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Statistical Analysis of Gold Assay and Other Trace-Element Data



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Report of Investigations 7495

Statistical Analysis of Gold Assay and Other Trace-Element Data

**By Richard F. Link, George S. Koch, Jr.,
and John H. Schuenemeyer**



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STATISTICAL ANALYSIS OF GOLD ASSAY AND OTHER TRACE-ELEMENT DATA

by

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and John H. Schuenemeyer³

ABSTRACT

The principal procedures for the statistical analysis of gold assay and other trace-element data depend on applying either the normal frequency distribution or the lognormal frequency distribution. The consequences of applying both methods are examined through reference to example data from United States and South African gold mines. It is concluded that for estimation of average grades, except for certain data with a very high coefficient of variation (ratio of standard deviation to mean), use of the lognormal distribution is ordinarily undesirable because bias may be introduced and little efficiency can be gained. However, if biased average grades can be tolerated, as in the examination of data from surveys in exploration geochemistry, use of the lognormal distribution may be desirable. Various statistical analyses are detailed, and the tables presented to implement the analyses are more complete than those previously published.

INTRODUCTION

This report compares and contrasts two statistical methods for extracting information from a set of numbers with the characteristic that compared to the average value, most of the numbers are small, but a few are very large. Examples of such data are the incomes of individuals, magnitudes of earthquakes, gold assay values from a mine, types of floods in a river, metal analyses from a survey of exploration geochemistry, and heights of buildings in New York City. In every one of these and many other examples, most of the values are small relative to the average, but a few are very large. The large values represent the millionaires, the catastrophic earthquakes, the bonanza pockets of gold, the disastrous floods, the geochemical anomalies, and the skyscrapers.

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For the summary and analysis of such data, statisticians make use of several different procedures, the most common of which are those based on normal theory (2-3, 14) and lognormal theory (1). Although lognormal theory is fundamentally more complicated than the normal statistical theory explained in elementary statistical textbooks, great gains in interpretation can be made by applying lognormal theory to suitable data, but risks also arise.

The principal purpose of this report is to compare and contrast the application of the normal and lognormal distributions to data of interest in the mineral industries, as exemplified by gold data, and to discuss the problems that arise. Familiarity with elementary applications of normal theory is assumed. However, the explanation of lognormal theory starts from elementary principles. Some problems of lognormal distribution are discussed in nonmathematical terms, and the mathematical functions required for interpreting lognormally distributed data are tabulated in more detail than in previously published tables. Furthermore, some pitfalls in applying the lognormal distribution to gold assay data are pointed out through the analysis of several example sets of gold assay data. Other purposes are to discuss transformations in general, to explain the use of logarithmic probability paper, and to present a Fortran computer program for implementing some of the tedious computations.

While this report presents the results of part of the Bureau of Mines program to investigate methods for sampling gold deposits, lognormal methods are also relevant to certain other metal assay data and other data found in both theoretical and applied geology and other disciplines. Any data representing chemical elements present in rocks in amounts of less than a few percent may be susceptible to lognormal interpretation; perhaps the outstanding data are those from exploration geochemistry.

Previous work is summarized and appraised throughout this report. In contrast to the work of some earlier investigators, the principal points stressed in this paper are that the use of lognormal theory may be dangerous and lead to wrong conclusions, particularly in mine evaluation, unless the application and interpretation are expertly done. Lognormal theory is a sharp-edged, dangerous tool. In estimating ore grades where bias cannot be tolerated, lognormal techniques should be applied with caution. However, where bias is less important, as in comparing data from exploration geochemical surveys, the possible increase in efficiency using lognormal distribution may outweigh the risk of bias.

Frequency Distributions

In order to explain and illustrate the kind of data treated in this report, this subsection presents data for two kinds of frequency distributions. Table 1 and figure 1 show the frequency distribution and histogram for 224 phosphate assays from the Phosphoria Formation near Fort Hall, Idaho. This frequency distribution closely corresponds to the normal frequency distribution of theoretical statistics with values symmetrically disposed around a central value. Table 2 and figure 2 show the frequency distribution and histogram for 900 gold assays from the Homestake mine, Lead, S. Dak. Similarly, table 3 and figure 3 show the frequency distribution and histogram for 1,536 gold assays from the City Deep mine, Republic of South Africa. The frequency distributions for this gold data closely correspond to the lognormal frequency distribution with many low values and a few high values. The normal and lognormal distributions are only two of many distributions that have been studied by statisticians, but they are part of the small number of distributions commonly applied to real data.

TABLE 1. - Frequency distribution of 224 phosphate assays from Phosphoria Formation near Fort Hall, Idaho

Assay interval, percent P_2O_5	Interval midpoint, w	Frequency, f	Relative frequency, percent	Cumulative frequency	Relative cumulative frequency, percent
14-16	15	1	0.45	1	0.45
16-18	17	1	.45	2	.90
18-20	19	8	3.57	10	4.47
20-22	21	21	9.37	31	13.84
22-24	23	44	19.64	75	33.48
24-26	25	54	24.12	129	57.60
26-28	27	56	25.00	185	82.60
28-30	29	30	13.39	215	95.99
30-32	31	7	3.12	222	99.11
32-34	33	2	.89	224	100.00

TABLE 2. - Frequency distribution of 900 gold assays from 1-foot-long intervals of EX diamond-drill core from the Homestake mine, Lead, S. Dak.

Assay interval, ppm	Frequency	Cumulative frequency	Relative cumulative frequency, percent
0- 5	632	632	70.22
5- 10	91	723	80.33
10- 15	58	781	86.78
15- 20	31	812	90.22
20- 25	18	830	92.22
25- 30	17	847	94.11
30- 35	14	861	95.67
35- 40	3	864	96.00
40- 45	7	871	96.78
45- 50	3	874	97.11
50- 55	5	879	97.67
55- 60	2	881	97.89
60- 65	2	883	98.11
65- 70	6	889	98.78
70- 75	1	890	98.89
75- 80	1	891	99.00
80- 85	1	892	99.11
85- 90	0	892	99.11
90- 95	0	892	99.11
95-100	1	893	99.22
100-105	1	894	99.33
115-120	2	896	99.56
120-125	1	897	99.67
135-140	1	898	99.78
200-205	1	899	99.89
255-260	1	900	100.00

TABLE 3. - Frequency distribution of 1,536 gold assays from stopes in a part of the City Deep mine, Republic of South Africa

Assay interval, dwt/short ton	Frequency	Cumulative frequency	Relative cumulative frequency, percent
0- 5	910	910	59.24
5- 10	208	1,118	72.79
10- 15	118	1,236	80.47
15- 20	80	1,316	85.68
20- 25	54	1,370	89.19
25- 30	33	1,403	91.34
30- 35	24	1,427	92.90
35- 40	13	1,440	93.75
40- 45	14	1,454	94.66
45- 50	8	1,462	95.18
50- 55	8	1,470	95.71
55- 60	10	1,480	96.36
60- 65	4	1,484	96.62
65- 70	4	1,488	96.88
70- 75	3	1,491	97.07
75- 80	1	1,492	97.14
80- 85	1	1,493	97.20
85- 90	4	1,497	97.46
90- 95	1	1,498	97.53
95-100	7	1,505	97.99
100-105	3	1,508	98.18
105-110	2	1,510	98.31
110-115	3	1,513	98.51
120-125	2	1,515	98.63
125-130	1	1,516	98.70
130-135	5	1,521	99.03
145-150	1	1,522	99.09
150-155	1	1,523	99.16
155-160	3	1,526	99.35
180-185	1	1,527	99.42
190-195	2	1,529	99.56
205-210	2	1,531	99.68
215-220	1	1,532	99.72
245-250	1	1,533	99.81
305-310	1	1,534	99.87
420-425	1	1,535	99.93
620-625	1	1,536	100.00

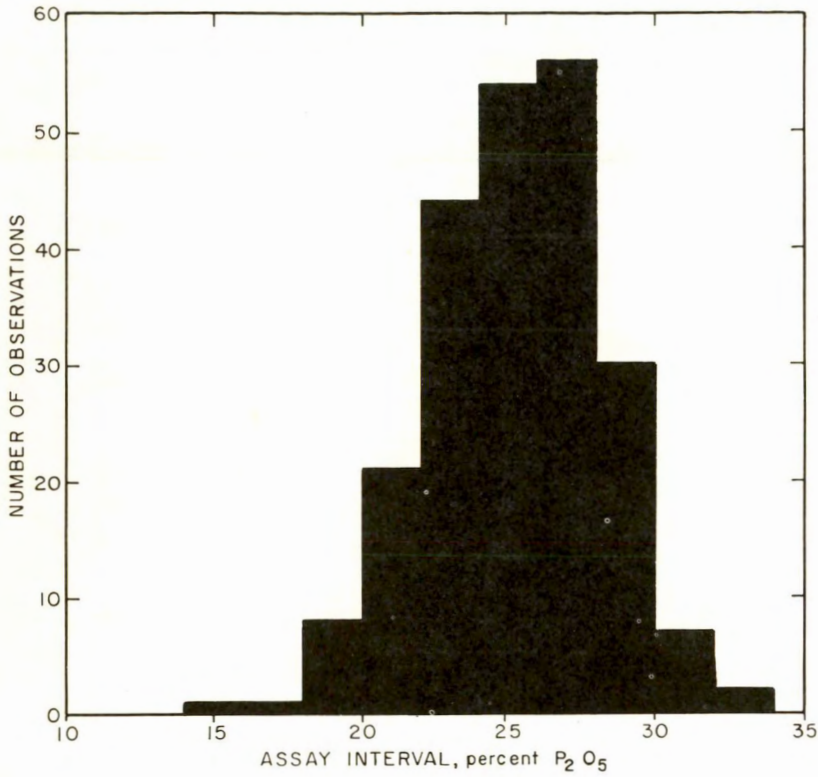


FIGURE 1. - Histogram Showing Distribution of 224 Phosphate Assays From Idaho.

