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EXAMINING SOCIOECONOMIC HEALTH DISPARITIES USING A RANK-DEPENDENT RÉNYI INDEX

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Abstract

The Rényi index (RI) is a one-parameter class of indices that summarize health disparities among population groups by measuring divergence between the distributions of disease burden and population shares of these groups. The *rank-dependent* RI introduced in this paper is a two-parameter class of health disparity indices that also accounts for the association between socioeconomic rank and health; it may be derived from a rank-dependent social welfare function. Two competing classes are discussed and the rank-dependent RI is shown to be more robust to changes in the distribution of either socioeconomic rank or health. The standard error and sampling distribution of the rank-dependent RI are evaluated using linearization and re-sampling techniques, and the methodology is illustrated using health survey data from the U.S. National Health and Nutrition Examination Survey and registry data from the U.S. Surveillance, Epidemiology and End Results Program. Such data underlie many population-based objectives within the U.S. Healthy People 2020 initiative. The rank-dependent RI provides a unified mathematical framework for eliciting various societal positions with regards to the policies that are tied to such wide-reaching public health initiatives. For example, if population groups with lower socioeconomic position were ascertained to be more likely to utilize costly public programs, then the parameters of the RI could be selected to reflect prioritizing those population groups for intervention or treatment.

Keywords and phrases

Social welfare function; Concentration curve; Rényi divergence; Statistical inference; Complex survey data; Health surveillance data

1. Introduction

The socioeconomic gradient in health outcomes and resulting health disparities are now well documented in the United States (U.S.) and elsewhere (Costa-Font and Hernández-Quevedo, 2012; Braveman *et al.*, 2010; Wilson, 2009; WHO-CSDH, 2008; Lynch *et al.*, 2004; Krieger, Williams and Moss, 1997). Public health programs can leverage social determinants of health to address health inequities and improve health outcomes, as discussed in a recent supplement to *Public Health Reports* (Dean, Williams and Fenton,

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2013). The U.S. Healthy People 2020 (HP2020) initiative emphasizes the importance of addressing the social determinants of health and eliminating disparities: two of its four overarching goals are to “create social and physical environments that promote good health for all” and “achieve health equity, eliminate disparities, and improve the health of all groups” (DHHS, 2014).

Improving overall population health while simultaneously striving to eliminate health disparities is a fundamental public health and social policy challenge, because interventions designed to improve the health of individuals may increase disparities between groups and, conversely, reducing a group’s burden of disease may have little impact on overall population health (Frohlich and Potvin, 2008; Mechanic, 2002; Rose, 1985). Therefore, it is imperative that measures of health disparities be explicit about the value judgments and trade-offs that are inherent to their methodology—e.g., choice of reference for evaluating disparities, relative versus absolute disparities, attainment (i.e., favorable outcomes) versus shortfall (i.e., adverse outcomes) inequalities, equally-weighted versus population-weighted groups, etc. (Lambert and Zheng, 2011; Harper *et al.*, 2010; Erreygers, 2009a; Keppel *et al.*, 2005; Mackenbach and Kunst, 1997).

In the context of socioeconomic disparities in health, the slope index of inequality (Pamuk, 1988, 1985), the classical concentration index (Wagstaff, Paci and van Doorslaer, 1991), and the health achievement index (Wagstaff, 2002) have provided the impetus for much of the literature on socioeconomic health inequality measures. For example, the partial concentration index removes the effect of covariates (e.g., age or sex) that may be correlated with both health and income but may be irrelevant to policy in that neither their direct effect on health nor their joint distribution with income can be altered (Gravelle, 2003). Further, an intuitive policy-oriented interpretation of the concentration index ensues from certain redistribution schemes (Koolman and van Doorslaer, 2004).

A slope index of inequality consists of the slope of the (weighted) least-squares regression of health outcomes onto socioeconomic ranking and is designed to summarize the association between health and socioeconomic status (SES). Similarly, the classical concentration index can be written as twice the covariance between socioeconomic rank and health shares. A health achievement index represents an equally-distributed level of health equivalent to the population average but such that all groups achieve the same average outcome. Those three indices are interrelated; they are reviewed in section 3 of this paper.

Even though the concentration index is widely used, due to its simple formulation and its appeal to policy makers, its shortcomings have come under intense scrutiny in recent years (Bleichrodt, Rohde and van Ourti, 2012) and various options for correcting its behavior, especially when measuring socioeconomic inequality in a binary health outcome variable, have been debated (Kjellsson and Gerdtham, 2013; Wagstaff, 2011; Erreygers, 2009b).

This paper is not intended as a critique of the concentration index. Instead, it builds on the differential weighting scheme for socioeconomic groups (Berrebi and Silber, 1981) that the concentration index utilizes and explores a two-parameter alternative to the concentration index that is derived from Rényi divergence and includes the entropy-based Rényi index of

Talih (2013b) as a special case. The proposed approach builds a bridge between the theory of rank-dependent social welfare functions and the information theoretic evaluation of divergence between probability distributions. On the one hand, there is an extensive statistical literature on discrepancy measures, with applications to goodness-of-fit tests, robust parameter estimation, and signal processing; see Talih (2013b) and references therein. On the other hand, social welfare theory provides a framework for the measurement and characterization of socioeconomic inequalities in health (Erreygers and van Ourti, 2011; Bleichrodt and van Doorslaer, 2006), though social justice principles remain foundational in socioeconomic inequality measurement (Bommier and Stecklov, 2002; Peter, 2001).

In parallel with the development of rank-dependent inequality indices, there is renewed interest in composite indices (Asada, Yoshida and Whipp, 2013), particularly for analyses and international comparisons of wellbeing, e.g., using the Human Development Index (Foster, McGillivray and Seth, 2013; Paruolo, Saisana and Saltelli, 2013). In the U.S., composite measures of health and health-related quality of life remain core tools for monitoring progress toward the HP2020 goals (DHHS, 2014). The focus on multidimensional analyses is also manifested in the development of indices for *multidimensional inequality* (Bennett and Mitra, 2013; Decancq and Lugo, 2009; Tsui, 1999; Maasoumi, 1986).

The Rényi index (RI), reviewed in section 2, is a class of inequality indices, $\{RI_\alpha : \alpha > 0\}$, that is derived from Rényi divergence (Talih, 2013b). The parameter $\alpha > 0$ is an inequality aversion parameter. The RI is invariant to the choice of the reference used for evaluating disparities. This invariance property is relevant to HP2020 and related public health initiatives because, as mentioned previously, the identification of a reference involves a value judgment and, moreover, can be affected by statistical reliability (NCHS, 2011). As discussed in section 2, the well-known generalized entropy (GE) class also can be modified for reference-invariance. Yet, the RI is more robust than its GE-based counterpart to changes in the distribution of the adverse health outcome.

Section 3 extends the RI to population groups that are ordered by family income, educational attainment, or other SES variables (or composites thereof) that contribute to the social determinants of health. A two-parameter rank-dependent RI is proposed in section 3.2, $\{RI_\alpha^{(\nu)} : \alpha \geq 0, \nu \geq 1\}$, where increased values of $\alpha > 0$ reflect an increased societal aversion to (pure) health inequality and increased values of $\nu > 1$ allow groups with lower SES to weigh more heavily than groups with higher SES. Section 3.3 shows how the rank-dependent RI can be derived from a rank-dependent social welfare function, relating the proposed index to the Makdissi-Yazbeck two-parameter classes of health achievement and inequality indices (Makdissi and Yazbeck, 2012); in turn, those extend the corresponding Wagstaff classes of indices (Wagstaff, 2002), reviewed in section 3.1. (In Appendix A, a “convenient regression” relates the rank-dependent RI to the slope index of inequality.) In section 3.4, the GE class of indices is modified for rank-dependence (and reference-invariance). Simulation results in section 4.1 provide empirical evidence that the rank-dependent RI is more robust than either of its Makdissi-Yazbeck or GE-based counterparts to changes in the distributions of SES or health outcomes.

Sections 4.2 and 4.3 illustrate the proposed methodology using data from the U.S. National Health and Nutrition Examination Survey (NHANES), CDC, NCHS, as well as data from the U.S. Surveillance, Epidemiology and End Results (SEER) Program, NIH, NCI. Such health survey and registry data are common for tracking population-based HP2020 objectives. The standard error and sampling distribution of the rank-dependent RI for these data are evaluated using linearization and resampling techniques. Even though progress has been made in understanding the asymptotic behavior of health inequality indices (Cowell, Davidson and Flachaire, 2011; Aaberge, 2005) and first-order linearization can be adapted for evaluating the sampling variability of such indices; see Appendix B, and Langel and Tillé (2013), Bor-rell and Talih (2012, 2011), Biewen and Jenkins (2006), and Kakwani, Wagstaff and van Doorslaer (1997), resampling methods remain most useful for evaluating statistical significance, especially with complex survey data (Talih, 2013b; Chen, Roy and Gotway Crawford, 2012; Cheng, Han and Gansky, 2008; Harper *et al.*, 2008; Rao, Wu and Yue, 1992; Rao and Wu, 1988).

2. Rényi index

For a population that is partitioned into M mutually exclusive groups of sizes n_1, n_2, \dots, n_M , with $n = \sum_{j=1}^M n_j$ and $n_j > 0$ for $j = 1, 2, \dots, M$, consider the distribution of a particular adverse health outcome y_{ij} for individual i in group j . Findings of health disparities between groups rest on the comparison of the aggregate health outcomes $y_{\cdot j} = \sum_{i=1}^{n_j} y_{ij}, j = 1, 2, \dots, M$, either to one another or to the total, $y_{\cdot\cdot} = \sum_{j=1}^M y_{\cdot j}$. Below, $y_{\cdot\cdot}$ is assumed to be positive (i.e., the outcome of interest is observed) and the average adverse health outcomes for the groups and the total population are denoted $\bar{y}_{\cdot j} = y_{\cdot j}/n_j$ and $\bar{y}_{\cdot\cdot} = y_{\cdot\cdot}/n$, respectively.

