}_{\beta\beta}(\boldsymbol{\theta})$ to $\mathbf{d}_{\beta\beta}(\boldsymbol{\theta})$, it follows from the arguments of Foutz (1977) that there exists a unique root of the score equation $\mathbf{U}_{MS}(\boldsymbol{\beta}, \widehat{\boldsymbol{\alpha}}) = \mathbf{0}$ that converges to $\boldsymbol{\beta}^*$.

Asymptotic Distribution of $\widehat{\boldsymbol{\beta}}_{MS}$

The background for the derivation of the asymptotic distribution of $\widehat{\boldsymbol{\beta}}_{MS}$ is presented in Section 2.2 of the main paper. We start here with the derivation of $\mathbf{U}^{(1)*}(\boldsymbol{\theta})$. As in Zucker and Spiegelman (2008), we use the argument of Lin and Wei (1989). In what follows, the symbol $\doteq$ will denote equality up to negligible terms.

Let us recall the counting process representation of $\mathbf{U}^{(1)}(\boldsymbol{\theta})$:

$$\mathbf{U}^{(1)}(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n}\int_0^{\tau}\{\omega_i\mathbf{X}_i + (1-\omega_i)\widehat{\mathbf{X}}_i(\boldsymbol{\alpha})\}dN_i(t) - \int_0^{\tau}\frac{\mathbf{S}_1(t,\boldsymbol{\theta})}{S_0(t,\boldsymbol{\theta})}d\overline{N}(t)$$

We now proceed to analyze this quantity. For brevity, we will omit the arguments t and $\boldsymbol{\theta}$. We can write $\mathbf{U}^{(1)}(\boldsymbol{\theta}) = \mathbf{U}_a^{(1)}(\boldsymbol{\theta}) - \mathbf{U}_b^{(1)}(\boldsymbol{\theta})$ with

$$\begin{aligned}
\mathbf{U}_a^{(1)}(\boldsymbol{\theta}) &= \frac{1}{n}\sum_{i=1}^{n}\int_0^{\tau}\left\{\omega_i\mathbf{X}_i + (1-\omega_i)\widehat{\mathbf{X}}_i(\boldsymbol{\alpha}) - \frac{\widehat{\mathbf{s}}_1}{\mathcal{E}_X}\right\}dN_i \\
\mathbf{U}_b^{(1)}(\boldsymbol{\theta}) &= \int_0^{\tau}\left(\frac{\mathbf{S}_1}{S_0} - \frac{\widehat{\mathbf{s}}_1}{\mathcal{E}_X}\right)d\overline{N} \doteq \int_0^{\tau}\left(\frac{\mathbf{S}_1}{S_0} - \frac{\widehat{\mathbf{s}}_1}{\mathcal{E}_X}\right)d\mathcal{N}
\end{aligned}$$

where $\widehat{\mathbf{s}}_1(t,\boldsymbol{\beta},\boldsymbol{\alpha}) = \widehat{\pi}\boldsymbol{\mathcal{E}}_{XX}(t,\boldsymbol{\beta},\boldsymbol{\alpha}) + (1-\widehat{\pi})\boldsymbol{\mathcal{E}}_{WX}(t,\boldsymbol{\beta},\boldsymbol{\alpha})$. Regarding $\mathbf{U}_a^{(1)}(\boldsymbol{\theta})$, after some manipulation we obtain

$$\begin{aligned}
\mathbf{U}_a^{(1)}(\boldsymbol{\theta}) = &\frac{1}{n}\sum_{i=1}^{n}\omega_i\left\{\int_0^{\tau}\left(\mathbf{X}_i - \frac{\mathcal{E}_{XX}}{\mathcal{E}_X}\right)dN_i - (1-\widehat{\pi})\int_0^{\tau}\frac{\mathcal{E}_{WX}-\mathcal{E}_{XX}}{\mathcal{E}_X}(dN_i - d\mathcal{N})\right\} \\
&+ \frac{1}{n}\sum_{i=1}^{n}(1-\omega_i)\left\{\int_0^{\tau}\left(\widehat{\mathbf{X}}_i - \frac{\mathcal{E}_{WX}}{\mathcal{E}_X}\right)dN_i + \widehat{\pi}\int_0^{\tau}\frac{\mathcal{E}_{WX}-\mathcal{E}_{XX}}{\mathcal{E}_X}(dN_i - d\mathcal{N})\right\}
\end{aligned}$$

We now turn to $\mathbf{U}_b^{(1)}(\boldsymbol{\theta})$. We need to work on the term $\mathbf{S}_1(t,\boldsymbol{\theta})/S_0(t,\boldsymbol{\theta})$. We can write

$$\begin{aligned}
\frac{\mathbf{S}_1}{S_0} - \frac{\widehat{\mathbf{s}}_1}{\mathcal{E}_X} &= S_0^{-1}\left(\mathbf{S}_1 - \frac{\widehat{\mathbf{s}}_1}{\mathcal{E}_X}S_0\right) \\
&\doteq \mathcal{E}_X^{-1}\left(\mathbf{S}_1 - \frac{\widehat{\mathbf{s}}_1}{\mathcal{E}_X}S_0\right) \\
&= \mathcal{E}_X^{-1}\left\{(\mathbf{S}_1 - \widehat{\mathbf{s}}_1) - \frac{\widehat{\mathbf{s}}_1}{\mathcal{E}_X}(S_0 - \mathcal{E}_X)\right\}
\end{aligned}$$

Now,

$$
\begin{aligned}
S_0 - \mathcal{E}_X &= S_{0a} + \left(\frac{S_{0a}}{S_{0c}}\right) S_{0b} - \mathcal{E}_X \\
&\doteq (S_{0a} - \widehat{\pi}\mathcal{E}_X) + \left(\frac{\mathcal{E}_X}{\mathcal{E}_W}\right)(S_{0b} - (1-\widehat{\pi})\mathcal{E}_W) + \widehat{\phi}(S_{0a} - \widehat{\pi}\mathcal{E}_X) - \widehat{\phi}\left(\frac{\mathcal{E}_X}{\mathcal{E}_W}\right)(S_{0c} - \widehat{\pi}\mathcal{E}_W) \\
&= \frac{1}{\widehat{\pi}}\left\{\frac{1}{n}\sum_{j=1}^{n}\omega_j(Y_j e^{\boldsymbol{\beta}^T\mathbf{X}_j} - \mathcal{E}_X)\right\} + \left(\frac{\mathcal{E}_X}{\mathcal{E}_W}\right)\left\{\frac{1}{n}\sum_{j=1}^{n}(1-\omega_j)(Y_j e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_j} - \mathcal{E}_W)\right\} \\
&\qquad - \widehat{\phi}\left(\frac{\mathcal{E}_X}{\mathcal{E}_W}\right)\left\{\frac{1}{n}\sum_{j=1}^{n}\omega_j(Y_j e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_j} - \mathcal{E}_W)\right\}
\end{aligned}
$$

By a similar argument,

$$
\begin{aligned}
\mathbf{S}_1 - \widehat{\mathbf{s}}_1 \doteq\ & \frac{1}{n}\sum_{j=1}^{n}\omega_j(Y_j\mathbf{X}_j e^{\boldsymbol{\beta}^T\mathbf{X}_j} - \boldsymbol{\mathcal{E}}_{XX}) + \frac{1}{n}\sum_{j=1}^{n}(1-\omega_j)(Y_j\widehat{\mathbf{X}}_j e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_j} - \boldsymbol{\mathcal{E}}_{WW}) \\
& + \frac{\widehat{\phi}}{n}\sum_{j=1}^{n}\omega_j(Y_j\widehat{\mathbf{X}}_j e^{\boldsymbol{\beta}^T\mathbf{X}_j} - \boldsymbol{\mathcal{E}}_{WX}) - \frac{\widehat{\phi}}{n}\sum_{j=1}^{n}\omega_j(Y_j\widehat{\mathbf{X}}_j e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_j} - \boldsymbol{\mathcal{E}}_{WW}) \\
& + \mathcal{E}_W^{-1}(\boldsymbol{\mathcal{E}}_{WX} - \boldsymbol{\mathcal{E}}_{WW})\left\{\frac{1}{n}\sum_{j=1}^{n}(1-\omega_j)(Y_j e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_j} - \mathcal{E}_W) - \frac{\widehat{\phi}}{n}\sum_{j=1}^{n}\omega_j(Y_j e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_j} - \mathcal{E}_W)\right\}
\end{aligned}
$$