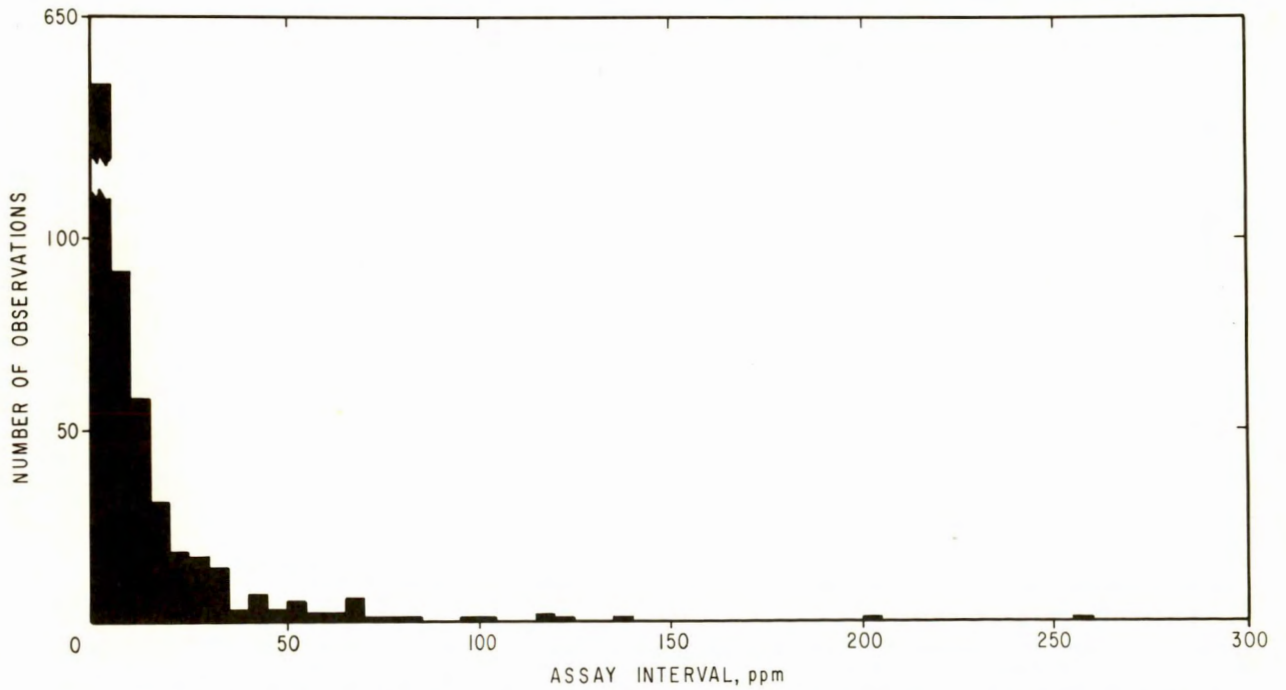


FIGURE 2. - Histogram Showing Distribution of 900 Gold Assays From the Homestake Mine, Lead, S. Dak.

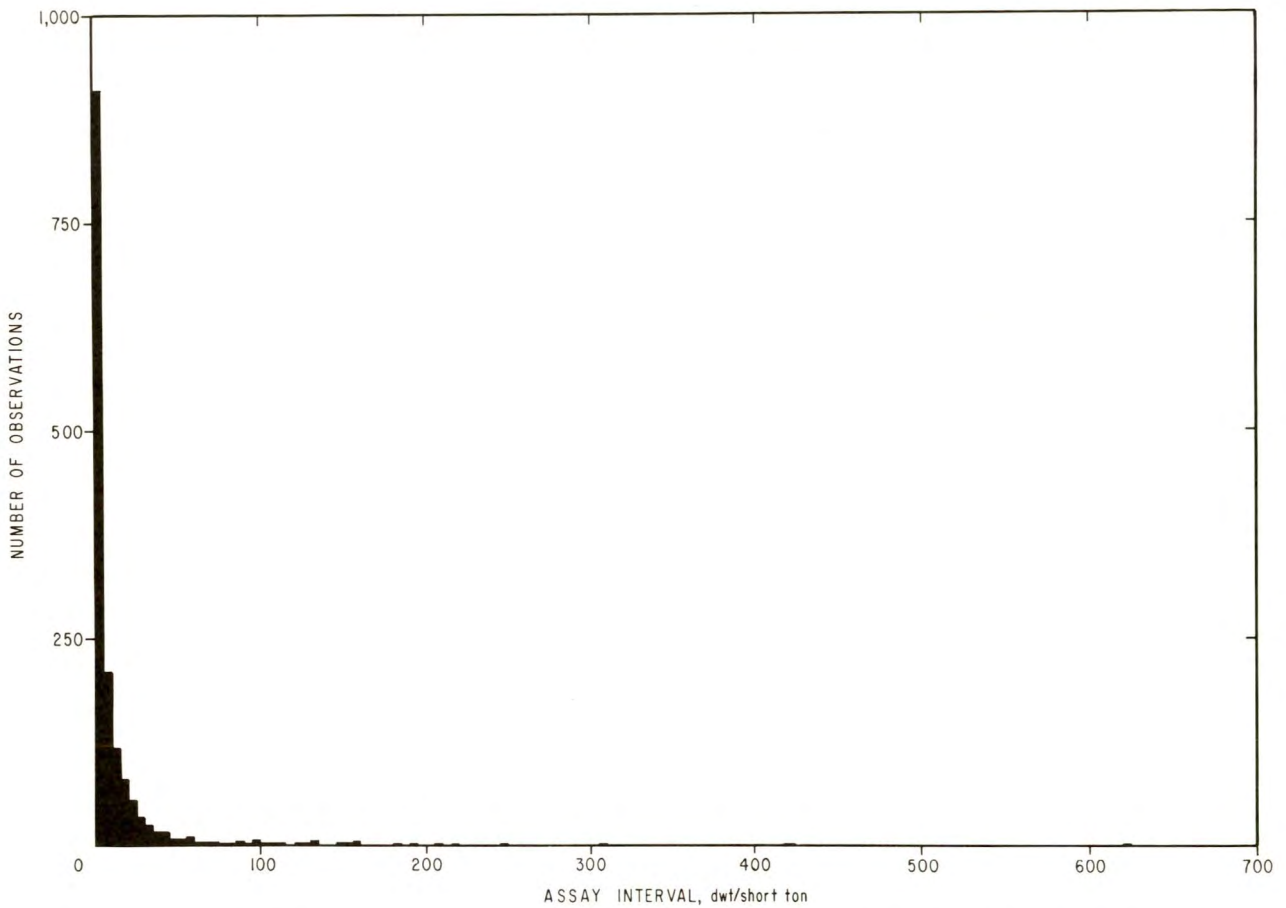


FIGURE 3. - Histogram Showing Approximately Lognormal Distribution of 1,536 dwt Gold Assay Values From the City Deep Mine, Republic of South Africa.

To illustrate that real data do not necessarily follow either one of these distributions, another set of gold assay data is introduced. Table 4 and figure 4 portray 956 gold assay values from the Getchell mine, Humboldt County, Nev. Although this distribution is skewed, it is not lognormal as it does not contain enough high observations, or it contains too many small observations. This distribution illustrates that real data may not exactly follow a theoretical distribution that is recognizable. This leads to the crucial problem of applying a theoretical distribution when it is inappropriate to do so.

TABLE 4. - Frequency distribution of 956 gold assays from the
Getchell mine, Humboldt County, Nev.

Assay interval, ounces/ton	Frequency	Cumulative frequency	Relative cumulative frequency, percent
0.00-0.02	130	130	13.60
.02- .04	97	227	23.74
.04- .06	73	300	31.38
.06- .08	74	374	39.12
.08- .10	69	443	46.34
.10- .12	51	494	51.67
.12- .14	55	549	57.43
.14- .16	49	598	62.55
.16- .18	40	638	66.74
.18- .20	45	683	71.44
.20- .22	32	715	74.79
.22- .24	29	744	77.82
.24- .26	27	771	80.65
.26- .28	23	794	83.05
.28- .30	19	813	85.04
.30- .32	17	830	86.82
.32- .34	11	841	87.97
.34- .36	14	855	89.44
.36- .38	8	863	90.27
.38- .40	14	877	91.74
.40- .42	18	895	93.62
.42- .44	10	905	94.67
.44- .46	7	912	95.40
.46- .48	8	920	96.23
.48- .50	3	923	96.55
.50- .52	5	928	97.07
.52- .54	2	930	97.28
.54- .56	1	931	97.38
.56- .58	6	937	98.01
.58- .60	1	938	98.12
.60- .62	3	941	98.43
.62- .64	0	941	98.43
.64- .66	3	944	98.74
.66- .68	3	947	99.06
.68- .70	1	948	99.16
.70- .72	0	948	99.16
.72- .74	2	950	99.37
.74- .76	0	950	99.37
.76- .78	1	951	99.48
.78- .80	0	951	99.48
.80- .82	1	952	99.58
.82- .84	0	952	99.58
.84- .86	1	953	99.69
1.08-1.10	1	954	99.79
1.22-1.24	1	955	99.90
1.30-1.32	1	956	100.00

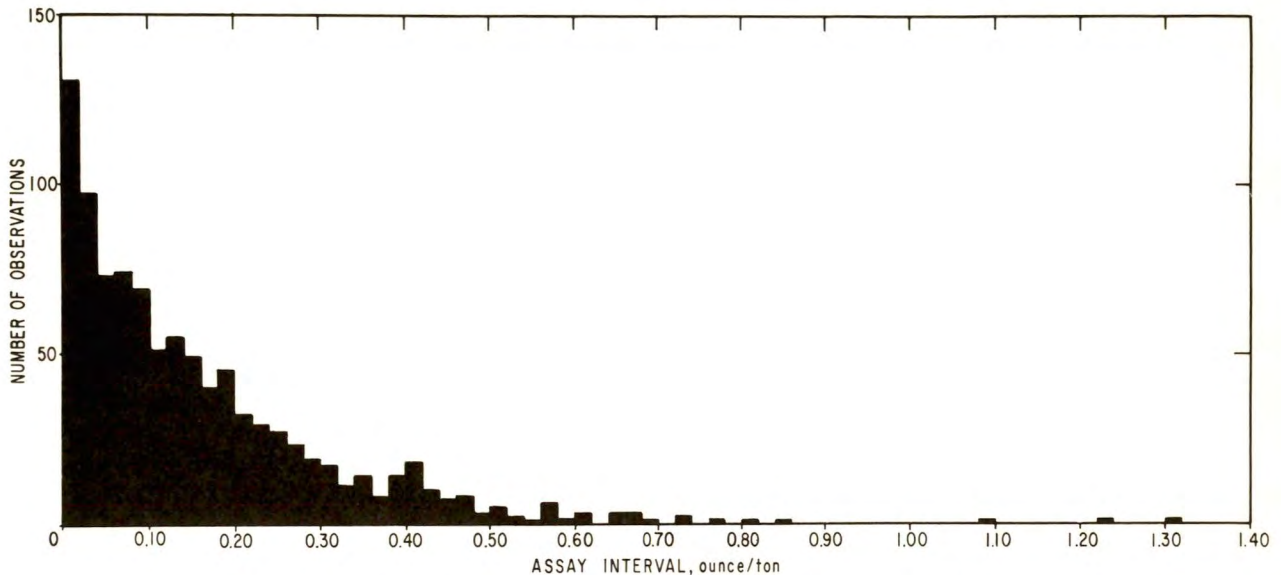


FIGURE 4. - Histogram Showing Distribution of 956 Gold Assays From the Getchell Mine, Humboldt County, Nev.

