A Procedure for Examining Time Changes in Rates of Readmission to Psychiatric Hospitals

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INTEREST has been growing in recent years in estimating the rates of readmission to institutions from which patients have been discharged, particularly when the periods since discharge are varied. Readmission rates for prisons (*1*) and psychiatric hospitals (*2–6*) have been studied. Current estimating procedures based on data collected over a rather long timespan generally incorporate the assumption that no significant changes occurred during that time. We have developed methods showing how to examine this assumption.

A patient discharged from a hospital for the first time risks a second admission. He is in a risk group where readmissions can occur. Patients are classified by discharge rather than admission because only after they have been discharged do they enter the risk group.

Because each patient in the risk group has belonged to the group for different periods of time, difficulties are encountered in estimating readmission rates. In this paper we have defined rate of readmission as the probability of return to the institution for each patient after he enters the risk group. Because of the varying periods of time the patient may be in the risk group, the rate of readmission cannot be estimated by the ratio of the number of patients readmitted for a second time to the number of discharged patients.

Dorn (*7*) developed a procedure for estimating the readmission rate when this rate did not change over time, but he did not illustrate the technique. Unfortunately, this procedure does not seem to be widely known although it is the appropriate basic method for estimating readmission rates.

To demonstrate our procedure, we used data on male narcotic drug addicts readmitted to the Public Health Service Hospital in Lexington, Ky., from 1935 to 1966. The analysis to detect time trends was done with the same data. Hospital patients were considered in two groups: those with one admission and those with a readmission, or admitted for a second time. The technique, with minor modifications, also can be used to estimate two or more readmissions.

Moon and Patton (*6*) have discussed the disadvantages of several crude measures of readmission rates, such as the percentage of readmissions among all admissions. They offer instead a readmission index that they define as the number of readmissions in any fiscal year per 1,000 patients discharged during the previous 3 fiscal years. This index, however, has two

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serious disadvantages: (*a*) patients in the denominator have not been at risk for the same length of time and (*b*) some patients in the numerator, specifically those who have been away from the hospital for more than 3 years, are not part of the group in the denominator. It is desirable that a readmission rate express the number of patients readmitted to the hospital to the number of patients at risk, adjusted for the varying lengths of time at risk.

Our method does not have these disadvantages. As in other methods, uncontrollable factors, such as death or admission to another institution, are not directly considered in the estimating scheme, but if usable information existed concerning these factors, such information could be readily used.

Estimating Procedure (No Time Trends)

To obtain the desired probability of a second admission, P, it is necessary to define some variables. These variables also are useful for discussing the techniques of extracting information on time trends. Let

p_0=probability that a patient will have a second admission within 1 year of his first discharge

p_1=probability that a patient will have a second admission within the second year following his first discharge, and generally

p_i=probability that a patient will have a second admission within the $(i+1)$ year following his first discharge; $i = 0, 1, 2, \ldots, 31$

From simple laws of probability, it follows that

$$P=p_0+p_1+p_2+\ldots+p_{31}+\ldots$$

To compute the values of the p's, the hospital discharge records can conveniently be condensed into a table in which the patients readmitted for a second time are classified by year of first discharge and length of time between discharge and second admission. A condensation of the Lexington data is in table 1. For each year j of hospital operation ($j = 1935, 1936, \ldots, 1966$), we know the number of patients discharged that year. Of those discharged, we know the number readmitted i years after discharge, where $i = 0, 1, 2, \ldots, (1966-j)$. In 1938, for example, 934 male patients were discharged. Of this number 144 were readmitted within 1 year of discharge, 46 were readmitted during the year following the year of discharge, 33 were readmitted during the second year following the year of discharge, and so on. By the end of 1966, a total of 355 of the 934 patients had a second admission.

The probability p_0 is estimated by dividing the total number of patients discharged into the total number of patients readmitted within 1 year following discharge. Symbolically,

$$\hat{p}_0=\frac{\sum_{j=1935}^{1966} n_{0_j}}{\sum_{j=1935}^{1966} n_j}$$

where

Table 1. Number of male addicts readmitted to the Lexington Hospital, by years at risk and year of discharge