As a result, we can write $\mathbf{U}^{(1)}(\boldsymbol{\theta}^*) \doteq \mathbf{U}^{(1)*}(\boldsymbol{\theta}^*)$ with

$$
\mathbf{U}^{(1)*}(\boldsymbol{\theta}^*) = \widehat{\pi}\left\{\frac{1}{m}\sum_{i=1}^{n}\omega_i\mathbf{Z}_i^{(11)}\right\} + (1-\widehat{\pi})\left\{\frac{1}{n-m}\sum_{i=1}^{n}(1-\omega_i)\mathbf{Z}_i^{(12)}\right\}
$$

where

$$
\begin{aligned}
\mathbf{Z}_i^{(11)} =\ & \int_0^\tau \left(\mathbf{X}_i - \frac{\boldsymbol{\mathcal{E}}_{XX}}{\mathcal{E}_X}\right) dN_i - (1-\widehat{\pi})\int_0^\tau \frac{\mathcal{E}_{WX} - \mathcal{E}_{XX}}{\mathcal{E}_X}(dN_i - d\mathcal{N}) - \int_0^\tau (Y_i\mathbf{X}_i e^{\boldsymbol{\beta}^T\mathbf{X}_i} - \boldsymbol{\mathcal{E}}_{XX})\mathcal{E}_X^{-1} d\mathcal{N} \\
& - \widehat{\phi}\int_0^\tau (Y_i\widehat{\mathbf{X}}_i e^{\boldsymbol{\beta}^T\mathbf{X}_i} - \boldsymbol{\mathcal{E}}_{WX})\mathcal{E}_X^{-1} d\mathcal{N} + \widehat{\phi}\int_0^\tau (Y_i\widehat{\mathbf{X}}_i e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i} - \boldsymbol{\mathcal{E}}_{WW})\mathcal{E}_X^{-1} d\mathcal{N} \\
& - \widehat{\phi}\int_0^\tau \mathcal{E}_W^{-1}(\boldsymbol{\mathcal{E}}_{WX} - \boldsymbol{\mathcal{E}}_{WW})(Y_i e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i} - \mathcal{E}_W)\mathcal{E}_X^{-1} d\mathcal{N} + \int_0^\tau \left(\frac{\widehat{\mathbf{s}}_1}{\widehat{\pi}\mathcal{E}_X}\right)(Y_i e^{\boldsymbol{\beta}^T\mathbf{X}_i} - \mathcal{E}_X)\mathcal{E}_X^{-1} d\mathcal{N} \\
& - \int_0^\tau \left(\frac{\widehat{\phi}\widehat{\mathbf{s}}_1}{\mathcal{E}_W}\right)(Y_i e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i} - \mathcal{E}_W)\mathcal{E}_X^{-1} d\mathcal{N}
\end{aligned}
$$

and

$$\mathbf{Z}_i^{(12)} = \int_0^\tau \left(\widehat{\mathbf{X}}_i - \frac{\boldsymbol{\mathcal{E}}_{WX}}{\mathcal{E}_X}\right) dN_i + \widehat{\pi}\int_0^\tau \frac{\mathcal{E}_{WX}-\mathcal{E}_{XX}}{\mathcal{E}_X}(dN_i - d\mathcal{N}) + \int_0^\tau (Y_i\widehat{\mathbf{X}}_i e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i} - \boldsymbol{\mathcal{E}}_{WW})\mathcal{E}_X^{-1} d\mathcal{N}$$
$$- \int_0^\tau \mathcal{E}_W^{-1}(\boldsymbol{\mathcal{E}}_{WX} - \boldsymbol{\mathcal{E}}_{WW})(Y_i e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i} - \mathcal{E}_W)\mathcal{E}_X^{-1} d\mathcal{N} + \int_0^\tau \frac{\widehat{\mathbf{s}}_1}{\mathcal{E}_W}(Y_i e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i} - \mathcal{E}_W)\mathcal{E}_X^{-1} d\mathcal{N}$$

In computing $\widehat{\mathbf{C}}$ as described in the main paper, we substitute $\widehat{\boldsymbol{\theta}}$ for $\boldsymbol{\theta}^*$, $d\overline{N}(u)$ for $d\mathcal{N}(u)$, S_0 for $\mathcal{E}_X$, $\mathbf{S}_1$ for $\widehat{\mathbf{s}}_1$, $S_{0b}+S_{0c}$ for $\mathcal{E}_W$, $\widehat{\pi}^{-1}\mathbf{S}_{1a}$ for $\boldsymbol{\mathcal{E}}_{XX}$, $\widehat{\pi}^{-1}\mathbf{S}_{1d}$ for $\boldsymbol{\mathcal{E}}_{WX}$, and $\mathbf{S}_{1b}+\mathbf{S}_{1c}$ for $\boldsymbol{\mathcal{E}}_{WW}$.

We now turn to the derivation of the relevant components of Jacobian matrix $\mathbf{D}(\boldsymbol{\theta})$. The derivative of $-\mathbf{U}^{(1)}(\boldsymbol{\theta})$ with respect to $\boldsymbol{\beta}$ is given by (8). Let $\widehat{\mathbf{X}}_i^{\bullet}$ denote the matrix whose (r,t) entry equals $\partial\widehat{X}_{ir}/\partial\alpha_t$. The matrix $\widehat{\mathbf{X}}_i^{\bullet}$ is a $p\times(p+1)p_1$ matrix whose first p_1 rows are $\mathbf{I}_{p_1}\otimes\overline{\mathbf{w}}_i^T$ and whose remaining rows are filled with 0's. We then have

$$\mathbf{D}_{\beta\alpha} = -\frac{\partial\mathbf{U}^{(1)}(\boldsymbol{\theta})}{\partial\boldsymbol{\alpha}} = -\frac{1}{n}\sum_{i=1}^n \delta_i(1-\omega_i)\widehat{\mathbf{X}}_i^{\bullet} + \frac{1}{n}\sum_{i=1}^n \delta_i\left\{\frac{\mathbf{H}_2}{S_0} - \left(\frac{\mathbf{S}_1}{S_0}\right)\left(\frac{\mathbf{H}_1}{S_0}\right)\right\}$$

where

$$\mathbf{H}_1 = \frac{\partial S_0}{\partial\boldsymbol{\alpha}} = \left(\frac{S_{0a}}{S_{0c}}\right)\left\{\sum_{i=1}^n(1-\omega_i)Y_i\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i^{\bullet}e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i}\right\} - \left(\frac{S_{0a}S_{0b}}{S_{0c}^2}\right)\left\{\sum_{i=1}^n\omega_iY_i\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i^{\bullet}e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i}\right\}$$

and

$$\mathbf{H}_2 = \frac{\partial\mathbf{S}_1}{\partial\boldsymbol{\alpha}} = \frac{1}{n}\sum_{i=1}^n(1-\omega_i)Y_i\widehat{\mathbf{X}}_i^{\bullet}e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i} + \frac{1}{n}\sum_{i=1}^n(1-\omega_i)Y_i\widehat{\mathbf{X}}_i(\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i^{\bullet})e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i}$$
$$+ S_{0c}^{-1}(\mathbf{S}_{1d}-\mathbf{S}_{1c})\left\{\frac{1}{n}\sum_{i=1}^n(1-\omega_i)Y_i(\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i^{\bullet})e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i}\right\}$$
$$+ \left(\frac{S_{0b}}{S_{0c}}\right)\left\{\frac{1}{n}\sum_{i=1}^n\omega_iY_i\widehat{\mathbf{X}}_i^{\bullet}e^{\boldsymbol{\beta}^T\mathbf{X}_i} - \frac{1}{n}\sum_{i=1}^n\omega_iY_i\widehat{\mathbf{X}}_i^{\bullet}e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i} - \frac{1}{n}\sum_{i=1}^n\omega_iY_i\widehat{\mathbf{X}}_i(\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i^{\bullet})e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i}\right\}$$
$$- \left(\frac{S_{0b}}{S_{0c}^2}\right)(\mathbf{S}_{1d}-\mathbf{S}_{1c})\left\{\frac{1}{n}\sum_{i=1}^n\omega_iY_i(\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i^{\bullet})e^{\boldsymbol{\beta}^T\widehat{\mathbf{X}}_i}\right\}$$