Bias and Efficiency

Background on the statistical concepts used in this investigation can be found in elementary textbooks (14), but the following explanation of the concepts of bias and efficiency is included because of their particular relevance to the paper. Desirable characteristics of a statistic are that it be unbiased and efficient, but often these characteristics are difficult to obtain together. An unbiased statistic has the property that the mean of its sampling distribution is equal to the parameter that the statistic is estimating. The value of the statistic fluctuates from sample to sample, and the sampling distribution of the statistic exhibits the values that the statistic may assume for all possible samples. Thus, if the mean value of the statistic for all possible samples is equal to the value of the parameter, the statistic is unbiased. An efficient statistic is one whose value is closer to that of the parameter as measured by the mean square error of the statistic than that of any other statistic for the same size sample. (The mean square error of a statistic is the average squared deviation of the statistic measured relative to the parameter being estimated; that is,

$$\text{MSE} = \int_{-\infty}^{\infty} (s - \theta)^2 f(s) ds,$$

where MSE is mean square error, s is the statistic being measured, and θ is the parameter being estimated.) If a statistic is unbiased, its mean square error is the variance of its sampling distribution. The relative efficiencies of two unbiased statistics which both estimate a certain parameter may be compared by comparing the variances of their respective sampling distributions. If the sampling distribution of a certain statistic is known to have the smallest possible mean square error for a particular sample size, the efficiency of

another statistic with that statistic may be made in terms of absolute relative efficiency, often simply stated as efficiency. For instance, the statement that a statistic is 90 percent efficient indicates that 110 observations using the inefficient statistic are required to achieve the precision obtained by 100 observations using the efficient statistic. For observations drawn from a normal population, the sample mean is the efficient statistic to estimate the population mean, and the sample variance is the efficient statistic to estimate the population variance.

A simple example may make these terms clearer. Suppose that long term records at a weather station show that 50 percent of the days are clear and 50 percent cloudy. One weather forecaster might make predictions by tossing a coin, and predicting clear for heads and cloudy for tails. His predictions would be unbiased but inefficient. A second weather forecaster might make use of worldwide data to make predictions so that he was nearly always right but have a built-in prejudice toward sunshine so that he would predict 5 percent too many clear days. His predictions would be efficient but biased. In deciding which forecaster to rely on, the user of the reports would base his decision on the use to which he would put the information. The concepts of efficiency and bias are explained in elementary textbooks; Dixon and Massey (3, pp. 74-75) give a particularly clear explanation.

If a frequency distribution is skewed, bias and efficiency become particularly important. To estimate the population mean of such a distribution, one can use the ordinary arithmetic average, or a statistic computed by lognormal theory, or a statistic computed in still another way. The ordinary arithmetic average is unbiased, regardless of the form of the distribution, but it may be inefficient. On the other hand, the mean computed by lognormal theory may be more efficient, but it may also be seriously biased. Thus, a decision must be made based on the user's purpose and on an examination of the particular data of interest, whether bias or efficiency is more important. In gold mining, bias is generally of greatest interest; in other kinds of work, such as exploration geochemistry, efficiency may be more important.

Some Characteristics of Gold Assay Data

The fundamental difficulty in estimation of gold quantity comes from the fact that gold ore contains a very small amount of the metal. A deposit with 1 troy ounce (20 pennyweights) of gold per short ton contains only 34 parts per million by weight (34 grams per metric ton, or 0.0034 percent) of gold. Moreover, the particles of gold making up these 34 parts per million are rarely, if ever, spread evenly throughout the ore. For example, the gold in a 1-ounce-per-ton deposit might be in small but easily visible particles 1 cubic millimeter in size corresponding to cubes 0.04 inch on a side. Then, each particle would weigh 0.019 gram, and a metric ton of ore (1.1 short tons) would contain only about 1,800 particles, or six out of every million particles. Although a 1-ounce-per-ton illustration was furnished, the actual amount of gold present may be much less; many lode mines work ore with a gold content of 5 to 10 parts per million or even less, and some placer mines work gravel with a gold content of 1/4 part per million by weight or 1 per 28 million by volume.

As is to be expected in an industry of ancient origin, gold assay data are reported in many different units. There would be some advantage to converting all values to the simplest units, those of the metric system. However, to do so in this report might lead to confusion for those familiar with other units, and therefore, most data in this report are given in the original units. These original units are grams per metric ton (g/T) or the equivalent of parts per million (ppm), troy ounces per short ton (oz/T), and pennyweights per short ton (dwt). At a gold price of \$35 per troy ounce, 1 gram of gold is equal to \$1.021, and 1 dwt is equal to \$1.750.

Some data are reported in an assay-width unit, obtained by multiplying assay by ore body width, to yield units such as inch-pennyweights (in.-dwt), meter-grams, and foot-ounces. Using an assay-width unit in narrow, tabular ore bodies provides an unbiased estimate of ore grade as mined in almost all cases is well known to engineers and geologists concerned with such ore bodies. This fact, which is not recognized by some workers familiar with ore bodies of other shapes, has been argued exhaustively elsewhere (7) and is not reviewed in this report.

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The authors gratefully acknowledge the assistance of the various mining companies who supplied the assay data used in this report and the engineers and geologists of these companies who discussed the significance of these data and criticized the authors' interpretations. Special acknowledgments are made to the Homestake Mining Company, Lead, S. Dak., for arranging drilling by the Bureau in the Homestake mine and to the City Deep Mine Ltd., Johannesburg, Republic of South Africa, for permission to publish the results of our findings. P. C. Pirow of Rand Mines Ltd. and D. G. Krige of Anglo-Transvaal Consolidated Investment Co. Ltd. provided invaluable suggestions while computing was in an early stage and made detailed reviews and comments during the presentation of an earlier manuscript in South Africa. While Drs. D. G. Krige and H. S. Sichel reviewed and criticized this report, they disagree with some of the conclusions.

LOGARITHMIC TRANSFORMATIONS AND LOGARITHMIC PROBABILITY PAPER

A transformation is a mathematical function of an observation that defines a new observation; that is,

$$u = f(w),$$

where u is the new observation, f is the transforming function, and w is the original observation. A familiar example is the linear transformation of troy ounces to pennyweights by multiplying by the constant 20. Less familiar are nonlinear transformations, that is, those for which a plot of u , w is not a straight line; for example, the logarithmic transformation, whereby each observation is transformed to its logarithm, according to the formula,

$$u = \ln(w).$$

The basic purpose of a nonlinear transformation is to change the shape of a frequency distribution. Thus, the shape of the gold frequency distribution in figure 2 can be changed to more or less the shape of the phosphate distribution in figure 1 by a logarithmic transformation. In contrast, a linear transformation does not change the shape of a distribution but only the scale.

In gold data analysis, there are two reasons for changing from one frequency distribution to another by a nonlinear transformation. The first is to transform the observations so as to follow a normal distribution. This has two advantages. First, if observations are normally distributed, the sample mean is an efficient estimate of the population mean which is not the case in an untransformed, highly skewed lognormal distribution for which the sample mean is a rather inefficient estimate of the population mean. Secondly, most statistical procedures for confidence intervals and hypotheses tests assume underlying normality. However, for these procedures, departures of data from normality are seldom serious because many of the procedures are quite robust and work rather well even with observations that are not exactly normally distributed, and the mean is likely to be a quite efficient estimator except for frequency distribution with long tails. The rather robust techniques are t and F as used in the analysis of variance, and any statistic based on sample means. Those less robust statistics are Chi-square and F as used to compare variances.

The second reason for using a nonlinear transformation is to stabilize the variance which may be explained as follows: An observation w may be written as a mean value μ plus a random fluctuation e ; that is, $w = \mu + e$. This expression implies that the mean value of e is 0, but its variance is not necessarily constant; constant variance is the requirement for stability. However, if the variance of e , σ_e^2 depends upon μ in some fashion, it may be possible to find a transformation,

$$u = f(w),$$

such that in the transformed equation,

$$u = \mu^* + e^*,$$

where the variance of e^* is a constant independent of the value of μ^* . Constant variance is also named homoscedasticity. As an example of nonconstant variance, consider the fluctuations associated with low-grade and high-grade gold assays. The variability of gold assays from low-grade ore is much less than the variability of gold assays from high-grade ore. Therefore, the average size of the random fluctuation e depends upon the grade of the ore. For some gold ores, a constant variance may be obtained by taking the logarithms of the observations.

Transformations should never be made as a matter of course, because disadvantages arise that may be serious. The most serious is that transformations may lead to biased estimates. For example, in estimating the mean of lognormally distributed gold assays, greater efficiency may be obtained by transforming the observations to logarithms, but a bias is introduced because,

as explained below, the antilogarithm of \bar{u} is not, on the average, equal to the population mean μ . For observations that are distributed exactly log-normally, the bias may be removed, because the lognormal and normal distributions are well understood. For observations following other distributions, such bias may also be removed in principle, but practical methods may not have been devised. A second disadvantage of transformations is that their use always requires more calculation. Even if an appropriate transformation is known, the additional work may not be economically justified, because the efficiency of an estimate based on original untransformed data may be adequate.

The Logarithmic Transformation

In the logarithmic transformation, an original observation w is transformed to its logarithm u by the formula,

$$u = \ln(w).$$

Either natural logarithms (base e), common logarithms (base 10), or logarithms to any other base can be used. It makes no difference which type of logarithm is used, because the logarithmic variables u are merely linear transformations of each other. For example, the natural logarithm of w equals 2.303 times the common logarithm of w . In this report, the calculations are carried out for natural logarithms, and the reader should keep this in mind, or he will be confused by details of arithmetic.

Logarithmic Probability Paper

Normally distributed observations plotted on normal probability paper will define a straight line. For example, the phosphate observations of figure 1 and table 1 are plotted in figure 5 on normal probability graph paper. Note that since one of the scales is percent-relative-cumulative frequency, a frequency distribution (see table 1) must be made before plotting.

Although the straight line fitted by eye to the phosphate data does not exactly fit the points, it is close enough to indicate that they are approximately normally distributed. The mean of the observations is estimated by reading the 50-percent point on the straight line; estimated in this way, the mean is 24.9 which is in good agreement with the mean 25.23 computed from the classed data in table 1. The standard deviation of the observations is estimated by subtracting the 16-percent point from the 84-percent point and dividing the result by 2. For the phosphate data, the 16-percent point is 22.2, the 84-percent point is 28.1, the difference is 5.9, and the estimated standard deviation is 2.95 which agrees with the computed value of 3.03. If, however, observations are not normally distributed, their plot on normal probability graph paper will not define a straight line.