Year of discharge	Number discharged	Years at risk: 0	1	2	3	4	→	28	29	30	31	Total readmitted
1935	80	9	8	1	4	2		0	0	0	0	35
1936	773	101	42	32	29	18		1	0	0	----	315
1937	759	124	43	23	16	17		0	1	----	----	292
1938	934	144	46	33	28	27		0	----	----	----	355
↓												
1963	1,098	107	57	28	13	----		----	----	----	----	205
1964	1,053	101	45	13	----	----		----	----	----	----	159
1965	982	121	30	----	----	----		----	----	----	----	151
1966	1,075	64	----	----	----	----		----	----	----	----	64
Total	33,479	5,169	2,350	1,319	891	636		1	1	0	0	11,674

n_{0j}=number of second-admission patisent readmitted within 1 year who were discharged during year j

n_j=number of patients discharged for the first time in year j

p_0=estimate of p_0

Using the data in table 1,

$$\hat{p}_0=\frac{5,169}{33,479}=0.15440$$

To estimate p_1, the total number of patients discharged for at least 1 year were divided into the number of patients who returned within 2 years but were at risk at least 1 full year. That is,

$$\hat{p}_1=\frac{\sum_{j=1935}^{1965} n_{1j}}{\sum_{=1935}^{1965} n_j}$$

where

n_{1j}=number of patients readmitted for a second time within 1 year following a 1-year risk period who were discharged in year j.

For the Lexington hospital,

$$\hat{p}_1=\frac{2,350}{32,404}=0.07252$$

The 1,075 patients discharged in 1966 were not included in the denominator of $\hat{p}_1$ because they were at risk for less than 1 year.

Similarly, p_i (probability of a readmission within the i^{th} year following discharge) is estimated by the ratio

$$\hat{p}_i=\frac{\sum_{j=1935}^{1966-i} n_{ij}}{\sum_{=1935}^{1966-i} n_j}$$

where

n_{ij}=number of second admission patients readmitted within 1 year following i years at risk who were discharged in year j.

Using this formula on the data in table 1, the following estimates were obtained:

$$\hat{p}_2=\frac{1,319}{31,422}=0.04198$$

$$\hat{p}_3=\frac{891}{30,369}=.02934$$

$$\downarrow$$

$$\hat{p}_{28}=\frac{1}{2,546}=.00039$$

$$\hat{p}_{29}=\frac{1}{1,612}=.00062$$

$$\hat{p}_{30}=\hat{p}_{31}=0$$

Using the above estimates, the estimate of the probability of a second admission, denoted $\hat{P}$, is the sum $\hat{p}_0+\hat{p}_1+\hat{p}_2+\ldots\hat{p}_{31}$. Thus, $\hat{P}=0.15440+0.07252+0.04198+\ldots+0.00039+0.00062=0.3814$.

The estimated standard deviation of $\hat{P}$ or standard error of $\hat{P}$, can be calculated from the following formula. (See box p. 878 for derivation of formula.)

$$S_{\hat{P}}=\text{S.E. of }\hat{P}=\sqrt{\sum_{j=1935}^{1966} n_j\left[\sum_{i=0}^{1966-j}\frac{\hat{p}_i}{m_i^2}-\left(\sum_{k=0}^{1966-j}\frac{\hat{p}_i}{m_i}\right)^2\right]}$$

where

$$m_i=\sum_{j=1935}^{1966-i} n_j$$

For testing hypotheses and constructing confidence intervals, it can be assumed that $\frac{\hat{P}-P}{S^P}$ has a standard normal distribution. For the Lexington data, $S_{\hat{P}}=0.0030$; therefore, a 95 percent confidence interval is from 0.3755 to 0.3873.

Examining Time Trends

In the above estimation procedure, one important assumption is that the probability of a second admission is constant over the period of time that the hospital was in operation. As i approaches 31, the estimates of probability of readmission within the i^{th} year following discharge are based on events occurring over several decades. For example, the estimate $\hat{p}_{25}$ is based on those patients who were first discharged from 1935 to 1941 and returned during 1960 to 1966. By using data without any time adjustment, there is an implicit assumption that the underlying probability of a patient's readmission is constant from 1935 to 1966.

A simple method of looking for changes over time is to consider, by year (or years) of discharge, the percentage of patients readmitted after 2 years or less following the year of discharge. With this percentage, one can estimate the probability that a patient will return within 2 years following discharge; the estimate should be stable in time if the patients are readmitted at the same rate with equal times of risk. The results of this procedure are presented in table 2.

A rather significant time trend does, in fact, exist in these data; an analysis of the reasons underlying the trend exhibited in this table points to an area of research in the future. In fact, the trend could well be of more interest than the readmission rate previously calculated.

If it is suspected that the underlying probability of readmittance is changing, then the readmission rates can be calculated for each year. However, these rates are not directly comparable because of the different times a patient is at risk. For each year j we can calculate the probability of readmission as

$$\hat{\pi}_j = \frac{\sum_{i=0}^{1966-j} n_{ij}}{n_j}$$

where the n_{ij} and n_j have the same meaning as before. However, while $\hat{\pi}_{1940}$ is based on patients who had a 26-year risk period, $\hat{\pi}_{1962}$ is based on only a 4-year risk period.

