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## Conditions for valid estimation of causal effects on prevalence in cross-sectional and other studies

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### Abstract

**Purpose**—Causal effects in epidemiology are almost invariably studied by considering disease incidence even when prevalence data are used to estimate the causal effect. For example, if certain conditions are met, a prevalence odds ratio can provide a valid estimate of an incidence rate ratio. Our purpose and main result are conditions that assure causal effects on prevalence can be estimated in cross-sectional studies, even when the prevalence odds ratio does not estimate incidence.

**Methods**—Using a general causal effect definition in a multivariate counterfactual framework, we define causal contrasts that compare prevalences among survivors from a target population had all been exposed at baseline with that prevalence had all been unexposed. Although prevalence is a measure reflecting a moment in time, we consider the time sequence to study causal effects.

**Results**—Effects defined using a contrast of counterfactual prevalences can be estimated in an experiment and, with conditions provided, in cross-sectional studies. Proper interpretation of the effect includes recognition that the target is the baseline population, defined at the age or time of exposure.

**Conclusions**—Prevalences are widely reported, readily available measures for assessing disabilities and disease burden. Effects on prevalence are estimable in cross-sectional studies but only if appropriate conditions hold.

### Keywords

Prevalence; Causal effects; Validity; Survey; Cross-sectional studies; Target population

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## Introduction

A now-common way to define causal effects in epidemiology uses a counterfactual framework [1–5]. Expanding on this approach, Flanders and Klein [6] presented a general definition of causal effects as a contrast of parameters of the distribution of multivariate counterfactual outcomes for the same population under two exposure conditions.

This general approach shows that causal effects can be defined using contrasts of prevalences [6,7]. Nevertheless, prevalences are infrequently used to define or estimate effects. For example in cross-sectional studies, prevalence odds ratios are often not viewed as causal measures unless they are a proxy for an incidence rate ratio [8–10]. Grabovschi et al. [11] seemingly echo this view, stating “The reviewed research studies also have some important methodological limitations related mostly to their reliance on survey data, which could preclude causal interpretation and only measure statistical associations and tendencies.”

Although some have defined [6] and others estimated [7] causal effects using prevalence contrasts, the conditions needed for valid estimation in cross-sectional studies have not yet been discussed. Therefore, our purpose here is to discuss valid estimation of causal effects when interest includes disease prevalence itself rather than just disease occurrence. We review the definition of a causal prevalence difference, provide examples, and discuss interpretation. Our main, new, and novel results are the presentation and discussion of assumptions that, when true, assure that causal effects on prevalence can be validly estimated in cross-sectional studies.

## Notation and definitions

We assume exposure (E) is dichotomous and occurs at an early age  $a_0$ , if at all. Disease (D) can occur at any age, can resolve, in which case we say D is not present, and can recur in people in whom it had resolved.

### Notation

The outcome-vector  $[D_{i,a}, S_{i,a}]$  encodes disease status and survival: disease component  $D_{i,a}$  is 1 if individual  $i$  is alive with disease and 0 otherwise and survival component  $S_{i,a}$  is 1 if individual  $i$  is alive and 0 otherwise, both at age  $a$ .  $E_j$  is 1 if individual  $i$  was exposed at age  $a_0$  and 0 otherwise. Parentheses denote counterfactual outcomes [1,12]: the vector  $[D_{i,a}(e), S_{i,a}(e)]$  is the value of  $[D_{i,a}, S_{i,a}]$  if  $E_j$  had been set to  $e$  at age  $a_0$ , for  $e = 0, 1$ . In particular, disease component  $D_{i,a}(e)$  is 0 if individual  $i$  would have died before age  $a$  after setting  $E_j$  to  $e$ , but other definitions are possible [6,13,14].

Because an individual cannot have been both exposed and unexposed at age  $a_0$ , one of the outcome vectors  $[D_{i,a}(1), S_{i,a}(1)]$  or  $[D_{i,a}(0), S_{i,a}(0)]$  is counterfactual.

### Definitions

Clear effect definitions require several components [2,6,15–17], including specification of the target; relevant ages including those when exposure is to be set and the outcome

measured (e.g., follow-up periods); how the exposure will be set to the levels considered; and the contrast (e.g., difference or ratio).

Assuming these components are specified, the effect of E on presence of D at age  $a$  for individual  $i$  can be defined as  $D_{i,a}(1) - D_{i,a}(0)$ , Table 1. Because D encodes being alive with disease at age  $a$ , this effect is on the joint (composite) outcome having disease and being alive. Possible values are  $-1$ ,  $0$ , and  $+1$ . For example,  $-1$  corresponds to being alive with disease presence D at age  $a$  if unexposed and either not having disease or not being alive if exposed. This definition differs from the typical one wherein those dying without disease are treated as incomplete observations; however, it is akin to the approach of Fine and Grey [18] wherein those dying without disease are treated as nondiseased, complete observations. Disease could potentially have changed several times between exposure at age  $a_0$  and age  $a$ ; focusing on age  $a$  summarizes net changes. The effect of exposure can depend on disease status at age  $a_0$ .

To define a population average effect of E on disease presence at age  $a$ , we must specify the target population ( $P_0$ ) at age  $a_0$  when exposure is set. Then, we define the causal prevalence difference ( $cPD$ ) at age  $a$  for  $P_0$  as the prevalence in  $P_0$  if all had been exposed at age  $a_0$ , compared with that prevalence if all had been unexposed. In equation form,  $cPD$  is

$$cPD = \frac{\sum_{i \in P_0} D_{i,a}(1)}{\sum_{i \in P_0} S_{i,a}(1)} - \frac{\sum_{i \in P_0} D_{i,a}(0)}{\sum_{i \in P_0} S_{i,a}(0)} \quad (1)$$

The vectors  $[\sum_{i \in P_0} D_{i,a}(e), \sum_{i \in P_0} S_{i,a}(e), e = 0, 1]$  whose components appear in Equation 1 are parameters (means) of the distributions of counterfactual outcome vectors. If exposure affects survival, that would be part of the causal pathway and reflected in  $cPD$ . Importantly, the target is  $P_0$ , not the subpopulation that survives to age  $a$ .

Of note, Flanders and Klein [6] previously defined causal effects using prevalence ratios rather than differences. Then, the target population was  $P_f$ , the full population at baseline. Now the target population ( $P_0$ ) coincides with  $P_f$ , provided the survey population  $P_1$  consists of all survivors from  $P_f$ . Thus, apart from changing from ratios to differences, the previous [6] and present definitions essentially coincide.

