AJAE Appendix for

By Ounce or By Calorie: The Differential Effects of Alternative Sugar-Sweetened Beverage Tax Strategies

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Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE)
In this appendix, we derive the formulas for FMDM price elasticities. The STATA codes for estimating the fixed-effect 2SLS conventional DM, quasi-FMDM, and FMDM models and SAS codes for generating the analysis file, calculating elasticities, and conducting counterfactual simulations are posted online as supplementary data.

**FMDM Elasticities**

For brevity of notation, we drop the market and time subscripts \( h \) and \( t \) from the FMDM elasticities. The Marshallian price elasticity conditional on total beverage expenditures is

\[
\eta_i = \frac{d \ln q_i}{d \ln p_j} = -\delta_i + \frac{d \ln w_i}{d \ln p_j} = -\delta_i + \frac{1}{w_i} \left\{ \gamma_{ij} + \sum_{r \neq i} \ln p_r \frac{\partial \gamma_{ij}}{\partial \ln p_j} - \beta_i \frac{\partial \ln p_r}{\partial \ln p_j} \right\},
\]

where \( q_i \) is the quantity of product \( i \); \( \delta_i = 1 \) for \( i = j \), and 0 otherwise. Note that, following the approach in Green and Alston (1990),

\[
\frac{\partial \gamma_{ir}}{\partial \ln p_j} = \sum_{m=1}^{M} d_m \left[ w_{mir}^* \frac{\partial w_i}{\partial \ln p_j} + w_i \frac{\kappa_{mir}}{\omega_{mi}} \frac{\partial w_r}{\partial \ln p_j} - w_i w_r \frac{\kappa_{mir}}{\omega_{mi}^2} \frac{\partial \omega_{mi}}{\partial \ln p_j} \right],
\]

\[
\frac{\kappa_{mir}}{\omega_{mi}^2} \frac{\partial \omega_{mi}}{\partial \ln p_j} = \frac{\kappa_{mir}}{\omega_{mi}^2} \left\{ \frac{\partial w_i}{\partial \ln p_j} + \sum_{k \in N} \frac{\kappa_{mk}}{\omega_{mi}^2} \frac{\partial \omega_{mk}}{\partial \ln p_j} \right\}
\]

\[
= \frac{\kappa_{mir}}{\omega_{mi}} \left\{ \frac{w_i}{\omega_{mi}} \frac{\partial \ln w_i}{\partial \ln p_j} + \sum_{k \in N} \frac{w^*_{mk}}{\omega_{mi}} \frac{\partial \ln w_k}{\partial \ln p_j} \right\},
\]

and

\[
\frac{\partial \ln p_r}{\partial \ln p_j} = w_j + \sum_{r \in N} w_i \ln p_r \frac{\partial \ln w_i}{\partial \ln p_j} = w_j + \sum_{r \in N} w_i \ln p_j (\eta_{ij} + \delta_{ij}).
\]

Substituting (A2) through (A4) into (A1) gives
\[ \eta_{ij} = -\delta_{ij} + \frac{1}{w_i} \left[ \gamma_{ij} + \sum_r \sum_{m=1}^M d_{m} w_{mij} \ln p_r \left[ \eta_{ij} + \delta_{ij} + \eta_{ij} + \delta_{ij} - (w_i/\omega_m)(\eta_{ij} + \delta_{ij}) \right] \right] \\
- \beta_i \left[ w_j + \sum_r w_r \ln p_r (\eta_{ij} + \delta_{ij}) \right] \]

(A5)

\[ \eta_{ij} = -\delta_{ij} + \frac{1}{w_i} \left[ \gamma_{ij} + \sum_r \sum_{m=1}^M d_{m} w_{mij} \ln p_r \left[ \sum_k w_{mk}^* (\eta_{ij} + \delta_{ij}) + \eta_{ij} + \delta_{ij} \right] \right] \\
- \beta_i \left[ w_j + \sum_r w_r \ln p_r (\eta_{ij} + \delta_{ij}) \right] \]

