Nomograms for Simplified Demographic Calculations

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PUBLIC HEALTH personnel untrained in demography are now being asked to undertake population programs. Since demography was not part of their education, they find it difficult to acquire facility in handling the basic demographic calculations which are necessary for practical planning and decision making.

The important demographic relationships are relatively simple and predictable although based on sophisticated calculations. The public health worker needs only to develop a practical set of working tools without necessarily knowing their complex origins. He is faced with a bewildering range of family planning proposals. Targets have been set more from a sense of the urgency of what should be done than from sound judgment that the goals are feasible.

When officials of a developing country talk about cutting their birth rate in half in 15 years they demonstrate little understanding of what this involves. They usually extrapolate from the rapid fall of death rates to the wishful hope that fertility would respond as rapidly. The shifting age distribution of their populations in itself will make such a target manifestly impossible, because of the transitional increase in fertile women as the present large proportions of children reach adulthood. To simply and clearly describe probable trends and to project reasonable goals, those technically involved must learn how to handle the basic calculations.

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This paper presents a series of nomograms to simplify such calculations. The major relationships between quantitative variables are shown graphically. By simply drawing lines between appropriate scales it is possible to derive projections and correlates quite accurately. A sufficient range of differences in numerical inputs is provided to meet most situations encountered in practical work. Fieldworkers can use known items of information to derive roughly accurate measures of growth determinants and the extent to which they can be changed.

Although this paper concentrates on quantitative variables, the ultimate success of family planning is in fact mainly influenced by the less-tangible dynamic forces underlying family decisions. Motivational and educational factors are difficult to quantify or reduce to charts. By using these nomograms, however, the population worker can at least begin with a clear understanding of the relevant quantitative variables.

Rates of Population Increase

The fundamental quantitative relationship in demographic calculations is the balance between birth rates and death rates, each of which are about 40 per 1,000 population per year in technologically underdeveloped societies. One of the first effects of economic development is to produce a dramatic decline in mortality; modern

Equations Used to Construct Graphs

Before using the figures, one should understand the assumptions underlying their construction. It will be useful, therefore, to have statements of the equations used to depict the relationships shown, using the following symbols:

Y_t: births in population in tth year
Y_o: births in population in initial or current year
B: mean number of live births per family over entire generation
M: mean number of children per family over entire generation
M_o: initial or present value of M
M_r: reduced value of M contemplated
F_m: proportion of females among survivors to age 15
G: intergeneration span in years
G_o: initial or present value of G
G_i: increased value of G contemplated
T: number of years into future being considered
BR: crude birth rate
DR: crude death rate
N: natural rate of annual increase in population
R: intrinsic rate of annual increase in population
S_c: probability that a child born will survive to age 15
S_m: average annual probability that women aged 15–49 will survive
H: halfspan—mean number of years between mother's age at first delivery and intergeneration span
I: mean interval in years between births
A_1: age at which a mother bears her first child

Since T/G represents some number of generations, say K, the association

$$Y_t = Y_o(MF_m)^k \tag{1}$$

can be restated as

$$Y_t = Y_o(MF_m)^{T/G} \tag{2}$$

This is the equation that underlies figure 2. Currently M is M_o and G is G_o. We seek to modify M, holding G at G_o or alternatively to modify G, holding M at M_o. Mathematically we require

$$Y_t = Y_o(M_rF_m)^{T/G_o} \tag{3}$$

or

$$Y_t = Y_o(M_oF_m)^{T/G_i}. \tag{4}$$

Combining (3) and (4) and canceling Y_o, we obtain

$$(M_rF_m)^{T/G_o} = (M_oF_m)^{T/G_i}. \tag{5}$$

Raising each side to the power G_i/T produces

$$(M_rF_m)^{G_i/G_o} = M_oF_m \tag{6}$$

Then, for a given value of F_m, we can plot (6) for selected values of M_o, M_r (and thus M_r/M_o) and ratios G_i/G_o. We have chosen $F_m = 0.49$.

Figure 3 simply depicts the relationship

$$N = BR - DR \tag{7}$$

The scale of number of years to double the population is based upon the equation

$$(1+R)^T = 2, \tag{8}$$

approximated by

$$(1+N)^T = 2. \tag{9}$$

The nomograms in figures 4 and 5 incorporate five different relationships. The first, scales I, II, and III, stems from the equation

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health measures contribute their effect also so that death rates may go down to about 10 per 1,000. Rapid population growth occurs during the period when birth rates remain considerably higher than death rates.

The attempt to understand more clearly the various components and ramifications of population growth can be approached in three ways: (*a*) through a precise mathematical formulation and analysis of the given conditions, (*b*) by a projection of experience through computer simulations in lieu of precise mathematical analysis, and (*c*) through application of the analytical approach to a simplified model which approximates the more complex circumstances of primary interest.

The first approach is normally not feasible, and although the simulation approach has proved useful in a number of respects (*1*, *2*), it is limited by the need for rather elaborate computers and programing. The third approach, therefore, is geared more to field conditions in which one initially needs a quick, admittedly crude "feel" for the special characteristics and critical concerns of the local situation.

A simplified model naturally carries with it the danger of being an inadequate approximation to reality. We must begin, therefore, by making explicit the underlying assumptions of our model, namely that the populations of concern are stable, though not necessarily stationary (*3*), and unaffected by migration.

Equations Used to Construct Graphs—Continued

$$(1+R)^T=\frac{Y_t}{Y_o}. \qquad (10)$$

The element T has been placed on scale I. The right side of the equation appears in scale II as

$$100(Y_t/Y_o-1). \qquad (11)$$

Scale III shows R as a percentage.

Next we recognize that the geometric expansion of families over several generations is a function of the number of females who survive to the childbearing years, their survival rate during those years, and the fertility rates of the survivors. Specifically at this stage we have treated births as if they occurred at mother's age G. This gives us

$$F_m M S_m^{G-15}=(1+R)^G, \qquad (12)$$

or

$$F_m M=\frac{(1+R)^G}{S_m^{G-15}}. \qquad (13)$$

From the previously described development of scale III, it effectively handles the right side of (13) with $S_m=1.00$. To depict the case of high female mortality we constructed scale IV with $S_m=0.99$. Scales V and VI show the M component of (13) for the same two extremes of S_m. The G factor appears on scale VII. The remaining element, F_m, has again been taken as 0.49.

