AJAE Appendix for

By Ounce or By Calorie: The Differential Effects of Alternative Sugar-Sweetened Beverage Tax Strategies

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In this appendix, we derive the formulas for FMDM price elasticities. The STATA codes for estimating the fixed-effect 2SLS conventional DM, quasi-FMDM, and FMDM models and SAS codes for generating the analysis file, calculating elasticities, and conducting counterfactual simulations are posted online as supplementary data.

FMDM Elasticities

For brevity of notation, we drop the market and time subscripts h and t from the FMDM elasticities. The Marshallian price elasticity conditional on total beverage expenditures is

$$\text{(A1)} \qquad \eta_{ij} = \frac{d \ln q_i}{d \ln p_j} = -\delta_{ij} + \frac{d \ln w_i}{d \ln p_j} = -\delta_{ij} + \frac{1}{w_i}\left\{ \gamma_{ij} + \sum_{r \neq i} \ln p_r \frac{\partial \gamma_{ir}}{\partial \ln p_j} - \beta_i \frac{\partial \ln p_b}{\partial \ln p_j} \right\},$$

where q_i is the quantity of product i; $\delta_{ij} = 1$ for $i = j$, and 0 otherwise. Note that, following the approach in Green and Alston (1990),

$$\text{(A2)} \qquad \frac{\partial \gamma_{ir}}{\partial \ln p_j} = \sum_{m=1}^{M} d_m \left\{ w_{mir}^{*} \frac{\partial w_i}{\partial \ln p_j} + w_i \frac{\kappa_{mir}}{\omega_{mi}} \frac{\partial w_r}{\partial \ln p_j} - w_i w_r \frac{\kappa_{mir}}{\omega_{mi}^2} \frac{\partial \omega_{mi}}{\partial \ln p_j} \right\},$$

$$\text{(A3)} \qquad \begin{aligned} \frac{\kappa_{mir}}{\omega_{mi}^2} \frac{\partial \omega_{mi}}{\partial \ln p_j} &= \frac{\kappa_{mir}}{\omega_{mi}^2} \left\{ \frac{\partial w_i}{\partial \ln p_j} + \sum_{k \in N} \kappa_{mik} \frac{\partial w_k}{\partial \ln p_j} \right\} \\ &= \frac{\kappa_{mir}}{\omega_{mi}} \left\{ \frac{w_i}{\omega_{mi}} \frac{\partial \ln w_i}{\partial \ln p_j} + \sum_{k \in N} w_{mik}^{*} \frac{\partial \ln w_k}{\partial \ln p_j} \right\} \\ &= \frac{\kappa_{mir}}{\omega_{mi}} \left\{ \frac{w_i}{\omega_{mi}} \left(\eta_{ij} + \delta_{ij} \right) + \sum_{k \in N} w_{mik}^{*} \left(\eta_{kj} + \delta_{kj} \right) \right\} \end{aligned} \quad ,$$

and

$$\text{(A4)} \qquad \frac{\partial \ln p_b}{\partial \ln p_j} = w_j + \sum_{r \in N} w_r \ln p_r \frac{\partial \ln w_r}{\partial \ln p_j} = w_j + \sum_{r \in N} w_r \ln p_r \left(\eta_{rj} + \delta_{rj} \right).$$

Substituting (A2) through (A4) into (A1) gives

(A5)
$$\eta_{ij} = -\delta_{ij} + \frac{1}{w_i}\left\{\begin{array}{l} \gamma_{ij} + \sum_r \sum_{m=1}^{M} d_m w_i w_{mir}^* \ln p_r \left[\begin{array}{l} \eta_{ij} + \delta_{ij} + \eta_{rj} + \delta_{rj} - (w_i / \omega_{mi})(\eta_{ij} + \delta_{ij}) \\ -\sum_k w_{mik}^* (\eta_{kj} + \delta_{kj}) \end{array}\right] \\ -\beta_i \left[w_j + \sum_r w_r \ln p_r (\eta_{rj} + \delta_{rj}) \right] \end{array}\right\}$$
$$= -\delta_{ij} + \frac{1}{w_i}\left\{\begin{array}{l} \gamma_{ij} + \sum_r \sum_{m=1}^{M} d_m w_i w_{mir}^* \ln p_r \left[\begin{array}{l} \sum_k w_{mik}^* (\eta_{ij} + \delta_{ij}) + \eta_{rj} + \delta_{rj} \\ -\sum_k w_{mik}^* (\eta_{kj} + \delta_{kj}) \end{array}\right] \\ -\beta_i \left[w_j + \sum_r w_r \ln p_r (\eta_{rj} + \delta_{rj}) \right] \end{array}\right\}$$

Equation (A5) can be written in matrix form as

(A6)
$$\mathrm{H} = A + \left[\sum_{m=1}^{M} B_m\right](\mathrm{H} + I) + \left[\sum_{m=1}^{M} D_m\right](\mathrm{H} + I) - \left[\sum_{m=1}^{M} C_m F_m\right](\mathrm{H} + I) - (UV)(\mathrm{H} + I)$$

where the matrix elements are $\mathrm{H}_{ij} = \eta_{ij}$ in H ($n \times n$ matrix), $A_{ij} = -\delta_{ij} + \gamma_{ij} / w_i - \beta_i w_j / w_i$ in A ($n \times n$ matrix), $B_{mii} = \sum_r d_m w_{mir}^* \ln p_r \sum_k w_{mik}^*$ in B_m ($n \times n$ diagonal matrix), $D_{mij} = d_m w_{mij}^* \ln p_j$ in D_m ($n \times n$ matrix), $C_{mii} = \sum_r d_m w_{mir}^* \ln p_r$ in C_m ($n \times n$ diagonal matrix), $F_{mij} = w_{mij}^*$ in F_m ($n \times n$ matrix), $U_i = \beta_i / w_i$ in U ($n \times 1$ vector), $V_j = w_j \ln p_j$ in V ($1 \times n$ vector), and I is a $n \times n$ identity matrix.

