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SUPPLEMENT TO: A REFERENCE-INVARIANT HEALTH DISPARITY INDEX BASED ON RÉNYI DIVERGENCE—ADDITIONAL CASE STUDY FROM NHANES

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Talih (2012) illustrates the use of the symmetrized Rényi index with National Health and Nutrition Examination Survey (NHANES) 2001–04 data on prevalence of moderate or severe periodontitis among U.S. adults aged 45–74. These binary individual-level data track Oral Health objective OH-5 in Healthy People 2020 (HP2020). NHANES is the data source for about 1 in 7 population-based objectives in HP2020. Close to one half of the (approximately) 1,200 objectives in HP2020 are population-based, and most population-based HP2020 objectives track a proportion or rate where the underlying individual-level variable has a binary outcome. This supplement provides further illustration of the proposed methodology in Talih (2012) using continuous individual-level data on total blood cholesterol levels from NHANES 2005–08. These data track Heart Disease and Stroke objective HDS-8 in HP2020, which aims to reduce the mean total blood cholesterol level of adults aged 20 and over.

Disparities in mean total blood cholesterol level among U.S. adults aged 20 and over. Healthy People 2020 (HP2020) objective HDS-8 in the Heart Disease and Stroke Topic Area aims to reduce the mean total blood cholesterol level of adults aged 20 and over. Table 1 presents age-adjusted estimates of mean total blood cholesterol levels (in μg/dL) among U.S. adults aged 20 and over, as estimated from NHANES 2005–08. Those are the baseline data for HP2020 objective HDS-8 and the final data for Healthy People 2010 objective 12-13. As seen in Table 1, the groups with the lowest mean total cholesterol levels are non-Hispanic black adults, males, those with at least

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a college degree, those with a family income 400–499 percent of the federal poverty level, and those born in the U.S. However, as documented in National Center for Health Statistics (2011; chapter 12), disparities relative to the group with the best rate remain generally either smaller than 10 percent or statistically non-significant.

< Insert Table 1 about here. >

To illustrate the utility of the symmetrized Rényi index (SRI) in investigating age group-specific disparities, we examine crude estimates of mean total blood cholesterol levels that are stratified by the age groups 25–44, 45–54, 55–64, 65–74, 75–79, and 80 and over; see Table 2. In Figure 1, we present only the standardized SRI values for the analysis by sex, where it is seen that the group weighting scheme (population-weighted vs. equally-weighted) has little to no effect. Further, the steady increase in the between-group SRI by sex for older age groups is documented in Matthews *et al.* (2009). However, overall, a large proportion of total or aggregate disparity in total blood cholesterol levels remains unexplained.

< Insert Table 2 and Figure 1 about here. >

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TABLE 1

Total blood cholesterol levels (in μg/dL) in U.S. adults aged 20 and over (age-adjusted), 2005–08.[1]

	Mean[2]	SE[3]	95% CI[4]	
Total	197.7	0.548	196.6	198.8
Sex				
Male	194.8	0.792	193.2	196.4
Female	200.0	0.670	198.6	201.4
Race/Ethnicity				
White only, non-Hispanic	198.2	0.698	196.8	199.6
Black only, non-Hispanic	192.0	0.972	190.0	193.9
Mexican-American	201.0	1.420	198.1	203.9
Other[5]	198.4	1.440	195.5	201.3
Educational attainment[6]				
Less than high school	200.7	1.141	198.4	203.0
High school graduate	200.3	1.095	198.1	202.6
Some college or AA degree	200.8	0.971	198.8	202.7
College graduate or above	198.7	1.165	196.3	201.1
Family income (percent FPL[7])				
Less than 100	198.6	1.269	196.0	201.2
100–199	199.2	1.090	196.9	201.4
200–399	197.2	0.865	195.4	199.0
400–499	195.3	1.233	192.8	197.8
500 or above	198.0	1.309	195.4	200.7
N/A[8]	197.9	2.341	193.1	202.6
Country of birth				
U.S.	197.3	0.597	196.1	198.5
Outside U.S.	200.3	1.145	197.9	202.6