## Estimation

A natural estimator of the population average causal effect of E on presence of D in population  $P_0$  is the observed difference:

$$\widehat{cPD} = \overline{D}_{1,a_1} - \overline{D}_{0,a_1} \quad (2)$$

where an overbar indicates the average;  $\overline{D}_{j,a_1} = \sum_{i \in P_0} E_i = j D_{i,a_1} / N_{j,a_1} = \sum_{i \in P_0: E_i = j} D_{i,a_1} / \sum_{i \in P_0: E_i = j} S_{i,a_1}$  is the observed prevalence of D at age  $a = a_1$  in exposure group  $j$  ( $j = 1$  if exposed,  $0$  otherwise); and  $N_{j,a_1}$  is the observed number alive with exposure equal to  $j$  at age  $a_1$ .

In results, we present assumptions that suffice for this estimator to be unbiased in cross-sectional studies; in Appendix 1, we justify this claim, and in Appendix 2, we discuss why the estimator is unaffected by collider bias from conditioning on survival.

Of note, this estimator is simple and involves directly observable variables. As in the definition, deaths including those from “competing risks” are treated realistically, as part of the causal process affecting disease prevalence.

## Results

Our goal and main novel result is to state assumptions sufficient for expression [2] to be a valid estimator of causal prevalence differences in cross-sectional surveys. We first motivate the approach by briefly considering experiments.

### Randomized experiments

The effects of exposure on disease presence at a specified time after exposure can be estimated in an experiment. Briefly, one identifies and enrolls subjects, say at age  $a_0$ . For simplicity, we focus throughout on a specific age at exposure ( $a_0$ ), although one could include different age groups and calculate a summary measure or model age patterns. We may optionally measure baseline presence of disease (age  $a_0$ ) and then expose a random subgroup to E or placebo. We follow the cohort to age  $a_1$  and measure disease presence, assuming no dropouts or loss to follow-up.

Sufficient conditions under which estimator 2 validly estimates causal effects in randomized experiments are exchangeability (Table 1) for disease presence and survival [ $D_{i,a}(e), S_{i,a}(e)$ ]; independence between people (stable unit treatment value assumption, SUTVA [19,20]); and counterfactual model consistency [4] (Table 1; e.g.,  $D_{i,a}(j) = D_{i,a}$  if  $E_j = j$ ). The target is the baseline cohort, and all subjects remain under observation, unless death intervenes. These assumptions describe good randomization (exchangeability), no intersubject interference (SUTVA), complete follow-up, and conceptual clarity (consistency), respectively. They should hold in a well-conducted experiment, possibly apart from SUTVA which can depend on characteristics of the exposure and outcome. Example 1 illustrates how causal effects might be estimated using prevalence contrasts. Additional examples are provided by community intervention trials, often randomized, that frequently use prevalence contrasts to estimate causal effects [21]. Cohort studies involve similar assumptions, with the important caveat that, absent randomization, exchangeability is not necessarily expected (see Appendix 1).

Exchangeability need hold only conditional on measured baseline covariates (C; Table 1). Moreover, the target need not be disease free (at baseline). However, the assumption of exchangeability for nonrandomized studies could be implausible if the prevalence at baseline differed by exposure as noted in the discussion.

### Example 1

We illustrate estimation of effects on prevalence in an experiment. Kuller et al. [22] studied the effect of a healthy lifestyle intervention on women’s low-density lipoprotein (LDL)

cholesterol. Women had similar cholesterol levels at baseline, but 4.5 years after randomization, 27% of the women in the lifestyle intervention group had LDL cholesterol <100 mg/dL (“optimal”) compared with 16% in the assessment only group. Many measures and comparisons were additionally reported (e.g., average between-group differences), but results included these prevalences illustrating that prevalences can be relevant and informative.

From their results, the prevalence difference is  $27\% - 16\% = 9\%$ . This contrast is not only descriptive, but with our assumptions that should often be plausible in a randomized experiment (previously mentioned), can also be interpreted as estimating the causal effect of the intervention on prevalence of optimal LDL cholesterol. Because prevalence reflects both incidence and duration after onset [23], effect can be on both—as summarized in the prevalence difference. Follow-up was high (95%) with one death, but if the intervention had affected mortality, then  $\widehat{cPD}$  would nevertheless estimate the causal effect on prevalence in the randomization cohort, measured in survivors.

### Cross-sectional studies

The assumptions that assure unbiasedness of estimator 2 are less straightforward in a cross-sectional survey. We assume that survey participants are randomly selected from a well-defined population of living people  $P_1$  at age  $a_1$  (no surrogates for the deceased). The presence (or absence) of disease at age  $a_1$  ( $D_{i,a_1}$ ) and prior exposure at age  $a_0 < a_1$  are accurately assessed.

Perhaps the main challenge is defining the target population needed to clearly define causal effects and for which a survey of population  $P_1$  is expected to yield valid estimates of causal prevalence differences. Because effects require time to occur, the target must have been enumerable at age  $a_0$  before measuring the outcome. We can clearly specify the target if population  $P_1$  consists of all survivors from a larger population  $P_0$  that was alive at the time of potential exposure, age  $a_0$ . Population  $P_0$  should be definable by observable, contemporaneous factors. If we can specify  $P_0$ , then measurement of disease prevalence in the survey provides just the information needed to estimate  $cPD$  and had a cohort study of  $P_0$  been done. In particular, summation over  $i \in P_0$  appearing in estimator 2 can be replaced by summation over  $i \in P_1$  because summands  $D_{i,a_1}$  and  $S_{i,a_1}$  are 0 for individuals who die between age  $a_0$  and  $a_1$ . If the survey involves a 100% sample, prevalences in the sample coincide with those from a cohort study of  $P_0$ , and if less than 100%, prevalences are valid estimators of them because we assume exposure-specific prevalences in survey participants represent those in  $P_1$  (Appendix 1).

The other assumptions needed for unbiased estimation coincide with those for experiments and cohort studies. Specifically, we need exchangeability for target population  $P_0$  (Table 1;  $D_{i,a_1}(e) \perp\!\!\!\perp E_i$  and  $S_{i,a_1}(e) \perp\!\!\!\perp E_i$  for  $i \in P_0$  and SUTVA). These assumptions mean that comparison of disease status among exposed and unexposed survivors informs what would have happened if exposure had been randomized in population  $P_0$  at age  $a_0$  and the cohort followed to age  $a_1$ .