The middle nomogram depicts the difference between births and surviving adults as a function of childhood mortality. We have

$$M=BS_c, \qquad (14)$$

and these factors are entered on scales VIII, IX, and X. For convenience, childhood survival has been translated into mortality percentages.

The purpose of the lowest nomogram in figures 4 and 5 is to estimate the intergeneration span (G) as a function of the age (A_1) of the mother at birth of her first child and the halfspan (H) over which her childbearing takes place. The nomogram further relates H to the total number of live births (B) and the interval (I) between each.

The mother gives birth to children at ages

$$A_1,\ A_2,\ A_3,\ \ldots\ A_B \qquad (15)$$

$$[A_1],\ [A_1+I],\ [A_1+2I],\ \ldots\ [A_1+(B-1)I], \qquad (16)$$

assuming equal spacing between births and disregarding fetal wastage. Observe that the j^{th} term in (16) employs the multiplier $(j-1)$, that is,

$$A_j=A_1+(j-1)I. \qquad (17)$$

Since the median age is defined as

$$A_{\frac{B+1}{2}}=A_1+\left(\frac{B+1}{2}-1\right)I=A_1+\left(\frac{B-1}{2}\right)I, \qquad (18)$$

this can be used as a satisfactory estimate of G. Thus

$$G=A_1+\left(\frac{B-1}{2}\right)I=A_1+H, \qquad (19)$$

where

$$H=\left(\frac{B-1}{2}\right)I. \qquad (20)$$

Equation (20) forms the basis for scales XI, XII, and XIII. The product association forces the use of logarithmic scales in contrast to the arithmetic scales employed in XIV, XV, and XVI, which depict equation (19).

A stable population is one in which constant age-specific fertility and mortality rates have persisted for a period—a generation or more—sufficient for the residual effects of previous fertility and mortality patterns to have disappeared. Under such circumstances population growth thereafter will be at a fixed, intrinsic rate. Moreover, the relative proportion of the total population in each age group will be unchanging.

At any point in time, of course, the actual rate of population growth can be measured as the difference between the crude birth rate and the crude death rate. This observed difference, the natural rate of increase, is likely to differ somewhat from the intrinsic rate, upon which the succeeding nomograms necessarily are based.

Superficially, it may appear unreasonable to apply the notion of intrinsic growth rates associated with stable populations to situations in which conscious efforts are being made to alter fertility rates. The practical effects, however, can be appreciated only after examination of actual situations, such as that of Costa Rica which is used to illustrate much of the following.

Costa Rica's 1963 population and mortality figures (*4*) are shown by age and sex in table 1. Death rates to the age of 5 were similar to those of United Nations mortality level 80. Thereafter, the data corresponded well to level 85 for males and level 90 for females. (The U.N.

model life tables for the various mortality levels are given in publication ST/SOA/Series A, Population Studies No. 25. Their method of derivation is explained both in that study and at greater length in No. 22 of the same series. From a study of sex and age patterns of mortality in 158 different life tables of many countries at different periods of time, it was possible to assemble sets of death rates for the whole of life which would reproduce given values of life expectancies at birth. U.N. mortality levels 80, 85, and 90 respectively reproduce expectation of life values 60.4, 63.2, and 65.8 years as a composite for both sexes.) Data on births by age of mother provided estimates of age-specific fertility rates.

Ten successive 5-year population projections were then based on these fertility and mortality rates. As the existing population grows older, these constant age-specific rates can be expected to make varying contributions to the total growth rate. Nevertheless, line A of figure 1 shows that the average annual percentage increase in population during each 5-year interval would never differ substantially from the intrinsic rate of 4 percent that eventually would be sustained.

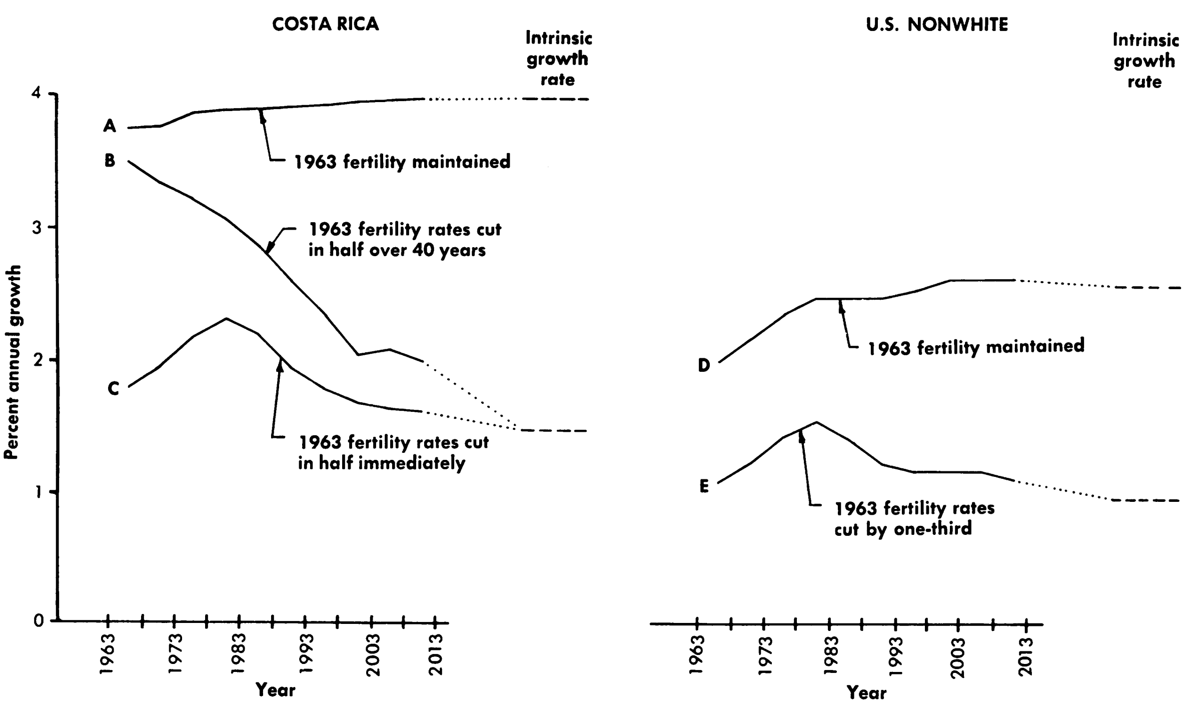
Table 1. Basic population data, Costa Rica, 1963

Age group (years)	Population (thousands)		Live births by age of mother	Deaths	
	Male	Female		Male	Female
Under 1	26. 0	25. 2	----------	2, 523	1, 933
1–4	100. 4	97. 4	----------	714	762
5–9	110. 3	106. 9	----------	180	125
10–14	86. 1	84. 4	64	79	54
15–19	↑	66. 0	7, 829	↑	60
20–24		52. 9	17, 760		62
25–29		43. 5	15, 623		79
30–34	345. 4	38. 6	11, 315	2, 552	82
35–39		34. 2	7, 671		87
40–44		26. 7	2, 649		99
45–49		22. 5	372		117
50 and over	↓	67. 8	----------	↓	1, 758
Total	668. 2	666. 1	63, 283	6, 048	5, 218

Source of basic data: reference 4.