Finally,

$$\mathbf{D}_{\alpha\alpha} = -\frac{\partial\mathbf{U}^{(2)}(\boldsymbol{\theta})}{\partial\boldsymbol{\alpha}} = \frac{1}{n}\sum_{i=1}^n\omega_i(\mathbf{I}_{p_1}\otimes\overline{\mathbf{w}}_i\overline{\mathbf{w}}_i^T)$$

Table S1: Simulation results for the single-covariate common disease case with independent measurement error. β^* is the true value of β. Bias(%) is the relative bias, *i.e.* Bias(%)=$100 \times (\hat{\beta} - \beta^*)/\beta^*$. IQR is 0.74 times the interquartile range of the $\hat{\beta}$ values. SE is the mean of the estimated standard error of $\hat{\beta}$. SD is the empirical standard deviation of the $\hat{\beta}$ values. CR is the empirical coverage rate of the asymptotic 95% confidence interval. Methods considered: MS = modified score, CH = Chen, RC = regression calibration, CC = complete case, NA = naive.

				Mean		Median					
Corr(X, W)	$\exp(\beta^*)$	β^*	Method	$\hat{\beta}$	Bias(%)	$\hat{\beta}$	Bias(%)	IQR	SE	SD	CR
0.90	1.5	0.4055	MS	0.4036	-0.5	0.4023	-0.8	0.1005	0.0982	0.1001	0.938
			CH	0.4042	-0.3	0.3999	-1.4	0.1016	0.1045	0.1089	0.938
			RC	0.4020	-0.9	0.4017	-0.9	0.0998	0.0966	0.0986	0.934
			CC	0.3927	-3.1	0.3847	-5.1	0.1394	0.1472	0.1503	0.945
			NA	0.3541	-12.7	0.3503	-13.6	0.0849	0.0858	0.0875	0.902
0.70	1.5	0.4055	MS	0.4053	-0.0	0.3997	-1.4	0.1125	0.1133	0.1147	0.926
			CH	0.4028	-0.7	0.3995	-1.5	0.1197	0.1225	0.1278	0.938
			RC	0.3991	-1.6	0.3969	-2.1	0.1087	0.1088	0.1102	0.930
			CC	0.3927	-3.1	0.3847	-5.1	0.1394	0.1472	0.1503	0.945
			NA	0.2484	-38.7	0.2458	-39.4	0.0659	0.0713	0.0723	0.406
0.50	1.5	0.4055	MS	0.4042	-0.3	0.3944	-2.7	0.1333	0.1281	0.1299	0.945
			CH	0.3988	-1.7	0.3936	-2.9	0.1295	0.1334	0.1393	0.930
			RC	0.3931	-3.0	0.3923	-3.2	0.1227	0.1202	0.1216	0.942
			CC	0.3927	-3.1	0.3847	-5.1	0.1394	0.1472	0.1503	0.945
			NA	0.1445	-64.4	0.1445	-64.4	0.0530	0.0538	0.0545	0.000
0.90	2.5	0.9163	MS	0.9175	0.1	0.9101	-0.7	0.1002	0.1129	0.1147	0.934
			CH	0.9184	0.2	0.9060	-1.1	0.1171	0.1178	0.1261	0.922
			RC	0.8965	-2.2	0.8855	-3.4	0.0911	0.1032	0.1050	0.930
			CC	0.9380	2.4	0.9259	1.0	0.1389	0.1605	0.1576	0.953
			NA	0.7847	-14.4	0.7759	-15.3	0.0854	0.0910	0.0934	0.664
0.70	2.5	0.9163	MS	0.9256	1.0	0.9229	0.7	0.1550	0.1361	0.1412	0.949
			CH	0.9213	0.5	0.9078	-0.9	0.1409	0.1370	0.1469	0.938
			RC	0.8695	-5.1	0.8659	-5.5	0.1224	0.1132	0.1163	0.914
			CC	0.9380	2.4	0.9259	1.0	0.1389	0.1605	0.1576	0.953
			NA	0.5212	-43.1	0.5206	-43.2	0.0706	0.0722	0.0751	0.000
0.50	2.5	0.9163	MS	0.9295	1.4	0.9218	0.6	0.1683	0.1546	0.1635	0.961
			CH	0.9197	0.4	0.9081	-0.9	0.1474	0.1462	0.1555	0.922
			RC	0.8469	-7.6	0.8381	-8.5	0.1295	0.1198	0.1233	0.895
			CC	0.9380	2.4	0.9259	1.0	0.1389	0.1605	0.1576	0.953
			NA	0.2902	-68.3	0.2924	-68.1	0.0537	0.0530	0.0548	0.000
0.90	4.0	1.3863	MS	1.3883	0.1	1.3739	-0.9	0.1345	0.1396	0.1404	0.949
			CH	1.3819	-0.3	1.3669	-1.4	0.1417	0.1404	0.1472	0.934
			RC	1.3159	-5.1	1.3068	-5.7	0.1092	0.1149	0.1203	0.887
			CC	1.4206	2.5	1.4002	1.0	0.1770	0.1896	0.1802	0.941
			NA	1.1412	-17.7	1.1310	-18.4	0.1041	0.0986	0.1057	0.324
0.70	4.0	1.3863	MS	1.4098	1.7	1.3971	0.8	0.1781	0.1662	0.1757	0.938
			CH	1.3858	-0.0	1.3684	-1.3	0.1612	0.1605	0.1708	0.914
			RC	1.2394	-10.6	1.2362	-10.8	0.1249	0.1218	0.1272	0.750
			CC	1.4206	2.5	1.4002	1.0	0.1770	0.1896	0.1802	0.941
			NA	0.7101	-48.8	0.7097	-48.8	0.0777	0.0736	0.0790	0.000
0.50	4.0	1.3863	MS	1.4181	2.3	1.4000	1.0	0.2068	0.1971	0.2083	0.953
			CH	1.3865	0.0	1.3718	-1.0	0.1780	0.1679	0.1787	0.918
			RC	1.1926	-14.0	1.1892	-14.2	0.1259	0.1240	0.1301	0.633
			CC	1.4206	2.5	1.4002	1.0	0.1770	0.1896	0.1802	0.941
			NA	0.3807	-72.5	0.3810	-72.5	0.0543	0.0524	0.0534	0.000

Table S2: Simulation results for the single-covariate common disease case with dependent measurement error. β^* is the true value of β. Bias(%) is the relative bias, *i.e.* Bias(%)=$100 \times (\hat{\beta} - \beta^*)/\beta^*$. IQR is 0.74 times the interquartile range of the $\hat{\beta}$ values. SE is the mean of the estimated standard error of $\hat{\beta}$. SD is the empirical standard deviation of the $\hat{\beta}$ values. CR is the empirical coverage rate of the asymptotic 95% confidence interval. Methods considered: MS = modified score, CH = Chen, RC = regression calibration, CC = complete case, NA = naive.