Some examples may help illustrate conceptualization of population  $P_0$  (Table 2). Suppose survey respondents, perhaps like respondents to the National Health and Nutrition Examination Survey or Behavioral Risk Factor Surveillance System, are representative of all (noninstitutionalized) U.S. residents, say at age  $a_1$ , and that we focus on exposure at age  $a_0$ . We exclude recent immigrants from  $P_1$  because they were not in the resident population at younger ages. Population  $P_0$  is then the population of residents at the younger age  $a_0$ ,  $a_1 - a_0$  years earlier.  $P_1$  should be (or represent) all in  $P_0$  who survive to  $a_1$ . Emigration from  $P_0$  is permissible if independent of disease, survival, and exposure.

Figure 1 summarizes causal relationships that, if correct, assure the needed exchangeability assumptions for target population  $P_0$  and representativeness of the surveyed population ( $P_1$ ) using a directed acyclic graph (DAG). Rules for constructing and interpreting DAGs are reviewed in detail elsewhere [24–26]. The DAG shows that exposure E is (being assumed) independent of other causes of disease ( $U_0^*$ ), and of other causes ( $U_0$ ) of disease and survival ( $S_1$ ). Membership in  $P_1$  depends on survival, not emigrating and other factors, but not directly on E. Participation depends on  $P_1$ , other factors  $U_1$ , but not directly on E. Under these causal patterns relationships, we expect exchangeability for target population  $P_0$  and representativeness survey population  $P_1$ . Figure 2 illustrates situations wherein additional causal effects are present (dotted line) with both  $S_1$  and  $D_1$  affecting membership in  $P_1$ . We now expect bias as survey population  $P_1$  may not represent all survivors from target population  $P_0$ .

If the exposure-specific prevalences in survey participants do “not” represent those in  $P_1$  (i.e., all  $P_0$ -survivors), estimator [2] is likely biased. In this case, however, prevalent disease should be associated with survey participation, or with emigration or loss from the surviving population suggesting a common cause (e.g.,  $C_0$  in Fig. 3). If we can control for all such common causes, prevalent disease should then be independent of participation, emigration, and loss (assuming no conditioning on a collider), exposure-specific prevalences in participants should consistently estimate those in survivors, and the conditional estimator should be consistent for the effect in [1].

## Example 2

Example 2 illustrates the use of prevalence contrasts for estimating effects in a survey (cross-sectional study). Our goal is to estimate the effect of starting smoking at age 18 years versus never starting or starting later on the prevalence of poor or fair health 20 years later at age 38 years. Self-rated health status is of interest partly because it consistently and strongly predicts subsequent mortality, even after control for multiple other health-status indicators [27]. To estimate this effect, we use data from the National Health and Nutrition Examination Survey for the years 2007–2010. To increase sample size, we include respondents 35–39 years old when surveyed (instead of just 38 years). The exposed group is smokers who started regular smoking between ages 18 and 21 years, using a wider age interval to increase sample size, and the unexposed are all who never started or started after age 21 years. The outcome is self-reported poor or fair health at interview (age, 35–39 years). After adjusting for gender and race, the prevalence odds ratio was 1.7, indicating a

70% estimated higher prevalence odds of poorer health among those who started regular smoking at ages 18–21 years, compared with never or later starting smokers.

The baseline population  $P_0$  consisted of U.S. residents who were 18–21 years old about 20 years before interview. If the exposed and unexposed were exchangeable conditional on controlled covariates, and participants were representative of all U.S. residents aged  $38 \pm 2$  years during this time period—the prevalence odds ratio should consistently estimate the effect of taking up regular smoking at about age 18 years on having self-reported poor health 20 years later. Some U.S. residents died between ages 18 and 38 years possibly because of smoking. Because our interest is in the effect of smoking on prevalence, these deaths do not represent bias because of competing risks but rather are part of the defined effect of smoking [6] on subsequent disease prevalence among survivors.

## Discussion

When epidemiologists consider disease causation they almost invariably consider it in terms of disease onset (i.e., incidence). Rothman et al. developed causal concepts as follows ([5], p.6): “To begin, we need to define cause. One definition is the cause of a specific disease occurrence is an antecedent event, condition, or characteristic that was necessary for the occurrence of the disease at the moment it occurred, given that other conditions are fixed.” Although their interest in that definition is on disease onset, for effects on disease prevalence, we can similarly consider an antecedent event, condition, or characteristic that was necessary for an individual having disease at a particular point in time, given that other conditions are fixed. Effects on survival, disease onset, or disease duration in this context are part of the causal pathway.

A cohort study is often viewed as a natural design to estimate disease incidence, and a cross-sectional study as a natural design to estimate prevalence as prevalence is a measure reflecting a moment in time. However, to study causal effects, the time sequence must be considered. So, to study causal effects on prevalence, a cohort study would be a natural design. Our assumptions provide conditions wherein observations from a cross-sectional study provide information that adequately approximates information from a cohort study for estimating effects on prevalence.

We have defined causal contrasts that compare the prevalence among survivors from the target population had all in the target been exposed at baseline with that prevalence had they been unexposed. The definition requires specification of the target population, exposure, ages, and other factors [2·6·28]. The assumptions needed for valid estimation are strong and require critical review to assess their validity. Each of these issues merits separate discussion.

A key assumption is that the baseline, target population be clearly defined and potentially enumerable. Although explicitly defined in experiments and cohort studies, in cross-sectional studies, this baseline population may require conceptualization as an earlier “parent” population defined so that the population surveyed would consist of all (represent) survivors from the parent population. Identification of a parent population with the needed



characteristics creates a situation wherein observations from a cross-sectional study can reproduce those that would have been obtained if a cohort study of that population had been done. Many surveys will not permit clear delineation of the parent population; if not, associated causal-effect definitions may be unsatisfactory. Other assumptions for validity of the observed PD as an estimator of the causal PD in cross-sectional studies are equally important. In particular, exchangeability must be evaluated and, as in cohort studies, can be suspect. It is not expected to hold if confounding is present, perhaps due to causes of disease that are also associated with exposure. If the target is not disease free at baseline, exchangeability can also be suspect if the prevalence at baseline differs between the exposed and unexposed subgroups. In cross-sectional studies involving prevalence contrasts, exchangeability can be threatened by factors that affect either disease duration or risk and are associated with exposure. As with other observational studies, some assumptions are unverifiable, and sensitivity analyses may be useful.

The ages at exposure and at disease measurement must be clearly specified. These ages are critical for several reasons. First, age is a potential confounder, for example, if it affected prevalence and was associated with exposure. Second, age at exposure could be an effect measure modifier. Third, the age and time intervals between exposure (or nonexposure) and outcome measurement can also affect prevalence.