To project the potential impact of family planning programs we must next consider the effect of a dramatic reduction in fertility rates. The distortion in age distribution that ensues will cause the natural and the intrinsic rates to diverge. To investigate this result, we postulated a sudden halving of each age-specific fertility rate in 1963 followed by a retention of the

Figure 1. Natural rates of increase, 1963 data



reduced level. Although such a reduction is far more abrupt than that which could be realistically anticipated, it would produce natural rates of increase that vary between only 1.7 and 2.3 percent. As shown in line C of figure 1, the natural rate would fall immediately to 1.8 percent, very close to the level at which it would eventually stabilize.

A normal result of the demographic transition in birth and death rates is a shift in age distribution. A decline in childhood mortality is followed by an increase in the number of adults who reach reproductive ages. Line C of figure 1 shows that the transitory increased fertility produced by this age shift is not major when the birth rate is halved abruptly. When the birth rate falls more gradually, as is normally the case, the effect is even less pronounced. This is shown by the gently sloping line B which projects a halving of age-specific fertility rates in regular intervals over a 40-year span.

To further illustrate transitions in growth rates we projected the U.S. nonwhite population over the half-century beginning in 1963 (*5*). Line D in figure 1 depicts results based upon 1963 age-specific fertility rates, whereas line E reflects an immediate reduction by one-third, which would bring the crude birth rate virtually to the level of the U.S. white population. The patterns followed by lines D and E are quite similar to those of lines A and C respectively.

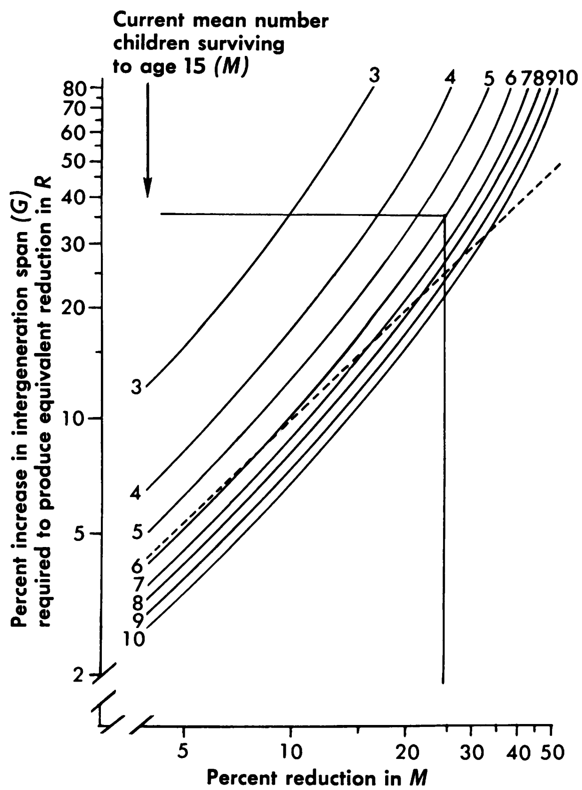
Thus, one can generally deal with the convenient natural rates of increase which are easily calculated from the crude rates usually available. To develop the formulations and graphs for this analysis, however, it was necessary to employ the more refined intrinsic rate of increase as a measure of population growth.

Primary Determinants of Growth Rate

Although public health personnel normally are concerned mainly with reducing mortality rates, in family planning these are accepted as exogenous and given. Attention currently is focused specifically upon possible mechanisms for reduction of fertility. Two major determinants which are the final common pathways for the effects of other variables are (*a*) a decreased number of children per family and (*b*) an increased intergeneration span. Reduced fertility does not correspond directly to the acceptance rate for a particular family planning device, for many women accept the device merely as a convenient substitute for their present form of contraception.

An appreciation of the relative strength under varying situations of each of the two basic determinants can be gained from figure 2. Each of the curved lines represents the number of children per family surviving to age 15, assuming that 49 percent are girls. The abscissa and ordinate scales then permit a comparison of the relative effectiveness of the two alternative approaches to fertility reduction. This is essentially a reformulation of an equation used by Coale and Tye (*6*), who in turn had re-examined the work of Dublin and Lotka (*7*). To depict the basic association as simply as possible, we ignored the somewhat greater risk of mortality to the mother that accompanies an increase in the average age of childbearing. (Admittedly, this

Figure 2. Alternative ways to achieve a given reduction in intrinsic rate [1]



[1] Assuming that 49 percent of the children are girls.

tends to underestimate the benefits to be gained from an increase of the intergeneration span. The effects are likely to be slight, however, as can be seen from subsequent comparisons which do include the mortality factor.)

Suppose that the mean number of surviving children in a population is currently six. The vertical and horizontal lines of figure 2 indicate that a reduction by 25 percent in mean number of surviving children would reduce the pace of population growth by exactly as much as a 36 percent increase in mean intergeneration span. These percentages can be translated easily into numbers of children and years of intergeneration span by using the relationships

$$M_r = M_o(1-P_r)$$

and

$$G_i = G_o(1+P_i),$$

where

M_o is the initial mean number of surviving children,

M_r is the reduced mean number of surviving children that results from the proportional reduction P_r,

G_o is the initial mean intergeneration span,

G_i is the increased mean intergeneration span that results from the proportional increase P_i.

Thus, if in the preceding illustration the initial intergeneration span is 25 years, we have

$$M_r = 6(1-0.25) = 4.5,$$
$$G_i = 25(1+0.36) = 34.$$

This means that a reduction in mean number of surviving children from 6 to 4.5 is equivalent in effect to an increase in the intergeneration span from 25 to 34 years.