Definition—Let relative health disparities r_j be proportional to the groups' average adverse health outcomes: $r_j \propto \bar{y}_{\cdot j}$. For any positive group weights p_j , define $\bar{p}_j = p_j/\sum_k p_k$ and $r_j^- = r_j/\sum_k p_k \bar{r}_k$. The Rényi index, which takes values in $[0, +\infty]$, is given by

$$RI_\alpha = \begin{cases} -\frac{1}{1-\alpha} \ln \left(\sum_{j=1}^M \bar{p}_j \bar{r}_j^{1-\alpha} \right), & \text{for } \alpha \neq 1, \alpha \geq 0 \\ -\sum_{j=1}^M \bar{p}_j \ln \bar{r}_j, & \text{for } \alpha = 1. \end{cases} \quad (2.1)$$

Thus, $RI_\alpha = 0$ if $\bar{y}_{\cdot j} = \sum_k p_k \bar{y}_{\cdot k}$. The expression in (2.1) is that of the Rényi divergence between the two probability mass functions \bar{p}_j and $q_j^- := p_j \bar{r}_j^-$ (Talih, 2013b; Rényi, 1960).

Remarks—The p_j are positive weights that are assigned to each group. Groups are equally weighted ($p_j = 1/M$), population weighted ($p_j = n_j/n$), or, otherwise, reflect a preference ordering, such as the socioeconomic weights of section 3. The r_j are relative health disparities, where the reference is the population average ($r_j = \bar{y}_{\cdot j}/\bar{y}_{\cdot\cdot}$), the least adverse health outcome ($r_j = \bar{y}_{\cdot j}/\min_k \bar{y}_{\cdot k}$), or, otherwise, any fixed reference such as a HP2020 target ($r_j = \bar{y}_{\cdot j}/y_{\text{target}}$). Due to the scale-invariance of the RI, the r_j need only be proportional to the groups' average adverse health outcomes $\bar{y}_{\cdot j}$ (Talih, 2013b).

When $p_j = n_j/n$, the standardized Rényi index, with values in $[0, 1]$, is the (between-group) Atkinson index (Atkinson, 1970), obtained from

$$A_\alpha = 1 - e^{-\text{RI}_\alpha}. \quad (2.2)$$

The RI increases with α . With infinite inequality aversion $\alpha \rightarrow \infty$, the RI is dominated by the population group with the least adverse health outcome:

$$\lim_{\alpha \rightarrow \infty} \text{RI}_\alpha = -\ln \left(\min_{1 \leq k \leq M} \bar{r}_k \right) =: \text{RI}_\infty.$$

Because $0 < A_\alpha < A_\infty < 1$, an alternative standardization to that in (2.2) emerges:

$$\frac{A_\alpha}{A_\infty} = \frac{1 - e^{-\text{RI}_\alpha}}{1 - e^{-\text{RI}_\infty}}.$$

Some of the most commonly used (between-group) health inequality indices belong to the generalized entropy (GE) class, with values in $[0, +\infty]$,

$$\text{GE}_\alpha = \sum_{j=1}^M p_j g_\alpha(r_j),$$

where $p_j = n_j/n$, $r_j = y_{.j}/y_{..}$, and

$$g_\alpha(r) = \begin{cases} \frac{1-r^{1-\alpha}}{1-\alpha}, & \alpha \neq 1, \alpha \geq 0, \\ -\ln r, & \alpha = 1; \end{cases} \quad (2.3)$$

see Talih (2013b) and literature review therein. When $\alpha = 1$, the Rényi and GE indices are equal. When $\alpha > 1$, $\alpha < 0$, these indices are related as follows:

$$\text{RI}_\alpha = -\frac{1}{1-\alpha} \ln[1 - (1-\alpha)\text{GE}_\alpha]. \quad (2.4)$$

An important result from Talih (2013b) is that $\text{RI}_\alpha > \text{GE}_\alpha$ for $\alpha > 1$, which entails that, for $\alpha > 1$, the RI is more robust than the GE index to changes in the distribution of health outcomes. For example, consider the hypothetical populations in Table 1, which are studied in section 4.1, below. With the commonly used parameter value $\alpha = 2$, the RI increases 35% between populations 1 and 3, from 0.257 to 0.348, whereas the GE increases 42%, from 0.293 to 0.417, 1.2 times the rate of increase of the RI. With $\alpha = 4$, the RI increases 26% between populations 1 and 3, from 0.567 to 0.717, whereas the GE increases 70%, from 1.494 to 2.534, over 2.6 times the rate of increase of the RI. Figure 2 further illustrates the lack of robustness of the rank-dependent GE compared with the rank-dependent RI for a range of parameter values. Robustness is especially important for less common adverse

health outcomes because even small absolute differences between groups can translate into very large relative disparities r_j and, therefore, large index values. Harper *et al.* (2010) provide an excellent outline of the debate regarding absolute versus relative disparities.

3. Rank-dependence and differential weighting

The crucial difference between a rank-dependent health disparity index and a health disparity index that is not rank-dependent is that the former accounts for the association between an exposure (e.g., SES) and an outcome (e.g., late-stage uterine cervical cancer), whereas the latter accounts only for inequalities in the outcome variable.

Let the population groups be ranked from lowest to highest SES, with $n_j > 0$. For $j = 1, \dots, M - 1$, define rank variables R_j as follows:

$$R_1 = \frac{1}{2} \frac{n_1}{n} \quad \text{and} \quad R_{j+1} = \frac{1}{2} \frac{n_{j+1}}{n} + \sum_{k=1}^j \frac{n_k}{n}. \quad (3.1)$$

By construction, $0 < R_j < R_{j+1} < 1$. For scalar $\nu > 1$, define

$$w_\nu(R_j) = \nu(1 - R_j)^{\nu-1}.$$

The rank-dependent Rényi index proposed in this paper is derived from (2.1) using the socioeconomic weights $p_j^{(\nu)} = w_\nu(R_j)p_j$ instead of just p_j , as seen in section 3.2, below. For $\nu > 1$, the initial weights $p_j^{(1)} = p_j$ are rescaled according to the rank of each group: groups with lower SES are weighted more heavily. In particular,

$$w_\nu(R_j) = \left(\frac{1 - R_j}{R_j} \right)^{\nu-1} \times w_\nu(1 - R_j). \quad (3.2)$$

For example, suppose groups are equally weighted to start, i.e., $p_j \equiv 1/M$. Then, for $\nu = 2$, the socioeconomic weight for a group at the first quintile of the SES distribution (i.e., with $R_j = 0.20$) would be 4 times the socioeconomic weight for the corresponding group at the fourth quintile of the SES distribution. With $\nu = 3$, this factor grows to $4^{\nu-1} = 4^2 = 16$. When groups are population weighted initially, i.e., $p_j = n_j/n$, the effect of increasing the value of the parameter ν is not as clear cut. Still, Figure 1 shows, for example, that moving from $\nu = 1$ to $\nu = 3$ triples the relative weight of the “poor” and more than doubles the relative weight of the “near poor,” while rendering the weight on the “high income” group negligibly small (these groups are defined in Table 1). The selection of the parameter ν , in practice, will vary according to the context and data. The analyst is advised to explore different scenarios and, if required, select the parameter ν that most closely reflect his/her expectation.

As seen next, the slope index of inequality, the classical concentration index, and the extended concentration and health achievement indices all utilize such differential SES weighting as in (3.2), either implicitly or explicitly.

3.1. Concentration and health achievement indices

Consider the Q-Q plot of the cumulative distribution of health burden y_j against the SES rank variables R_j defined previously. The classical concentration index is defined as twice the area between the resulting Q-Q curve and the diagonal; equivalently, it can be written as twice the covariance between SES rank and health burden, which directly relates it to the slope index of inequality; see Wagstaff, Paci and van Doorslaer (1991), Pamuk (1988, 1985), as well as Appendix A.

With $p_j = n_j/n$, $r_j = \bar{y}_j/\bar{y}$, and a normalizing constant $W = \sum_j(1 - R_j)p_j$, the classical concentration index, with values in $[-1, +1]$, can be written as

$$C = 1 - \frac{2}{W} \sum_{j=1}^M (1 - R_j) p_j r_j.$$

The index C takes the value 0 when the aforementioned Q-Q curve coincides with the diagonal (i.e., when the covariance between SES rank and health is 0).

Even though the concentration index C initially appears value neutral, this latest expression reveals that C is value laden: all else being equal, the relative disparities r_j for groups with lower SES (i.e., lower rank R_j) are weighted more heavily than those for groups with higher SES; specifically, C uses $\nu = 2$ in (3.2).

To enable the analyst to account more explicitly for such a value judgment with respect to the differential weighting of the groups, Wagstaff (2002) introduced the extended concentration index, defined for $\nu \geq 1$ as

$$C(\nu) = 1 - \frac{\nu}{W(\nu)} \sum_{j=1}^M (1 - R_j)^{\nu-1} p_j r_j,$$

with normalizing constant $W(\nu) = \sum_j (1 - R_j)^{\nu-1} p_j$. As previously, increasing the value of ν results in increasingly larger weights placed on the groups with lower SES, whereas groups with higher SES are assigned increasingly smaller weights. Thus, the parameter ν reflects a degree of socioeconomic inequality aversion.