				Mean		Median					
Corr(X, W)	$\exp(\beta^*)$	β^*	Method	$\hat{\beta}$	Bias(%)	$\hat{\beta}$	Bias(%)	IQR	SE	SD	CR
0.90	1.5	0.4055	MS	0.4033	-0.5	0.4005	-1.2	0.0957	0.0981	0.1003	0.938
			CH	0.4038	-0.4	0.3961	-2.3	0.1012	0.1041	0.1091	0.934
			RC	0.4008	-1.2	0.4000	-1.3	0.0968	0.0958	0.0982	0.930
			CC	0.3927	-3.1	0.3847	-5.1	0.1394	0.1472	0.1503	0.945
			NA	0.3556	-12.3	0.3512	-13.4	0.0847	0.0857	0.0877	0.902
0.70	1.5	0.4055	MS	0.4050	-0.1	0.4002	-1.3	0.1168	0.1132	0.1153	0.930
			CH	0.4024	-0.8	0.3965	-2.2	0.1158	0.1224	0.1284	0.934
			RC	0.3984	-1.8	0.3960	-2.3	0.1091	0.1082	0.1105	0.934
			CC	0.3927	-3.1	0.3847	-5.1	0.1394	0.1472	0.1503	0.945
			NA	0.2470	-39.1	0.2466	-39.2	0.0659	0.0706	0.0725	0.387
0.50	1.5	0.4055	MS	0.4040	-0.4	0.3919	-3.4	0.1359	0.1280	0.1308	0.945
			CH	0.3981	-1.8	0.3937	-2.9	0.1313	0.1332	0.1393	0.922
			RC	0.3930	-3.1	0.3873	-4.5	0.1284	0.1201	0.1222	0.938
			CC	0.3927	-3.1	0.3847	-5.1	0.1394	0.1472	0.1503	0.945
			NA	0.1447	-64.3	0.1439	-64.5	0.0508	0.0537	0.0550	0.000
0.90	2.5	0.9163	MS	0.9167	0.0	0.9109	-0.6	0.1036	0.1161	0.1175	0.938
			CH	0.9167	0.0	0.9079	-0.9	0.1147	0.1180	0.1285	0.922
			RC	0.8875	-3.1	0.8845	-3.5	0.0916	0.1015	0.1044	0.922
			CC	0.9380	2.4	0.9259	1.0	0.1389	0.1605	0.1576	0.953
			NA	0.7805	-14.8	0.7763	-15.3	0.0840	0.0893	0.0942	0.641
0.70	2.5	0.9163	MS	0.9237	0.8	0.9157	-0.1	0.1507	0.1378	0.1421	0.949
			CH	0.9205	0.5	0.9106	-0.6	0.1444	0.1365	0.1477	0.934
			RC	0.8652	-5.6	0.8605	-6.1	0.1239	0.1116	0.1161	0.910
			CC	0.9380	2.4	0.9259	1.0	0.1389	0.1605	0.1576	0.953
			NA	0.5135	-44.0	0.5146	-43.8	0.0768	0.0700	0.0765	0.000
0.50	2.5	0.9163	MS	0.9281	1.3	0.9248	0.9	0.1760	0.1558	0.1658	0.938
			CH	0.9188	0.3	0.9030	-1.5	0.1421	0.1457	0.1556	0.926
			RC	0.8473	-7.5	0.8441	-7.9	0.1318	0.1200	0.1244	0.902
			CC	0.9380	2.4	0.9259	1.0	0.1389	0.1605	0.1576	0.953
			NA	0.2912	-68.2	0.2938	-67.9	0.0565	0.0525	0.0560	0.000
0.90	4.0	1.3863	MS	1.3860	-0.0	1.3705	-1.1	0.1471	0.1575	0.1600	0.953
			CH	1.3778	-0.6	1.3625	-1.7	0.1409	0.1410	0.1486	0.918
			RC	1.2942	-6.6	1.2843	-7.4	0.1202	0.1126	0.1207	0.824
			CC	1.4206	2.5	1.4002	1.0	0.1770	0.1896	0.1802	0.941
			NA	1.1260	-18.8	1.1180	-19.4	0.1058	0.0953	0.1092	0.250
0.70	4.0	1.3863	MS	1.4024	1.2	1.3717	-1.1	0.1896	0.1800	0.1861	0.941
			CH	1.3846	-0.1	1.3658	-1.5	0.1594	0.1597	0.1694	0.914
			RC	1.2325	-11.1	1.2298	-11.3	0.1239	0.1198	0.1279	0.715
			CC	1.4206	2.5	1.4002	1.0	0.1770	0.1896	0.1802	0.941
			NA	0.6987	-49.6	0.7012	-49.4	0.0825	0.0705	0.0838	0.000
0.50	4.0	1.3863	MS	1.4177	2.3	1.3872	0.1	0.2124	0.2077	0.2240	0.941
			CH	1.3853	-0.1	1.3715	-1.1	0.1725	0.1673	0.1783	0.922
			RC	1.1963	-13.7	1.1970	-13.7	0.1274	0.1247	0.1311	0.660
			CC	1.4206	2.5	1.4002	1.0	0.1770	0.1896	0.1802	0.941
			NA	0.3856	-72.2	0.3868	-72.1	0.0552	0.0519	0.0559	0.000

Table S3: Simulation results for the multiple-covariate common disease case with independent covariates and independent measurement error. β^* is the true value of β. Bias(%) is the relative bias, *i.e.* Bias(%)=$100 \times (\hat{\beta} - \beta^*)/\beta^*$. IQR is 0.74 times the interquartile range of the $\hat{\beta}$ values. SE is the mean of the estimated standard error of $\hat{\beta}$. SD is the empirical standard deviation of the $\hat{\beta}$ values. CR is the empirical coverage rate of the asymptotic 95% confidence interval. Methods considered: MS = modified score, CH = Chen, RC = regression calibration, CC = complete case, NA = naive.