For a greater reduction in growth rate, such as could be achieved by a 30 percent reduction in mean number of surviving children, the alternative of an increase in intergeneration span would be even more difficult to achieve—close to 50 percent. Thus, even if the intergeneration span were as low as 25 years, it would have to be raised unreasonably to 37 in order to achieve the same effect as a reduction in mean number from 6 to 4.2 surviving children.

On the contrary, if families are very large initially, the effect of an increase in intergeneration span is relatively large. For example, if the initial mean number of surviving children is eight, a 25 percent reduction in mean number (to 6) is equivalent in effect on intrinsic rate to a 28 percent increase in intergeneration span (say from 25 to 32). This is to be compared to the required increase in intergeneration span to 34 years when the mean number of surviving children was six.

If the cost of each 1 percent reduction in mean number could be equated to the cost or difficulty of achieving a certain increase in intergeneration span, then a diagonal could be drawn in figure 2 that would help to choose between the two alternatives. If, for example, each 1 percent reduction in mean number were considered to be as difficult as an equivalent increase in intergeneration span, we would draw a line of 45° slope from the origin of figure 2. Then whenever our use of the graph led us to a position above this diagonal, we would favor the alternative which encourages a reduction in family size. Most of the points on the curves would indeed lie above a 45° line.

Of course, it is not simple to compare the relative costs and difficulties of the two approaches. Nevertheless, the implications of given values of mean number of surviving children and intergeneration span can be made quite explicit and assessed accordingly. Suppose, for example, that in the population of concern the typical woman bears six children beginning at age 20 and continuing at equal intervals during the ensuing 10 years. The intergeneration span can be considered roughly to equal the age at which childbearing normally begins plus one-half the total childbearing period (later referred to as the "halfspan"). In this illustration, then, intergeneration span is approximately $20 + \frac{1}{2} \times 10 = 25$. It follows that an increase in intergeneration span to 34 years would require (*a*) a postponement of childbearing until ages 29 to 39, (*b*) an increased spacing of births so that childbearing takes place during ages 20 to 48, or (*c*) some combination of postponed initiation of childbearing and increased spacing.

Further, a reduction in completed family size from six children to 4.5 would itself force a reduction in intergeneration span by 1 year unless accompanied by postponed marriage or increased spacing.

Thus we can gain insight indirectly into the costs of alternative approaches in terms of the required changes in behavior patterns. This is perhaps the best we can hope for, particularly in light of existing cultural blocks and complex administrative restraints that may overbalance these straightforward demographic considerations.

From the preceding appraisal of figure 2, we can make the following generalization. The use of approaches such as delaying the age of marriage in order to increase the intergeneration span will be relatively more effective when families are large initially, and where one must settle for modest reductions in the birth rate. It is doubtful, however, whether ambitious programs in family planning can be successful in the long run without giving primary attention to reducing family size. In practice, of course, limiting family size by starting to have children at the same rate as before and then abruptly stopping childbearing will have a negative effect on intergeneration span by reducing the average age of maternal childbearing. Apart from these subtle but important relationships between mean number of surviving children and intergeneration span, it must be realized that most family planning programs rely upon the combined effect of both approaches. Still it is helpful to identify the probable relative effects of a different balance of determinants.

Procedure for Evaluating Determinants

Although an understanding of the relative influence of intergeneration span and mean number of surviving children is of general interest, the fieldworker must work with more detailed

Table 2. Calculations based on basic data of table 1, Costa Rica

Age group (years)	Basic data in thousands					Age-specific calculations		
	Population		Live births by age of mother (C)	Deaths		Birth rates $\frac{C}{B}$	Survival rates	
	Male (A)	Female (B)		Male (D)	Female (E)		Male $1-\frac{D}{A}$	Female $1-\frac{E}{B}$
Under 1	26.0	25.2		2,523	1,933		0.9030	0.9233
1–4	100.4	97.4		714	762		.9929	.9922
5–9	110.3	106.9		180	125		.9984	.9988
10–14	86.1	84.4	64	79	54	0.0008	.9991	.9994
15–19	↑	66.0	7,829	↑	60	.1186		.9991
20–24		52.9	17,760		62	.3357		.9988
25–29		43.5	15,623		79	.3591		.9982
30–34	345.4	38.6	11,315	2,552	82	.2931		.9979
35–39		34.2	7,671		87	.2243		.9975
40–44		26.7	2,649		99	.0992		.9963
45–49		22.5	372		117	.0165		.9948
50 and over	↓	67.8		↓	1,758			
Total	668.2	666.1	63,283	6,048	5,218	1.4473		

The following are additional calculations to estimate relevant parameters.

Crude birth rate:

$$\frac{63{,}283}{668.2+666.1}=47.4$$

Survival—

Crude death rate:

$$\frac{6{,}048+5{,}218}{668.2+666.1}=8.4$$

Male child survival: $0.9030\times0.9929^4\times0.9984^5\times0.9991^5=0.867$

Female child survival: $0.9233\times0.9922^4\times0.9988^5\times0.9994^5=0.887$

Child survival rate: $0.51\times0.867+0.49\times0.887=0.877$

Female adult survival (ages 15–49): $(0.9991\times0.9988\times0.9982\times0.9979\times0.9975\times0.9963\times0.9948)^5=0.9166$

Average annual female adult survival rate:

$$\sqrt[35]{0.9166}=0.9975$$

Births—

Live births per woman: $1.4473\times5=7.236=7.2$

Mean number surviving children: $7.2\times0.877=6.3$

Intergeneration span:

$$25.0+\frac{1.4473/2-0.4551}{0.3591}\times5=28.7$$

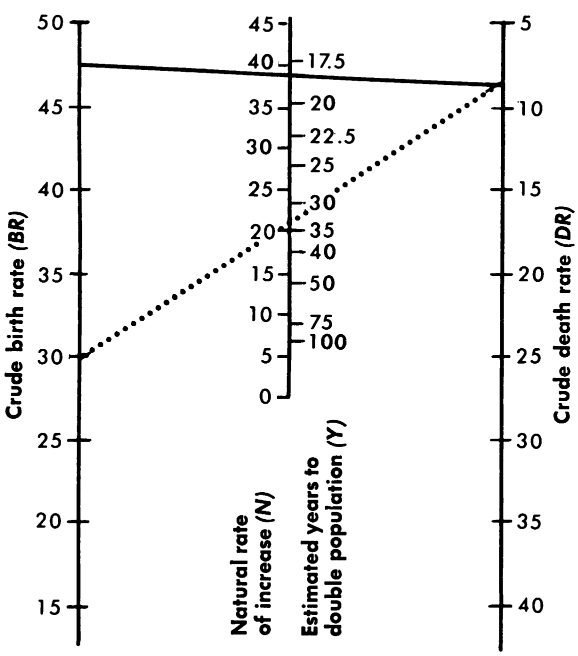
NOTES: If relevant life tables were available, child survival rate would be more simply and accurately calculated as $l_{15}\div100{,}000$, and female adult survival would be more simply and accurately calculated as $l_{50}\div l_{15}$. Live births per woman ignores the small influence of female mortality during childbearing years. The median is used to estimate the intergeneration span.