It is possible to calculate the logarithm of each lognormally distributed observation and plot on normal probability graph paper. This is quite tedious, and a much easier method is to plot lognormally distributed observations on logarithmic probability graph paper. Figure 6 plots the dwt stope observations of the City Deep mine. Since the points of this figure approximately

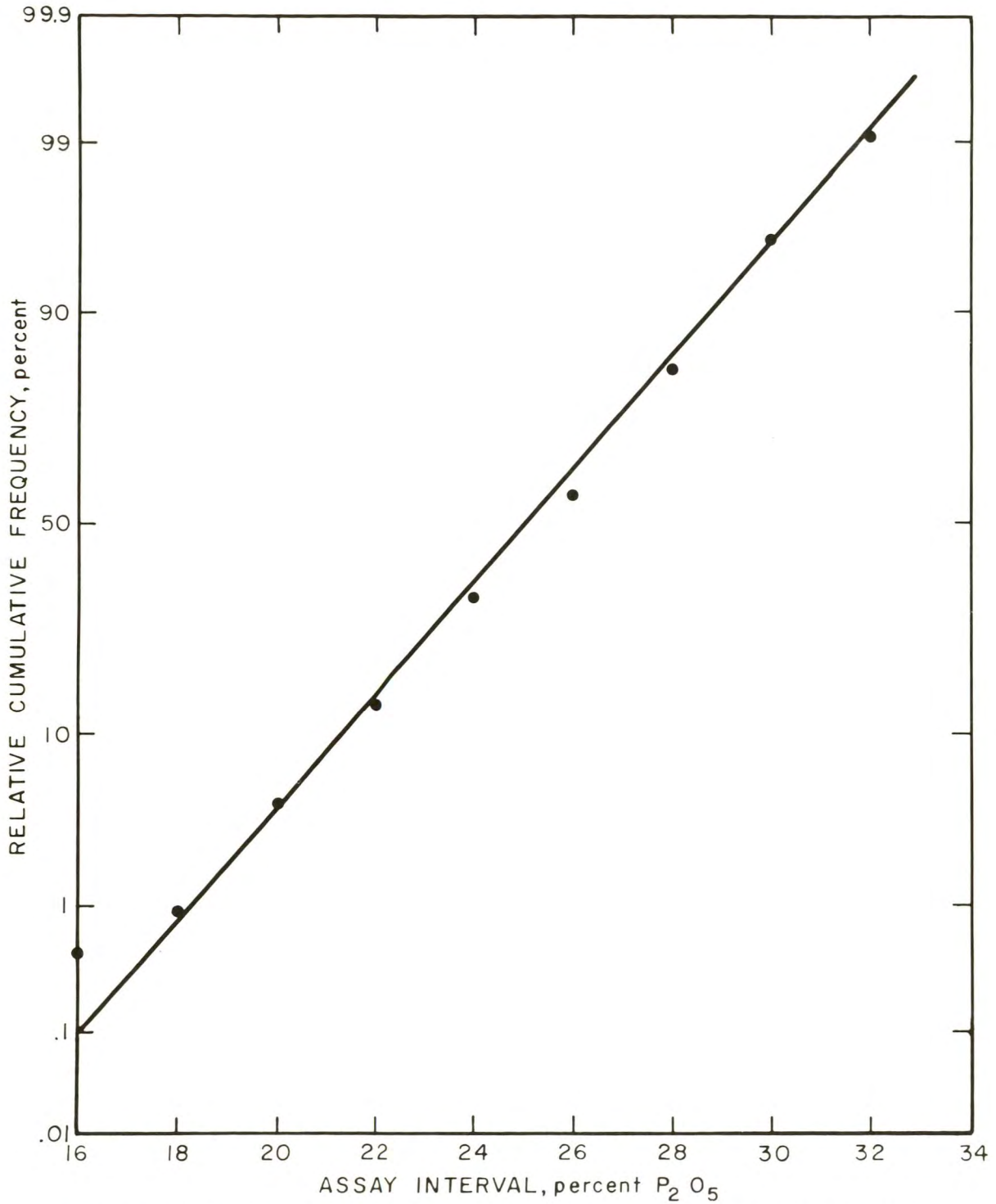


FIGURE 5. - Plot on Normal Probability Paper of Phosphate Assays From Table 1.

define a straight line, the observations are nearly lognormally distributed. A straight line indicates that the observations must be lognormally distributed. The reader should note that the arithmetic mean and standard deviation

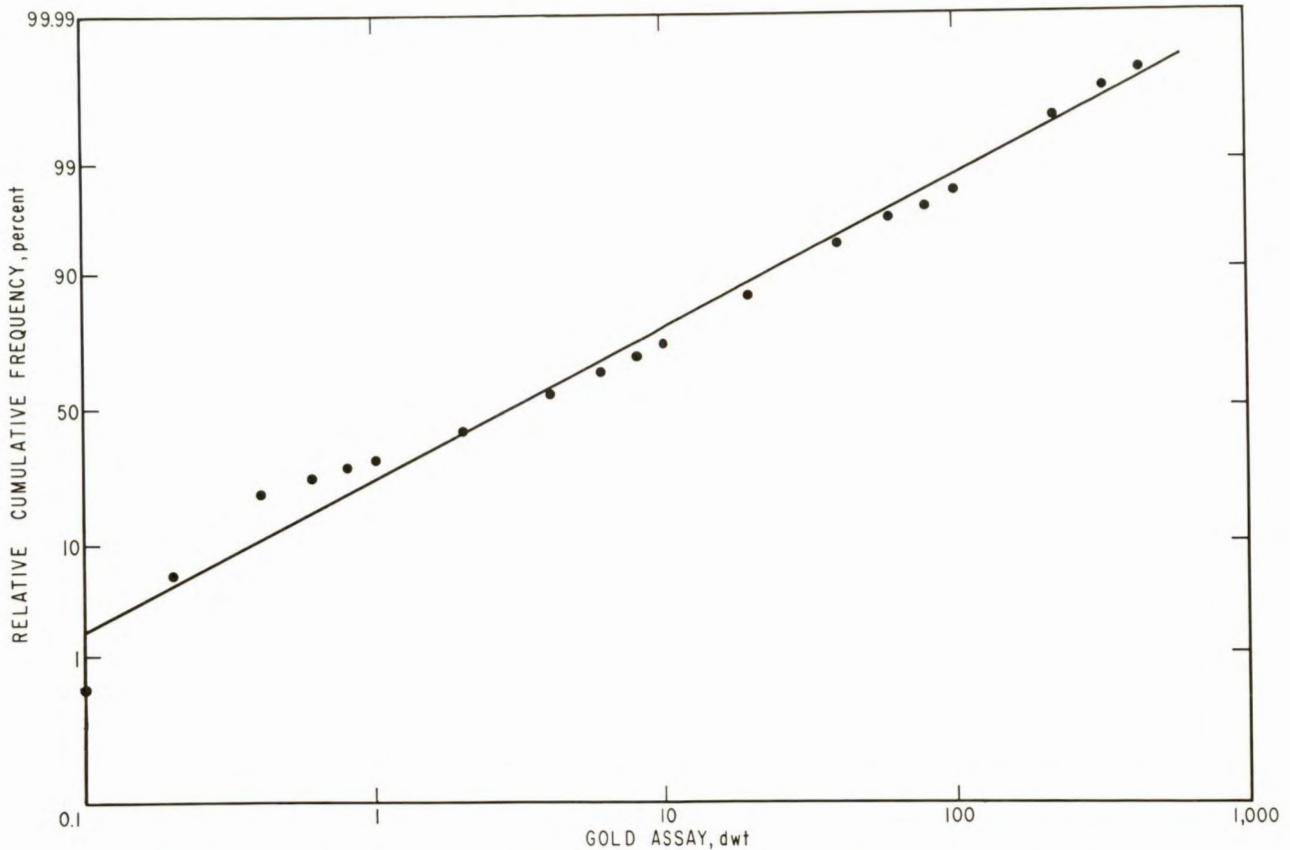


FIGURE 6. - Plot on Logarithmic Probability Paper of City Deep Stope Data.

of the untransformed observations cannot be obtained from a graph on logarithmic probability paper. For example, the 50-percent point from figure 6 provides the mean of the logarithms, 2.8 dwt. The antilogarithm is 16.4 dwt, while the arithmetic mean for these observations is 12.0 dwt. Despite this restriction, lognormal probability paper is a useful tool to determine if observations are lognormally distributed.

ESTIMATION OF MEANS FOR LOGNORMALLY DISTRIBUTED OBSERVATIONS

In the gold mining industry, the main interest in lognormally distributed data is usually in estimating the mean grade of the gold. Accordingly, estimation of means is emphasized in this report. In scientific work, estimation of other quantities, such as the variance, a measure of variability of the observations, may also be of as much or equal interest, and therefore, some attention is given to it.

Description of the Lognormal Distribution

The lognormal distribution is characterized by the property that the logarithms of the observations follow a normal distribution. The formula for the lognormal frequency distribution is

$$f(w) = \frac{1}{w\beta\sqrt{2\pi}} \exp \left(- \frac{(\ln w - \alpha)^2}{2\beta^2} \right),$$

where the two parameters α and β^2 are the mean and variance, respectively, of the natural logarithm of the observation w . Note that for clarification the term \exp may be used to represent the exponential instead of e . Thus,

$$e^x = \exp (x).$$

The mean of w is given by the formula

$$\mu = \exp \left(\alpha + \frac{1}{2}\beta^2 \right)$$

and the variance of w , by the formula

$$\sigma^2 = \mu^2 (e^{\beta^2} - 1).$$

For consistency in relating this report to previous reports on sampling, the parameters α , β^2 , μ , and σ^2 are defined in the opposite way from the definition of Aitchison and Brown (1). Table 5 compares the two systems of notation.

TABLE 5. - Comparison of notation for lognormal distribution in this report with that of Aitchison and Brown

Notation adopted	Observation	Mean of observation	Variance of observation	Logarithm of observation	Mean of logarithms	Variance of logarithms	Coefficient of variation	Multiplying factor for the geometric mean	Multiplying factor for the variance
This report...	w	μ, m^1	σ^2, V^2	u	α, \bar{u}	β^2, s_u^2	γ, C	ψ_n	ϕ_n
Aitchison and Brown	x	α, a_1	β^2, b_1^2	y	μ, \bar{y}	σ^2, v_y^2	η	ψ_n	χ_n

¹Where two symbols are given, the first is the parameter, the second is the statistic.