If we divide the $\hat{\pi}_j$ by the estimate of the probability of readmission in the $1967-j$ years following first discharge calculated from the entire set of data, we have an index of the return rate for the j^{th} year:

Table 2. Number and percent of male addicts readmitted for first time to Lexington Hospital during year of discharge or following 2 years, by selected groups of years

Year of discharge	Number discharged	Number readmitted	Percent
1935–40	4, 112	1, 025	24. 9
1941–46	4, 170	1, 318	31. 6
1947–52	7, 553	2, 586	34. 2
1953–58	8, 113	2, 050	25. 3
1959–64	7, 474	1, 489	19. 9
Total	31, 422	8, 468	26. 9

$$I_j = \frac{\pi_j}{\sum_{i=0}^{1966-j} \hat{p}_i} \qquad j=1935, \ldots, 1966$$

With the assumption that there are no time trends, the I_j differs from unity by randomness. Unfortunately, I_j is a ratio of random variables, and its exact variance could not be obtained. However, since the denominator is relatively stable when compared to the numerator, an approximation of the variance

$$s_j^2 = \frac{1-\sum_{i=0}^{1966-j} \hat{p}_i}{\left(\sum_{i=0}^{1966-j} \hat{p}_i\right) n_j}$$

can be obtained by assuming the denominator of I_j is fixed. If we are willing to assume that I_j follows a normal distribution, then we may test the null hypothesis that the expected value of I_j is unity.

We can also assess the effect of time trends on the natural randomness of the I_j by performing a "runs test" (*8*) on the sequence of numbers $\{I_j-1\}$. For the data in table 3, the probability of observing nine or fewer runs is 0.005 if the sequence of 26 numbers is random. Because there are only four runs, the data support the hypothesis of a time trend.

The data from the Lexington Hospital are excellent for illustrating these techniques. We presented in table 3 the n_j, $\hat{\pi}_j$, I_j, and estimates of probability of second admission, indexed by year and adjusted for varying times of risk. These estimates were obtained by simply multiplying the index I_j by the overall probability $\hat{P}$. The 95 percent confidence intervals were constructed by calculating $\hat{P}I_j \pm 1.96\,\hat{P}s_j$.

Discussion

Because there are only 32 years of data, an obvious restriction to the estimating technique is that the probability of returning for a second admission after 31 years have elapsed since the first discharge cannot be estimated. However, it is unlikely that enough patients will be readmitted for the first time after 31 years to alter the estimate seriously. Of the patients who had a second admission, more than 99 percent

Table 3. Unadjusted and adjusted probabilities of readmittance, indices of readmittance, and 95 percent confidence interval of adjusted probability of readmittance of male addicts, Lexington Hospital, 1935–66

Year j	Number discharged n_i	Unadjusted probability $\hat{\pi}_i$	Index of readmittance I_i	Adjusted probability $\hat{P}I_i$	95 percent confidence interval on $\hat{P}I_i$	
					Low	High
1935	80	0.4375	1.1470	0.4375	0.331	0.544
1936	773	.4075	1.0684	.4075	.373	.442
1937	759	.3847	1.0086	.3847	.350	.419
1938	934	.3803	.9971	.3803	.349	.411
1939	655	.4047	1.0622	.4051	.368	.442
1940	911	.4898	1.2865	.4907	.459	.522
1941	627	.4960	1.3045	.4975	.459	.536
1942	721	.4591	1.2075	.4605	.425	.496
1943	783	.4393	1.1560	.4409	.407	.475
1944	679	.4713	1.2418	.4736	.437	.510
1945	673	.4116	1.0854	.4140	.377	.451
1946	687	.4137	1.0932	.4169	.380	.453
1947	910	.4660	1.2328	.4702	.438	.502
1948	858	.4907	1.2988	.4954	.463	.528
1949	1,066	.5179	1.3733	.5238	.494	.553
1950	1,722	.4285	1.1390	.4344	.411	.458
1951	1,668	.4096	1.0919	.4165	.393	.440
1952	1,329	.3958	1.0588	.4038	.377	.430
1953	1,521	.3622	.9739	.3714	.346	.396
1954	1,335	.3550	.9615	.3667	.340	.393
1955	1,149	.3359	.9177	.3500	.321	.379
1956	1,322	.3435	.9491	.3620	.335	.389
1957	1,341	.3468	.9689	.3695	.342	.397
1958	1,445	.3315	.9436	.3599	.333	.387
1959	1,363	.3023	.9031	.3444	.316	.373
1960	1,460	.2390	.7121	.2716	.244	.299
1961	1,262	.2559	.7895	.3011	.271	.331
1962	1,238	.2287	.7389	.2818	.250	.314
1963	1,098	.1866	.6423	.2450	.210	.280
1964	1,053	.1509	.5718	.2181	.180	.257
1965	982	.1537	.6846	.2611	.217	.305
1966	1,075	.0595	.3856	.1471	.094	.200