basic data (or perhaps crude estimates) such as those in table 1, which illustrates the following procedure for quickly relating such data to intergeneration span and mean number of surviving children and in turn to the population growth rate.

Establish existing baselines. The reasonableness of desired objectives of population control can be judged only in the light of current conditions. It is necessary, therefore, to estimate the current crude birth rate, crude death rate, mean number of surviving children, mean intergeneration span, anticipated average annual survival rate for women during the childbearing period, and anticipated percentage of live-born children who will survive to age 15.

The calculations in table 2, applied to the data of table 1, supply these estimates for Costa Rica. The values of crude birth and death rates are obtained directly from the reported births, deaths, and population size. The child survival rate is the product of the conditional probabilities of survival during each year from birth until adulthood. We consider this to be age 15, at which time the surviving females become subject to age-specific fertility and adult mortality rates. Where appropriate life tables are available, survival values for children and women can be obtained more directly from them.

Figure 3. Relation between birth rate, death rate, and population growth, Costa Rica



Subsequent use of the nomograms requires female survival during the childbearing years to be stated as an average annual rate. In this instance the appropriate average is the geometric mean of the annual probabilities of survival during the entire 15–49 age span (*8*).

To determine the total number of live births per woman we use a cohort of 10,000 ten-year-old girls subject to the prospective fertility rates in table 2 and immune from death. They can be expected to produce eight children in each of the first 5 years, 1,186 in each of the next 5, and so on. By the age of 50 they will have produced $8\times5+1{,}186\times5\ \ldots\ +165\times5=14{,}473\times5=72{,}365$ children in all, or 7.2 per woman. About 87.7 percent of these children, or 6.3, could be expected to survive to age 15.

The method used to estimate intergeneration span is the standard statistical procedure for calculating a median from grouped data (*8*). We define

$$Md=L+\frac{N/2-S}{F}\times I,$$

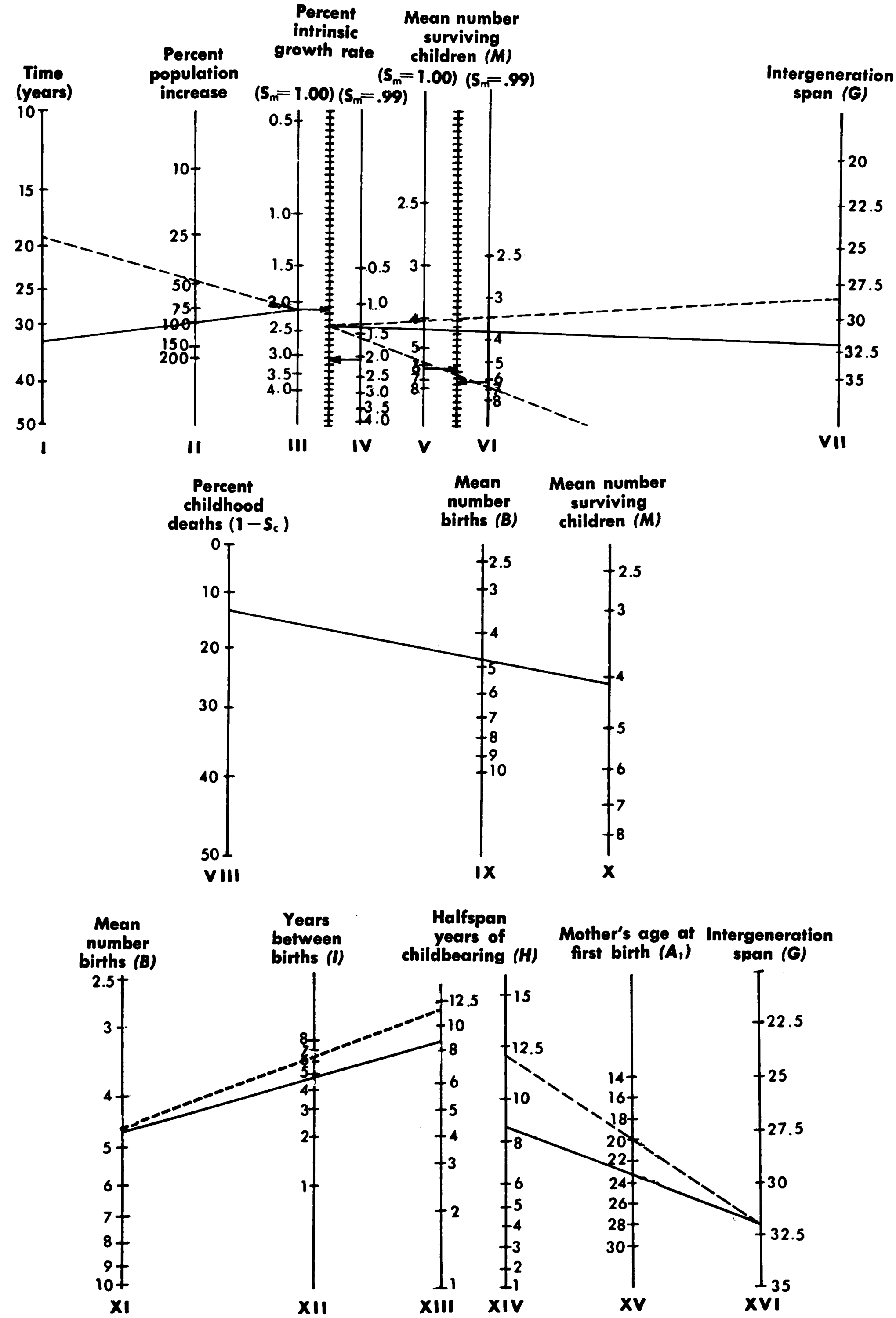
where

Md is the median to be calculated,
N is the total number of observations,
L is the lower limit of the median class, the one that contains the median value,
S is the cumulative number of observations in all classes below the median class,
F is the number of observations in the median class,
I is the width of the median class interval.