Between-group disparities, as well as the socioeconomic weighting of the groups, are sensitive to the implicit (or explicit) value judgments underlying the classical (or extended) concentration index. Moreover, assessing disparity based solely on an average health burden fails to account for that burden's association with SES and the extent of inequality between the lower and higher SES groups. Wagstaff (2002) introduced a (rank-dependent) health

achievement index to quantify this trade-off between improving population health and reducing health inequality. The Wagstaff health achievement index is defined for $\nu \geq 1$ as

$$H(\nu) = \frac{\nu}{W(\nu)} \sum_{j=1}^M (1 - R_j)^{\nu-1} p_j \bar{y}_{\cdot j}.$$

With $p_j = n_j/n$ and $r_j = \bar{y}_{\cdot j}/\bar{y}_{\cdot\cdot}$, $H(1) = \bar{y}_{\cdot\cdot}$, the population average, and the concentration and health achievement indices are related as follows:

$$H(\nu) = [1 - C(\nu)] \times \bar{y}_{\cdot\cdot} \quad (3.3)$$

As before, consider a particular adverse health outcome y_{ijk} for individual i in SES group j and population k , e.g., late-stage uterine cervical cancer by SES within racial/ethnic population groups in the U.S. If SES was not accounted for (e.g., $\nu = 1$ and $C(1) = 0$), then only the population means would be compared; for example, the mean $\bar{y}_{\cdot\cdot 1}$ for population 1 might be higher than the mean $\bar{y}_{\cdot\cdot 2}$ for population 2, signifying a higher cancer burden for population 1 than for population 2 (e.g., 9.0 versus 6.4 per 100,000). On the other hand, if SES was accounted for (e.g., $\nu > 1$ and $|C(\nu)| > 0$), and it was ascertained that the two populations had the same value of $H(\nu)$ (e.g., 8.64 per 100,000 for both populations) then this could occur because, say, population 1 had a more equal distribution across SES groups (Q-Q curve closer to the diagonal, e.g., $C(\nu) = 0.04$), whereas population 2 had a higher burden of disease for the lower SES groups (Q-Q curve farther from the diagonal, e.g., $C(\nu) = -0.35$).

Similarly for comparisons over time for a single population; the mean $\bar{y}_{\cdot\cdot}$ could remain unchanged, yet the health achievement could become worse due to a shift in the SES distribution of health burden. Incidentally, Chen *et al.* (2013) caution against causal inference from socioeconomic health inequality indices such as the slope index of inequality or the concentration index precisely for this reason. Nonetheless, such indices remain useful for descriptive as well as comparative analyses in large indicator initiatives, where resource limitations do not always permit in-depth causal analyses; the HP2020 initiative, for example, houses over 1,200 health indicators (DHHS, 2014).

3.2. Rank-dependent Rényi index

As stated previously, the rank-dependent RI is derived from (2.1) using the socioeconomic weights $p_j^{(\nu)} = w_\nu(R_j)p_j$. To better highlight its connection to social evaluation functions in section 3.3, we introduce appropriate notation, here, and re-express the rank-dependent RI accordingly. In addition, to simplify the remainder of this paper, the p_j will, henceforth, denote the population-weighted group weights $p_j = n_j/n$. However, identical derivations follow for equally-weighted groups as well as any other group weights as a starting point $p_j^{(1)}$.

Notation—For $r > 0$, let f_α denote the power transform and f_α^{-1} its inverse

$$f_\alpha(r) = \begin{cases} \frac{r^{1-\alpha}}{1-\alpha} & \alpha \neq 1, \alpha \geq 0, \\ \ln r & \alpha = 1. \end{cases} \quad f_\alpha^{-1}(s) = \begin{cases} [(1-\alpha)s]^{\frac{1}{1-\alpha}}, & \alpha \neq 1, \alpha \geq 0, \\ e^s, & \alpha = 1. \end{cases} \quad (3.4)$$

For $\alpha > 0$, the function f_α is the generalized logarithm. Define

$$W_1(\nu) = \sum_{j=1}^M w_\nu(R_j) p_j, \quad W_2(\nu) = \sum_{j=1}^M w_\nu(R_j)^2 p_j$$

$$\bar{w}_\nu(R_j) = \frac{w_\nu(R_j)}{W_1(\nu)},$$

$$S(\nu, \alpha) = \sum_{j=1}^M \bar{w}_\nu(R_j) p_j f_\alpha(r_j).$$

Let $\bar{p}_j^{(\nu)} = p_j^{(\nu)} / \sum_k p_k^{(\nu)}$ and $\bar{r}_j^{(\nu)} = r_j / \sum_k \bar{p}_k^{(\nu)} r_k$. Using this notation, we have

$$\bar{p}_j^{(\nu)} = \bar{w}_\nu(R_j) p_j \quad \text{and} \quad \bar{r}_j^{(\nu)} = \frac{r_j}{S(\nu, 0)},$$

and the rank-dependent Rényi index from (2.1) is expressed for all $\alpha \geq 0$ and $\nu \geq 1$ as

$$RI_\alpha^{(\nu)} = -\ln \left\{ \frac{f_\alpha^{-1}[S(\nu, \alpha)]}{S(\nu, 0)} \right\}. \quad (3.5)$$

3.3. Rank-dependent social evaluation function

A two-parameter social evaluation function is given in aggregate form by

$$S^*(\nu, \alpha) = \sum_{j=1}^M \bar{w}_\nu(R_j) p_j f_\alpha(\bar{y}_{\cdot j}), \quad (3.6)$$

where $f_\alpha(\bar{y}_{\cdot j})$, $\alpha > 0$, represents society's evaluation of the group's health burden $\bar{y}_{\cdot j}$ and $\bar{w}_\nu(R_j) p_j = \bar{p}_j^{(\nu)}$ is the group's socioeconomic weight (Makdissi and Yazbeck, 2012).

Remark—The asterisk in $S^*(\nu, \alpha)$ is to distinguish it from the *relative* measure $S(\nu, \alpha)$ defined previously, where the social evaluation function f_α was evaluated at the relative disparities r_j instead of the average health outcomes $\bar{y}_{\cdot j}$.

In the above, two components of societal evaluation of health are featured:

- i. A *pure health inequality* component, driven by society's evaluation of a group's health burden irrespective of its SES rank—the function f_α in (3.4) has constant

relative-inequality aversion $\alpha = -yf''_{\alpha}(y)/f'_{\alpha}(y)$ (Cowell and Gardiner, 1999; Pratt, 1964). In particular, $\nu = 1$ results in a social preference function that is indifferent to SES (at least explicitly, since, directly or indirectly, SES remains a determinant of health).

- ii. A *socioeconomic health inequality* component, driven by the rank-dependent weighting function $w_{\nu}(R_j)$ —the parameter ν is a socioeconomic health inequality aversion parameter, with hyperbolic absolute-inequality aversion $(\nu - 2)/(1 - R) = -w''_{\nu}(R)/w'_{\nu}(R)$ when $\nu > 2$. Here, $\alpha = 0$ results in a social preference function that is indifferent to pure health inequalities (again, at least explicitly), quantifying solely the distribution of the adverse health outcome along the SES gradient, as in the extended concentration index of Wagstaff (2002).

A rank-dependent health achievement index is obtained from $H^* = f_{\alpha}^{-1}(S^*)$ in (3.6); it represents an equally-distributed equivalent level of health such that S^* is equivalent to $f_{\alpha}(H^*)$ —i.e., a hypothetical society in which all groups achieve an average outcome $\bar{y}_{.j}$ equal to H^* . The Makdissi and Yazbeck (2012) health achievement index is expressed as:

$$H^*(\nu, \alpha) = \begin{cases} \left[\sum_{j=1}^M \bar{w}_{\nu}(R_j) p_j \bar{y}_{.j}^{1-\alpha} \right]^{\frac{1}{1-\alpha}}, & \text{for } \alpha \neq 1, \alpha \geq 0, \\ \exp \left[\sum_{j=1}^M \bar{w}_{\nu}(R_j) p_j \ln \bar{y}_{.j} \right], & \text{for } \alpha = 1. \end{cases} \quad (3.7)$$

For example, $H^*(1, 0)$ is the population average outcome $\sum_{j=1}^M \bar{p}_j \bar{y}_{.j} = \bar{y}_{..}$ (when $p_j = n_j/n$), whereas $H^*(\nu, 0)$ is the SES-weighted population average outcome $\sum_{j=1}^M \bar{p}_j^{(\nu)} \bar{y}_{.j}$. In addition, the two limiting cases $\alpha \rightarrow \infty$ and $\nu \rightarrow \infty$ are important for interpretation:

$$H^*(\nu, \infty) := \lim_{\alpha \rightarrow \infty} H^*(\nu, \alpha) = \min_{1 \leq k \leq M} \bar{y}_{.k} \quad \text{and} \quad H^*(\infty, \alpha) := \lim_{\nu \rightarrow \infty} H^*(\nu, \alpha) = \bar{y}_{.k^*}, \quad (3.8)$$

where $k^* = \arg \min_{1 \leq k \leq M} R_k$ is the group with the lowest SES rank.

As $\nu > 1$ increases, more weight is given to the group with the lowest SES. If the SES gradient in health is positive when groups are ranked from highest to lowest SES, then the group with the lowest SES will also have the worst health outcome $\bar{y}_{.k^*} \equiv \max_k \bar{y}_{.k}$. Thus, when $\nu \rightarrow \infty$ society's health achievement becomes only as good as that of its socioeconomically most disadvantaged (Rawls, 1999). On the other hand, holding the parameter ν constant, health achievement can only be improved at a progressively steeper cost of nonintervention, as reflected by increasing $\alpha > 0$. In a society that is infinitely averse to inequality (and that has unlimited resources), all groups achieve the best group rate $H^*(\nu, \infty) = \min_k \bar{y}_{.k}$.