1. Data are from the National Health and Nutrition Examination Survey (NHANES) 2005–06 and 2007–08. Total cholesterol is a combination of high-density lipoproteins, low-density lipoproteins, and very-low density lipoproteins. It is measured enzymatically in a series of coupled reactions, as described in the NHANES laboratory procedures for total cholesterol measurement.
2. Estimates are age-adjusted (by the direct method) to the year 2000 U.S. population using the age groups 20–29, 30–39, 40–49, 50–59, 60–69, 70–79, and 80 and over.
3. Designed-based standard errors (SE) obtained via Taylor linearization (e.g., SUDAAN or R 'survey' package).
4. Lower and upper confidence limits, respectively, for a 95 percent confidence interval (CI).
5. The category *Other* consists of Hispanic or Latino other than Mexican-American and non-Hispanic of races other than black and white, including multiracial adults. The category *Other* is listed to provide a complete partition of the population into mutually exclusive groups, but it is not part of the HP2020 population template for objectives monitored using NHANES 1999 and later.
6. Educational attainment is for adults aged 25 and over. Age-adjustment groups are 25–29, 30–39, 40–49, 50–59, 60–69, 70–79, and 80 and over.
7. Family income as a percent of the federal poverty level (FPL), also known as the poverty income ratio (PIR).
8. Adults whose family PIR is not available (N/A), listed to maintain a complete partition of the population.

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(Footnotes for Table 2.)

1. Data are from the National Health and Nutrition Examination Survey (NHANES) 2005–06 and 2007–08.
2. Designed-based standard errors (SE) obtained via Taylor linearization (e.g., SUDAAN or R 'survey' package).
3. Lower and upper confidence limits, respectively, for a 95 percent confidence interval (CI).
4. The category *Other* consists of Hispanic or Latino other than Mexican-American and non-Hispanic of races other than black and white, including multiracial adults. The category *Other* is listed to provide a complete partition of the population into mutually exclusive groups, but it is not part of the HP2020 population template for objectives monitored using NHANES 1999 and later.
5. Educational attainment is for adults aged 25 and over.
6. Family income as a percent of the federal poverty level (FPL), also known as the poverty income ratio (PIR).
7. Adults whose family PIR is not available (N/A), listed to maintain a complete partition of the population.
8. Data are statistically unreliable (DSU) due the sample size being less than 30.

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TABLE 2. *Total blood cholesterol levels (μg/dL) in U.S. adults aged 20 years and over (stratified by age), 2005–08.*[1]

	Ages 20–24				Ages 25–44				Ages 45–54				Ages 55–64			
	Mean	SE[2]	95% CI[3]		Mean	SE	95% CI		Mean	SE	95% CI		Mean	SE	95% CI	
Total	176.5	1.892	172.6	180.3	195.9	0.961	193.9	197.9	206.7	1.308	204.1	209.4	208.0	1.640	204.7	211.4
Sex																
Male	173.2	1.890	169.3	177.1	199.0	1.249	196.4	201.5	204.6	1.788	200.9	208.2	199.3	1.863	195.5	203.1
Female	179.6	3.009	173.5	185.8	193.0	1.151	190.6	195.3	208.9	1.444	206.0	211.9	215.8	2.509	210.6	220.9
Race/Ethnicity																
White only, non-Hispanic	177.7	2.615	172.3	183.0	196.5	1.380	193.7	199.3	207.7	1.639	204.3	211.0	207.4	2.185	203.0	211.9
Black only, non-Hispanic	175.7	2.451	170.7	180.7	187.0	1.942	183.1	191.0	196.4	2.217	191.9	201.0	205.6	2.236	201.1	210.2
Mexican-American	176.9	2.492	171.8	182.0	199.1	1.789	195.4	202.7	210.3	2.493	205.2	215.4	210.3	2.963	204.3	216.4
Other[4]	170.2	4.506	161.0	179.4	198.1	2.159	193.7	202.5	209.2	5.160	198.7	219.7	214.4	4.110	206.1	222.8
Educational attainment[5]																
Less than high school	—	—	—	—	196.2	1.655	192.8	199.5	207.3	2.351	202.5	212.1	212.0	2.498	206.9	217.1
High school graduate	—	—	—	—	197.4	1.788	193.8	201.1	208.6	2.232	204.0	213.1	205.4	3.482	198.3	212.5
Some college or AA degree	—	—	—	—	194.9	1.636	191.6	198.3	206.7	2.515	201.5	211.8	209.7	2.764	204.1	215.4
College graduate or above	—	—	—	—	195.6	2.198	191.1	200.1	204.8	2.258	200.2	209.4	206.9	2.047	202.8	211.1
Family income (% FPL[6])																
Less than 100	175.7	3.193	169.1	182.2	194.5	1.877	190.7	198.3	211.2	3.913	203.2	219.1	205.4	4.043	197.2	213.7
100–199	176.2	2.784	170.5	181.9	194.5	1.494	191.4	197.5	209.3	2.659	203.9	214.7	216.3	3.201	209.8	222.8
200–399	178.8	2.649	173.4	184.2	196.8	1.539	193.7	199.9	203.4	2.340	198.6	208.1	205.6	2.620	200.3	211.0
400–499	172.3	5.904	160.2	184.3	194.3	2.452	189.3	199.3	203.1	2.769	197.4	208.7	208.2	3.088	201.9	214.5
500 or above	177.9	5.551	166.6	189.2	197.4	1.797	193.8	201.1	207.4	1.586	204.2	210.7	206.6	2.266	202.0	211.2
N/A[7]	171.3	6.107	158.8	183.7	196.9	4.868	187.0	206.9	209.8	3.531	202.6	217.0	210.3	4.618	200.9	219.7
Country of birth																
U.S.	176.4	2.144	172.0	180.8	195.4	1.061	193.2	197.6	205.8	1.378	203.0	208.6	207.8	1.821	204.1	211.5
Outside U.S.	176.7	2.824	170.9	182.4	197.9	1.426	194.9	200.8	212.0	3.438	205.0	219.1	210.4	2.545	205.2	215.6