In our case we can consider the birth rates as relative frequencies within the various classes, that is, age groups. The median "observation" is $N/2$, or 0.72365 in the array; this is one of the 0.3591 "observations" contained in age group 25–29. It follows that $S=0.0008+0.1186+0.3357=0.4551$.

Determine the natural rate of increase. As mentioned earlier, the natural rate of increase is a simple approximation to the intrinsic rate and is derived by subtracting death rate from birth rate. Its derivation is facilitated by figure 3, where the appropriate values of birth and death rates are connected by a line. The point of intersection with the middle scales determines the

Figure 4. Illustration of association of elements of population growth, Costa Rica [1]



[1] Assuming that 49 percent of the children are girls.

natural rate of increase. The intersection also identifies the estimated number of years required for this rate of increase to double the population. In Costa Rica the natural rate of increase is 39 per 1,000 population, which suggests an annual growth rate of 3.9 percent and a doubling of the population within 18 years.

Specify an acceptable growth rate. Turning from a view of present conditions to future objectives, suppose that the basic family planning goal is to double the population in no less than one-third of a century. In the nomograms of figure 4 we located the value 33 years on scale I, the value 100 percent on scale II, and connected the two points with a line that extends to scale III. The intersection at scale III reveals a target growth rate of 2.1 percent per year. The broken line in figure 3 shows that, neglecting any change in the crude death rate, such an achievement would require a reduction in the crude birth rate to just under 30. Other implications of the goal can be seen from figure 4; for example, a population increase of 50 percent in approximately 20 years.

Adjust for female mortality during the childbearing years. The next step is to analyze the relation of the prescribed growth rate to family size and intergeneration span. In doing so, however, we must consider the level of adult female mortality. Scale III assumes this factor to be inconsequential, whereas scale IV is based upon an average annual mortality rate of 1 percent during the childbearing period (fig. 4). The actual circumstances are likely to be between these two extremes, thus requiring interpolation. With adult female mortality in Costa Rica estimated to be one-fourth of 1 percent ($S_m=$ 0.9975), we have located the point that is one-fourth of the distance between 2.1 on scale III and the corresponding point on scale IV (indicated by arrows).

Establish reasonable values of mean number of surviving children and intergeneration span. Any line extending from the prescribed point on scale III–IV to scales V–VI and VII indicates a potential combination of M and G values that would produce the desired growth rate (fig. 4). In determining the M value applicable to a given case, interpolation between scale V–VI must be accomplished in the same manner as it was on scale III–IV.

It is unlikely that all of the lines emanating from the value 2.1 on scale III–IV will represent viable alternatives. At one extreme we might suppose that the intergeneration span would remain at 28.7 years, thereby forcing M to be reduced unreasonably from 6.3 to less than 4. At the other extreme we might consider the number of surviving children to remain 6.3, thereby forcing G to increase to a nonsensical 50 years.

Feasible alternatives are likely to lie within the funnel formed by these two extremes. From figure 4, for example, the family planner might reason that the intergeneration span might be extended to 32 years but not beyond, in which case M would have to be reduced to roughly 4.1. This result is shown by the solid line extending to the center of scale III–IV and across V–VI and VII.

Assess the implications of the level of mean number of surviving children selected. For further insight into the reasonableness of the level of M derived above, we analyzed the restrictions it imposes upon the average number of live births per woman. This is a function of survival rates during childhood. With S_c estimated to be 0.877 in Costa Rica, we might postulate no foreseeable improvement and record a childhood death rate of 12 percent on scale VIII, transfer the M value of 4.1 from scale V–VI to scale X, and connect the selected points on scales VIII and X to intersect scale IX at $B=4.7$ (fig. 4). The family planner now knows that he has to reduce the number of live births per woman from 7.2 to 4.7, or perhaps further if childhood mortality rates are reduced.

Assess the implications of the levels of mean number of births and intergeneration span selected. The first set of scales (I–VII) establishes the overall relationship between R, M, and G (fig. 4). Scales VIII–X review the ramifications of the M value selected. Now the selected G value must be transferred from scale VII to scale XVI and the consequences reviewed. As mentioned earlier, intergeneration span is the sum of the mother's age (A_1) at the birth of her first child and the halfspan (H) of her period of actual childbearing. When the halfspan and the total number of live births (B) per woman have been specified, the average interval (I) between births is fixed; therefore the value of B derived from scale IX must be entered on scale XI in order to analyze its relation to I and H and ultimately to G.

Suppose, for example, that mothers typically bear their first children at age 20. This information would be inserted on scale XV and, coupled with the G value of 32 from scale XVI, would produce on scale XIV a required H of 12 years. Since scale XIV is an arithmetic scale, H must also be entered on the logarithmic scale XIII so that it may be related to $B=4.7$ on scale XI to show (scale XII) that a 6.5 year interval between births has been imposed.

Since an interval of this length would probably be considered unrealistic, the family planner would seek to modify some of the other conditions. To illustrate, he might decide that the most that could be expected would be a spacing of 4.5 years. The resulting line across scales XI, XII, and XIII would yield a halfspan of less than 9 years. If he continues to deal with an intergeneration span of 32 years, the first birth must be postponed until after the mother has become 23 years old.

To summarize, one way to reduce the Costa Rican growth rate from 3.9 percent to 2.1 percent annually would be to induce the typical woman to marry late enough that her first child would not be born before she was 23 years of age. She must also be motivated to space subsequent births at 4- to 5-year intervals until she had borne a total of five children, four of whom could be expected to live to adulthood.