Just as there are many normal distributions depending on the values of the parameters μ and σ^2 , so there are many lognormal distributions depending on the values of these parameters or upon the values of the corresponding parameters α and β^2 . The lognormal distribution is always skewed to the right. The amount of skewness depends only on the value of β^2 , the variance of the logarithms of the observations. If the value of β^2 is small, so is the skewness, and the frequency distribution is nearly normal. Figure 7 graphs three lognormal distributions for α equals 0, with different values of β^2 . Although the distribution with β^2 equal to 0.1 is already noticeably skewed, it is more nearly normal than the other distributions with larger values of β^2 .

In data for such constituents as the silica content of glass sands, in which the constituent analyzed makes up most of the material, the negative-skew

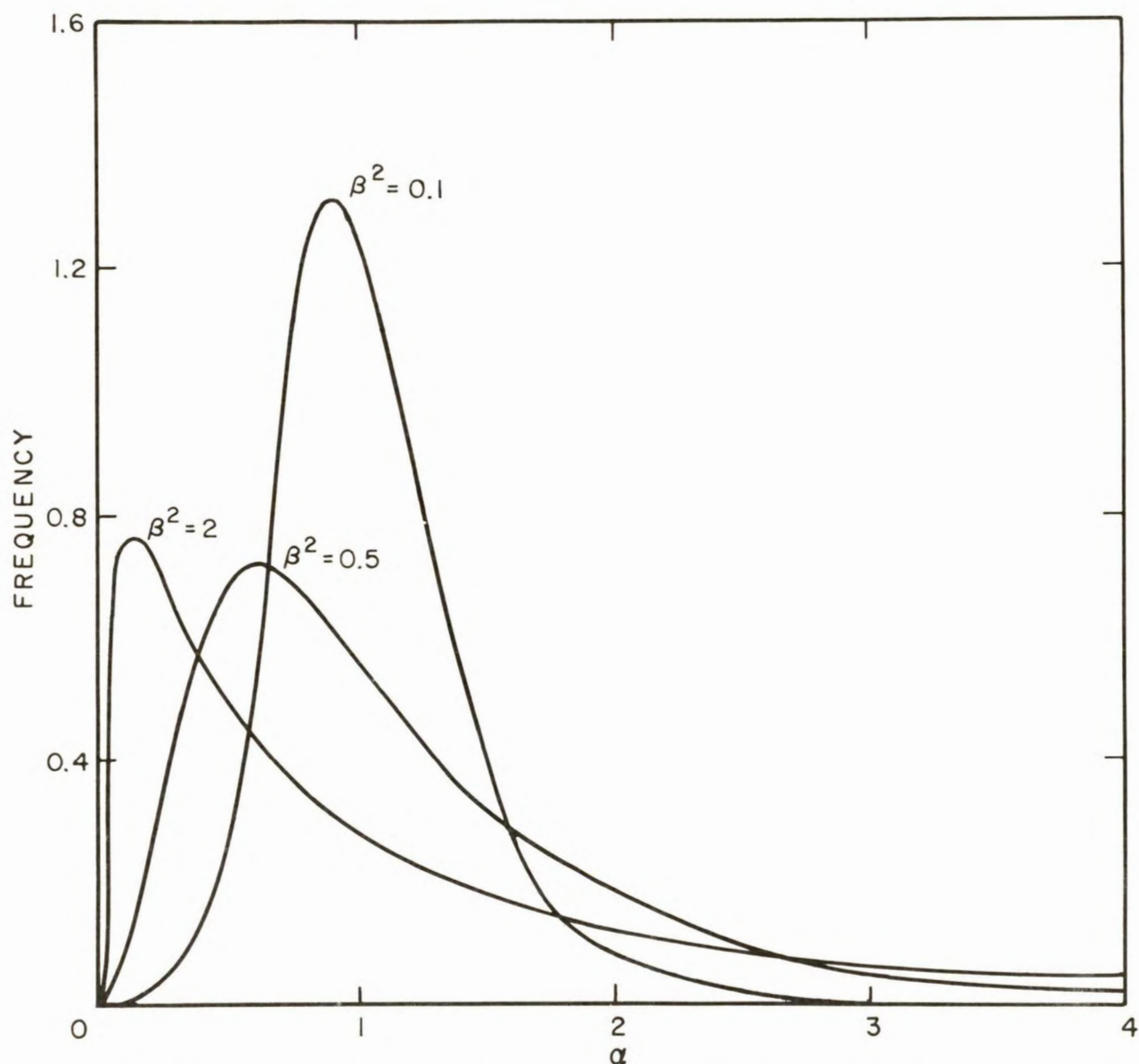


FIGURE 7. - Frequency Curves of the Lognormal Distribution for Three Values of β^2
(After Aitchison and Brown).

lognormal distribution, a reversed lognormal distribution, is also met. Everything in this report can readily be related to the negative-skew lognormal distribution; for details see Aitchison and Brown (1).

The first problem in applying lognormal statistics is to estimate the parameters μ and σ^2 . Because for any frequency distribution, the mean of the sampling distribution of sample means is equal to the population mean, the sample mean \bar{w} is an unbiased estimate of μ . Similarly, because the mean of the sampling distribution of sample variances is equal to the population variance, the sample variance s^2 is an unbiased estimate of σ^2 . However, these estimates are not the most efficient ones.

The most efficient estimates are obtained by first estimating α and β^2 and then estimating μ and σ^2 . The estimate of α is the average value of the natural logarithms of the observations,

$$\alpha = \bar{u} = \frac{\sum \ln(w)}{n},$$

which is efficient because the logarithms of the observations are normally distributed, and the sample mean is an efficient estimate of the population mean for observations so distributed. The estimate of β^2 is the variance of the logarithms of the observations,

$$s_u^2 = \frac{\sum (u - \bar{u})^2}{n - 1},$$

where $u = \ln(w)$. Now the unbiased efficient estimate m of μ may be obtained from the formula,

$$m = e^{\bar{u}} \Psi_n \left(\frac{1}{2} s_u^2 \right),$$

and the unbiased efficient estimate V^2 of σ^2 may be obtained from the formula,

$$V^2 = e^{2\bar{u}} \left(\Psi_n(2 s_u^2) - \Psi_n \left(\frac{n-2}{n-1} s_u^2 \right) \right) = e^{2\bar{u}} (\Phi_n(s_u^2)),$$

where Ψ_n and Φ_n are functions defined in the following paragraph.

To implement these formulas, values of Ψ_n , the multiplying factor for the geometric mean, must be tabulated, and it is convenient to have values of Φ_n , the multiplying factor for the variance, tabulated. In appendix tables B-1 and B-2, values of Ψ_n and Φ_n are tabulated for the range of values appropriate for gold assay data. Less detailed and less extensive tables are presented by Aitchison and Brown (1). For values not presented in the tables, linear interpolation will yield a close approximation. Values of Ψ_n and Φ_n presented in the text and tables are calculated by the following formulas:

$$\Psi_n(t) = 1 + \frac{n-1}{n} t + \frac{(n-1)^3}{n^2(n+1)} \frac{t^2}{2!} + \frac{(n-1)^5}{n^3(n+1)(n+3)} \frac{t^3}{3!} + \dots,$$

$$\Phi_n(t^*) = \Psi_n \left(\frac{n-2}{n-1} t^* \right),$$

where t in $\Psi_n(t)$ is $\frac{1}{2}s_u^2$; t^* in $\Phi_n(t^*)$ is s_u^2 , and n represents the sample size.

Estimates of the Mean and Variance

To illustrate parameter estimation for the lognormal distribution, we use 13 gold assay values from boreholes at the Welkom mine, Republic of South Africa, from Krige's work (13). The first calculation illustrates use of the formulas for m and V^2 . In table 6, column 1 lists the 13 original in.-dwt values, column 2 lists the natural logarithms, and calculations for mean and

variance are given below the data list. The note at the bottom of the table verifies that the calculations can be done in either natural or common logarithms. In table 7, \bar{u} and s_u^2 are repeated from table 6, and m is calculated. The value of $e^{\bar{u}}$ is the natural antilogarithm of \bar{u} . Although this value was calculated by computer, an approximate value may be obtained from appendix table B-3 using linear interpolation. Line 4 is the value of 13 for n , and the next line is the value for the argument $\frac{1}{2}s_u^2$ used to determine the value of ψ_n . By a similar calculation, the value of V^2 is obtained.

TABLE 6. - Calculation of means and variances of observations and logarithms of observations for 13 gold assay values from the Welkom mine, Republic of South Africa

DATA					
w		u = ln(w)		v = log(w)	
154		5.03695		2.18752	
525		6.26340		2.72016	
1,560		7.35245		3.19312	
1,252		7.13247		3.09760	
377		5.93225		2.57634	
70		4.24850		1.84510	
308		5.73010		2.48855	
109		4.69135		2.03743	
1,221		7.10743		3.08672	
15		2.70806		1.17609	
48		3.87121		1.68124	
237		5.46806		2.37475	
68		4.21951		1.83251	
CALCULATED VALUES					
Notation	Value	Notation	Value	Notation	Value
Σw	5,944	Σu	69.76174	Σv	30.29713
\bar{w}	457	\bar{u}	5.36629	\bar{v}	2.33055
$(\Sigma w)^2$	35,331,136	$(\Sigma u)^2$	4,866.70036783	$(\Sigma v)^2$	917.91608624
$(\Sigma w)^2/n$	2,717,780	$(\Sigma u)^2/n$	374.361567	$(\Sigma v)^2/n$	70.608930
Σw^2	6,108,382	Σu^2	398.155209	Σv^2	75.096706
SS_w	3,390,602	SS_u	23.793642	SS_v	4.487776
s_w^2	282,550	s_u^2	1.98280	s_v^2	.37398
s_w	531	s_u	1.4081	s_v	.6115

Note: $(2.302585)\bar{v} = \bar{u}$.
 $(2.302585)^2 s_v^2 = s_u^2$.