Ideally, exposure would have occurred, if at all, at age  $a_0$ , the age of the target population at baseline, analogous to recommendations that follow-up in cohort studies begins at or before exposure [29–31]. To illustrate how we might define exposure and the target in practice, suppose our goal is to estimate the effect of hormone replacement therapy (HRT) on cardiovascular disease (CVD) prevalence. A population-based survey of 60 year olds is available that included questions about age at starting HRT. We could define “exposure” as having started HRT by a specific age, say 50 years (10 years before the survey) and “nonexposure” as not having started by that age (including never starting).  $P_0$  should consist of people who were 50 years old 10 years before the survey and should be defined so that  $P_1$  consists of all survivors from  $P_0$ .  $P_0$  may be easier to define if the survey is population-based (e.g., all 60-year-old, U.S. residents in 2010 excluding recent immigrants), so that it might be defined as the corresponding population, 10 years earlier (e.g., all 50-year-old, U.S. residents in 2000). See Appendix 2 for additional discussion. If the timing and ages are not specified, then the effect may not be clearly defined, and confusion can ensue. Importantly, similar issues can arise in cohort studies. A possible example concerns side effects of HRT, thought by many to be protective for CVD. After randomized trials showed HRT to increase rather than decrease CVD risk, reanalyses of one observational study suggested that better control for time since initiation and confounding could lead to better agreement with the randomized trial. Thus, clear specification of age at, and time since, exposure are important in cohort as well as cross-sectional studies [29–31] with exposure, if it occurs, being at or around the start of follow-up. Here, we restrict to specific ages for simplicity, but a stratified approach with calculation of summary measures or model-based estimation would directly extend our results.

A comparison of prevalences may seem wanting as a causal measure because prevalence is affected by disease incidence and duration (see Appendix 2) [23]. A harmful exposure could



increase the prevalence by increasing the rate of disease onset or decrease the prevalence by increasing the case fatality rate. Because the causal prevalence difference is a summary effect, use of additional contrasts such as differences in risk or duration can be helpful, sometimes vital. Nevertheless, prevalence itself is a widely reported and readily available epidemiologic measure for assessing disabilities, disease burden, and frequency, particularly for chronic, incurable diseases with long duration and unclear timing of onset [23]. Causal inference from prevalence has usually been considered as a proxy for incidence. Our purpose here and main novel result is to provide conditions for validly estimating causal effects in cross-sectional studies. However, we also discuss interpretation and related conceptual issues as the use of prevalence contrasts for defining and estimating causal effects is uncommon and involves relatively new considerations.

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## Appendix 1

### Experiments

We argue that the estimator (expression 2) is unbiased under our assumptions for experiments. By completeness of follow-up,  $N_{j,a_1} = \sum_{i \in P_0: E_{i,j}} S_{i,a_1}$  for  $j = 0$  or  $1$ , where  $S_{i,a_1} = 1$  if subject  $i$  is alive at age  $a_1$  and  $0$  otherwise. By counterfactual model consistency for both  $D_{i,a_1}(e)$  and  $S_{i,a_1}(e)$ ,  $\sum_{i \in P_0: E_{i,j}} D_{i,a_1}(j) / \sum_{i \in P_0: E_{i,j}} S_{i,a_1}(j) = \sum_{i \in P_0: E_{i,j}} D_{i,a_1}(j) /$

$i \in P_0: E_{i \neq j} S_{i,a_1}(j)$ . By exchangeability,  $E[ i \in P_0: E_{i \neq j} D_{i,a_1}(j) ] / E[ i \in P_0: E_{i \neq j} S_{i,a_1}(j) ] = i \in P_0 D_{i,a_1}(j) / i \in P_0 S_{i,a_1}(j)$ . Slutsky's theorem [32] now implies that expression [2] is unbiased (technically, consistent) for the causal effect (Equation 1).

### Cohort studies

To estimate the *cPD* in a cohort study, we select a cohort  $P_0$ , some of whom were exposed at baseline (age  $a_0$ ), others not. The assumptions needed for unbiasedness of estimator 2 are the same as those for an experiment. However, exchangeability, which should hold in an experiment with good randomization, needs to be critically evaluated. In particular, we must verify that collider bias [4·5·33·34], if any, induced by cohort selection at baseline is negligible. We follow the cohort to age  $a_1$  and assess disease presence.

Our claim, that estimator [2] is unbiased given our assumptions, follows from the preceding arguments as the design and assumptions closely parallel those for an experiment. The key difference for cohort studies is that the assumption of exchangeability is not expected to hold by design, at least absent restriction, stratification or adjustment, and must be evaluated with particular care using all available information.

### Cross-sectional studies

Finally, we argue that estimator 2 is consistent under our assumptions for cross-sectional studies. By assumption, the sample is representative of population  $P_1$ , so

$\widehat{cPD} \approx (\sum_{i \in P_1: E_i=1} D_{i,a} / N_{1,a_1} - \sum_{i \in P_1: E_i=0} D_{i,a} / N_{0,a_1})$ , where the summation is over subjects in population  $P_1$ . Also, by assumption, a larger, enumerable population  $P_0$  exists such that  $P_1$  consists of all surviving members of  $P_0$ . Because  $D_{i,a}$  and  $S_{i,a}$  are both 0 for the deceased, we have in expectation,

$$\widehat{cPD} \approx (\sum_{i \in P_0: E_i=1} D_{i,a} / \sum_{i \in P_0: E_i=1} S_{i,a} - \sum_{i \in P_0: E_i=0} D_{i,a} / \sum_{i \in P_0: E_i=0} S_{i,a}).$$

This last expression is the same as estimator 2 for the target cohort  $P_0$  if it was the baseline population in a cohort study followed from age  $a_0$  to age  $a_1$ . But estimator 2 is unbiased by our assumptions and arguments mentioned previously for the cohort  $P_0$ .

At times, an alternative estimator that accounts for baseline prevalence may be unbiased even if estimator 2 is biased.

$$\widehat{cPD} = (\overline{D}_{1,a_1} - \overline{D}_{0,a_1}) - (\overline{D}_{1,a_0} - \overline{D}_{0,a_0}) \quad (3)$$

For example, the prevalence at baseline may differ between exposed and unexposed because of factors associated with exposure but unassociated with changes in disease thereafter. Measurement error can affect the decision to further account or adjust for baseline disease status as discussed by Glymour et al. [35].