Additional "solutions" could be developed from the nomograms, which incorporate the interrelationships among all the variables cited, so that inconsistencies are made obvious. Application of a certain amount of trial and error to

Table 3. U.S. nonwhite population estimates, 1963

Age group (years)	Basic data in thousands					Age-specific calculations		
	Population		Live births by age of mother (C)	Deaths		Birth rates $\frac{C}{B}$	Survival rates	
	Male (A)	Female (B)		Male (D)	Female (E)		Male $1-\frac{D}{A}$	Female $1-\frac{E}{B}$
Under 1	328	324	----------	15.3	11.9	----------	0.9533	0.9633
1–4	1,265	1,264	----------	2.4	2.0	----------	.9981	.9984
5–9	1,429	1,429	----------	.4	.4	----------	.9997	.9997
10–14	1,231	1,229	4.9	1.5	1.0	0.0040	.9988	.9992
15–19	↑	955	133.6	↑	.8	.1399	----------	.9992
20–24		770	214.1		.9	.2781	----------	.9988
25–29		699	147.6		1.3	.2112	----------	.9981
30–34	6,457	719	92.7	103.6	2.2	.1289	----------	.9969
35–39		701	48.3		3.1	.0689	----------	.9956
40–44		697	14.6		4.5	.0209	----------	.9935
45–49		552	.8		5.0	.0014	----------	.9909
50 and over	↓	2,028	----------	↓	65.8	----------	----------	----------
Total	10,710	11,367	656.6	123.2	98.9	.8533	----------	----------

The following are additional calculations to estimate relevant parameters.

Crude birth rate:

$$\frac{656{,}600}{10{,}710+11{,}367}=29.7$$

Survival—

Crude death rate:

$$\frac{123{,}200+98{,}900}{10{,}710+11{,}367}=10.1$$

Male child survival: $0.9533\times0.9981^4\times0.9997^5\times0.9988^5=0.939$

Female child survival: $0.9633\times0.9984^4\times0.9997^5\times0.9992^5=0.952$

Child survival rate: $0.51\times0.939+0.49\times0.952=0.945$

Female adult survival (ages 15–49): $(0.9992\times0.9988\times0.9981\times0.9969\times0.9956\times0.9935\times0.9909)^5=0.8734$

Average annual female adult survival rate:

$$\sqrt[35]{0.8734}=0.9960$$

Births—

Live births per woman: $0.8533\times5=4.266=4.3$

Mean number surviving children: $4.3\times0.945=4.1$

Intergeneration span:

$$25.0+\frac{0.8533/2-0.4228}{0.2112}\times5=25.1$$

NOTE: If relevant life tables were available, child survival rate would be more simply and accurately calculated as $l_{15}\div100{,}000$, and female adult survival would be more simply and accurately calculated as $l_{50}\div l_{15}$. Live births per woman ignores the small influence of female mortality during childbearing years. The median is used to estimate the intergeneration span.

SOURCE OF BASIC DATA: reference 6.

the nomograms should quickly replace difficult constraints with those that can be attained with relative ease. The result should be a reasonable, consistent set of specific objectives for a family planning program.

Application to U.S. Nonwhite Data

Family planning programs must deal with a set of interrelated variables, some of which are targeted in advance to reach some desired level. The function of the nomograms, then, is to identify the resulting impact upon the remaining variables. Since the target variables will not always be the same, it follows that the steps taken in probing the reasonableness of the program proposal will not always be in the order described for Costa Rica.

To illustrate this, we consider the prospect of altered fertility patterns among the U.S. nonwhite population. In particular, what would happen if the average nonwhite woman were motivated to bear a total of only three children spaced 4 years apart beginning at age 22. Presumably such a proposal would be based upon estimates of existing conditions, as recorded in table 3, and experienced judgments concerning the prospects for change.

The nomograms of figure 4 are reproduced in figure 5 to show the appropriate entries relevant to the U.S. nonwhite population. The analysis of proposal ramifications begins by locating the point $B=3$ on scale XI, the point $I=4$ on scale XII, and extending to scale XIII a line connecting these points. This shows a halfspan of 4 years, a result which is also noted on scale XIV, and associated with the age $A_1=22$ on scale XV to produce a line which extends to the point $G=26$ on scale XVI. Thus, the proposal yields an intergeneration span of 26 years. (In this instance the result was so obvious that it could have been obtained by some quick mental arithmetic. The nomogram approach is also fast, however, even when a more complex set of numbers is used.)

Because it is not feasible to introduce mortality improvements into our calculations, we can anticipate, from table 3, that 6 percent of the births will lead to death (including neonatal death) before age 15, and we enter this prospect on scale VIII with the value $B=3$ on scale IX. The line that cuts these points and extends to scale X produces an M value of 2.8.