The rank-dependent RI in (3.5) provides a unified mathematical framework for engaging in the aforementioned considerations. The index $RI_{\alpha}^{(\nu)}$ and the standardized index $A_{\alpha}^{(\nu)}$ are

$$RI_{\alpha}^{(\nu)} = -\ln \left[\frac{H^*(\nu, \alpha)}{H^*(\nu, 0)} \right] \quad \text{and} \quad A_{\alpha}^{(\nu)} = 1 - \frac{H^*(\nu, \alpha)}{H^*(\nu, 0)}. \quad (3.9)$$

In other words, what equations (3.9), as well as Figure 1, below, show is that, for each given value of $\nu > 1$, the standardized rank-dependent RI expresses the relative change that would be required to “move the needle” from the *status quo* (e.g., the reference achievement level $H^*(\nu, 0)$, an SES-weighted population average health burden) to a level of health achievement that is compatible with societal expectations (achievement level $H^*(\nu, a)$ for aversion parameter value a).

Two-parameter extended concentration index—As in (3.3), when $p_j = n_j/n$, the two-parameter Makdissi and Yazbeck (2012) extended concentration index

$$C(\nu, \alpha) = 1 - \frac{H^*(\nu, \alpha)}{H^*(1, 0)} = 1 - \frac{H^*(\nu, \alpha)}{\bar{y}_{..}} \quad (3.10)$$

compares the requisite equally-distributed equivalent health level $H^*(\nu, a)$ to the population average health outcome $\bar{y}_{..}$. $C(\nu, a)$ corresponds to the standardized index $\tilde{A}_{\alpha}^{(\nu)}$ that would be obtained if one used $\bar{r}_j^{(1)} = \bar{y}_{.j} / \sum_k \bar{p}_k y_{.k} \equiv \bar{y}_{.j} / \bar{y}_{..}$ instead of $\bar{r}_j^{(\nu)} = \bar{y}_{.j} / \sum_k \bar{p}_k^{(\nu)} \bar{y}_{.k}$ in (2.1). However, unlike the standardized index $A_{\alpha}^{(\nu)}$ in (3.9), $C(\nu, a)$ does not remain nonnegative. $C(\nu, 0)$ and $C(2, 0)$ are the extended $[C(\nu)]$ and classical (C) health concentration indices, respectively; see section 3.1. Instead of the population average outcome $\bar{y}_{..} = H^*(1, 0)$ as reference for health achievement, the standardized index $A_{\alpha}^{(\nu)}$ in (3.9) uses the SES-weighted average $H^*(\nu, 0)$. The relationship between the standardized rank-dependent RI, the two-parameter extended concentration index, and the extended concentration index, is as follows:

$$1 - A_{\alpha}^{(\nu)} = \frac{1 - C(\nu, \alpha)}{1 - C(\nu, 0)}.$$

Achievement versus capacity to achieve—As noted in section 2, the standardization in (2.2) is not fully satisfactory in that $A_{\alpha} \rightarrow 1$ only if $RI_{\alpha} \rightarrow \infty$. Thus, for the rank-dependent RI, the following standardization may be preferable:

$$\frac{A_{\alpha}^{(\nu)}}{A_{\infty}^{(\nu)}} = \frac{H^*(\nu, 0) - H^*(\nu, \alpha)}{H^*(\nu, 0) - H^*(\nu, \infty)}. \quad (3.11)$$

Holding the parameter ν constant, $A_{\alpha}^{(\nu)} / A_{\infty}^{(\nu)}$ is the proportion of the maximum potential improvement in health achievement $[H^*(\nu, 0) - H^*(\nu, \infty)]$ that would be attained at nonintervention cost $a > 0$ if all groups were to achieve $\bar{y}_{.j} \equiv H^*(\nu, a)$ instead of only $\bar{y}_{.j} \equiv H^*(\nu, 0)$.

Figure 1 illustrates the notion of health achievement relative to the population’s “capacity to achieve” using data for hypothetical population 1 in Table 1, with socioeconomic health inequality parameter $\nu = 1$ (rank-neutral group weights) and $\nu = 3$ (weights favorable to groups with low income level). The reference ‘achievement’ level $H(\nu, 0)$, i.e., the income-weighted population proportion in fair or poor health, is higher for larger ν , resulting in a larger gap relative to the best rate $H^*(\nu, \infty)$. A larger ν results in a larger a —i.e., a higher ‘cost of nonintervention’—for about the same achievement level $H(\nu, a) \approx 8\%$.

3.4. Rank-dependent generalized entropy class

As stated earlier, the GE index (2.3) also can be modified for rank-dependence. Originating in the study of likelihood ratio tests (Chernoff, 1952), the GE class is tied to important axiomatic properties in inequality measurement (Cowell and Kuga, 1981) and remains widely used in the economic analysis of income inequalities; see Talih (2013b) for a review of relevant literature.

Definition—As before, let the relative health disparities r_j be proportional to the groups’ average adverse health outcomes: $r_j \propto \bar{y}_j$. For any positive group weights p_j , define $p_j^- = p_j / \sum_k p_k$ and $r_j^- = r_j / \sum_k p_k \bar{r}_k$. A reference-invariant GE index is given by

$$GE_\alpha = \begin{cases} \frac{1}{1-\alpha} \left(1 - \sum_{j=1}^M \bar{p}_j \bar{r}_j^{1-\alpha} \right), & \text{for } \alpha \neq 1, \alpha \geq 0, \\ -\sum_{j=1}^M \bar{p}_j \ln \bar{r}_j, & \text{for } \alpha = 1. \end{cases} \quad (3.12)$$

As before, a rank-dependent reference-invariant GE index is derived from (3.12) using the socioeconomic weights $p_j^{(\nu)} = w_\nu(R_j)p_j$ instead of p_j . Using the previous notation, the rank-dependent reference-invariant GE index is expressed for all $\nu \geq 1$ and $\alpha \geq 1, \alpha \geq 0$, as

$$GE_\alpha^{(\nu)} = \frac{1}{1-\alpha} \left\{ 1 - \left[\frac{f_\alpha^{-1}[S(\nu, \alpha)]}{S(\nu, 0)} \right]^{1-\alpha} \right\}. \quad (3.13)$$

When $\alpha = 1$, $GE_1^{(\nu)} = RI_1^{(\nu)}$. For $\alpha \geq 1$, the rank-dependent GE index is obtained from the (standardized) rank-dependent RI as follows, similarly to (2.4):

$$GE_\alpha^{(\nu)} = \frac{1}{1-\alpha} \left\{ 1 - \left[1 - A_\alpha^{(\nu)} \right]^{1-\alpha} \right\}.$$

As noted earlier, an important result from Talih (2013b) is that, for $\nu \geq 1$ and $\alpha \geq 1$, $RI_\alpha^{(\nu)} \leq GE_\alpha^{(\nu)}$. The inequality is reversed for $0 < \alpha < 1$. Thus, the rank-dependent RI is more conservative and, therefore, more robust to changes in the distribution of either SES or health burden than its GE-based counterpart for $\alpha > 1$.

4. Empirical findings

4.1. Simulation studies

I compare the rank-dependent RI (3.5) with the Makdissi-Yazbeck concentration index (3.10) and the rank-dependent reference-invariant GE index (3.13) for hypothetical populations studied by Keppel *et al.* (2005); see Table 1.

In Figure 2, the rank-dependent reference-invariant GE index (top row) is standardized as in (2.2), so that it takes values in $[0,1]$. In addition, because the Makdissi-Yazbeck index in (3.10) may be negative, only its absolute value is plotted (bottom row). For $\nu = 1$, the rank-dependent RI and the Makdissi-Yazbeck index are equal; therefore only $\nu = 2$ and $\nu = 3$ are shown. By construction, the rank-dependent RI and rank-dependent reference-invariant GE index are equal 0 for $\alpha = 0$; the Makdissi-Yazbeck index is not. Conversely, the latter may be zero for positive values of α , whereas the RI and GE index remain strictly positive unless $r_j \equiv 1$ for all j . Shown in the bottom row of Figure 2 with $\alpha = 0$, the class $C(\nu, 0)$ is the Wagstaff class $C(\nu)$ of extended concentration indices and $C(2, 0)$ is the classical health concentration index C ; see section 3.1.

The relative ranking of the three hypothetical populations in Table 1 changes when the parameters of either of the three indices displayed in Figure 2 are modified. For example, for the Wagstaff class $C(\nu, 0)$, setting $\nu = 2$ yields the ranking $2 < 1 < 3$ of the three populations from lowest to highest inequality, whereas $\nu = 3$ results in the ranking $1 < 2 < 3$. (Setting $\nu = 4$, not shown, results in the ranking $1 < 3 < 2$.) The Makdissi-Yazbeck index $C(\nu, \alpha)$ further suffers from lack of smoothness as the pure health inequality aversion parameter α increases, with inequality in some populations assessed to be zero even for larger values of α . This results in yet further permutations of the relative ranking of the three hypothetical populations considered. In contrast, both the rank-dependent RI and rank-dependent reference-invariant GE index remain smooth functions of the parameter α . Even though the relative rankings resulting from the use of either of those two classes of indices are usually in agreement, the rank-dependent RI is more conservative than its GE-based counterpart for all values of $\alpha > 1$, as known from the inequality at the end of section 3.4. As a result, the rankings induced from those two classes of indices may differ, especially for larger values of α . In addition, the rank-dependent RI is less affected than its GE-based counterpart by changes to either the health (population 1 vs. population 2) or income (population 1 vs. population 3) distributions.

4.2. NHANES case study

During the past 20 years, there was an increase in obesity in the U.S. Although rates have leveled off in recent years, they remain at historically high levels. Between 1988–1994 and 2009–2010, the obesity rate increased 69% among children and adolescents aged 2–19 years, from 10.0% to 16.9% (Ogden *et al.*, 2012).