				Mean		Median					
Corr(X, W)	$\exp(\beta^*)$	β^*	Method	$\hat{\beta}$	Bias(%)	$\hat{\beta}$	Bias(%)	IQR	SE	SD	CR
0.90	1.5	0.4055	MS	0.4186	3.3	0.4124	1.7	0.1025	0.1016	0.0991	0.961
			CH	0.4244	4.7	0.4225	4.2	0.1182	0.1084	0.1102	0.914
			RC	0.4148	2.3	0.4082	0.7	0.1003	0.0986	0.0972	0.949
			CC	0.4381	8.0	0.4545	12.1	0.1439	0.1538	0.1609	0.930
			NA	0.3631	-10.5	0.3538	-12.7	0.0875	0.0875	0.0849	0.933
0.70	1.5	0.4055	MS	0.4264	5.2	0.4196	3.5	0.1220	0.1188	0.1214	0.949
			CH	0.4324	6.6	0.4438	9.4	0.1355	0.1266	0.1397	0.926
			RC	0.4138	2.1	0.4084	0.7	0.1163	0.1110	0.1142	0.942
			CC	0.4381	8.0	0.4545	12.1	0.1439	0.1538	0.1609	0.930
			NA	0.2508	-38.1	0.2478	-38.9	0.0782	0.0727	0.0728	0.410
0.50	1.5	0.4055	MS	0.4342	7.1	0.4337	7.0	0.1365	0.1357	0.1426	0.942
			CH	0.4357	7.5	0.4458	9.9	0.1411	0.1372	0.1549	0.898
			RC	0.4127	1.8	0.4154	2.5	0.1241	0.1228	0.1290	0.938
			CC	0.4381	8.0	0.4545	12.1	0.1439	0.1538	0.1609	0.930
			NA	0.1432	-64.7	0.1426	-64.8	0.0576	0.0550	0.0562	0.000
0.90	2.5	0.9163	MS	0.9463	3.3	0.9359	2.1	0.1267	0.1218	0.1284	0.938
			CH	0.9480	3.5	0.9377	2.3	0.1233	0.1243	0.1339	0.922
			RC	0.9151	-0.1	0.9096	-0.7	0.1127	0.1067	0.1125	0.938
			CC	0.9717	6.1	0.9652	5.3	0.1740	0.1708	0.1793	0.938
			NA	0.7982	-12.9	0.7980	-12.9	0.0936	0.0938	0.0967	0.734
0.70	2.5	0.9163	MS	0.9679	5.6	0.9511	3.8	0.1598	0.1536	0.1642	0.926
			CH	0.9639	5.2	0.9519	3.9	0.1551	0.1442	0.1668	0.895
			RC	0.8887	-3.0	0.8803	-3.9	0.1193	0.1176	0.1271	0.914
			CC	0.9717	6.1	0.9652	5.3	0.1740	0.1708	0.1793	0.938
			NA	0.5265	-42.5	0.5254	-42.7	0.0731	0.0745	0.0790	0.000
0.50	2.5	0.9163	MS	0.9830	7.3	0.9738	6.3	0.2044	0.1792	0.1888	0.933
			CH	0.9716	6.0	0.9559	4.3	0.1660	0.1534	0.1786	0.895
			RC	0.8697	-5.1	0.8677	-5.3	0.1382	0.1252	0.1338	0.906
			CC	0.9717	6.1	0.9652	5.3	0.1740	0.1708	0.1793	0.938
			NA	0.2907	-68.3	0.2896	-68.4	0.0554	0.0547	0.0582	0.000
0.90	4.0	1.3863	MS	1.4460	4.3	1.4218	2.6	0.1712	0.1734	0.1863	0.957
			CH	1.4281	3.0	1.4114	1.8	0.1431	0.1498	0.1722	0.918
			RC	1.3443	-3.0	1.3401	-3.3	0.1254	0.1193	0.1306	0.902
			CC	1.4615	5.4	1.4410	3.9	0.2077	0.1979	0.2049	0.957
			NA	1.1643	-16.0	1.1631	-16.1	0.1002	0.1028	0.1131	0.398
0.70	4.0	1.3863	MS	1.4990	8.1	1.4768	6.5	0.2355	0.2573	0.2608	0.933
			CH	1.4524	4.8	1.4352	3.5	0.1781	0.1715	0.2012	0.914
			RC	1.2666	-8.6	1.2594	-9.2	0.1334	0.1280	0.1392	0.812
			CC	1.4615	5.4	1.4410	3.9	0.2077	0.1979	0.2049	0.957
			NA	0.7236	-47.8	0.7202	-48.0	0.0810	0.0768	0.0875	0.000
0.50	4.0	1.3863	MS	1.5228	9.8	1.5014	8.3	0.2518	0.2963	0.2777	0.965
			CH	1.4645	5.6	1.4493	4.5	0.1984	0.1793	0.2080	0.910
			RC	1.2217	-11.9	1.2226	-11.8	0.1386	0.1309	0.1400	0.719
			CC	1.4615	5.4	1.4410	3.9	0.2077	0.1979	0.2049	0.957
			NA	0.3865	-72.1	0.3822	-72.4	0.0615	0.0546	0.0605	0.000

Table S4: Simulation results for the multiple-covariate common disease case with independent covariates and dependent measurement error. β^* is the true value of β. Bias(%) is the relative bias, *i.e.* Bias(%)$=100 \times (\hat{\beta} - \beta^*)/\beta^*$. IQR is 0.74 times the interquartile range of the $\hat{\beta}$ values. SE is the mean of the estimated standard error of $\hat{\beta}$. SD is the empirical standard deviation of the $\hat{\beta}$ values. CR is the empirical coverage rate of the asymptotic 95% confidence interval. Methods considered: MS = modified score, CH = Chen, RC = regression calibration, CC = complete case, NA = naive.

				Mean		Median					
Corr(X,W)	$\exp(\beta^*)$	β^*	Method	$\hat{\beta}$	Bias(%)	$\hat{\beta}$	Bias(%)	IQR	SE	SD	CR
0.90	1.5	0.4055	MS	0.4187	3.3	0.4117	1.5	0.0987	0.1016	0.0990	0.961
			CH	0.4244	4.7	0.4217	4.0	0.1053	0.1078	0.1086	0.918
			RC	0.4141	2.1	0.4078	0.6	0.1005	0.0979	0.0959	0.949
			CC	0.4381	8.0	0.4545	12.1	0.1439	0.1538	0.1609	0.930
			NA	0.3655	-9.9	0.3558	-12.2	0.0860	0.0875	0.0844	0.941
0.70	1.5	0.4055	MS	0.4259	5.0	0.4191	3.4	0.1296	0.1183	0.1199	0.945
			CH	0.4327	6.7	0.4413	8.8	0.1316	0.1263	0.1382	0.910
			RC	0.4136	2.0	0.4108	1.3	0.1209	0.1106	0.1128	0.942
			CC	0.4381	8.0	0.4545	12.1	0.1439	0.1538	0.1609	0.930
			NA	0.2505	-38.2	0.2451	-39.6	0.0745	0.0723	0.0716	0.406
0.50	1.5	0.4055	MS	0.4339	7.0	0.4386	8.2	0.1330	0.1358	0.1412	0.945
			CH	0.4356	7.4	0.4484	10.6	0.1417	0.1369	0.1537	0.902
			RC	0.4131	1.9	0.4163	2.7	0.1160	0.1229	0.1282	0.942
			CC	0.4381	8.0	0.4545	12.1	0.1439	0.1538	0.1609	0.930
			NA	0.1440	-64.5	0.1435	-64.6	0.0590	0.0550	0.0555	0.000
0.90	2.5	0.9163	MS	0.9503	3.7	0.9345	2.0	0.1300	0.1328	0.1473	0.945
			CH	0.9463	3.3	0.9366	2.2	0.1226	0.1240	0.1340	0.914
			RC	0.9084	-0.9	0.9011	-1.7	0.1108	0.1050	0.1104	0.938
			CC	0.9717	6.1	0.9652	5.3	0.1740	0.1708	0.1793	0.938
			NA	0.7973	-13.0	0.7970	-13.0	0.0917	0.0924	0.0962	0.722
0.70	2.5	0.9163	MS	0.9655	5.4	0.9485	3.5	0.1592	0.1564	0.1685	0.930
			CH	0.9626	5.1	0.9445	3.1	0.1473	0.1435	0.1659	0.887
			RC	0.8868	-3.2	0.8823	-3.7	0.1179	0.1161	0.1242	0.926
			CC	0.9717	6.1	0.9652	5.3	0.1740	0.1708	0.1793	0.938
			NA	0.5222	-43.0	0.5202	-43.2	0.0699	0.0728	0.0775	0.000
0.50	2.5	0.9163	MS	0.9808	7.0	0.9628	5.1	0.1805	0.1838	0.1895	0.949
			CH	0.9694	5.8	0.9496	3.6	0.1571	0.1528	0.1782	0.902
			RC	0.8717	-4.9	0.8723	-4.8	0.1309	0.1255	0.1331	0.906
			CC	0.9717	6.1	0.9652	5.3	0.1740	0.1708	0.1793	0.938
			NA	0.2933	-68.0	0.2901	-68.3	0.0575	0.0544	0.0574	0.000
0.90	4.0	1.3863	MS	1.4540	4.9	1.4104	1.7	0.1767	0.1815	0.1713	0.960
			CH	1.4231	2.7	1.4146	2.0	0.1571	0.1495	0.1741	0.918
			RC	1.3277	-4.2	1.3207	-4.7	0.1241	0.1166	0.1299	0.878
			CC	1.4615	5.4	1.4410	3.9	0.2077	0.1979	0.2049	0.957
			NA	1.1553	-16.7	1.1552	-16.7	0.1097	0.1000	0.1146	0.340
0.70	4.0	1.3863	MS	1.4954	7.9	1.4553	5.0	0.2283	0.2383	0.2426	0.945
			CH	1.4496	4.6	1.4369	3.6	0.1841	0.1705	0.2019	0.910
			RC	1.2648	-8.8	1.2581	-9.2	0.1354	0.1257	0.1380	0.804
			CC	1.4615	5.4	1.4410	3.9	0.2077	0.1979	0.2049	0.957
			NA	0.7174	-48.3	0.7147	-48.4	0.0858	0.0742	0.0883	0.000
0.50	4.0	1.3863	MS	1.5150	9.3	1.4847	7.1	0.2765	0.2870	0.2953	0.960
			CH	1.4610	5.4	1.4452	4.3	0.2029	0.1786	0.2093	0.914
			RC	1.2282	-11.4	1.2285	-11.4	0.1309	0.1315	0.1412	0.750
			CC	1.4615	5.4	1.4410	3.9	0.2077	0.1979	0.2049	0.957
			NA	0.3935	-71.6	0.3962	-71.4	0.0666	0.0542	0.0603	0.000