## Appendix 2

### Alternative approach to defining prevalence effects

Here we consider an approach to defining prevalence effects that provides more details, still rooted in the general causal-effect definition which contrasts parameters of the multivariate counterfactual-outcome distributions presented by Flanders and Klein [1]. The idea is to separate and incorporate the three components of prevalence: disease onset, disease duration, and survival.  $D_{i,a}$  defined in the main text can be viewed as an infinite-dimensional vector,  $\mathbf{D}_i$  that traces the disease and survival status of individual  $i$  over each moment of the period of interest. The  $a^{\text{th}}$  component of  $\mathbf{D}_i = D_{i,a}$  encodes disease presence, as defined in the main text, for each age  $a \geq 0$ . Similarly, the  $a^{\text{th}}$  components of  $\mathbf{D}_i(e)$ ,  $\mathbf{S}_i(e)$  and  $\mathbf{S}_i$  are  $D_{i,a}(e)$ ,  $S_{i,a}(e)$  and  $S_{i,a}$  respectively. Using  $\mathbf{D}_i(e)$ , a matrix of counterfactual outcomes consisting of three vectors can be derived:

$$\mathbf{T}_i(e) = (t_{i,1}(e), t_{i,2}(e), \dots, t_{i,n_i(e)-1}(e), t_{i,n_i(e)}^\delta(e))$$

$$\mathbf{U}_i(e) = (u_{i,1}(e), u_{i,2}(e), \dots, u_{i,n_i(e)-1}(e), u_{i,n_i(e)}^\delta(e))$$

$$\mathbf{M}_i(e) = (m_i^\delta(e))$$

where  $t_{i,k}(e)$  represents the time from baseline (time 0 or age  $a_0$ ) to the  $k^{\text{th}}$  disease episode of subject  $i$ , and  $n_i(e)$  is the total number of his/her episodes during the observation period, if  $E_i$  were set to  $e$  at baseline. If subject  $i$  has disease at baseline then  $t_{i,1}(e)$  is zero.  $u_{i,k}(e)$  represents the duration of the  $k^{\text{th}}$  episode of subject  $i$ , if  $E_i$  were  $e$ . If subject  $i$  has disease at baseline then  $u_{i,1}(e)$  is the duration of the first episode from baseline.  $m_i^\delta(e)$  represents survival time from baseline. The superscript  $\delta$  (in  $t_{i,n_i(e)}^\delta(e)$ ,  $u_{i,n_i(e)}^\delta(e)$  and  $m_i^\delta(e)$ ) is 1 if information is censored and 0 otherwise.

This formulation includes details about disease onset and duration for potentially multiple disease episodes over time. With it, causal effects of exposure on prevalence can be traced over time since baseline. Additionally, this information allows consideration of the causal effect of exposure on disease onset, survival, cumulative disease duration, average duration per episode, proportion of time with the disease or condition and the number of episodes.

#### Comparison of causal prevalence differences (Equation 1) with causal conditional risk differences

One of the examples used by Flanders and Klein to illustrate their general, multivariate definition of causal effects was the causal conditional risk difference (cCRD) [1]. Here we compare the cCRD with the causal Prevalence Difference (cPD) given by Equation (1). The

cCRD for the risk of an outcome during a risk period for the target  $P_0$ , conditional on survival to age  $a$ , can be defined for the target  $P_0$  by:

$$\text{cCRD} = \frac{\sum_{i \in P_0} I_{i,a}(1)}{\sum_{i \in P_0} S_{i,a}(1)} - \frac{\sum_{i \in P_0} I_{i,a}(0)}{\sum_{i \in P_0} S_{i,a}(0)} \quad (1A)$$

where  $I_{i,a}(e)$  is 1 if the outcome of interest occurs between age  $a$  and before the end of the risk period and as 0 otherwise. This expression is like Expression 1 of the main text, but  $I_{i,a}(e)$  replaces  $D_{i,a}(e)$  and is defined differently. The definition of cPD is similar, but involves presence of disease at age  $a$  (reflected in  $D_{i,a}(e)$ ), rather than occurrence of disease in the risk period starting at age  $a$  (reflected in  $I_{i,a}(e)$ ).

### The causal prevalence difference estimator (expression 2) and potential collider bias

Since the denominator of Estimator 2 equals the number in the baseline population (target  $P_0$ ) who survive to age  $a_1$ , one could, and a reviewer did, ask if the estimator in Equation 2 might be affected by collider bias due to conditioning on survival to age  $a_1$ . A theoretical justification that the estimator in Equation 2 is consistent is outlined in Appendix 1. Here, we provide alternative, less technical arguments. (Briefly, that justification uses the assumptions in the main text including exchangeability in the target  $P_0$  and SUTVA to show that  $(\sum_{i \in P_0: E=e} D_{i,d} / N_{P_0,e}, \sum_{i \in P_0: E=e} S_{i,d} / N_{P_0,e})$  is an unbiased estimator of the population-average, multivariate effect of  $E$  on outcome vector  $(D_i, S_i)$ , where  $N_{P_0,e}$  is the number in  $P_0$  with  $E = e$ . Slutsky's theorem then shows consistency for the ratio contrasts—the prevalence difference.) Collider bias for one target (e.g.,  $P_1$ ) but not another (e.g.  $P_0$ ), is also discussed elsewhere [1].

First, we emphasize that the target population is selected at baseline (time 0), and the exposure is independent of risk factors for disease and survival in this population (exchangeability assumption, perhaps conditional on common causes). Thus, selection (collider bias) from selection of the target is not an issue, by assumption. Furthermore, the final estimator (equation 2) is merely an algebraic manipulation of the multivariate estimator  $(\sum_{i \in P_0: E=e} D_{i,d} / N_{P_0,e}, \sum_{i \in P_0: E=e} S_{i,d} / N_{P_0,e})$ . But this multivariate estimator involves no exclusions, stratification, or control (except for stratification by exposure which is exchangeable), and so involves no collider bias.

Second, we note reassuringly that the cPD (equation 1) is directly estimable in a well-conducted experiment (using estimator 2), in which exposure is randomized at baseline in the target  $P_0$ . The causal prevalence difference addresses the question – “What is the population average effect of exposure on the target population, as measured by disease prevalences at age  $a_1$ ?” Of course other question can and typically should be asked, such as, “What is the population average effect of exposure on the target population, as measured by survival at age  $a_1$ ?” or “What is the population average effect of exposure on the target population, as measured by disease incidence through age  $a_1$ ?” Although other questions exist, by randomizing exposure at baseline, following the exposed and unexposed groups to age  $a_1$ , and then accurately measuring disease presence and contrasting the prevalences, one

can consistently estimate the effect of exposure on prevalence (via estimator 2). Thus, the defined effect (expression 1) is directly observable using simple, well-defined procedures that end by using estimator 2 in a randomized experiment.