	Ages 65–74				Ages 75–79				Ages 80 and over			
	Mean	SE	95% CI		Mean	SE	95% CI		Mean	SE	95% CI	
Total	197.5	1.284	194.9	200.1	192.5	1.842	188.8	196.3	193.9	1.901	190.1	197.8
Sex												
Male	183.6	1.576	180.4	186.8	183.0	2.616	177.7	188.4	176.0	1.850	172.2	179.8
Female	208.7	1.729	205.1	212.2	200.5	2.500	195.4	205.6	204.9	2.330	200.2	209.7
Race/Ethnicity												
White only, non-Hispanic	197.5	1.565	194.3	200.7	193.1	2.071	188.9	197.3	193.4	2.093	189.2	197.7
Black only, non-Hispanic	199.0	2.886	193.1	204.9	185.6	4.887	175.6	195.6	199.4	5.370	188.4	210.3
Mexican-American	197.5	3.923	189.5	205.5	191.0	4.764	181.3	200.7	210.5	10.233	189.6	231.3
Other	194.4	8.222	177.6	211.1	191.8	8.738	174.0	209.6	187.0	7.740	171.2	202.8
Educational attainment												
Less than high school	199.5	2.967	193.5	205.6	186.3	3.376	179.4	193.2	196.1	3.339	189.3	202.9
High school graduate	198.0	3.029	191.8	204.2	187.1	2.441	182.1	192.1	189.6	2.708	184.1	195.1
Some college or AA degree	199.5	4.063	191.3	207.8	203.5	4.851	193.6	213.4	199.7	4.221	191.0	208.3
College graduate or above	192.9	2.469	187.9	197.9	200.1	6.440	187.0	213.3	189.2	5.501	178.0	200.4
Family income (% FPL)												
Less than 100	207.2	5.704	195.5	218.8	198.8	4.416	189.8	207.8	195.7	5.732	184.0	207.3
100–199	202.6	3.706	195.0	210.2	189.7	3.151	183.3	196.1	199.1	3.061	192.9	205.3
200–399	196.9	2.691	191.5	202.4	191.7	3.490	184.6	198.9	191.6	3.828	183.7	199.4
400–499	195.9	4.761	186.2	205.6	DSU[8]	—	—	—	DSU	—	—	—
500 or above	191.3	3.034	185.1	197.5	197.0	7.731	181.2	212.8	191.7	6.269	178.9	204.5
N/A	187.8	6.715	174.1	201.5	193.9	6.766	180.1	207.7	188.4	5.026	178.1	198.6
Country of birth												
U.S.	197.2	1.314	194.5	199.8	192.5	1.883	188.7	196.3	194.2	2.013	190.1	198.3
Outside U.S.	200.7	4.650	191.2	210.1	193.0	6.304	180.2	205.9	191.3	5.090	181.0	201.7

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References.

Matthews, K. A., Crawford, S. L., Chae, C. U., Everson-Rose, S. A., Sowers, M. F., Sternfeld, B. and Sutton-Tyrrell, K. (2009). Are Changes in cardiovascular disease risk factors in midlife women due to chronological aging or to the menopausal transition? *Journal of the American College of Cardiology* **54** 2366–73. doi:10.1016/j.jacc.2009.10.009.

National Center for Health Statistics, (2011). *Healthy People 2010 Final Review*. National Center for Health Statistics, Hyattsville, MD.

Talih, M. (2012). A Reference-invariant health disparity index based on Rényi divergence. *Under Review*.