Table S5: Simulation results for the multiple-covariate common disease case with dependent covariates and independent measurement error. β^* is the true value of β. Bias(%) is the relative bias, *i.e.* Bias(%)=$100 \times (\hat{\beta} - \beta^*)/\beta^*$. IQR is 0.74 times the interquartile range of the $\hat{\beta}$ values. SE is the mean of the estimated standard error of $\hat{\beta}$. SD is the empirical standard deviation of the $\hat{\beta}$ values. CR is the empirical coverage rate of the asymptotic 95% confidence interval. Methods considered: MS = modified score, CH = Chen, RC = regression calibration, CC = complete case, NA = naive.

				Mean		Median					
Corr(X, W)	$\exp(\beta^*)$	β^*	Method	$\hat{\beta}$	Bias(%)	$\hat{\beta}$	Bias(%)	IQR	SE	SD	CR
0.90	1.5	0.4055	MS	0.4022	-0.8	0.4006	-1.2	0.1087	0.1039	0.1091	0.945
			CH	0.4060	0.1	0.3987	-1.7	0.1267	0.1108	0.1264	0.926
			RC	0.3915	-3.4	0.3891	-4.0	0.1016	0.0984	0.1041	0.941
			CC	0.4070	0.4	0.3852	-5.0	0.1816	0.1560	0.1700	0.930
			NA	0.3435	-15.3	0.3441	-15.1	0.0877	0.0878	0.0911	0.894
0.70	1.5	0.4055	MS	0.4095	1.0	0.4019	-0.9	0.1353	0.1234	0.1353	0.922
			CH	0.4095	1.0	0.3981	-1.8	0.1561	0.1291	0.1581	0.914
			RC	0.3789	-6.5	0.3747	-7.6	0.1164	0.1091	0.1193	0.918
			CC	0.4070	0.4	0.3852	-5.0	0.1816	0.1560	0.1700	0.930
			NA	0.2316	-42.9	0.2295	-43.4	0.0737	0.0715	0.0765	0.296
0.50	1.5	0.4055	MS	0.4156	2.5	0.4012	-1.1	0.1677	0.1422	0.1587	0.914
			CH	0.4100	1.1	0.3875	-4.4	0.1682	0.1394	0.1702	0.895
			RC	0.3685	-9.1	0.3563	-12.1	0.1332	0.1192	0.1320	0.895
			CC	0.4070	0.4	0.3852	-5.0	0.1816	0.1560	0.1700	0.930
			NA	0.1302	-67.9	0.1277	-68.5	0.0495	0.0532	0.0583	0.000
0.90	2.5	0.9163	MS	0.9387	2.4	0.9467	3.3	0.1396	0.1258	0.1316	0.945
			CH	0.9396	2.5	0.9450	3.1	0.1455	0.1254	0.1432	0.906
			RC	0.8892	-3.0	0.9009	-1.7	0.1174	0.1059	0.1100	0.926
			CC	0.9491	3.6	0.9331	1.8	0.1795	0.1716	0.1775	0.949
			NA	0.7758	-15.3	0.7780	-15.1	0.1019	0.0931	0.0953	0.652
0.70	2.5	0.9163	MS	0.9678	5.6	0.9463	3.3	0.1760	0.1705	0.1848	0.949
			CH	0.9493	3.6	0.9445	3.1	0.1720	0.1451	0.1720	0.914
			RC	0.8432	-8.0	0.8512	-7.1	0.1349	0.1154	0.1262	0.852
			CC	0.9491	3.6	0.9331	1.8	0.1795	0.1716	0.1775	0.949
			NA	0.4981	-45.6	0.5005	-45.4	0.0885	0.0727	0.0808	0.000
0.50	2.5	0.9163	MS	0.9831	7.3	0.9618	5.0	0.2018	0.1902	0.2150	0.961
			CH	0.9518	3.9	0.9454	3.2	0.1817	0.1539	0.1780	0.914
			RC	0.8140	-11.2	0.8263	-9.8	0.1456	0.1220	0.1353	0.809
			CC	0.9491	3.6	0.9331	1.8	0.1795	0.1716	0.1775	0.949
			NA	0.2702	-70.5	0.2715	-70.4	0.0628	0.0527	0.0603	0.000
0.90	4.0	1.3863	MS	1.4391	3.8	1.4287	3.1	0.1483	0.1761	0.1747	0.937
			CH	1.4202	2.4	1.4186	2.3	0.1710	0.1473	0.1801	0.914
			RC	1.3154	-5.1	1.3210	-4.7	0.1338	0.1176	0.1273	0.848
			CC	1.4249	2.8	1.4102	1.7	0.1951	0.1956	0.2143	0.914
			NA	1.1400	-17.8	1.1446	-17.4	0.1145	0.1016	0.1119	0.347
0.70	4.0	1.3863	MS	1.4924	7.7	1.4610	5.4	0.2357	0.2558	0.2441	0.964
			CH	1.4272	3.0	1.4171	2.2	0.1994	0.1681	0.2064	0.914
			RC	1.2138	-12.4	1.2131	-12.5	0.1398	0.1259	0.1424	0.703
			CC	1.4249	2.8	1.4102	1.7	0.1951	0.1956	0.2143	0.914
			NA	0.6903	-50.2	0.6886	-50.3	0.1010	0.0750	0.0928	0.000
0.50	4.0	1.3863	MS	1.5228	9.9	1.4622	5.5	0.2639	0.2956	0.2898	0.972
			CH	1.4291	3.1	1.4199	2.4	0.1894	0.1757	0.2104	0.918
			RC	1.1601	-16.3	1.1596	-16.4	0.1368	0.1292	0.1469	0.562
			CC	1.4249	2.8	1.4102	1.7	0.1951	0.1956	0.2143	0.914
			NA	0.3618	-73.9	0.3617	-73.9	0.0647	0.0529	0.0650	0.000

Table S6: Simulation results for the multiple-covariate common disease case with dependent covariates and dependent measurement error. β^* is the true value of β. Bias(%) is the relative bias, *i.e.* Bias(%)=$100\times(\hat{\beta}-\beta^*)/\beta^*$. IQR is 0.74 times the interquartile range of the $\hat{\beta}$ values. SE is the mean of the estimated standard error of $\hat{\beta}$. SD is the empirical standard deviation of the $\hat{\beta}$ values. CR is the empirical coverage rate of the asymptotic 95% confidence interval. Methods considered: MS = modified score, CH = Chen, RC = regression calibration, CC = complete case, NA = naive.