Third, we note that effects of exposure, if any, on death before age  $a_1$ , are part of the defined effect on prevalence (Expression 1), and appropriately reflected in defining the effect and calculating the measure that estimates it. For example, one way in which exposure could cause a reduction in disease prevalence at age  $a_1$  would be to differentially reduce survival among those who had developed disease. Such a prevalence reduction would be an expected part of an effect on prevalence and correctly estimated, in a randomized experiment or other study under our assumptions. Through use of multivariate outcomes and effects, as described above, a more complete characterization of the exposure's impacts can be obtained.

### Supplemental Example 1

Supplemental example 1 illustrates use of prevalence contrasts for estimating effects in a cohort study. To estimate effects of early-life factors on sedentary lifestyle in adolescents, Hallal et al. [2] conducted a cohort study of all children born in-hospital, during 1993 in Pelotas, Brazil. They found a sedentary-lifestyle prevalence of 53.5% among 10–12 year olds whose mother had low education, compared to 63.2% among those whose mother had high education. The prevalence difference (9.7%), if exchangeability and our other assumptions are adequately approximated, is interpretable as the effect of maternal education on prevalence of sedentary lifestyle in adolescents and illustrates use and estimation of prevalence contrasts in a cohort study.

### Additional considerations in defining exposures for potentially-ongoing exposures

To further illustrate issues that can arise in defining exposure contrasts and the target  $P_0$ , consider the effects of starting alcohol use at a young age, say age 15, on prevalence of hepatic disease at age 35. We might define exposure as having started regular heavy alcohol use by age 15, and for comparison an “unexposed” group as those who had not started regular, heavy drinking by age 15. The resulting contrast, just as it would be in a cohort study, actually compares the effect of starting alcohol use early (age 15) versus later or never. The presence of people who later became a regular heavy drinker would, just as in a cohort study, reduce the expected effects of heavy drinking—compared to a completely unexposed population of, say, never drinkers. But, even in a cohort study—accounting for changes in exposure might require G-computation or related method [3]—if those changes reflect time-varying confounding. If a population-based survey of 35 year olds is available,  $P_0$  should be defined, if possible, as those who were 15 years old about 20 years before the survey, in a way that the survey population represents all survivors from  $P_0$ .

