Let $B$ denote the behavioral pattern and let $N$ denote the number of sexual partners. Additionally, let $C$ denote a vector of partnership characteristics that includes the numbers of sexual acts with HIV-infected partners who are not virally suppressed, $n_{jk}$, where $j$ is an index for sexual partner and $k$ is an index for type of sexual act (not to be confused with behavioral pattern). The model relies on the following probabilities:

1. $P(B)$
2. $P(N|B)$
3. $P(C|N, B)$

The modeling exercise estimates the probability distributions using various data sources, as explained in the Data subsection.

Each individual’s conditional probability for infection is a function of $N$ and $C$:

$$P(I = 1|N, C) = (1 - \prod_j \prod_k (1 - \pi_k)^{n_{jk}})(1 - \epsilon)^\delta$$

Here, $j$ is an indicator for sexual partner, $k$ is an index for type of sexual contact, $I$ is a Bernoulli-distributed random variable with a value of 1 if the individual becomes infected, $\pi_k$ is the per-act probability of infection associated with type $k$, $n_{jk}$ is the number of acts of type $k$ with partner $j$, $\epsilon$ is PrEP efficacy, and $\delta$ is an indicator taking a value of 1 if the individual accessed PrEP in the last year and 0 otherwise. The expression assumes independence across all acts, that the per-act probability of infection is constant within type, and that PrEP efficacy is constant.

By definition, $P(I|N, C) = P(I|N, C, B)$. Thus, the product of (1), (2), (3), and (4) is the joint distribution $P(I, C, N, B)$. Summation over margins of the joint distribution provides the numerators and denominators of the quantities of interest:

$$P(B = b|I = 1) = \frac{P(I = 1, B = b)}{P(I = 1)}$$

For example, the numerator is the sum of $P(I, C, N, B)$ over all $C$ and $N$. 