

SUPPLEMENTAL INFORMATION

If  $Y \sim ZINB(\omega, \mu)$ , then  $E(Y) = (1 - \omega)\mu$ . Define  $f(\omega, \mu) = (1 - \omega)\mu$ . Then, applying the delta method (Cooch and White 2006), we obtain

$$\begin{aligned}\widehat{E(Y)} &= E(f(\hat{\omega}, \hat{\mu})) \\ &\approx f[E(\hat{\omega}), E(\hat{\mu})] \\ &= (1 - E(\hat{\omega}))E(\hat{\mu})\end{aligned}$$

and

$$\begin{aligned}\text{Var}[\widehat{E(Y)}] &= \text{Var}(f(\hat{\omega}, \hat{\mu})) \\ &\approx \left[ \frac{\partial f}{\partial \theta} \right]^T \cdot \hat{\Sigma} \cdot \left[ \frac{\partial f}{\partial \theta} \right]_{\hat{\theta}},\end{aligned}$$

where

$$\theta = [\underline{\omega}^T, \underline{\mu}^T]^T \quad \text{and} \quad \hat{\Sigma} = \text{cov}(\hat{\theta}).$$

In the regression model,

$$\mu_i = \exp(Y_0 + Y_1 Z_{i1} + Y_2 Z_{i2} + \dots + Y_q Z_{iq}) \equiv g_{1i}(\underline{Y} | \underline{Z}_i) \quad (S1)$$

and

$$\omega_i = P_i = \frac{\exp(\beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots + \beta_p X_{ip})}{1 + \exp(\beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots + \beta_p X_{ip})} \equiv g_{2i}(\underline{\beta} | \underline{X}_i), \quad (S2)$$

where  $Y_0, Y_1, Y_2, \dots, Y_q$ , and  $\beta_0, \beta_1, \beta_2, \dots, \beta_p$ , are unknown coefficients of the count and “zero-inflated” components of the model, respectively, and  $Z_1, Z_2, \dots, Z_{iq}$  and  $X_1, X_2, \dots, X_{ip}$  are known covariates of the count and “zero-inflated” components of the model, respectively. Thus, the estimated expected counts and standard errors of the expected counts are, respectively,

$$\begin{aligned}\widehat{E(Y)} &= E[f(g_2(\hat{\beta}), g_1(\hat{Y}))] \\ &\approx f[g_2(E(\hat{\beta})), g_1(E(\hat{Y}))]\end{aligned} \quad (S3)$$

and

$$\begin{aligned}\text{Var}[\widehat{E(Y)}] &= \text{Var}[f(g_2(\hat{\beta}), g_1(\hat{Y}))] \\ &\approx \left[ \frac{\partial(f \circ g)}{\partial \xi} \right]^T \cdot \hat{\Gamma} \cdot \left[ \frac{\partial(f \circ g)}{\partial \xi} \right]_{\hat{\xi}},\end{aligned} \quad (S4)$$

where

$$\xi = [\underline{Y}^T, \underline{\beta}^T]^T \quad \text{and} \quad \hat{\Gamma} = \text{cov}(\hat{\xi}).$$