Low income children and adolescents are more likely to be obese than their higher income counterparts (Ogden *et al.*, 2010). In 2009–2010, those with family incomes at or above 500% of the poverty threshold had the lowest obesity rate, 11.5% (Table 2). Rates that differed significantly from the lowest rate at the 0.05 level of significance for children and

adolescents with lower family incomes were: 21.6% for those under the poverty threshold, nearly twice the lowest rate; 17.4% for those with family incomes at 100–199% of the poverty threshold, about one and a half times the lowest rate; and 15.7% for those with family incomes at 200–399% of the poverty threshold, almost one and a half times the lowest rate.

The rank variables R_j are computed according to (3.1) and shown in Table 2. Figure 3 displays the estimated rank-dependent RI together with its bootstrapped 95% confidence interval (using $B = 1000$ bootstrap samples) for the prevalence of obesity among children and adolescents by family income, for NHANES 2009–2010 and the combined cycles 2001–2004 and 2005–2008. For illustration, values of the socioeconomic health inequality parameter shown in Figure 3 are $\nu = 1$ (rank-neutral group weights; top panel) and $\nu = 3$ (weights favorable to those with low family income; bottom panel). Values of the pure health inequality aversion parameter shown are $\alpha = 0.5, 1, 2, 4,$ and 8 . With $\nu = 3$, a slight increase in the rank-dependent RI over time is observed, irrespective of α . However, the relative ranking of the three survey periods changes with ν , as observed in the simulation studies of section 4.1 as well as in Figure 3 for $\nu = 1$. Furthermore, for all combinations of ν and α shown, none of the observed differences in the rank-dependent RI between survey periods are statistically significant at the 0.05 level of significance.

Notes—Obesity for children and adolescents is defined as body mass index (BMI) at or above the sex- and age-specific 95th percentile from the 2000 CDC Growth Charts for the U.S. (Troiano and Flegal, 1998; Kuczmarski *et al.*, 2002). HP2020 objective NWS-10.4 tracks the proportion of children and adolescents aged 2–19 years who are considered obese. Data for NWS-10.4 are from the National Health and Nutrition Examination Survey (NHANES), CDC, NCHS. Preferably four years of data, e.g., 2009–2012, are pooled (Johnson *et al.*, 2013), however, at the time of writing this paper, only the two-year data for 2009–2010 were available for analysis. The derivation of a Taylor linearization approximation of the standard error of the rank-dependent RI for the various combinations of the parameters ν and α is presented in Appendix B.1; those standard errors are used in significance testing for the differences in the rank-dependent RI between NHANES 2001–2004, 2005–2008, and 2009–2010. The approximate 95% confidence intervals shown in Figure 3 are based on the rescaled bootstrap, which allows the examination of the sampling distribution of quantities such as the rank-dependent RI in complex survey data without relying on normality or other distributional assumptions (Talih, 2013b; Cheng, Han and Gansky, 2008; Rao, Wu and Yue, 1992; Rao and Wu, 1988).

4.3. SEER case study

Even though incidence and death rates have declined in recent years for all cancers, cancer remains a leading cause of death in the U.S., second only to heart disease. The cancer objectives for HP2020 underscore the importance of: promoting evidence-based screening for cervical, colorectal, and breast cancer in accordance with U.S. Preventive Services Task Force recommendations; and monitoring the incidence of invasive (cervical and colorectal) cancer and late-stage breast cancer, which are intermediate markers of cancer screening success (DHHS, 2014).

For this case study, I examine a subset of the data used to monitor HP2020 objective C-10, to reduce invasive uterine cervical cancer. Incidence and treatment of cervical cancer show disparities by race and ethnicity, SES, and health care access (Saraiya *et al.*, 2013; Akers, Newman and Smith, 2007). The data in Table 3 are from the Surveillance, Epidemiology, and End Results (SEER) Program's 18 Regs Research Data, NIH, NCI (Surveillance Research Program, 2013; SEER Program, 2013; Young, Jr *et al.*, 2001), and may not be nationally representative for the U.S.; see notes, below. Nonetheless, for the cases included in Table 3, counties where the proportion of persons below the poverty threshold was lowest (0.00–8.91%) had the lowest incidence of invasive uterine cervical cancer, 6.2 cases per 100,000 population (age adjusted). Rates that differed significantly from the lowest rate at the 0.05 level of significance for counties with lower area-level SES were: 8.7 per 100,000 for counties with the highest proportion (18.87–56.92%) of persons below the poverty threshold, nearly one and a half times the lowest rate; 8.0 per 100,000 for counties with the second highest proportion (14.53–18.86%) of persons below the poverty threshold, nearly one and a half times the lowest rate; and 7.4 per 100,000 for counties with the third highest proportion (11.61–14.52%) of persons below the poverty threshold, 19% higher than the lowest rate.

The rank variables R_j are computed according to (3.1) and shown in Table 3. Figure 4 displays the estimated rank-dependent RI and the boxplot for its bootstrapped sampling distribution (using $B = 1000$ bootstrap samples) under the null hypothesis of independence for the incidence of invasive uterine cervical cancer by area SES, 2006–2010, from the SEER 18 Regs Research Data. For illustration, values of the socioeconomic health inequality parameter shown in Figure 4 are $\nu = 1$ (rank-neutral group weights; top panel) and $\nu = 3$ (weights favorable to groups with low area SES; bottom panel). Values of the pure health inequality aversion parameter shown are $\alpha = 1, 2, \text{ and } 4$. For all combinations of ν and α shown, the observed rank-dependent RI differs significantly from its expected value under the null hypothesis, indicating that the latter can be rejected. However, for all combinations of ν and α shown in Figure 4, none of the changes in the index over time are statistically significant at the 0.05 level of significance.

Notes—U.S. cancer registries do not track individual or family income; therefore, area-level socioeconomic characteristics, linking cancer cases to U.S. counties, are used to get a proxy for individual-level SES (Yin *et al.*, 2010; Harper *et al.*, 2008). In addition to using data from the SEER Program, HP2020 objective C-10 also uses data collected through the National Program of Cancer Registries (NPCR), CDC, NCCDPHP. However, NPCR data are not as readily linked to county-level attributes as the SEER research data are; the latter are processed via online queries submitted securely using the SEER*Stat software (Surveillance Research Program, 2013). Further, because cases are linked to counties from the year 2000 U.S. Census, the analysis does not take into account changes in county boundaries and/or composition over time. For these reasons, data and results presented here may not be nationally representative for the U.S.; they are intended for illustration purposes only. The derivation of a Taylor linearization approximation of the standard error of the rank-dependent RI for the various combinations of the parameters ν and α is presented in Appendix B.2; those standard errors are used in significance testing for the differences in the

rank-dependent RI over time. Because of the assumption of a Poisson distribution for crude rates, random draws are readily generated under the null hypothesis of independence. The resulting bootstrapped null distribution for the rank-dependent RI for each year and combination of the parameters ν and α is summarized using a boxplot in Figure 4.

5. Discussion

The rank-dependent RI introduced in this paper is a two-parameter class of socioeconomic health inequality indices, $\{RI_{\alpha}^{(\nu)}: \alpha \geq 0, \nu \geq 1\}$, where $\alpha > 0$ is a constant relative-inequality aversion parameter and increasing values of the socioeconomic health inequality aversion parameter $\nu > 1$ allow groups with lower SES gradient to weigh more heavily than groups with higher SES. In relation to competing index classes such as the Makdissi-Yazbeck two-parameter extended concentration index and the rank-dependent reference-invariant GE class, the rank-dependent RI is more robust to changes in the distribution of either SES or adverse health outcomes. The proposed method is applicable to a wide range of public health measures and data, and statistical inference for the rank-dependent RI is readily implemented using standard statistical software.

The proposed methods are easily extended into a multivariate setting. As mentioned earlier in the context of the partial concentration index (Gravelle, 2003), it may be of interest to adjust for covariates when looking at disparities in health outcomes to rule out those parts of the SES disparity that might be considered ‘just’ or that, otherwise, cannot be amenable to policy. As an example, if communities in lower SES are relatively older and have higher rates of cancer for that reason, findings of socioeconomic disparities might be attenuated by age, so adjusting for age is of importance. Neighborhood-level or regional variation may be important for certain outcomes. For example, illnesses such as influenza outbreaks should adjust for region when measuring disparities if the outbreak is worse in certain areas of the country. The SEER data in section 4.3, above, are age adjusted. One could also apply the proposed methodology to adjusted rates obtained from log-linear or logistic regression models. For example, Rossen and Talib (2014) apply the (symmetrized) RI to population groups obtained from propensity score subclassification, accounting for demographic and contextual variables to examine disparities in weight among U.S. children and adolescents.

SES is a multidimensional construct that includes wealth, income, education, and occupation (Talib, 2013a; Krieger, Williams and Moss, 1997). Income and education are used in this paper as univariate SES measures only for illustration purposes. The proposed methods can also be applied to the ranking induced from any other SES measure, including composite SES measures. Nonetheless, the analyst should keep in mind that measuring occupation as an element of SES remains challenging. Historically, several approaches have been used, including the Nam-Powers occupational scale score (Boyd and Nam, 2004) which views occupation as a reflection of education, skill, income, and social status, as well as the National Opinion Research Center’s General Social Survey occupational prestige score (Nakao and Treas, 1990), which views occupation as an indicator of prestige. On the other hand, the O*NET work content model (O*NET, 2014) can provide relevant information for the measurement of socioeconomic status and whether certain occupations lead to reduced

workplace exposure, improved access to health care, or sick leave; see Baron (2012) for further discussion.