### References

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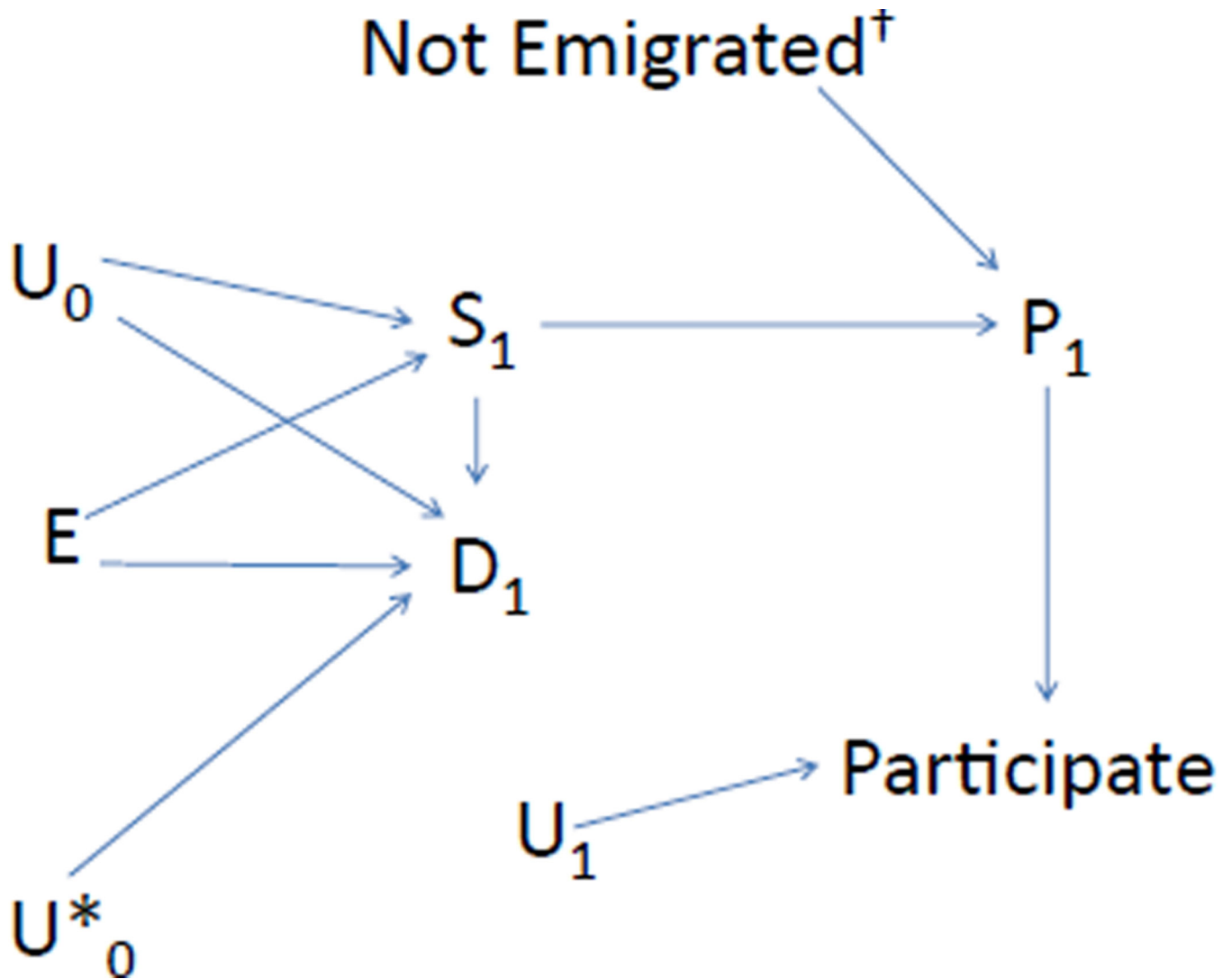
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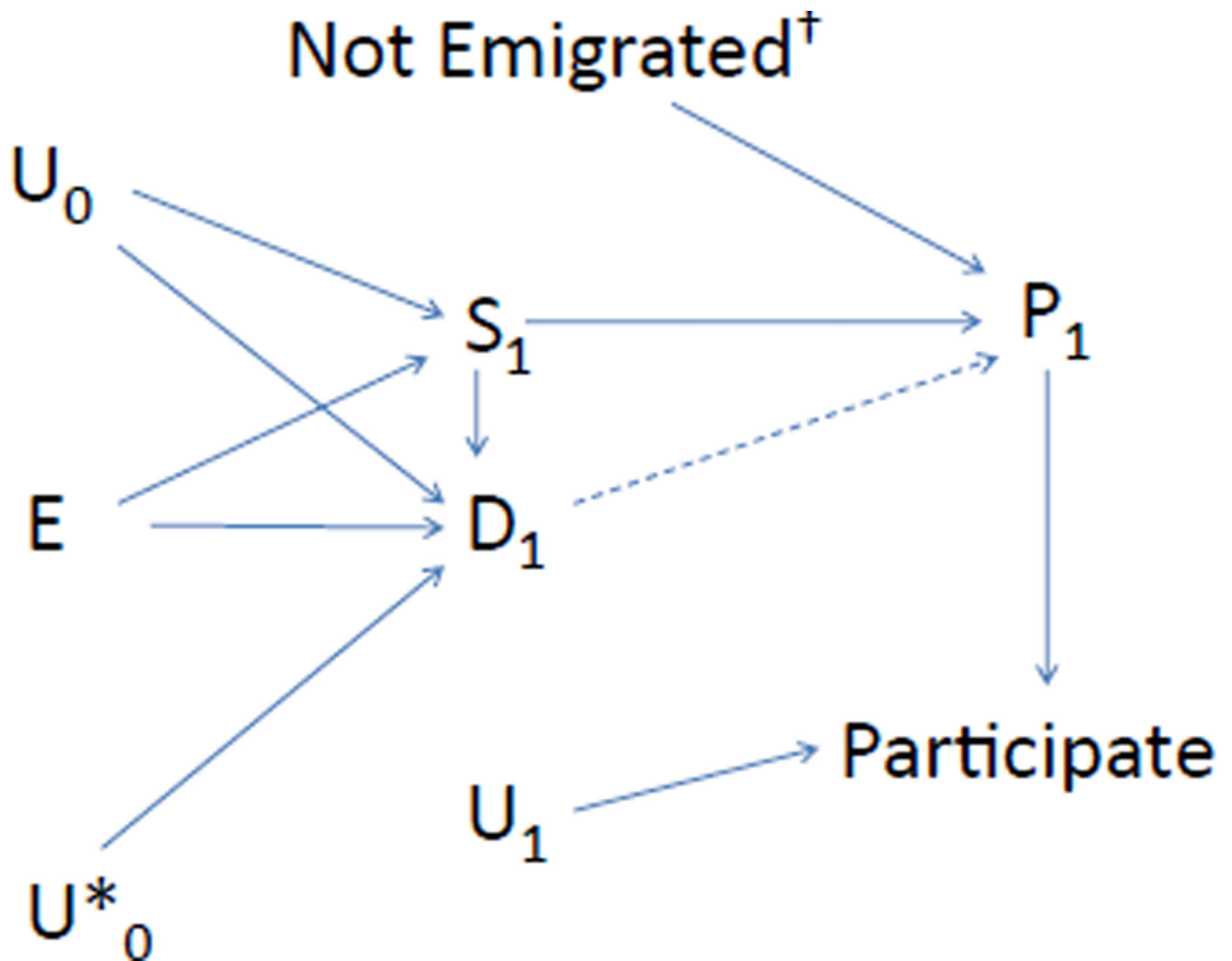
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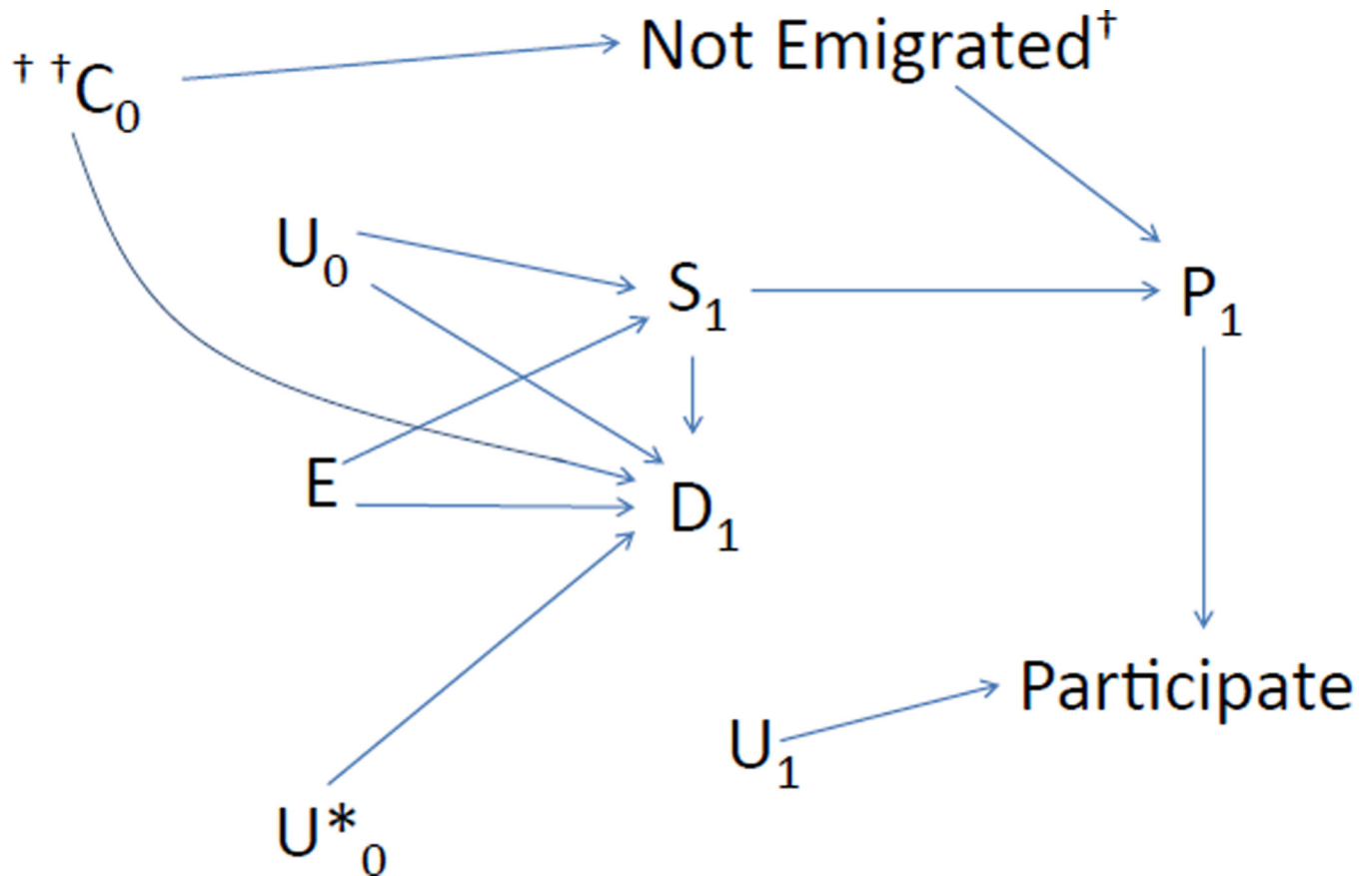