TABLE 7. - Calculation by lognormal theory of m , a statistic to estimate the mean μ , and V^2 , a statistic to estimate the variance σ^2 , for the data in table 6

CALCULATION OF m		CALCULATION OF V^2	
Notation	Value	Notation	Value
\bar{u}	5.36629	\bar{u}	5.36629
s_u^2	1.98280	s_u^2	1.98280
$e^{\bar{u}}$	214	$e^{2\bar{u}}$	45,825
n	13	n	13
$\frac{1}{2}s_u^2$.99	$\Phi_n(s_u^2)$	16.27
$\Psi_n(\frac{1}{2}s_u^2)$	2.370	V^2	745,573
m	507	V	863

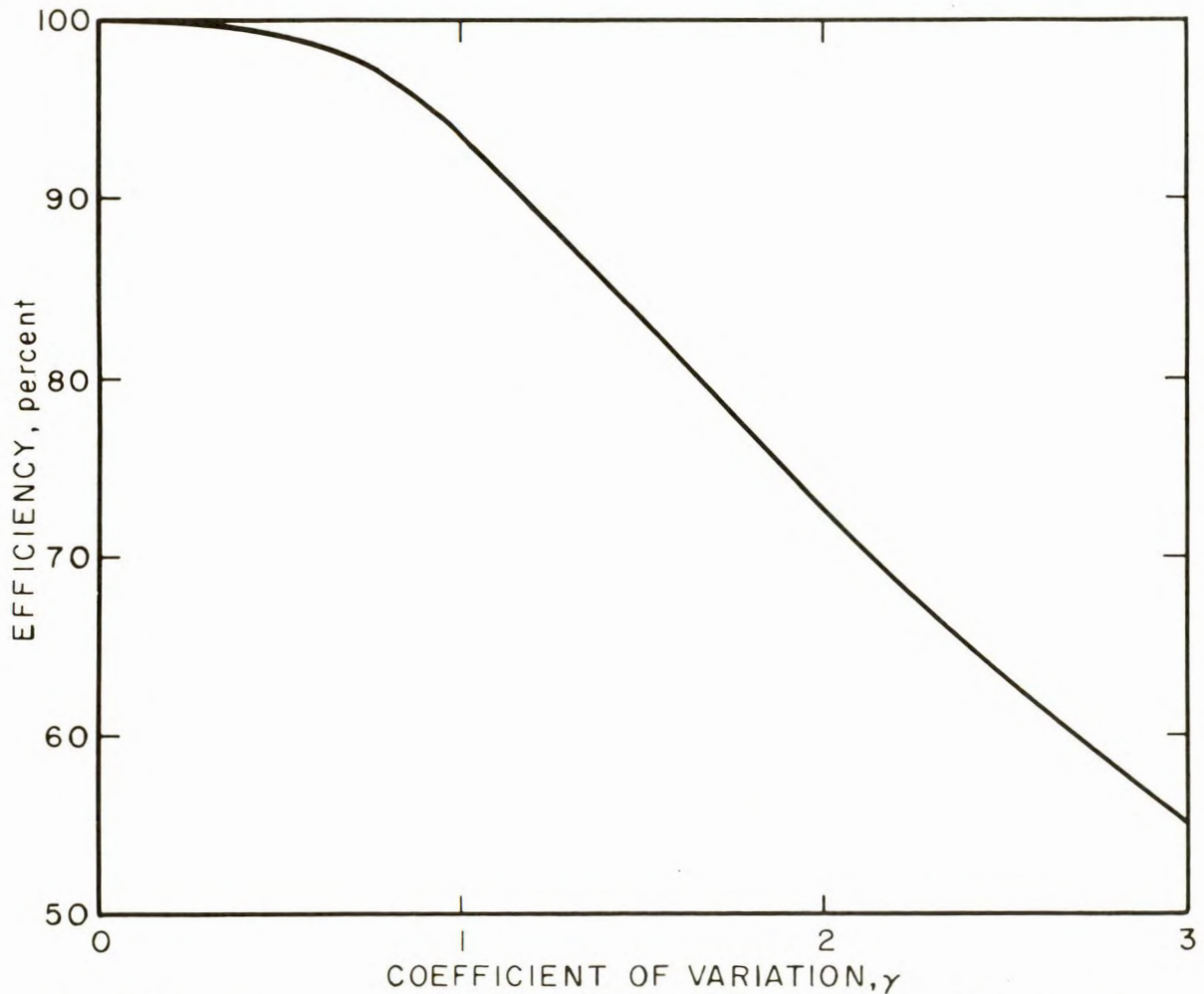


FIGURE 8. - Efficiency of the Untransformed Mean, \bar{w} , as a Function of the Coefficient of Variation for Lognormally Distributed Observations (Data From Finney).

The preceding calculations of mean and variance of observations that follow a lognormal distribution are quite complicated. Moreover, if the observations do not closely follow this distribution, the estimates may be seriously biased. Finney (4) showed that the mean, \bar{w} , of the untransformed observations is more than 90 percent efficient as an estimate of μ , provided that the coefficient of variation, λ , (ratio of standard deviation to mean, $\gamma = \sigma/\mu$, $C = s/\bar{w}$) is less than 1.2, corresponding to a variance of the logarithms β^2 of about 0.9. Figure 8, based on Finney's work, plots the efficiency of \bar{w} as a function of the coefficient of variation. If the coefficient of variation for the untransformed observations is less than 1.2, it is recommended that \bar{w} be used to estimate μ , because little efficiency can be lost, considerable arithmetic is saved, and bias cannot arise.

Because of the importance of the coefficient of variation, several ways to estimate it are reviewed. Table 8 gives the calculations as applied to the Welkom data. The first method, column 1, uses the estimate of the variance of the logarithms s_u^2 to find C. If the square root of the expression σ^2 in the first paragraph of this subsection is substituted in the equation

$$\gamma = \sigma/\mu,$$

the result obtained is

$$\gamma = \frac{\mu \sqrt{e^{\beta^2} - 1}}{\mu} = \sqrt{e^{\beta^2} - 1}.$$

When the calculations in column 1 are performed using the value of s_u^2 , C is found to equal 2.50. Another estimate of s_u^2 , based on a large number of assays from the whole mine, may be used in place of the estimate based on only 13 assays. Such an estimate, based on 33,031 assay values from Krige, is 1.05 which yields a much smaller C of 1.36 (column 4). A second way to estimate C is to divide the estimated standard deviation of the original observations s_w by estimated mean of the original observation \bar{w} ; the result is 1.16 (column 2). A third way is to divide V by m which yields 1.70 (column 3).

TABLE 8. - Methods to calculate C, the coefficient of variation, for lognormally distributed observations

Source of data.....	Calculations			
	Table 6 (1)	Table 6 (2)	Table 7 (3)	Krige (9) (4)
Sample size.....	13	13	13	33,031
Method of calculation	$\gamma = \sqrt{e^{\beta^2} - 1}$	$\gamma = \sigma/\mu$	$\gamma = \sigma/\mu$	$\gamma = \sqrt{e^{\beta^2} - 1}$
Calculations.....	$s_u^2 = 1.9828$ $e^{s_u^2} = 7.26305$ $e^{s_u^2} - 1 = 6.26305$	$s_w = 531$ $\bar{w} = 457$	$V = 863$ $m = 507$	$s_u^2 = 1.05$ $e^{s_u^2} = 2.85765$ $e^{s_u^2} - 1 = 1.85765$
C.....	2.50	1.16	1.71	1.36

All of the values for C in table 8 are different. Based on a large number of mine samples, the value in column 4 should be reliable for the mine as a whole; whether it is applicable to the 13 assay values depends on whether they were made on samples of typical ore or, in statistical terms, whether they are a random sample from the same population. Because the first three values for C are based on only 13 observations with a large standard deviation, they are different from one another, as well as from the value in column 4. The conclusion is that the coefficient of variation cannot be reliably estimated from so few lognormally distributed observations.

Calculation of Confidence Limits for the Population Mean

The next step is to calculate confidence limits for the population mean by one of the several methods available. The final choice of method depends on the number of observations available, background information on consistency and nature of mineralization, selection of the confidence limits whether both upper and lower confidence limits are required or whether only lower confidence limits that set a lower bound on estimated ore grade are acceptable, and the exact purpose. The different methods are explained by applying them to the Welkom data.

If a population variance of logarithms β^2 has been established from a large number of measurements and if the sample in question is known to come from this population, confidence limits can be calculated for the logarithms and then transformed into confidence limits for the original observations. As shown in table 9 for the Welkom assays, the initial data (lines 1 and 2) are a value of β^2 based on some 33,000 observations (table 8) and a value of \bar{u} calculated from the 13 observations. In lines 3 to 8 of table 9, confidence intervals are calculated for the logarithmic mean α using the standard method for normally distributed observations with the population variance known. In lines 9 and 10 the antilogarithms of the confidence intervals are computed. In lines 11, 12, and 13, a function of the variance of the logarithms is calculated. The values of the confidence limits for the untransformed mean μ are presented in lines 14 to 15. The method used to obtain these values is due to Finney (4). Finally, in lines 16 to 17, the point estimate m of μ is calculated, based on the β^2 value.

If the population variance of logarithms β^2 is unknown, it must be estimated. One of the several methods of calculating a confidence region for μ and σ^2 is explained by Mood (15, p. 229). This method is applied to the Welkom data in tables 10 to 12. Mood's method depends on the fact that because the distribution of the sample mean and the sum of squares are independent for samples from a normal distribution, a joint confidence region for the parameters α and β^2 can be obtained by finding the values of α and β^2 that satisfy the joint inequalities

$$|\bar{u} - \alpha| < 1.645 \beta / \sqrt{n},$$

$$\chi_{95\%}^2 < \frac{SS}{\beta^2} < \chi_{5\%}^2.$$

The solution produces an 81-percent (90-percent squared) confidence region. The first step, shown in table 10, is to find upper and lower limits for β^2 . The second is to apply these limits to find upper and lower limits for desired values of α between the upper and lower limits for β^2 . In table 11, eight particular values for α_L and α_U are calculated, and in figure 9, they are plotted. Additional values to complete the graph were plotted but are not listed in table 11.

