

Supplemental A

Particle Momentum Equations and Spherical Drag Law

Particle Momentum Equations

$$\frac{du_i^p}{dt} = \frac{(u_i - u_i^p)}{\tau} + \frac{(\rho_p - \rho)g_i}{\rho} + f_i^p$$

where:

u_i =fluid velocity

u_i^p =particle velocity

ρ_p =particle density

ρ =fluid density

f_i^p =forces acting on the fluid

τ =particle relaxation time, given by

$$\tau = \frac{4 \rho_p d_p^2}{3 \mu C_D Re^p}$$

where: μ =fluid viscosity

d_p =particle diameter

Re^p =particle Reynolds number= $d_p |u_i - u_i^p| / \mu$

C_D =drag coefficient (a function of Re)

Spherical Drag Law:

$$C_D = a_1 + \frac{a_2}{Re} + \frac{a_3}{Re^2}$$

Where a_1 , a_2 and a_3 are constants

L2 and R2 error Norms

L2 relative error norms examine the change in estimates between global solution error (GSE) tolerances within a given mesh density. These are computed from the following equation:

$$L_2 = \frac{\left[\sum_{position} (f_{lower} - f_{higher})^2 \right]^{0.5}}{\left[\sum_{position} (f_{lower})^2 \right]^{0.5}}$$

The pre-determined level of acceptability was a less than 5% change in each degree of freedom between sequentially smaller global solution error tolerances.

Three-mesh error norms (R_2) calculations allows for the examination of mesh-independence. The equation used to compute R_2 values is as follows:

$$R_2 = \left\| \varepsilon_{mid, fine} \right\|^2 / \left\| \varepsilon_{coarse, mid} \right\|^2$$

where:

$$\left\| \varepsilon \right\|^2 = \left(\sum \varepsilon_{j,k}^2 \right)^{0.5} \quad (S7)$$

and the term $\varepsilon_{j,k}$ is the difference between the coarser (j) and finer (k) mesh level values for a given degree of freedom (Roache, 1998).

Over regional positions of interest, using solutions from the same GSE tolerance, mesh independence was assured when values of R_2 were below 1.0 (Stern et al., 2002).

L_2 and R_2 error norms were investigated for the 0.2 m/s freestream velocity at moderate breathing for the mouth-breathing model at facing-the-wind orientation.

0.2 m/s freestream velocity, moderate mouth-breathing, 0° (facing-the-wind) orientation

L2 Error Norms							
Mesh	GSE Levels	U	V	W	KE	E	SP
Coarse (M1)	$10^{-3} - 10^{-4}$	0.25%	1.25%	2.94%	2.14%	1.75%	0.34%
	$10^{-4} - 10^{-5}$	0.08%	0.17%	0.50%	0.03%	0.11%	0.05%
Moderate (M2)	$10^{-3} - 10^{-4}$	0.54%	1.26%	4.13%	1.42%	3.87%	0.15%
	$10^{-4} - 10^{-5}$	0.14%	0.34%	0.62%	0.06%	0.06%	0.02%
Fine (M3)	$10^{-3} - 10^{-4}$	0.43%	2.19%	4.57%	5.12%	6.68%	0.44%
	$10^{-4} - 10^{-5}$	0.08%	0.21%	0.63%	0.10%	0.03%	0.03%
R2 Error Norms							
GSE level	Ux	Uy	Uz	KE	E	SP	

1.00E-03	0.172	0.642	0.372	0.374	0.073	0.010
1.00E-04	0.127	0.434	0.325	0.350	0.073	0.008
1.00E-05	0.120	0.426	0.316	0.327	0.074	0.008