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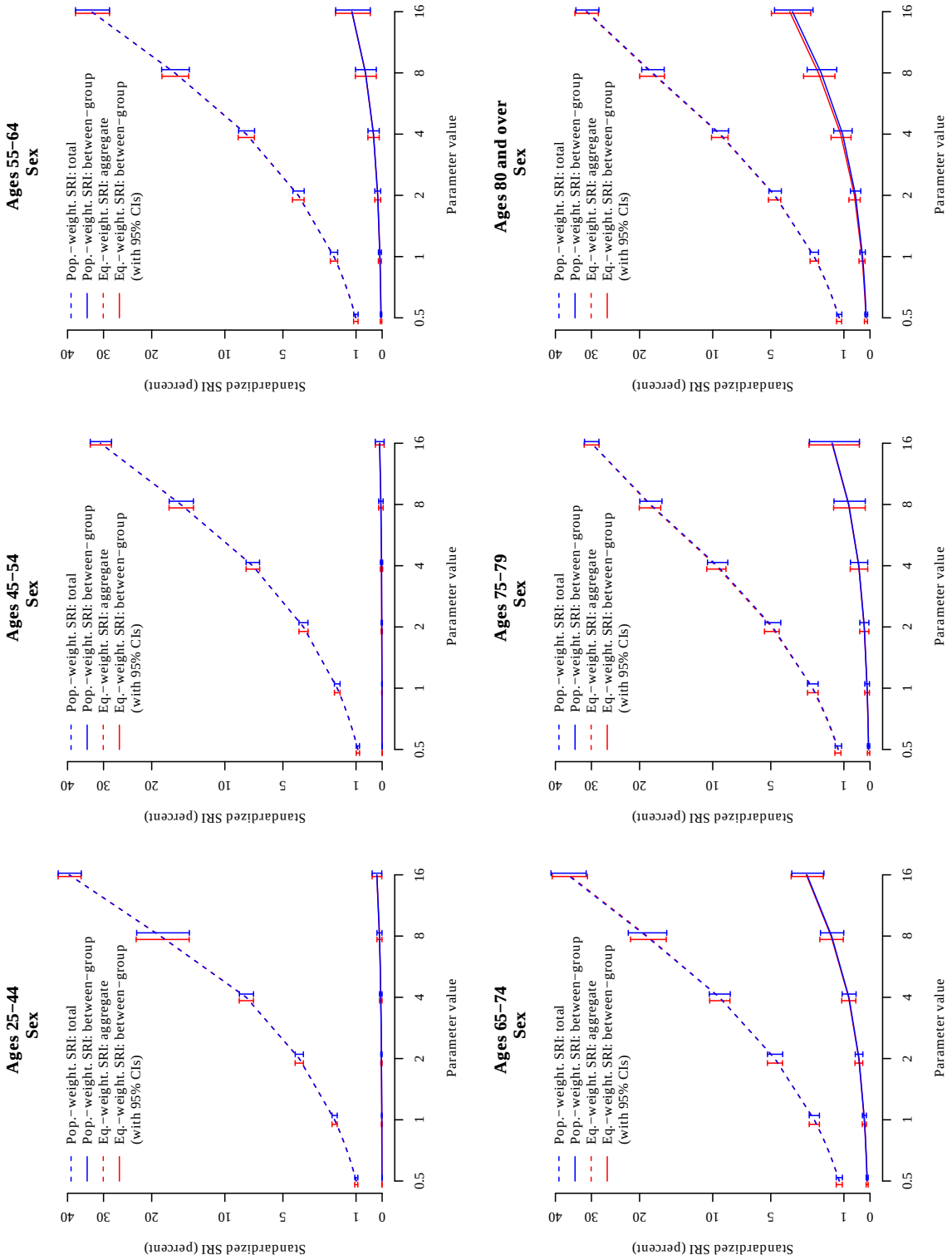

FIG 1. *Total or aggregate and between-group components of the standardized SRI by sex for the mean total blood cholesterol levels ($\mu g/dL$) in U.S. adults aged 25 and over, stratified by age, 2005–08. Due to symmetry of the SRI around the parameter value 0.5, only values of $\alpha \geq 0.5$ are shown. For values of $\alpha \geq 0.5$, the parameter α is a disparity aversion parameter for the standardized SRI: the standardized SRI is nondecreasing in α for $\alpha \geq 0.5$. The population-weighted SRI uses the estimated distributions for the relative shares of population ($p_j = n_j/n, p_{ij} = 1/n$) and of health outcome ($q_j = y_{\cdot j}/y_{\cdot\cdot}, q_{ij} = y_{ij}/y_{\cdot\cdot}$) in the symmetrized Rényi divergence $S_\alpha(p, q)$. The equally-weighted SRI uses $p_j = 1/m$. The population-weighted and equally-weighted SRIs are almost identical, here. The 95% confidence intervals (CIs) are design-based, obtained via Taylor linearization.*