Due to its derivation from a rank-dependent social evaluation function, as well as its origins as a measure of divergence between probability distributions, the rank-dependent RI provides a unified mathematical framework for modeling and/or eliciting various societal positions with regards to public health policy. Do we favor prioritizing population groups with lower SES (increasing $\nu > 1$) because, as it may be, those groups are more likely to utilize costly public programs? For a given priority ranking on the SES groups and a desired health achievement level for the population, what are the societal costs of nonintervention? Is it realistic to expect all groups to attain the best group rate ($\alpha \rightarrow \infty$)? Those policy-related questions are beyond the scope of this paper. Rather, the aim of this paper is to provide a platform that facilitates their discussion. Of course, public programs, whether costly or not, do not only benefit those groups with lower SES; they also benefit groups with higher SES. Thus, the aforementioned societal costs of nonintervention are not limited to deciding whether or not to have programs that impact those in lower SES. Further, there are other equity arguments outside of the cost and benefits of policies that also could be used to justify such differential weighting as in (3.2) when measuring socioeconomic health disparities (Wilson, 2009; Braveman, 2006). For instance, social justice principles remain foundational in socioeconomic inequality measurement (Bommier and Stecklov, 2002; Peter, 2001). The analyst should be advised that, while a cost-benefit justification does not commit him/her to an ethical theory *a priori*, cost-benefit analyses are inherently grounded in utilitarian principles.

Even though health disparity indices are useful in that they summarize the relationship between the distributions of disease burden and population shares, they do not replace in-depth scientific investigation into the complex causal pathways underlying various health outcomes. The value of health disparity indices such as the slope index of inequality, the concentration index, or the proposed rank-dependent Rényi index, is best appreciated when comparisons between different populations as well as between different time periods are desired, because the alternative option of tracking multiple pairwise between-group comparisons over time can be prohibitive—as mentioned earlier, large indicator initiatives such as HP2020 can house over 1,200 health indicators. As such, health disparity indices remain essential for tracking the nation's progress toward the overarching goal of achieving health equity. Like the slope and concentration indices, as well as competing index classes, the rank-dependent RI introduced in this paper accounts for the socioeconomic gradient in health outcomes. However, unlike competing index classes, the rank-dependent RI seems more stable relative to shifts in the underlying distributions. It also allows the analyst to be explicit about value judgment regarding the degree of societal aversion to health inequality and the differential weighting of groups relative to their socioeconomic rank.

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APPENDIX A: DERIVATION FROM WEIGHTED LEAST SQUARES

Using the notation from section 3 for $\nu > 1$, the weighted least-squares regression of the power-transformed outcomes $f_\alpha(\bar{y}_{.j})$ onto the standardized socioeconomic rankings $w_{\nu}(\bar{R}_j)$, with weights p_j , has slope and intercept parameters, respectively:

$$b(\nu, \alpha) = \frac{S^*(\nu, \alpha) - S^*(1, \alpha)}{\frac{W_2(\nu)}{W_1(\nu)^2} - 1} \quad \text{and} \quad a(\nu, \alpha) = S^*(1, \alpha) - b(\nu, \alpha),$$

where $S^*(\nu, \alpha)$ is the (weighted) product moment between the $f_\alpha(\bar{y}_{.j})$ and $w_{\nu}(\bar{R}_j)$, $S^*(1, \alpha)$ is the (weighted) mean of the $f_\alpha(\bar{y}_{.j})$, and the term $\frac{W_2(\nu)}{W_1(\nu)^2} - 1$ is the (weighted) variance of the $w_{\nu}(\bar{R}_j)$. (The weighted mean of the latter is 1.) From (3.5), it follows that

$$RI_\alpha^{(\nu)} = -\ln \left\{ \frac{f_\alpha^{-1} \left[a(\nu, \alpha) + \frac{W_2(\nu)}{W_1(\nu)^2} b(\nu, \alpha) \right]}{a(\nu, 0) + \frac{W_2(\nu)}{W_1(\nu)^2} b(\nu, 0)} \right\}.$$

The quantities $b(\nu, 0)$ and $b(2, 0)$ are akin to the extended and classical slope indices of inequality, respectively (Wagstaff, 2002; Wagstaff, Paci and van Doorslaer, 1991).

Remark

An even more “convenient regression” results in the direct interpretation of the $S^*(\nu, \alpha)$ as the slope of the line for the regression of the following linear transform onto the $w_{\nu}(\bar{R}_j)$:

$$\left(\frac{W_2(\nu)}{W_1(\nu)^2} - 1 \right) f_\alpha(\bar{y}_{.j}) + S^*(1, \alpha) w_{\nu}(\bar{R}_j).$$

APPENDIX B: SAMPLING VARIABILITY

B.1. NHANES data

Total statistics are defined as follows for any scalar a (Talih, 2013b):

$$U_{aj} = \sum_{s=1}^S \sum_{c=1}^{C_s} \sum_{i=1}^{I_{cs}} \delta_{icsj} \omega_{ics} y_{ics}^a \quad (\text{B.1})$$

$$U_a = \sum_{j=1}^M U_{aj}. \quad (\text{B.2})$$

In (B.1) and (B.2), S is the number of strata; C_s is the number of PSU's in stratum s ; l_{cs} is the number of sample observations in the PSU-stratum pair (c, s) ; w_{ics} is the sampling weight for observation i in the PSU-stratum pair (c, s) ; y_{ics} is the indicator of the adverse health outcome for observation i in the PSU-stratum pair (c, s) ; $\delta_{icsj} = 1$ when observation i [in PSU-stratum pair (c, s)] belongs to group j and $\delta_{icsj} = 0$ otherwise; and j ranges from 1 to M , where M is the number of groups in the population. Using the above notation, we have $n_j/n = U_{0j}/U_0$. and $\bar{y}_{\cdot j} = U_{1j}/U_{0j}$. Further, define

$$V_{0j} = \frac{U_{0j}}{2} + \sum_{\ell=j+1}^M U_{0\ell}.$$

Then $(1 - R_j) = V_{0j}/U_0$. Using these total statistics, the rank-dependent RI in (3.9) is re-expressed. For $\alpha > 1$,

$$RI_{\alpha}^{(\nu)} = \ln \underbrace{\left[\sum_{j=1}^M U_{1j} V_{0j}^{\nu-1} \right]}_{(I)} - \frac{1}{1-\alpha} \ln \underbrace{\left[\sum_{j=1}^M U_{0j} (U_{1j}/U_{0j})^{1-\alpha} V_{0j}^{\nu-1} \right]}_{(II)} + \frac{\alpha}{1-\alpha} \ln \underbrace{\left[\sum_{j=1}^M U_{0j} V_{0j}^{\nu-1} \right]}_{(III)}.$$

For $\alpha = 1$,

$$RI_1^{(\nu)} = \ln \left[\sum_{j=1}^M U_{1j} V_{0j}^{\nu-1} \right] - \underbrace{\frac{\sum_{j=1}^M U_{0j} \ln(U_{1j}/U_{0j}) V_{0j}^{\nu-1}}{\sum_{j=1}^M U_{0j} V_{0j}^{\nu-1}}}_{(IV)} - \ln \left[\sum_{j=1}^M U_{0j} V_{0j}^{\nu-1} \right].$$

Introduce an artificial variable σ_{icsk} that represents the variance contribution from each sample observation. The σ_{icsk} are obtained by taking the dot product of the vector of partial derivatives of the rank-dependent RI with the vector of summands in the total statistics U_{0k} and U_{1k} :

$$\sigma_{icsk} = \delta_{icsk} w_{ics} \left\{ \frac{\partial RI_{\alpha}^{(\nu)}}{\partial U_{0k}} + y_{ics} \frac{\partial RI_{\alpha}^{(\nu)}}{\partial U_{1k}} \right\}. \quad (B.3)$$

An estimate of the sample variance of $RI_{\alpha}^{(\nu)}$ is given by the sampling variance of the total statistic $\sum_{k=1}^M \sum_{i=1}^{l_{cs}} \sigma_{icsk}$. The latter is available using design-based estimation of variances of totals ('svytotal') in the R package 'survey' (Lumley, 2004, 2011; R Development Core Team, 2011).

Expressions for partial derivatives with respect to U_{0k} and U_{1k}

$$\frac{\partial V_{0j}}{\partial U_{0k}} = \begin{cases} 0 & \text{if } j > k, \\ 1/2 & \text{if } j = k, \\ 1 & \text{if } j < k, \end{cases} \quad \text{and} \quad \frac{\partial V_{0j}}{\partial U_{1k}} = 0.$$