were readmitted within 15 years of their discharge. In fact, as seen in table 1, more than 44 percent of the patients were readmitted within 1 year following discharge and more than 88 percent within 5 years following discharge.

Another factor is that deaths within the risk group are not taken into account because of no information on patients after they leave the hospital. Omitting the effects of deaths within the population at risk tends to understate the probability of readmission. This omission is probably not too serious for the first several years after the initial discharge, but it would become important in later years. Most second admissions, however, occur during the early years after discharge.

The Lexington data indicate that the initial assumption of no significant change in the probability of readmission over time is untenable. Both tables 2 and 3 show an increase in the probability of readmission from the late 1930's to about 1950 and a decrease since then. One reason for this change at the Lexington hospital is that the probability of readmission to a specific institution is affected by the availability of other institutions to which patients can return.

When the Lexington hospital opened in 1935 it was one of a few institutions to which addicts could be admitted. In 1938 the Fort Worth hospital was opened, and male addicts from west of the Mississippi were more likely to go there than to return to Lexington. In the 1950's, changes in the law increased the proportion of addicts who were imprisoned; later in the 1950's and in the 1960's, the number of treatment institutions to which addicts could go increased considerably. These facts contributed to the

402-975—70——3

changing probability of readmission to Lexington over the years.

Our procedure also can be used in cohort studies. If some theoretical formulation suggests that one type of patient should show higher readmission rates than another or that a treatment program in effect at one point in time should show a different rate of readmission than a program of another period, the procedure would furnish a method of making comparisons to test the hypotheses.

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Derivation of Standard Deviation of Readmission Rate

The estimate of P can be expressed as

$$\hat{P}=\sum_{i=0}^{31}\sum_{j=1935}^{1966-i}\frac{n_{ij}}{m_i}$$

where m_i is defined as

$$m_i=\sum_{j=1935}^{1966-i} n_j$$

For each year $j = 1935, 1936, \ldots, 1966$, the vectors of numbers $(n_{0j}, n_{1j}, \ldots, n_{1966-j})$ constitute a random sample from a multinomial population with parameters $(n_j, p_0, p_1, \ldots, p_{1966-j})$. If we now regard the n_{ij} as random variables rather than observed values, then the vectors are independently distributed for each j and from the properties of the multinomial distribution

Variance $(n_{ij})=n_j(p_i-p_i^2)$	for all i, j
Covariance $(n_{ij}, n_{kj})=-n_j p_i p_j$	for all $j, i\neq k$
Covariance $(n_{ij}, n_{st})=0$	for all $i, s, j\neq t$

The derivation of the variance of $\hat{P}$ is facilitated by interchanging the order of summation of $\hat{P}$ given above to

$$\hat{P}=\sum_{j=1935}^{1966}\left(\sum_{i=0}^{1966-j}\frac{n_{ij}}{m_i}\right)$$

Using the properties of the multinomial distribution,

$$\text{Variance}\left\{\sum_{i=0}^{1966-j}\frac{n_{ij}}{m_i}\right\}=n_j\left[\sum_{i=0}^{1966-j}\frac{\hat{p}_i}{m_i^2}-\left(\sum_{i=0}^{1966-j}\frac{\hat{p}_i}{m_i}\right)^2\right]$$

Thus the variance of $\hat{P}$ is

$$\sigma^4_{\hat{P}}=\sum_{j=1935}^{1966} n_j\left[\sum_{i=0}^{1966-j}\frac{\hat{p}_i}{m_i^2}-\left(\sum_{i=0}^{1966-j}\frac{\hat{p}_i}{m_i}\right)^2\right]$$

We denote by $S^2_{\hat{P}}$ an estimate of $\sigma^2_{\hat{P}}$, obtained by replacing the p_i in the expression for $\sigma^2_{\hat{P}}$ with their estimates.

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Tearsheet Requests

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World Travelers Given Cholera Notice

Cholera is occurring or is suspected widely throughout southeast and south Asia, the Middle East, and northeast Africa, according to a report from the Center for Disease Control, Health Services and Mental Health Administration, Atlanta, Ga.