TABLE 9. - Calculation of confidence limits for lognormally distributed observations with β^2 , the population variance of logarithms, assumed to be known

Line	Notation	Value	Calculation	Comments
1	β^2	1.05	From table 8	-
2	\bar{u}	5.36629	From table 6	-
3	$\beta^2/13$.0808	-	-
4	$\sqrt{\beta^2/13}$.2842	$\sqrt{.0808}$	Standard error of mean
5	-	1.645	Table value	5-percent point of standardized normal distribution
6	-	.4675	$1.645 \times .2842$	-
7	α_L	4.8988	$5.3663 - .4675$	Lower confidence limit for α
8	α_U	5.8338	$5.3663 + .4675$	Upper confidence limit for α
9	e^{α_L}	134	-	-
10	e^{α_U}	341	-	-
11	$(n-1)/2n$.4615	$(13-1)/26$	-
12	$[(n-1)/2n]\beta^2$.4846	-	-
13	$\exp\left(\frac{n-1}{2n}\beta^2\right)$	1.6235	-	-
14	u_L	218	134×1.6235	-
15	u_U	554	341×1.6235	-
16	e^{π}	214	From table 7	-
17	m	347	214×1.6235	-

TABLE 10. - Calculation of parameter estimates and joint confidence region for lognormally distributed observations by Mood's method: Part 1, calculation of upper and lower confidence limits for β^2

Line	Notation	Value	Calculation	Comments
1	\bar{u}	5.3663	From table 6	-
2	SS_u	23.793642	From table 6	-
3	n	13	From table 9	-
4	$\sqrt{\pi}$	3.6056	-	-
5	-	1.645	-	5-percent point of standardized normal distribution.
6	-	5.23	From χ^2 -table	95-percent point of χ^2 , d.f. = 12
7	-	21.03	From χ^2 -table	5-percent point of χ^2 , d.f. = 12
8	$SS_u/\chi_{95\%}^2$	1.1314	$23.793642/21.03$	β_L^2
9	$SS_u/\chi_{5\%}^2$	4.5495	$23.793642/5.23$	β_U^2
10	-	.4562	$1.645/\sqrt{13}$	-

TABLE 11. - Calculation of parameter estimates and joint confidence region for lognormally distributed observations by Mood's method: Part 2, calculation of upper and lower confidence limits for various values of α

β^2	β	$\frac{1.645\beta}{\sqrt{13}}$	$\alpha_L = \bar{u} - \frac{1.645\beta}{\sqrt{13}}$	$\alpha_U = \bar{u} + \frac{1.645\beta}{\sqrt{13}}$	$\frac{\beta^2}{2}$
1.1314	1.064	0.485	4.881	5.851	0.5657
4.5495	2.133	.973	4.393	6.339	2.2748
2.0000	1.414	.645	4.721	6.011	1.0000
3.0000	1.732	.790	4.576	6.156	1.5000

Because each α , β^2 point has a corresponding μ , σ point, the region in the α , β^2 coordinates can be transformed into a region in the μ , σ coordinates. In table 12, the required transformation is calculated for 8 points. Columns 1 and 2 give the α , β^2 values at the 8 points to be transformed. Columns 3, 5, 6, and 7 give intermediate terms combined according to the formulas at the beginning of this section to give the μ , σ values at the 8 points in columns 4 and 8. These points define the 81-percent confidence region graphed in figure 10. For comparison, the 64-percent confidence region

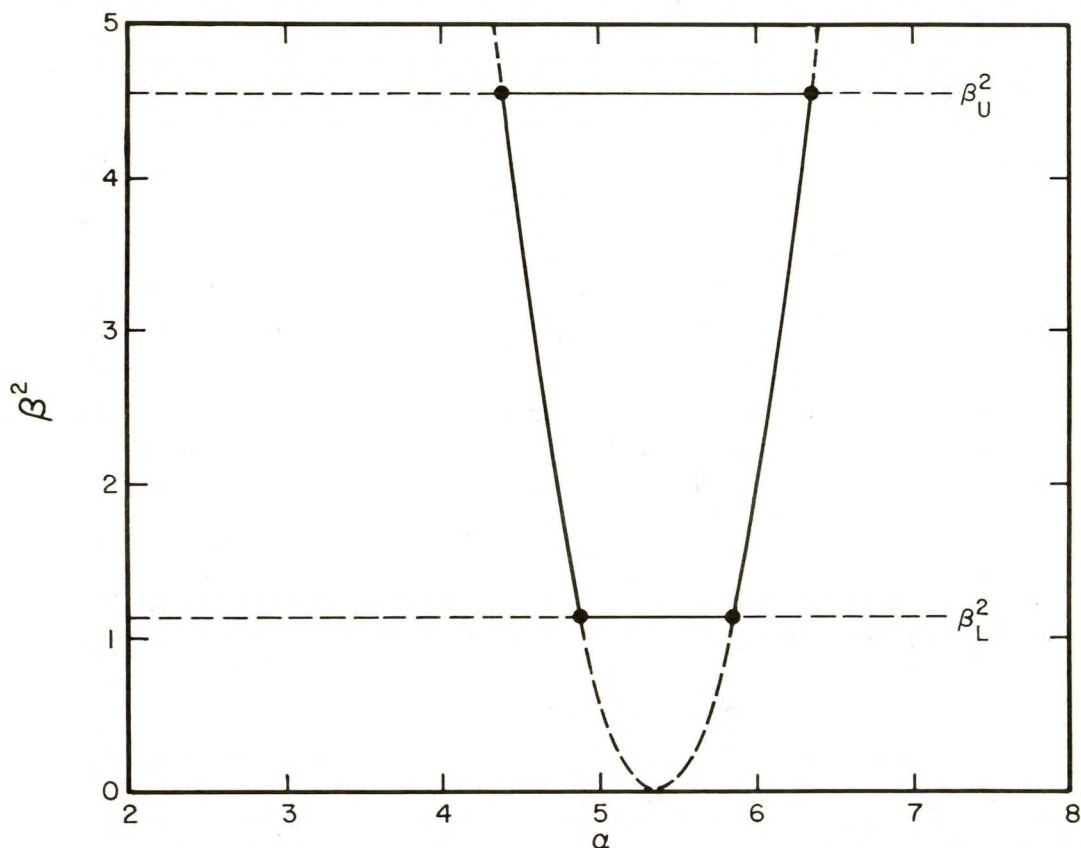


FIGURE 9. - Eighty-One-Percent Confidence Region in α , β^2 Space for Lognormally Distributed Observations From the Welkom Mine, Republic of South Africa (Data From Krige).

and the points \bar{w} , s and m , V are also graphed in figure 10. The interpretation of the 81-percent confidence region is as follows. With a confidence of about 90 percent, the expected value of μ is between 232 and 5,508; these are the μ values plotted as solid circles on the figure. Similarly, with a confidence of about 90 percent, the value of σ is between 336 and 53,824. With a confidence of exactly 81 percent, the point μ , σ is expected to lie within the confidence region.

TABLE 12. - Transformation of α , β^2 points to corresponding μ , σ^2 points

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	β^2	$\alpha + \frac{1}{2}\beta^2$	μ	μ^2	$e\beta^2 - 1$	σ^2	σ
4.881	1.1314	5.447	232	53,852	2.09999	113,100	336
5.851	1.1314	6.417	612	374,741	2.09999	786,900	887
4.393	4.5495	6.668	787	619,369	93.58511	57,963,700	7,613
6.339	4.5495	8.614	5,508	30,338,064	93.58511	2,839,191,000	53,284
4.721	2.0000	5.721	305	93,153	6.38906	595,200	771
6.011	2.0000	7.011	1,109	1,229,350	6.38906	7,854,000	2,802
4.576	3.0000	6.076	435	189,473	19.08554	3,616,000	1,902
6.156	3.0000	7.656	2,113	4,446,000	19.08554	85,240,000	9,233

It is not surprising that the point \bar{w} , s_w lies outside the confidence region, because this point is calculated only from a sample; however, the individual \bar{w} and s_w values lie within their respective confidence intervals. Notably, the upper part of the confidence region extends to extremely high values, reflecting the uncertainty about this part of the region based on the statistical analysis. Based on other information, one might wish to decrease the size of the region; for instance, if the user believed that σ could not be expected to exceed say 5,000, the region could be reduced in size by bounding it on the upper side by a line with the equation $\sigma = 5,000$. However, the confidence coefficient no longer has its nominal meaning.

A computer program to calculate the arithmetic mean, the mean of logarithms, and the joint confidence region for the mean and standard deviation by Mood's method is given in appendix A.

In both of the following methods, confidence limits are calculated by use of the t-distribution, proceeding as though the observations were normally distributed. The value of t for the 5-percent level (one tailed) with 12 degrees of freedom is tabled in any standard statistical text. In the first of these two methods, given in table 13, the mean and variance of the untransformed observations from table 6 are used.

In the second of these two methods, tabulated in table 14, the mean and variance of the transformed observations from table 6 are used. In the first eight lines, the 90-percent confidence limits for α are found, following the form in table 13. In lines 9 and 10, the powers to the base e of the values in lines 7 and 8 are calculated. Finally, the multiplier for the geometric mean, line 11, is applied to the values in lines 9 and 10 in order to obtain the confidence limits in lines 12 and 13.

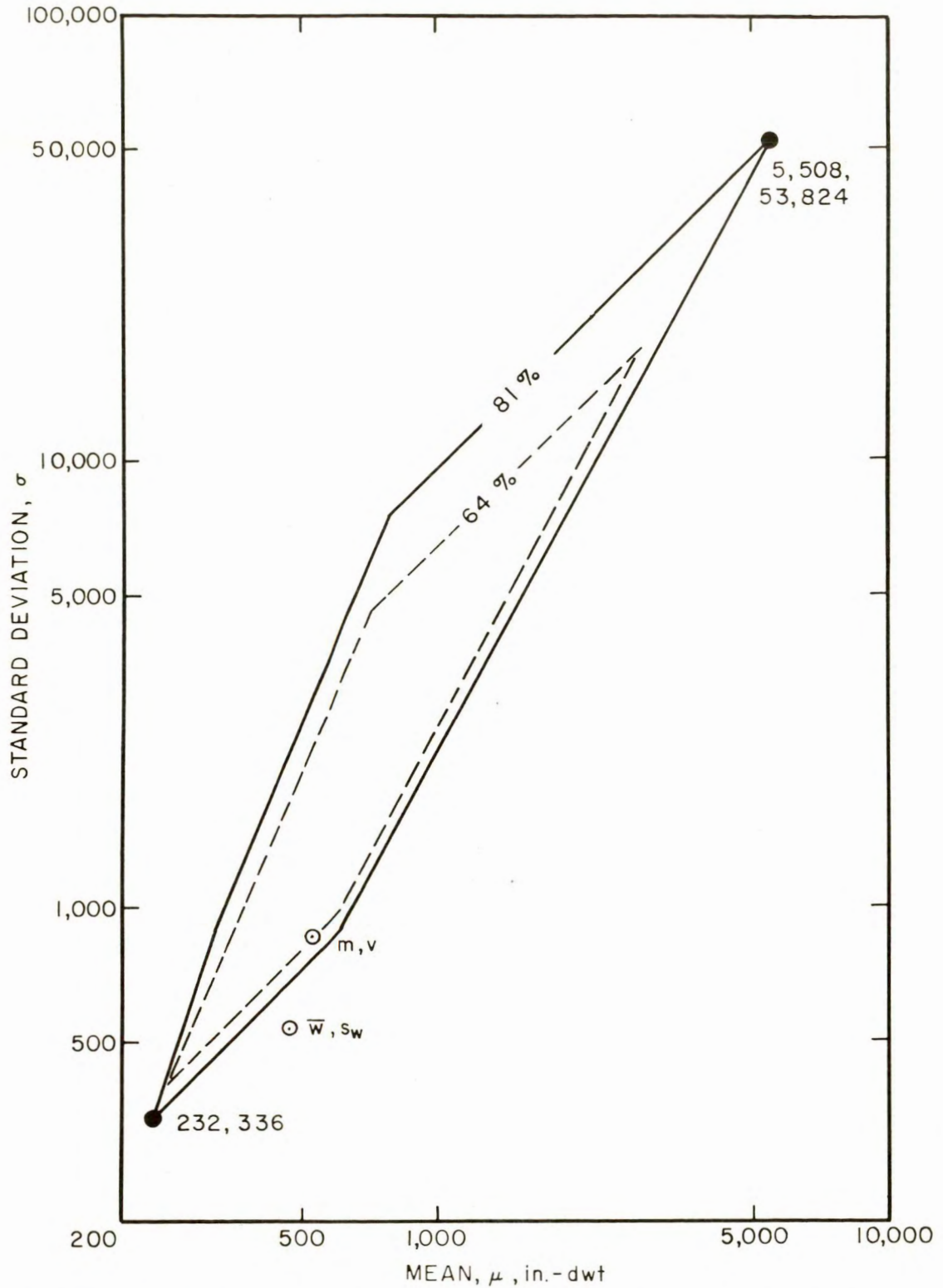


FIGURE 10. - Eighty-One-Percent Confidence Region in μ, σ Space for Lognormally Distributed Gold Observations From the Welkom Mine, Republic of South Africa.