**Fig. 1.**

The figure summarizes causal relationships using a DAG for the baseline population  $P_0$ . If correct, estimator 2 should be unbiased (see text).  $D_1$  represents disease presence at age  $a_1$ ,  $U_0^*$  other causes of disease,  $U_0$  other causes of survival ( $S_1$ ), and disease ( $D_1$ ).  $P_1$  is the survey population; membership depends deterministically on survival, and not emigrating or other loss, but not directly on exposure  $E$  or  $D_1$ . Participation depends on  $P_1$  and other factors  $U_1$ . <sup>†</sup>Emigration, other factors affect being in population  $P_1$ .

**Fig. 2.**

The figure illustrates a situation such as that in Figure 1, with an additional effect (dotted line) that could underlie bias in estimator 2. For example, bias is expected if  $S_1$  and  $D_1$  affect membership in  $P_1$  (see text).  $D_1$  represents disease presence at age  $a_1$ ,  $U_0^*$  other causes of disease,  $U_0$  other causes of survival ( $S_1$ ), and disease.  $P_1$  is the survey population; membership depends on survival, not emigrating, and  $D_1$ . Participation depends on  $P_1$  and other factors  $U_1$ . <sup>†</sup>Emigration, other factors affect being in population  $P_1$ .



**Fig. 3.**

The figure illustrates a situation such as that in Figure 1, but with a common cause ( $C_0$ ) of prevalent disease  $D_1$  and emigration. Exposure-specific prevalences in the survey population ( $P_1$ ) would be expected to differ from those in all survivors, and bias is expected.  $U_0^*$  represents other causes of disease,  $U_0$  other causes of survival ( $S_1$ ), and disease.  $P_1$  is the survey population; membership depends deterministically on survival, not emigrating or other loss, and  $D_1$ . Participation depends on  $P_1$  and other factors  $U_1$ .  $†$ Emigration, loss other factors affect being in population  $P_1$ .  $††C_0$  is common cause of prevalent disease and emigration, so prevalence in  $P_1$  expected to differ from that in all survivors.

Table 1

## Definitions and notation

Term	Brief definition
$[D_{i,a}(e), S_{i,a}(e)]$ counterfactual outcome vector	Components are counterfactual outcomes $D_{i,a}(e), S_{i,a}(e)$ defined next. $[D_{i,a}(e), S_{i,a}(e)] = [1,1]$ if subject $i$ is alive with disease at age $a$ ; $= [0,1]$ if subject $i$ is alive without disease at age $a$ ; $= [0,0]$ if subject $i$ is not alive at age $a$ ; all if exposure had been set to $e$
$D_{i,a}(e)$ —counterfactual disease outcome	First component of counterfactual-outcome vector $[D_{i,a}(e), S_{i,a}(e)]$ , defined previously
$S_{i,a}(e)$ —counterfactual survival, subject $i$	second component of counterfactual-outcome vector $[D_{i,a}(e), S_{i,a}(e)]$ , defined previously
$D_{i,a}(1) - D_{i,a}(0)$	Individual causal effect on disease presence at age $a$
$cPD$ —causal prevalence difference	Population-average effect of exposure at age $a_0$ on disease prevalence at $a$ , in defined target population (Equation 1),
Exchangeability—disease	The counterfactual outcome with $E$ set to $e$ , is independent of actual exposure: $D_{i,a}(e) \perp\!\!\!\perp E_i$ ; exchangeability can be conditional on covariates $C$ : $D_{i,a}(e) \perp\!\!\!\perp E_i   C$
Exchangeability—survival	The counterfactual outcome with $E$ set to $e$ , is independent of actual exposure: $S_{i,a}(e) \perp\!\!\!\perp E_i$ ; exchangeability can be conditional on covariates $C$ : $S_{i,a}(e) \perp\!\!\!\perp E_i   C$
Consistency	The observed outcome equals the counterfactual outcome if exposure were set to the actual exposure: $D_{i,a}(e) = D_{i,a}$ if $E_i = e$
Stable unit treatment value assumption	The outcome of individual $i$ is independent of the exposure status of all other individuals: $D_{i,a}(e) \perp\!\!\!\perp E_j$ for $i \neq j$

**Table 2**Examples of surveys for which the “parent” population  $P_0$  may be specifiable\*

Survey	Population $P_1$	Parent Population $P_0$
Population-based national survey (e.g. NHANES 3)	U.S. residents, noninstitutionalized, age $a_1$ —excluding immigrants between age $a_0$ and $a_1$	U.S. residents, noninstitutionalized, age $a_0$ , ( $a_0 < a_1$ )
Population-based statewide telephone surveys (e.g., BRFSS)	State residents, noninstitutionalized, age $a_1$ —excluding immigrants between age $a_0$ and $a_1$	State residents, noninstitutionalized, age $a_0$ , ( $a_0 < a_1$ )
Population-based national telephone survey (e.g., NHIS)	U.S. residents, noninstitutionalized, age $a_1$ —excluding immigrants between age $a_0$ and $a_1$	U.S. residents, noninstitutionalized, age $a_0$ , ( $a_0 < a_1$ )

BRFSS = Behavioral Risk Factor Surveillance System; NHANES = National Health and Nutrition Examination Survey; National Health Interview Survey (NHIS).

\*  $P_0$ : as in the main text,  $P_1$  is the population (age  $a_1$ ) sampled for the survey, and  $P_0$  is the parent population (age  $a_0$ ) defined so that  $P_1$  consists of all surviving members of  $P_0$ . To assure temporal precedence,  $a_0$  is less than  $a_1$ .