The Center advises, in its Weekly Morbidity Report released August 28, 1970, that the World Health Organization lists the following countries as reporting the presence of cholera: Indonesia, Philippines, Vietnam, Burma, India, Nepal, East Pakistan, Lebanon, Libya, South Korea, Syria, Trucial Oman Dubai, and U.S.S.R. In addition, as a protective measure against the spread of cholera, or because of fear that the disease may exist within their borders, the following countries are requiring evidence of vaccination or are quarantining persons entering their country: Guinea, Iran, Iraq, Israel, Jordan, Kuwait, Lebanon, Libya, Malaysia, Saudi Arabia, Syria, Tunisia, Turkey, and United Arab Republic.

The United States accepts a single dose of vaccine within 6 months as valid under International Sanitary Regulations. However, some countries concerned about further spread of the disease are requiring evidence of two doses of vaccine. Therefore, in order to facilitate travel, the Center for Disease Control advises that persons leaving the United States for southeast and south Asia, the Middle East, and northeast Africa receive two cholera shots, at least 1 week apart.

Physicians administering vaccine to travelers should emphasize that an International Certificate of Vaccination must be validated to be acceptable to quarantine authorities. Validation can be obtained at most city, county, and State health departments. Failure to secure validation can cause travelers to be revaccinated or quarantined during the course of travel. The certificate is valid for 6 months.

The traveler's best protection against cholera, as well as against many other enteric diseases, is to avoid potentially contaminated food and water. Persons following the usual tourist itinerary through countries reporting cholera and using standard accommodations run virtually no risk of acquiring cholera.

Persons returning to the United States from an area reporting cholera occurrence or suspected cholera occurrence will, upon arrival, receive a printed Health Alert Notice asking them to report any illness within a week and their itinerary abroad to their private physician or health department, whether or not they had had cholera vaccine.

Education Notes

Program in Health Planning and International Health Problems. A program in health planning leading to the degree of master of public health in health planning and international health problems is being offered by the University of Michigan School of Public Health.

The purpose is to train a cadre of comprehensive health planners capable of contributing to the development of effective health and related social services; build into the professional preparation of public health students a basic, but functional, understanding of the tools and processes of planning; and build a broad base of insights and a framework which will facilitate the students' development as policy planners. The program will seek to turn out graduates with a knowledge of the problems and institutions of society, an analytic approach to problem solving, an understanding of the health policy area, and operational appreciation of a wide range of pertinent techniques.

The program of study approaches the problems of health planning in a comprehensive manner, emphasizing the interrelationships between health and environment, urban problems, and public policy. The curriculum includes courses in basic concepts and methods of public health, mental health, economic analysis, public policy analysis, environmental health, urban-regional analysis and planning, quantitative methods for policy analysis, operations research and decision theory, and other courses offered by related departments and schools of the university. Students may elect to specialize in international health planning, focusing on such areas critical to international health activities as control of communicable diseases, epidemiology, population planning, industrial health, and planning for economic and social development.

Applicants holding a bachelor's degree with strong background in social science and high class standing will be given preference. Familiarity with statistics is essential. Students with no background in statistics will be required to remedy deficiencies. The school's general admission requirements will also apply to these students.

Public Health Service predoctoral traineeships are available, and awards provide full tuition, travel allowances, and annual living allowances of $2,400–$3,600 plus $500 for each dependent. Postdoctoral traineeships provide a maximum of $7,000. Other scholarship aid may be available for those who do not receive traineeships.

For additional information write to Program in Health Planning, School of Public Health, University of Michigan, Ann Arbor, Mich. 48104.

Doctoral Study in Social Work and Social Science. The University of Michigan offers an interdepartmental program which combines social work with economics, political science, psychology, or sociology and leads to a doctor of philosophy degree. Social psychology may be chosen as a field of concentration within psychology or sociology.

The program is designed to prepare students for careers in research, teaching, policy development, and administrative positions in the social welfare field. Candidates for a master's degree in social work in 1970–71, experienced social workers, holders of a master's degree in social science, or persons with a bachelor's degree only may apply.

Support for this advanced study is available from several sources, including the Children's Bureau and the Public Health Service. Stipends for the 8-month academic year range from $1,600 to $3,200 plus tuition and dependency allowances. Fellowship applications will be received up to January 10, 1971.

For detailed information and application forms write to Doctoral Program in Social Work and Social Science, University of Michigan School of Social Work, 1065 Frieze Building, Ann Arbor, Mich. 48104.

Announcements for publication should be forwarded to Public Health Reports 6 months in advance of the deadline date for application for admission or financial aid, whichever is earlier.