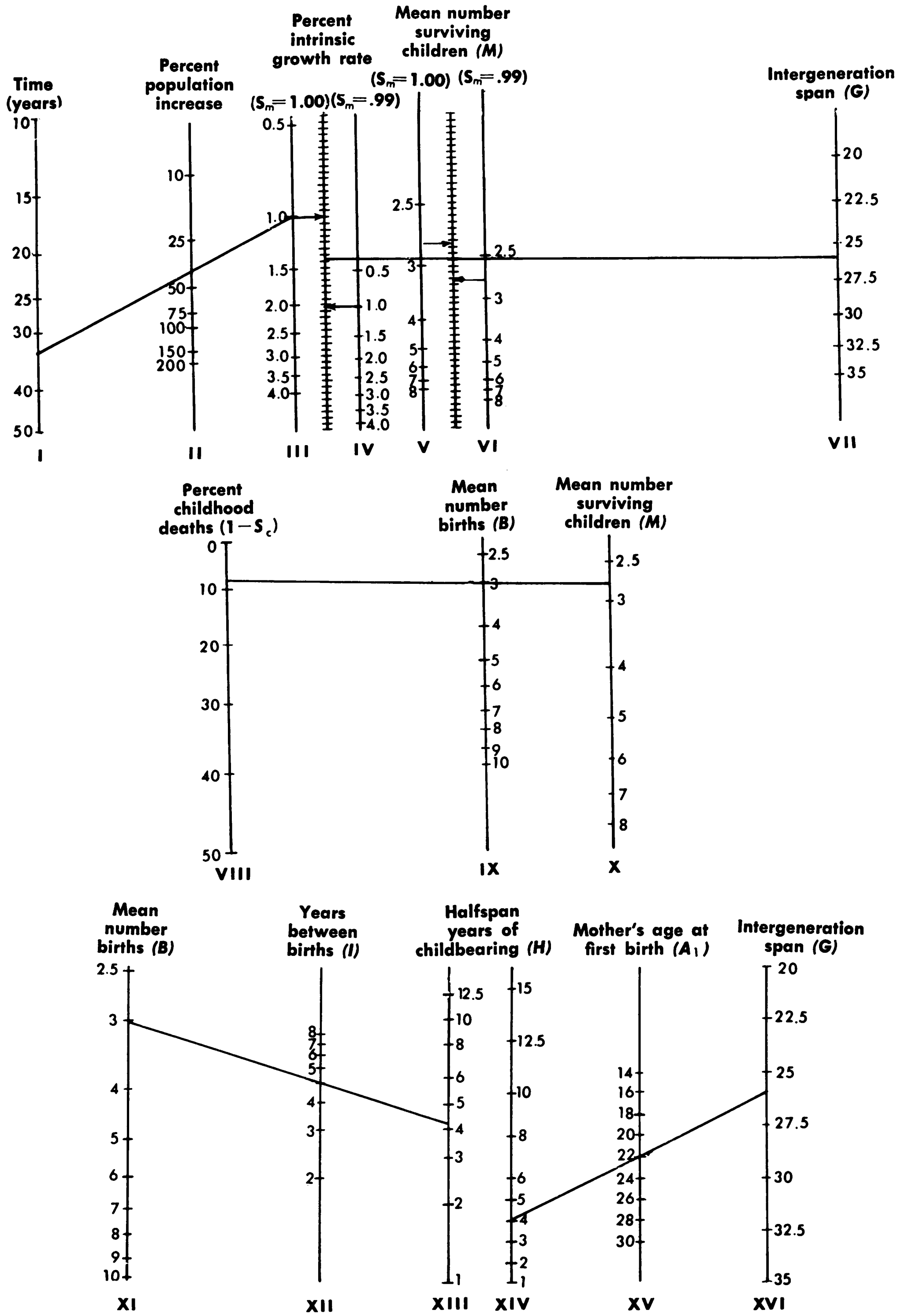
This leads to the crucial matter of deriving an intrinsic growth rate from the intergeneration span and the mean number of surviving children. We enter the value $G=26$ on scale VII and locate the level $M=2.8$ on scale V–VI, noting that $S_m=0.996$. On scale III–IV the intrinsic growth rate is 1 percent, which is approximately the current rate of increase of the U.S. white population.

Some interesting ramifications of this growth rate can be observed from scales I and II. For example, under these circumstances the nonwhite population will have increased by 36 percent by the year 2000 (in 31 years).

If the anticipated values of M and G are compared with the values shown in table 3, the proposal would reduce M by more than 30 percent (from 4.1 to 2.8), whereas G would be increased by only 4 percent (from 25.1 to 26.0). This may raise the question of whether more of the burden should be shifted to the intergeneration span through child-spacing. Figure 2 readily provides insight into this question. Here the substitute for a 30 percent reduction in M is so large an increase in G that it exceeds the limits of the graph (113 percent). Even a proposal to shift half (15 percent) of the reduction in M to an appropriate increase in G seems unsound, for figure 2 shows that G would have to be raised by 32 percent, from 26 to 34 years. Modifications that seem to be reasonable can be checked via the nomograms for their impact upon each of the relevant variables.

The relative ineffectiveness of postponed childbearing in the U.S. nonwhite population is especially interesting since U.S. nonwhite women bear their children at a young age ($G=25.1$ versus 28.7 in Costa Rica). The reason for the ineffectiveness of an increase in intergeneration span stems, paradoxically, from the lower level of mean number of surviving children in the U.S. nonwhite population. As we noted earlier in discussing figure 2, an increase in intergeneration span is most effective as a means of reducing population growth when families are very large and a moderate reduction in the growth rate is being proposed. Where the mean number of surviving children is six, as it is in Costa Rica, the effect of a 15 percent reduction

Figure 5. Illustration of association of elements of population growth, U.S. nonwhite population [1]



[1] Assuming that 49 percent of the children are girls.

in mean number of surviving children is equivalent to an 18 percent increase in intergeneration span. This is in sharp contrast to the 32 percent increase in intergeneration span which would be required as a substitute for a 15 percent reduction in mean number of surviving children from an initial level of four.

Conclusions

The nomograms can be used for a number of purposes other than those illustrated. They offer evidence of the practical ramifications of dissimilar conditions in different countries or in different segments of the same population. Hence they call attention to the fallacy of following a single approach everywhere. Moreover, they can be used to assess quickly the likely impact of certain anticipated changes in exogenous conditions, such as a reduction in infant mortality. Briefly, the graphs are designed for a quick review of general, quantitative relationships.

The graphs are by no means a substitute for simulations and other studies of transitional and dynamic effects of specific programs in individual countries. In particular, they do not deal with the important indirect association between changing rates of acceptance of a given form of contraception and the degree to which acceptance becomes reflected in reduced fertility or increased intergeneration span. The rate and effectiveness of acceptance depend upon parity, the extent to which other forms of contraception are being supplanted, and motivational factors, such as socioeconomic status, program publicity, and availability of family planning facilities. All of these factors are beyond the scope of this paper.

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Public Health Service Staff Appointment

Dr. Edward B. Cross, former director of the Department of Health, Education, and Welfare's Office of Health and Medical Care, has been promoted to the position of Assistant Surgeon General of the Public Health Service.

Dr. Cross, a Public Health Service commissioned officer since 1952, served in the former capacity from July 1968 until the time of his recent appointment. As a specialist in health service and medical care, he develops proposals for health policy and legislation and assists the Assistant Secretary for Health and Scientific Affairs in coordinating the activities of health services programs throughout the Department.

Dr. Cross is health affairs liaison officer to the Department of Defense, Office of Economic Opportunity, and the Veterans' Administration. He is also staff adviser to such Federal Government advisory committees as the Health Insurance Benefits Advisory Council, Medical Assistance Advisory Council, and Interagency Liaison Committee for Hospitals Systems Analysis (Department of Defense).

Dr. Cross has been active in efforts to improve the delivery of health services to disadvantaged groups in urban and semirural areas. Recently he served for 2 years as interim director of Cleveland's health department, reorganizing its functions to direct services to inner city residents. He has developed interagency funding and support for comprehensive maternal, children and youth programs in New Haven, Conn., and Jackson, Miss.

As Medical Director of the Peace Corps in Addis Ababa, Ethiopia, he had responsibility for a countrywide preventive health acute-medical care program providing services for 400 Peace Corps volunteers and staff as well as special projects providing health services to Ethiopian communities.