				Mean		Median					
Corr(X,W)	exp(β^*)	β^*	Method	$\hat{\beta}$	Bias(%)	$\hat{\beta}$	Bias(%)	IQR	SE	SD	CR
0.90	1.5	0.4055	MS	0.4026	-0.7	0.4013	-1.0	0.1090	0.1040	0.1093	0.945
			CH	0.4114	1.5	0.4089	0.8	0.1126	0.1112	0.1155	0.930
			RC	0.3909	-3.6	0.3876	-4.4	0.1005	0.0975	0.1036	0.937
			CC	0.4070	0.4	0.3852	-5.0	0.1816	0.1560	0.1700	0.930
			NA	0.3455	-14.8	0.3461	-14.6	0.0856	0.0876	0.0914	0.902
0.70	1.5	0.4055	MS	0.4093	0.9	0.3996	-1.5	0.1304	0.1233	0.1350	0.924
			CH	0.4151	2.4	0.4123	1.7	0.1365	0.1307	0.1382	0.926
			RC	0.3787	-6.6	0.3728	-8.1	0.1132	0.1085	0.1188	0.918
			CC	0.4070	0.4	0.3852	-5.0	0.1816	0.1560	0.1700	0.930
			NA	0.2308	-43.1	0.2329	-42.6	0.0681	0.0708	0.0765	0.277
0.50	1.5	0.4055	MS	0.4153	2.4	0.4037	-0.4	0.1535	0.1424	0.1575	0.914
			CH	0.4188	3.3	0.4164	2.7	0.1455	0.1420	0.1496	0.922
			RC	0.3694	-8.9	0.3598	-11.3	0.1327	0.1192	0.1314	0.898
			CC	0.4070	0.4	0.3852	-5.0	0.1816	0.1560	0.1700	0.930
			NA	0.1312	-67.7	0.1330	-67.2	0.0474	0.0531	0.0586	0.000
0.90	2.5	0.9163	MS	0.9388	2.5	0.9463	3.3	0.1355	0.1297	0.1353	0.961
			CH	0.9210	0.5	0.9106	-0.6	0.1253	0.1243	0.1331	0.926
			RC	0.8828	-3.7	0.8892	-3.0	0.1110	0.1041	0.1094	0.914
			CC	0.9491	3.6	0.9331	1.8	0.1795	0.1716	0.1775	0.949
			NA	0.7744	-15.5	0.7734	-15.6	0.0984	0.0915	0.0959	0.636
0.70	2.5	0.9163	MS	0.9588	4.6	0.9313	1.6	0.1864	0.1612	0.1765	0.953
			CH	0.9253	1.0	0.9115	-0.5	0.1627	0.1439	0.1567	0.922
			RC	0.8427	-8.0	0.8345	-8.9	0.1344	0.1139	0.1247	0.840
			CC	0.9491	3.6	0.9331	1.8	0.1795	0.1716	0.1775	0.949
			NA	0.4944	-46.0	0.4922	-46.3	0.0849	0.0708	0.0801	0.000
0.50	2.5	0.9163	MS	0.9773	6.7	0.9525	3.9	0.2090	0.1879	0.2051	0.965
			CH	0.9278	1.3	0.9166	0.0	0.1665	0.1535	0.1653	0.926
			RC	0.8186	-10.7	0.8215	-10.3	0.1376	0.1222	0.1344	0.809
			CC	0.9491	3.6	0.9331	1.8	0.1795	0.1716	0.1775	0.949
			NA	0.2746	-70.0	0.2748	-70.0	0.0591	0.0524	0.0601	0.000
0.90	4.0	1.3863	MS	1.4471	4.4	1.4253	2.8	0.1696	0.2125	0.2010	0.953
			CH	1.3876	0.1	1.3663	-1.4	0.1654	0.1468	0.1648	0.922
			RC	1.3006	-6.2	1.3027	-6.0	0.1277	0.1147	0.1257	0.836
			CC	1.4249	2.8	1.4102	1.7	0.1951	0.1956	0.2143	0.914
			NA	1.1322	-18.3	1.1365	-18.0	0.1152	0.0989	0.1124	0.304
0.70	4.0	1.3863	MS	1.4932	7.7	1.4481	4.5	0.2200	0.2822	0.2633	0.964
			CH	1.3971	0.8	1.3870	0.1	0.1907	0.1662	0.1876	0.910
			RC	1.2149	-12.4	1.2214	-11.9	0.1342	0.1236	0.1388	0.691
			CC	1.4249	2.8	1.4102	1.7	0.1951	0.1956	0.2143	0.914
			NA	0.6872	-50.4	0.6886	-50.3	0.1005	0.0727	0.0907	0.000
0.50	4.0	1.3863	MS	1.5150	9.3	1.4676	5.9	0.2638	0.3234	0.3050	0.980
			CH	1.4015	1.1	1.3830	-0.2	0.1983	0.1740	0.1942	0.926
			RC	1.1710	-15.5	1.1707	-15.5	0.1314	0.1298	0.1458	0.590
			CC	1.4249	2.8	1.4102	1.7	0.1951	0.1956	0.2143	0.914
			NA	0.3723	-73.1	0.3715	-73.2	0.0688	0.0527	0.0638	0.000

Table S7: Simulation results for the multiple-covariate intermediate disease rate case with independent covariates and independent measurement error. β^* is the true value of β. Bias(%) is the relative bias, *i.e.* Bias(%)=$100 \times (\hat{\beta} - \beta^*)/\beta^*$. IQR is 0.74 times the interquartile range of the $\hat{\beta}$ values. SE is the mean of the estimated standard error of $\hat{\beta}$. SD is the empirical standard deviation of the $\hat{\beta}$ values. CR is the empirical coverage rate of the asymptotic 95% confidence interval.

				Mean		Median					
Corr(X, W)	$\exp(\beta^*)$	β^*	Method	$\hat{\beta}$	Bias(%)	$\hat{\beta}$	Bias(%)	IQR	SE	SD	CR
0.90	4.0	1.3863	MS	1.4639	5.6	1.4283	3.0	0.2002	0.2186	0.2399	0.949
			CH	1.4547	4.9	1.4351	3.5	0.2079	0.1894	0.2186	0.910
0.70	4.0	1.3863	MS	1.5161	9.4	1.4683	5.9	0.2856	0.3223	0.3046	0.957
			CH	1.4820	6.9	1.4833	7.0	0.2436	0.2162	0.2528	0.938
0.50	4.0	1.3863	MS	1.5397	11.1	1.4899	7.5	0.3007	0.3626	0.3305	0.965
			CH	1.4956	7.9	1.4875	7.3	0.2706	0.2273	0.2633	0.933

Table S8: Simulation results for various sample sizes for the multiple-covariate common disease case with independent measurement error. β^* is the true value of β. NN is the sample size of the main study and NV is the sample size of the internal validation sample. Bias(%) is the relative bias, *i.e.* Bias(%)=$100 \times (\hat{\beta} - \beta^*)/\beta^*$. IQR is 0.74 times the interquartile range of the $\hat{\beta}$ values. SE is the mean of the estimated standard error of $\hat{\beta}$. SD is the empirical standard deviation of the $\hat{\beta}$ values. CR is the empirical coverage rate of the asymptotic 95% confidence interval. Methods considered: MS = modified score, CH = Chen, RC = regression calibration, CC = complete case, NA = naive.