Solving equation (A6) for H gives the conditional price elasticity matrix:

(A7)
$$\mathrm{H} = \left\{ I - \sum_{m=1}^{M} B_m - \sum_{m=1}^{M} D_m + \sum_{m=1}^{M} C_m F_m + UV \right\}^{-1} (A + I) - I .$$

The expenditure elasticities for the FMDM model are also more complicated than those for the conventional DM model because of the presence of current budget shares on the right-hand side of the demand equation. The expenditure elasticity is

$$\varepsilon_i = \frac{d\ln q_i}{d\ln x} = 1 + \frac{d\ln w_i}{d\ln x} = 1 + \frac{1}{w_i}\left\{\sum_{r\neq i}\ln p_r \frac{\partial \gamma_{ir}}{\partial \ln x} + \beta_i\left[1 - \frac{\partial \ln p_b}{\partial \ln x}\right]\right\}, \tag{A8}$$

where

$$\frac{\partial \gamma_{ir}}{\partial \ln x} = \sum_{m=1}^{M} d_m\left\{w^*_{mir}\frac{\partial w_i}{\partial \ln x} + w_i\frac{\kappa_{mir}}{\omega_{mi}}\frac{\partial w_r}{\partial \ln x} - w_i w_r \frac{\kappa_{mir}}{\omega^2_{mi}}\frac{\partial \omega_{mi}}{\partial \ln x}\right\}, \tag{A9}$$

$$\begin{aligned}\frac{\kappa_{mir}}{\omega^2_{mi}}\frac{\partial \omega_{mi}}{\partial \ln x} &= \frac{\kappa_{mir}}{\omega^2_{mi}}\left\{\frac{\partial w_i}{\partial \ln x} + \sum_{k\in N}\kappa_{mik}\frac{\partial w_k}{\partial \ln x}\right\}\\ &= \frac{\kappa_{mir}}{\omega_{mi}}\left\{\frac{w_i}{\omega_{mi}}\frac{\partial \ln w_i}{\partial \ln x} + \sum_{k\in N} w^*_{mik}\frac{\partial \ln w_k}{\partial \ln x}\right\},\\ &= \frac{\kappa_{mir}}{\omega_{mi}}\left\{\frac{w_i}{\omega_{mi}}(\varepsilon_i - 1) + \sum_{k\in N} w^*_{mik}(\varepsilon_k - 1)\right\}\end{aligned} \tag{A10}$$

and

$$\frac{\partial \ln p_b}{\partial \ln x} = \sum_{r\in N}\ln p_r \frac{\partial w_r}{\partial \ln x} = \sum_{r\in N} w_r \ln p_r(\varepsilon_r - 1). \tag{A11}$$

Substituting (A9) through (A11) into (A8) gives

$$\begin{aligned}\varepsilon_i &= 1 + \frac{1}{w_i}\left\{\begin{aligned}&\sum_{r\in N}\sum_{m=1}^{M} d_m w_i w^*_{mir}\ln p_r\left[\varepsilon_i - 1 + \varepsilon_r - 1 - (w_i/\omega_{mi})(\varepsilon_i - 1) - \sum_{k\in N} w^*_{mik}(\varepsilon_k - 1)\right]\\ &+ \beta_i\left[1 - \sum_{r\in N} w_r \ln p_r(\varepsilon_r - 1)\right]\end{aligned}\right\}\\ &= 1 + \frac{1}{w_i}\left\{\begin{aligned}&\sum_{r\in N}\sum_{m=1}^{M} d_m w_i w^*_{mir}\ln p_r\left[\sum w^*_{mik}(\varepsilon_i - 1) + \varepsilon_r - 1 - \sum_{k\in N} w^*_{mik}(\varepsilon_k - 1)\right]\\ &+ \beta_i\left[1 - \sum_{r\in N} w_r \ln p_r(\varepsilon_r - 1)\right]\end{aligned}\right\}\end{aligned}. \tag{A12}$$

Equation (A12) can be expressed in matrix form as

$$\text{(A13)} \qquad \mathrm{E} = \iota + U + \left[\sum_{m=1}^{M} B_m\right](\mathrm{E}-\iota) + \left[\sum_{m=1}^{M} D_m\right](\mathrm{E}-\iota) - \left[\sum_{m=1}^{M} C_m F_m\right](\mathrm{E}-\iota) - (UV)(\mathrm{E}-\iota)$$

where, in addition to matrices defined in equation (A5), the matrix elements are $\mathrm{E}_i = \varepsilon_i$ in E ($n \times 1$ vector), and ι is a $n \times 1$ vector of ones. Solving (A13) for the vector of expenditure elasticities E gives

$$\text{(A14)} \qquad \mathrm{E} = \left\{ I - \sum_{m=1}^{M} B_m - \sum_{m=1}^{M} D_m + \sum_{m=1}^{M} C_m F_m + UV \right\}^{-1} U + \iota \,.$$