$$\begin{aligned} \frac{\partial}{\partial U_{0k}} \text{(I)} &= \frac{\frac{\nu-1}{2} U_{1k} V_{0k}^{\nu-2} + (\nu-1) \sum_{j=0}^{k-1} U_{1j} V_{0j}^{\nu-2}}{\sum_{j=1}^M U_{1j} V_{0j}^{\nu-1}}, \\ \frac{\partial}{\partial U_{1k}} \text{(I)} &= \frac{V_{0k}^{\nu-1}}{\sum_{j=1}^M U_{1j} V_{0j}^{\nu-1}}; \\ \frac{\partial}{\partial U_{0k}} \text{(II)} &= \frac{\alpha (U_{1k}/U_{0k})^{1-\alpha} V_{0k}^{\nu-1} + \frac{\nu-1}{2} U_{0k} (U_{1k}/U_{0k})^{1-\alpha} V_{0k}^{\nu-2} + (\nu-1) \sum_{j=0}^{k-1} U_{0j} (U_{1j}/U_{0j})^{1-\alpha} V_{0j}^{\nu-2}}{\sum_{j=1}^M U_{0j} (U_{1j}/U_{0j})^{1-\alpha} V_{0j}^{\nu-1}}, \\ \frac{\partial}{\partial U_{1k}} \text{(II)} &= \frac{(1-\alpha) (U_{1k}/U_{0k})^{-\alpha} V_{0k}^{\nu-1}}{\sum_{j=1}^M U_{0j} (U_{1j}/U_{0j})^{1-\alpha} V_{0j}^{\nu-1}}; \\ \frac{\partial}{\partial U_{0k}} \text{(III)} &= \frac{V_{0k}^{\nu-1} + \frac{\nu-1}{2} U_{0k} V_{0k}^{\nu-2} + (\nu-1) \sum_{j=0}^{k-1} U_{0j} V_{0j}^{\nu-2}}{\sum_{j=1}^M U_{0j} V_{0j}^{\nu-1}}, \\ \frac{\partial}{\partial U_{1k}} \text{(III)} &= 0; \\ \frac{\partial}{\partial U_{0k}} \text{(IV)} &= \frac{[\ln(U_{1k}/U_{0k}) - 1] V_{0k}^{\nu-1} + \frac{\nu-1}{2} U_{0k} \ln(U_{1k}/U_{0k}) V_{0k}^{\nu-2} + (\nu-1) \sum_{j=0}^{k-1} U_{0j} \ln(U_{1j}/U_{0j}) V_{0j}^{\nu-2}}{\sum_{j=1}^M U_{0j} V_{0j}^{\nu-1}} \\ &\quad - \frac{[V_{0k}^{\nu-1} + \frac{\nu-1}{2} U_{0k} V_{0k}^{\nu-2} + (\nu-1) \sum_{j=0}^{k-1} U_{0j} V_{0j}^{\nu-2}] \left[\sum_{j=1}^M U_{0j} \ln(U_{1j}/U_{0j}) V_{0j}^{\nu-1} \right]}{\left(\sum_{j=1}^M U_{0j} V_{0j}^{\nu-1} \right)^2}, \\ \frac{\partial}{\partial U_{1k}} \text{(IV)} &= \frac{(U_{1k}/U_{0k})^{-1} V_{0k}^{\nu-1}}{\sum_{j=1}^M U_{0j} V_{0j}^{\nu-1}}. \end{aligned}$$

Notes

NHANES has a stratified multistage probability sampling design structure (Johnson *et al.*, 2013). While the sample weights provided in the NHANES public-use data files reflect the unequal probabilities of selection, they also reflect non-response adjustments and adjustments to independent population controls. Therefore, strictly speaking, they are not the true sampling weights w_{ics} in (B.1).

B.2. SEER data

Following SEER*Stat (Surveillance Research Program, 2013), crude rates are assumed to be distributed according to a Poisson distribution. In addition, age-adjusted rates are adjusted using the year 2000 U.S. standard population, with known age-adjustment weights ω_k and sizes n_{kj} . Thus, sample means and variances for the age-adjusted rates are as follows:

$$\hat{\mathbf{E}}[\bar{y}_{.j}] = \sum_{k=1}^K \omega_k \bar{u}_{.kj} \quad \text{and} \quad \widehat{\mathbf{Var}}[\bar{y}_{.j}] = \sum_{k=1}^K \omega_k^2 \bar{u}_{.kj} / n_{kj},$$

where the $\bar{u}_{.kj}$ are the underlying crude rates for age group k. Using the expression in (3.9), we have

$$\frac{\partial \text{RI}_\alpha^{(\nu)}}{\partial \bar{y}_{.j}} = \bar{w}_\nu(R_j) p_j \left[\frac{1}{H^*(\nu, 0)} - \frac{1}{\bar{y}_{.j}^\alpha H^*(\nu, \alpha)^{1-\alpha}} \right].$$

The Taylor series linearization approximation to the variance of the rank-dependent RI yields

$$\widehat{\text{Var}}[\text{RI}_\alpha^{(\nu)}] = \sum_{j=1}^M \left[\frac{\partial \text{RI}_\alpha^{(\nu)}}{\partial \bar{y}_{.j}} \right]^2 \widehat{\text{Var}}[\bar{y}_{.j}].$$

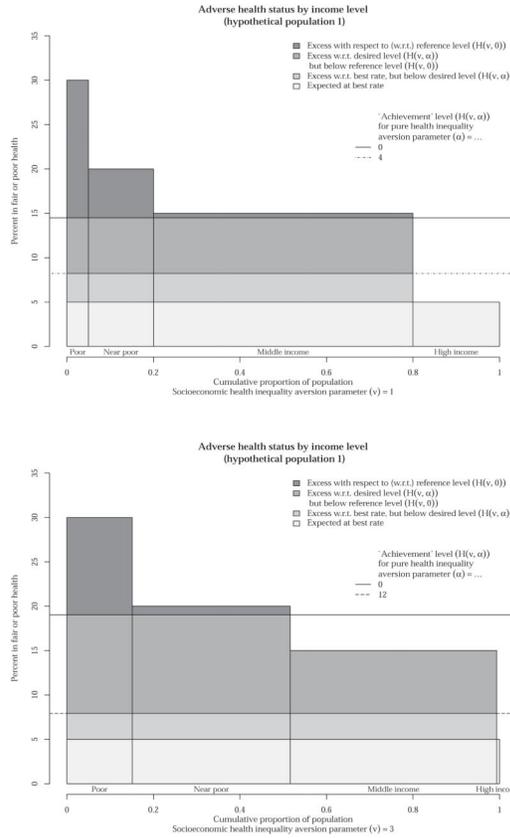


Fig. 1. Achievement versus capacity to achieve: Illustration using data for hypothetical population 1 in Table 1, with socioeconomic health inequality parameter $\nu = 1$ (rank-neutral group weights; top panel) and $\nu = 3$ (weights favorable to groups with low income level; bottom panel). The reference ‘achievement’ level $H(\nu, 0)$ (solid lines), i.e., the income-weighted population proportion in fair or poor health, is higher for larger ν , resulting in a larger gap relative to the best rate. A larger ν results in a larger α —i.e., a higher ‘cost of nonintervention’—for about the same achievement level $H(\nu, \alpha) \approx 8\%$ (dashed lines).

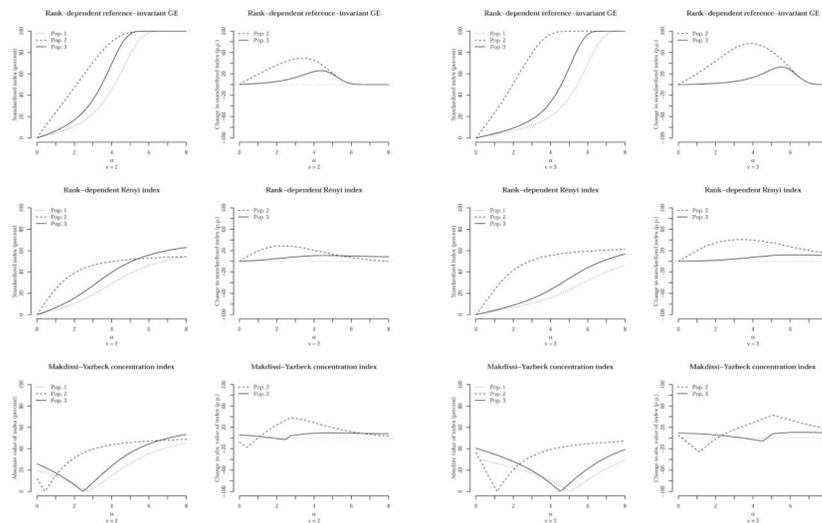


Fig. 2. Comparison of the rank-dependent Rényi index (3.5 and middle row) with the Makdissi-Yazbek concentration index (3.10 and bottom row) and the rank-dependent reference-invariant GE index (3.13 and top row) for hypothetical populations in Table 1. For $\nu = 1$, the rank-dependent Rényi index and the Makdissi-Yazbek concentration index are equal; therefore only $\nu = 2$ (two left columns) and $\nu = 3$ (two right columns) are shown here. By construction, the rank-dependent Rényi index and rank-dependent reference-invariant GE index are equal 0 for $\alpha = 0$; the Makdissi-Yazbek index is not. The class $C(\nu, 0)$ is the Wagstaff class of extended concentration indices. For $\nu = 2$, the index $C(2, 0)$ is the “classical” health concentration index.

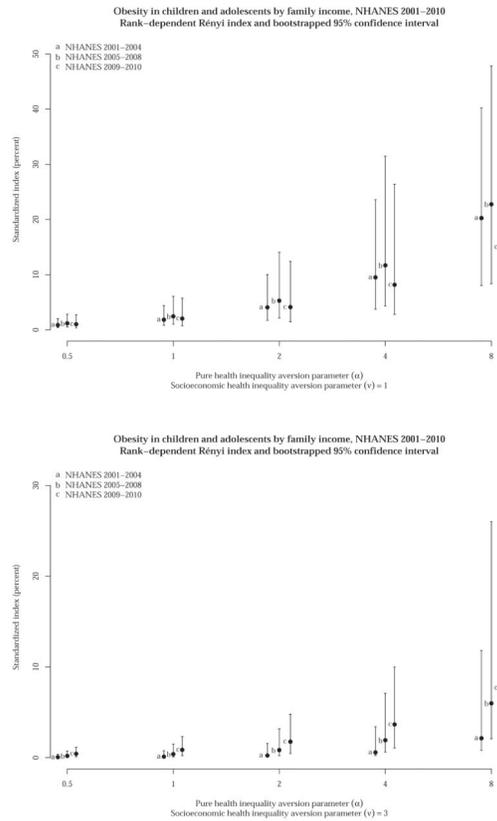


Fig. 3. Rank-dependent RI and its bootstrapped 95% confidence intervals ($B = 1000$) for the prevalence of obesity among children and adolescents aged 2–19 years by family income, from NHANES 2001–2010 (data in Table 2). For illustration, values of the socioeconomic health inequality parameter shown are $\nu = 1$ (rank-neutral group weights; top panel) and $\nu = 3$ (weights favorable to those with low family income; bottom panel). Values of the pure health inequality aversion parameter shown along the x-axis are $\alpha = 0.5, 1, 2, 4,$ and 8 . For all combinations of ν and α shown, observed differences between the three survey periods in the rank-dependent RI are not statistically significant.