				Mean		Median					
Corr(X, W)	$\exp(\beta^*)$	β^*	Sample Size	$\hat{\beta}$	Bias(%)	$\hat{\beta}$	Bias(%)	IQR	SE	SD	CR
0.90	1.5	0.4055	NN=500, NV=200	0.4186	3.3	0.4124	1.7	0.1025	0.1016	0.0991	0.961
			NN=2500, NV=1000	0.4084	0.7	0.4056	0.0	0.0466	0.0442	0.0468	0.938
			NN=5000, NV=2000	0.4070	0.4	0.4062	0.2	0.0306	0.0314	0.0314	0.949
0.70	1.5	0.4055	NN=500, NV=200	0.4264	5.2	0.4196	3.5	0.1220	0.1188	0.1214	0.949
			NN=2500, NV=1000	0.4107	1.3	0.4092	0.9	0.0514	0.0512	0.0543	0.930
			NN=5000, NV=2000	0.4096	1.0	0.4081	0.6	0.0332	0.0362	0.0368	0.949
0.50	1.5	0.4055	NN=500, NV=200	0.4342	7.1	0.4337	7.0	0.1365	0.1357	0.1426	0.942
			NN=2500, NV=1000	0.4130	1.9	0.4110	1.4	0.0584	0.0578	0.0618	0.922
			NN=5000, NV=2000	0.4122	1.7	0.4120	1.6	0.0379	0.0409	0.0415	0.949
0.90	2.5	0.9163	NN=500, NV=200	0.9463	3.3	0.9359	2.1	0.1267	0.1218	0.1284	0.938
			NN=2500, NV=1000	0.9229	0.7	0.9194	0.3	0.0507	0.0518	0.0521	0.945
			NN=5000, NV=2000	0.9226	0.7	0.9231	0.7	0.0393	0.0367	0.0379	0.938
0.70	2.5	0.9163	NN=500, NV=200	0.9679	5.6	0.9511	3.8	0.1598	0.1536	0.1642	0.926
			NN=2500, NV=1000	0.9271	1.2	0.9205	0.5	0.0620	0.0637	0.0657	0.926
			NN=5000, NV=2000	0.9253	1.0	0.9243	0.9	0.0448	0.0447	0.0472	0.942
0.50	2.5	0.9163	NN=500, NV=200	0.9830	7.3	0.9738	6.3	0.2044	0.1792	0.1888	0.933
			NN=2500, NV=1000	0.9304	1.5	0.9260	1.1	0.0714	0.0713	0.0740	0.918
			NN=5000, NV=2000	0.9276	1.2	0.9267	1.1	0.0483	0.0502	0.0527	0.930
0.90	4.0	1.3863	NN=500, NV=200	1.4460	4.3	1.4218	2.6	0.1712	0.1734	0.1863	0.957
			NN=2500, NV=1000	1.3993	0.9	1.3962	0.7	0.0622	0.0677	0.0679	0.945
			NN=5000, NV=2000	1.3995	1.0	1.3999	1.0	0.0442	0.0472	0.0444	0.977
0.70	4.0	1.3863	NN=500, NV=200	1.4990	8.1	1.4768	6.5	0.2355	0.2573	0.2608	0.933
			NN=2500, NV=1000	1.4085	1.6	1.3981	0.9	0.0758	0.0954	0.0963	0.938
			NN=5000, NV=2000	1.4045	1.3	1.3993	0.9	0.0547	0.0609	0.0591	0.957
0.50	4.0	1.3863	NN=500, NV=200	1.5228	9.8	1.5014	8.3	0.2518	0.2963	0.2777	0.965
			NN=2500, NV=1000	1.4096	1.7	1.4001	1.0	0.0878	0.0944	0.0972	0.957
			NN=5000, NV=2000	1.4071	1.5	1.4007	1.0	0.0667	0.0662	0.0669	0.942

Table S9: Simulation results for various sample sizes for the multiple-covariate common disease case with independent measurement error. β^* is the true value of β. NN is the sample size of the main study and NV is the sample size of the internal validation sample. Bias(%) is the relative bias, *i.e.* Bias(%)=$100 \times (\hat{\beta} - \beta^*)/\beta^*$. IQR is 0.74 times the interquartile range of the $\hat{\beta}$ values. SE is the mean of the estimated standard error of $\hat{\beta}$. SD is the empirical standard deviation of the $\hat{\beta}$ values. CR is the empirical coverage rate of the asymptotic 95% confidence interval. Methods considered: MS = modified score, CH = Chen, RC = regression calibration, CC = complete case, NA = naive.

				Mean		Median					
Corr(X,W)	$\exp(\beta^*)$	β^*	Sample Size	$\hat{\beta}$	Bias(%)	$\hat{\beta}$	Bias(%)	IQR	SE	SD	CR
0.90	1.5	0.4055	NN=10000, NV=200	0.4118	1.6	0.4101	1.1	0.0512	0.0684	0.0573	0.949
			NN=10000, NV=400	0.4053	-0.0	0.4024	-0.8	0.0470	0.0526	0.0464	0.965
			NN=50000, NV=1000	0.4070	0.4	0.4070	0.4	0.0260	0.0246	0.0243	0.960
			NN=100000, NV=2000	0.4059	0.1	0.4069	0.3	0.0173	0.0175	0.0163	0.959
0.70	1.5	0.4055	NN=10000, NV=200	0.4175	3.0	0.4067	0.3	0.0761	0.0860	0.0814	0.957
			NN=10000, NV=400	0.4086	0.8	0.4036	-0.5	0.0679	0.0726	0.0639	0.977
			NN=50000, NV=1000	0.4084	0.7	0.4053	-0.0	0.0392	0.0356	0.0352	0.960
			NN=100000, NV=2000	0.4056	0.0	0.4047	-0.2	0.0244	0.0252	0.0233	0.968
0.50	1.5	0.4055	NN=10000, NV=200	0.4300	6.1	0.4148	2.3	0.1105	0.1402	0.1340	0.952
			NN=10000, NV=400	0.4150	2.4	0.4008	-1.2	0.0960	0.1078	0.1090	0.980
			NN=50000, NV=1000	0.4109	1.3	0.4103	1.2	0.0543	0.0526	0.0514	0.976
			NN=100000, NV=2000	0.4062	0.2	0.4057	0.0	0.0316	0.0369	0.0354	0.953
0.90	2.5	0.9163	NN=10000, NV=200	0.9429	2.9	0.9289	1.4	0.1076	0.1272	0.1230	0.937
			NN=10000, NV=400	0.9247	0.9	0.9191	0.3	0.0655	0.0698	0.0644	0.969
			NN=50000, NV=1000	0.9214	0.6	0.9226	0.7	0.0399	0.0381	0.0389	0.938
			NN=100000, NV=2000	0.9193	0.3	0.9191	0.3	0.0260	0.0271	0.0259	0.961
0.70	2.5	0.9163	NN=10000, NV=200	0.9657	5.4	0.9398	2.6	0.1901	0.2015	0.2146	0.955
			NN=10000, NV=400	0.9447	3.1	0.9286	1.3	0.1133	0.1367	0.1250	0.968
			NN=50000, NV=1000	0.9292	1.4	0.9247	0.9	0.0680	0.0674	0.0696	0.953
			NN=100000, NV=2000	0.9223	0.7	0.9211	0.5	0.0394	0.0450	0.0415	0.968
0.50	2.5	0.9163	NN=10000, NV=200	1.0264	12.0	0.9356	2.1	0.3004	0.3131	0.3227	0.938
			NN=10000, NV=400	0.9689	5.7	0.9319	1.7	0.1828	0.2280	0.2173	0.960
			NN=50000, NV=1000	0.9396	2.5	0.9290	1.4	0.1003	0.1039	0.1077	0.970
			NN=100000, NV=2000	0.9278	1.3	0.9237	0.8	0.0553	0.0686	0.0645	0.965
0.90	4.0	1.3863	NN=10000, NV=200	1.4395	3.8	1.4175	2.3	0.1245	0.1333	0.1490	0.943
			NN=10000, NV=400	1.4273	3.0	1.4067	1.5	0.1153	0.1447	0.1605	0.952
			NN=50000, NV=1000	1.3985	0.9	1.3935	0.5	0.0663	0.0680	0.0686	0.949
			NN=100000, NV=2000	1.4008	1.0	1.3945	0.6	0.0449	0.0556	0.0465	0.966
0.70	4.0	1.3863	NN=10000, NV=200	1.5253	10.0	1.4410	3.9	0.3180	0.3329	0.3486	0.936
			NN=10000, NV=400	1.4776	6.6	1.4415	4.0	0.2077	0.2172	0.2123	0.972
			NN=50000, NV=1000	1.4172	2.2	1.4076	1.5	0.1195	0.1198	0.1253	0.944
			NN=100000, NV=2000	1.4073	1.5	1.4020	1.1	0.0706	0.0804	0.0780	0.957
0.50	4.0	1.3863	NN=10000, NV=200	1.5407	11.1	1.4472	4.4	0.4363	0.4511	0.4404	0.925
			NN=10000, NV=400	1.5108	9.0	1.4439	4.2	0.2793	0.3184	0.3242	0.966
			NN=50000, NV=1000	1.4499	4.6	1.4251	2.8	0.1724	0.1964	0.2048	0.957
			NN=100000, NV=2000	1.4214	2.5	1.4137	2.0	0.1028	0.1187	0.1212	0.964