Equation (A5) can be written in matrix form as

(A6) \[ H = A + \left[ \sum_{m=1}^M B_m \right] (H + I) + \left[ \sum_{m=1}^M D_m \right] (H + I) - \left[ \sum_{m=1}^M C_m F_m \right] (H + I) - (UV)(H + I) \]

where the matrix elements are \( H_{ij} = \eta_{ij} \) in \( H \) \( (n \times n) \) matrix, \( A_{ij} = -\delta_{ij} + \gamma_{ij}/w_i - \beta_i w_j/w_i \) in \( A \) \( (n \times n) \) matrix, \( B_{mij} = \sum_r d_{m} w_{mij}^* \ln p_r \sum_k w_{mk}^* \) in \( B_m \) \( (n \times n) \) diagonal matrix, \( D_{mij} = d_{m} w_{mij}^* \ln p_j \) in \( D_m \) \( (n \times n) \) matrix, \( C_{mij} = \sum_r d_{m} w_{mij}^* \ln p_r \) in \( C_m \) \( (n \times n) \) diagonal matrix, \( F_{mij} = w_{mij}^* \) in \( F_m \) \( (n \times n) \) matrix, \( U_i = \beta_i/w_i \) in \( U \) \( (n \times 1) \) vector, \( V_j = w_j \ln p_j \) in \( V \) \( (1 \times n) \) vector, and \( I \) is a \( n \times n \) identity matrix.

Solving equation (A6) for \( H \) gives the conditional price elasticity matrix:

(A7) \[ H = \left\{ I - \sum_{m=1}^M B_m - \sum_{m=1}^M D_m + \sum_{m=1}^M C_m F_m + UV \right\}^{-1} (A + I) - I . \]
The expenditure elasticities for the FMDM model are also more complicated than those for the conventional DM model because of the presence of current budget shares on the right-hand side of the demand equation. The expenditure elasticity is

(A8) \[ \epsilon_i = \frac{d \ln q_i}{d \ln x} = 1 + \frac{d \ln w_i}{d \ln x} = 1 + \frac{1}{w_i} \left\{ \sum_{r \neq i} \ln p_r \frac{\partial \gamma_{ir}}{\partial \ln x} + \beta \left[ 1 - \frac{\partial \ln p_b}{\partial \ln x} \right] \right\}, \]

where

(A9) \[ \frac{\partial \gamma_{ir}}{\partial \ln x} = \sum_{m=1}^{M} d_m \left\{ \frac{w_{mir}^*}{w_{im}} \right\} \cdot \frac{\partial w_{mir}}{\partial \ln x} + \frac{\kappa_{mir}}{\omega_{mi}} \frac{\partial w_{ir}}{\partial \ln x} - \frac{w_r}{w_i} \frac{\kappa_{mir}}{\omega_{mi}} \frac{\partial \omega_{mi}}{\partial \ln x} \],

and

(A10) \[ \frac{\kappa_{mir}}{\omega_{mi}} \frac{\partial \omega_{mi}}{\partial \ln x} = \frac{\kappa_{mir}}{\omega_{mi}} \left\{ \frac{\partial w_{ir}}{\partial \ln x} + \sum_{k \in N} \frac{\partial w_{ir}^*}{\partial \ln x} \right\} \]

Substituting (A9) through (A11) into (A8) gives

(A12) \[ \epsilon_i = 1 + \frac{1}{w_i} \left\{ \sum_{r \neq N} \sum_{m=1}^{M} w_{mir}^* \ln p_r \left[ \epsilon_i - 1 + \epsilon_r - 1 - \left( w_i / \omega_{mi} \right) (\epsilon_i - 1) - \sum_{k \in N} w_{rik}^* (\epsilon_k - 1) \right] \right\} + \beta \left[ 1 - \sum_{r \neq N} w_i \ln p_r (\epsilon_r - 1) \right]. \]
Equation (A12) can be expressed in matrix form as

\[(A13) \quad E = t + U + \left( \sum_{m=1}^{M} B_m \right)(E - t) + \left( \sum_{m=1}^{M} D_m \right)(E - t) - \left( \sum_{m=1}^{M} C_m F_m \right)(E - t) - (UV)(E - t) \]

where, in addition to matrices defined in equation (A5), the matrix elements are \( E_i = \epsilon_i \) in \( E \) \((n \times 1\) vector), and \( t \) is a \( n \times 1 \) vector of ones. Solving (A13) for the vector of expenditure elasticities \( E \) gives

\[(A14) \quad E = \left\{ I - \sum_{m=1}^{M} B_m - \sum_{m=1}^{M} D_m + \sum_{m=1}^{M} C_m F_m + UV \right\}^{-1} U + t. \]