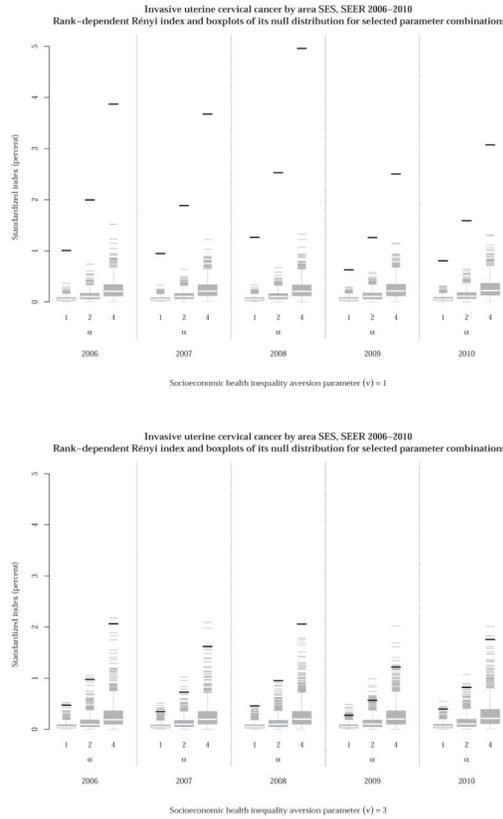


Fig. 4. Rank-dependent RI and boxplots of its bootstrapped sampling distribution ($B = 1000$) under the null hypothesis of independence for the incidence of invasive uterine cervical cancer by area SES, 2006–2010, from the SEER Program’s 18 Regs Research Data (data in Table 3). For illustration, values of the socioeconomic health inequality parameter shown are $\nu = 1$ (rank-neutral group weights; top panel) and $\nu = 3$ (weights favorable to groups with low area SES; bottom panel). Values of the pure health inequality aversion parameter shown are $\alpha = 1, 2, \text{ and } 4$. For all combinations of ν and α shown, the observed rank-dependent RI differs significantly from its expected value under the null hypothesis, indicating that the latter can be rejected. However, changes in the index over time are not statistically significant.

Table 1

Percentages in fair or poor health by income level for three hypothetical populations^a.

| Group (<i>j</i>) | Poor | Near poor | Middle income | High income |
|--|-------------|------------------|----------------------|--------------------|
| <i>Population 1</i> | | | | |
| Proportion of population (n_j/n) | 0.05 | 0.15 | 0.60 | 0.20 |
| Percent in fair or poor health (\bar{y}_j) | 30% | 20% | 15% | 5% |
| <i>Population 2</i> | | | | |
| Proportion of population (n_j/n) | 0.05 | 0.15 | 0.60 | 0.20 |
| Percent in fair or poor health (\bar{y}_j) | 30% | 20% | 5% | 15% |
| <i>Population 3</i> | | | | |
| Proportion of population (n_j/n) | 0.20 | 0.20 | 0.40 | 0.20 |
| Percent in fair or poor health (\bar{y}_j) | 30% | 20% | 15% | 5% |

^aIncome level is expressed as a percent of the poverty threshold. Here: poor = below 100%, near poor = 100–199%, middle income = 200–399%, and high income = at or above 400% of the poverty threshold.

Table 2
Prevalence of obesity in children and adolescents (aged 2–19 years) by family income, 2001–2010^{a,b}

| Income category (j) ^c (Family income expressed as percent of poverty threshold) | 1 (<100%) | 2 (100–199%) | 3 (200–399%) | 4 (400–499%) | 5 (>500%) |
|--|-----------|--------------|--------------|--------------|-----------|
| <i>NHANES 2001–2004</i> | | | | | |
| Prevalence (%) in group (\bar{y}_j) | 17.9 | 16.7 | 17.8 | 13.1 | 9.8 |
| Standard error (%) ^d | 1.295 | 1.249 | 1.182 | 1.691 | 1.600 |
| Population in group (p_j) ^e | 0.241 | 0.242 | 0.291 | 0.095 | 0.132 |
| Rank (R_j) ^f | 0.120 | 0.362 | 0.628 | 0.821 | 0.934 |
| <i>NHANES 2005–2008</i> | | | | | |
| Prevalence (%) in group (\bar{Y}_j) | 19.9 | 18.2 | 16.0 | 14.3 | 9.8 |
| Standard error (%) | 1.368 | 1.447 | 1.403 | 2.747 | 1.838 |
| Population in group (p_j) | 0.218 | 0.223 | 0.298 | 0.099 | 0.162 |
| Rank (R_j) | 0.109 | 0.329 | 0.590 | 0.788 | 0.919 |
| <i>NHANES 2009–2010^b</i> | | | | | |
| Prevalence (%) in group (\bar{Y}_j) | 21.6 | 17.4 | 15.7 | 14.2 | 11.5 |
| Standard error (%) | 1.306 | 1.428 | 1.437 | 2.686 | 2.591 |
| Population in group (p_j) | 0.232 | 0.235 | 0.274 | 0.088 | 0.171 |
| Rank (R_j) | 0.116 | 0.349 | 0.604 | 0.785 | 0.914 |

^a Obesity for children and adolescents aged 2–19 years is defined as body mass index (BMI) at or above the sex-and age-specific 95th percentile from the 2000 CDC Growth Charts for the U.S. (Troiano and Flegal, 1998; Kuczmarski *et al.*, 2002).

^b Data are available biennially and come from the National Health and Nutrition Examination Survey (NHANES), CDC, NCHS. Preferably four years of data are pooled for analysis when available (Johnson *et al.*, 2013), but two-year data are used as a placeholder to provide the latest data available.

^c Family income is expressed as a percent of the poverty threshold; missing values are not included in the analysis.

^d Standard error evaluated by Taylor linearization (RTI, 2012; SAS Institute, 2010).

^e Proportions are rounded for table display and may not add up to exactly 1.000; unrounded values are used in all calculations.

^f Rank of group in cumulative distribution of population computed according to (3.1).

Table 3

Incidence of invasive uterine cervical cancer (age adjusted, per 100,000) by area SES, SEER 2006–2010^a.

| County quintile group (j) ^b (Percentage of persons below poverty threshold in county) | 5 (18.87–56.92%) | 4 (14.53–18.86%) | 3 (11.61–14.52%) | 2 (8.92–11.60%) | 1 (0.00–8.91%) |
|--|------------------|------------------|------------------|-----------------|----------------|
| 2006 | | | | | |
| Incidence in group (\bar{Y}_j) ^c | 9.6 | 8.9 | 7.5 | 7.5 | 6.4 |
| Standard error ^d | 0.484 | 0.285 | 0.339 | 0.314 | 0.226 |
| Population in group (p_j) ^e | 0.104 | 0.267 | 0.159 | 0.179 | 0.292 |
| Rank (R_j) ^f | 0.052 | 0.237 | 0.450 | 0.619 | 0.854 |
| 2007 | | | | | |
| Incidence in group (\bar{Y}_j) | 9.0 | 9.0 | 8.1 | 6.9 | 6.5 |
| Standard error | 0.468 | 0.285 | 0.353 | 0.298 | 0.227 |
| Population in group (p_j) | 0.104 | 0.265 | 0.160 | 0.179 | 0.292 |
| Rank (R_j) | 0.052 | 0.237 | 0.449 | 0.618 | 0.854 |
| 2008 | | | | | |
| Incidence in group (\bar{Y}_j) | 9.7 | 9.0 | 8.0 | 7.3 | 6.2 |
| Standard error | 0.478 | 0.285 | 0.346 | 0.307 | 0.220 |
| Population in group (p_j) | 0.104 | 0.263 | 0.161 | 0.179 | 0.293 |
| Rank (R_j) | 0.052 | 0.236 | 0.448 | 0.618 | 0.854 |
| 2009 | | | | | |
| Incidence in group (\bar{Y}_j) | 8.5 | 8.4 | 7.0 | 7.7 | 6.4 |
| Standard error | 0.450 | 0.274 | 0.324 | 0.316 | 0.223 |
| Population in group (p_j) | 0.104 | 0.262 | 0.161 | 0.179 | 0.293 |
| Rank (R_j) | 0.052 | 0.236 | 0.447 | 0.617 | 0.853 |
| 2010 | | | | | |
| Incidence in group (\bar{Y}_j) | 8.7 | 8.0 | 7.4 | 6.4 | 6.2 |
| Standard error | 0.453 | 0.268 | 0.330 | 0.286 | 0.217 |
| Population in group (p_j) | 0.104 | 0.261 | 0.162 | 0.179 | 0.293 |
| Rank (R_j) | 0.052 | 0.235 | 0.447 | 0.617 | 0.853 |

^aData are from the SEER 18 Regs Research Data, NIH, NCI (SEER Program, 2013; Young, Jr *et al.*, 2001), and are age adjusted using the year 2000 U.S. standard population. Data shown here do not include the full set of registries used in HP2020 to track objective C-10; thus, data may not be nationally representative.

^bArea socioeconomic status (SES) is computed using county-level data for the 3,141 counties in the year 2000 U.S. Census. Cutpoints for each SES group, displayed in the table header row, correspond to the county quintiles when these are sorted according to the percentage of persons living below the poverty threshold in the county.

^cNew cases of invasive uterine cervical cancer (age adjusted) per 100,000 population in group.

^dStandard error evaluated by Taylor linearization (Surveillance Research Program, 2013).

^eProportions are rounded for table display and may not add up to exactly 1.000; unrounded values are used in all calculations.

^fRank of group in cumulative distribution of population computed according to (3.1).