TABLE 13. - Calculation of confidence limits for lognormally distributed observations by use of the t-distribution: Method 1, based on mean and variance of untransformed observations from table 6

Line	Notation	Value	Calculation
1	\bar{w}	457	From table 6
2	s_w	531	From table 6
3	\sqrt{n}	3.6056	$\sqrt{13}$
4	$t_{5\%}$	1.782	From t-table with 12 d.f.
5	s_w/\sqrt{n}	147	531/3.6056
6	$t_{5\%} s_w/\sqrt{n}$	262	1.782 x 147
7	μ_L	195	457 - 262
8	μ_U	719	457 + 262

TABLE 14. - Calculation of confidence limits for lognormally distributed observations by use of the t-distribution: Method 2, based on mean and variance of transformed observations from table 6

Line	Notation	Value	Calculation
1	\bar{u}	5.36629	From table 6
2	s_u	1.4081	From table 6
3	\sqrt{n}	3.6056	From table 10
4	s_u/\sqrt{n}	0.3905	-
5	$t_{5\%}$	1.782	From t-table with 12 d.f.
6	$t_{5\%} \frac{s_u}{\sqrt{n}}$.6959	-
7	$\bar{u} - t_{5\%} \frac{s_u}{\sqrt{n}}$	4.6704	-
8	$\bar{u} + t_{5\%} \frac{s_u}{\sqrt{n}}$	6.0622	-
9	$\exp\left(\bar{u} - t_{5\%} \frac{s_u}{\sqrt{n}}\right)$	106.7	-
10	$\exp\left(\bar{u} + t_{5\%} \frac{s_u}{\sqrt{n}}\right)$	429.3	-
11	$\psi_n\left(\frac{1}{2} s_u^2\right)$	2.370	From table 7
12	μ_L	253	-
13	μ_U	1,017	-

The last method for calculating confidence limits that is explained in this report was devised by Sichel (16-17), a pioneer worker in the application of statistical methods to the valuation of gold assay data from South African mines. In Sichel's method, outlined in table 15, the values of m and SS_u for the transformed variable u are obtained from tables 6 and 7. When the SS_u is divided by n (line 3 of table 15), a value of 1.83 is obtained. The next step is to enter Sichel's table B (17), using n and SS_u/n (in Sichel's notation, $SS_u/n = V$; this is not the variable V defined in this report) to obtain by interpolation two multiplying factors (lines 4 and 5), which are used to

obtain the confidence limits in lines 6 and 7 of table 15. For mathematical reasons explained in the section of this report on "Some Problems in the Application of the Lognormal Distribution to Gold Assay and Other Data," problems arise in the use of these tables.

TABLE 15. - Calculation of confidence limits for lognormally distributed observations by a method devised by Sichel

Line	Notation	Value	Calculation	Comments
1	m	507	From table 7	-
2	SS_u	23.793642	From table 6	-
3	SS_u/n	1.83	23.793642/13	V (in Sichel's notation) = SS_u/n
4	-	0.5124	From Sichel's table B ¹	Lower multiplying factor
5	-	4.015	From Sichel's table B ¹	Upper multiplying factor
6	u_L	260	507 × 0.5124	-
7	μ_U	2,036	507 × 4.015	-

¹Sichel, H. S. The Estimation of Means and Associated Confidence Limits for Small Samples From Lognormal Populations. Symp. on Mathematical Statistics and Computer Applications in Ore Valuation, Johannesburg, S. Afr. Inst. Min. and Met., 1966, pp. 106-122.

The estimated means and confidence limits obtained by the five methods are summarized in table 16, which shows that wide differences are obtained according to the method used. However, the lower bounds agree better than the upper ones, a fortunate circumstance for practical application in gold mining where the main concern is that minimum expected grade be estimated correctly. Among these five methods, the writers recommend the application of normal theory under the assumption that β^2 is known (table 9) and provided that a close estimate of β^2 is available from many observations and that a convincing case can be made that new observations in question come from the same statistical population. Otherwise, the writers recommend Mood's method of calculating a confidence region (tables 10 through 12). If the observations follow, or nearly follow, a lognormal distribution with β^2 larger than 0.9, the two methods that apply the t-distribution under the assumption that the observations follow a normal distribution (tables 13 and 14) are inefficient; however, if in fact the observations depart from a lognormal distribution, these methods yield conservative results because of their relative robustness. The authors believe that problems can arise in the use of Sichel's tables for reasons detailed in this report's section on Some Problems in the Application of Lognormal Distribution to Gold Assay and Other Data.

TABLE 16. - Comparison of confidence limits for lognormally distributed observations calculated by applying five different methods to example data from the Welkom mine, Republic of South Africa

Source table	Method	Estimated mean	Confidence limits	
			Lower	Upper
9	Apply normal theory, assume β^2 known....	347	218	554
12	Calculate confidence region.....	507	232	5,508
13	Assume \bar{w} normally distributed, use t based on \bar{w} and s_w^2	457	195	719
14	Assume m normally distributed, use t based on m and V^2	507	253	1,017
15	Use Sichel's approximation.....	507	260	2,036

GOLD IN RELATION TO THE LOGNORMAL DISTRIBUTION

Many natural substances commonly present in rocks in amounts of less than a few percent, such as gold, silver, uranium, and beryllium are distributed nearly or approximately lognormally. Because of the economic importance of gold, millions of gold assays have been made, and the distribution of gold has probably been studied more intensively than that of any other constituent of the earth's crust that is present in minute amounts. This section of the report considers the variability of gold assay data compared to variability of assay data for other metals, and some example data from two large gold mines are reviewed.

Variability of Gold Assay Data Compared to Variability of Assay Data for Other Metals

The variability of gold assay data, as measured by the coefficient of variation, is greater than that of base metal data (table 17). Base metal data are principally less variable, principally because the upper limit that corresponds to the metal content of the pure mineral (for example, 67 percent for sphalerite) is commonly less than 1,000 times the lower limit recorded (for example, 0.1 percent), and rock with a grade near the lower limit may not be sampled, because it can be rejected by visual inspection. The practical consequence of the high coefficient of variation for gold data, resulting from the highly skewed frequency distribution, is that it is expected to be more difficult to estimate grade in gold mines than in base metal deposits.

Considering gold data as a group, the coefficients of variation are also very different for different deposits. The less variable data presumably reflect less erratic mineralization. For example, at the Homestake mine, South Dakota, and at many gold mines in the Republic of South Africa, visible gold is common, and therefore, high-grade assays are to be expected. In contrast, no visible gold has ever been observed at the Getchell mine, Nevada. Obviously, the size of gold particles is related to geological conditions of formation, but discussion of this interesting question is beyond the scope of this report.

TABLE 17. - Coefficients of variation for assay data from selected ore deposits

Gold:	
Brown vein, Frisco mine, Chihuahua, Mexico.....	2.24
2137 vein, Fresnillo mine, Zacatecas, Mexico.....	1.24
Shamva, Southern Rhodesia.....	1.55
Mouat mine, Stillwater and Sweetgrass Counties, Mont.....	.37
Mt. View tungsten prospect, Hyder, Alaska.....	1.56
Manganese deposit, Phillipsburg district, Montana.....	1.81
Getchell mine, Nevada.....	1.15
Silver:	
Brown vein, Frisco mine, Chihuahua, Mexico.....	1.12
2137 vein, Fresnillo mine, Zacatecas, Mexico.....	1.07
Lead:	
Brown vein, Frisco mine, Chihuahua, Mexico.....	.57
2137 vein, Fresnillo mine, Zacatecas, Mexico.....	1.23
Zinc:	
Brown vein, Frisco mine, Chihuahua, Mexico.....	.85
2137 vein, Fresnillo mine, Zacatecas, Mexico.....	1.12

Example Data From the Homestake Mine, Lead, S. Dak.

The first group of example data was obtained from the Homestake mine, Lead, S. Dak., the largest gold mine of the United States. The Homestake mine has produced a total of about 96 million tons of ore containing some 28 million ounces of gold, an average of 0.29 ounce per ton (9.94 ppm). The ore is localized in parts of the Precambrian age Homestake Formation. The Homestake Formation, originally a low-grade iron formation, has been metamorphosed and intricately folded. Its present composition is mainly fluorite, cummingtonite (a calcium, magnesium amphibole), and quartz with subordinate ankerite, garnet, iron sulfide, and other minerals.

Ore in place, broken ore, millfeed, and mill products are extensively sampled at the Homestake mine. The ore in place is sampled by several different methods for the purposes of grade control and selective mining, including pillar location. The data discussed in this report come only from the diamond-drill sampling. Two groups of data are considered: a set of 219 assay values and a second set of 900 assay values.

The first group of 219 assay values was obtained from assays made on 56 feet of EX core from diamond-drill holes, ranging in length from 5 to 15 feet and bored by the company at Bureau of Mines expense for a designed sample experiment. The holes were drilled entirely in ore in the Homestake Formation at typical locations selected by G. T. Mathisrud, formerly assistant chief geologist, Homestake Mining Company. To compare the variability in gold content and gold distribution on a scale of a few inches with that on a scale of a few feet, three parallel holes spaced 3 inches apart were bored in one crosscut (No. 50, fig. 11), and two parallel holes spaced 6 inches apart were bored in an adjacent crosscut (No. 49, fig. 11).

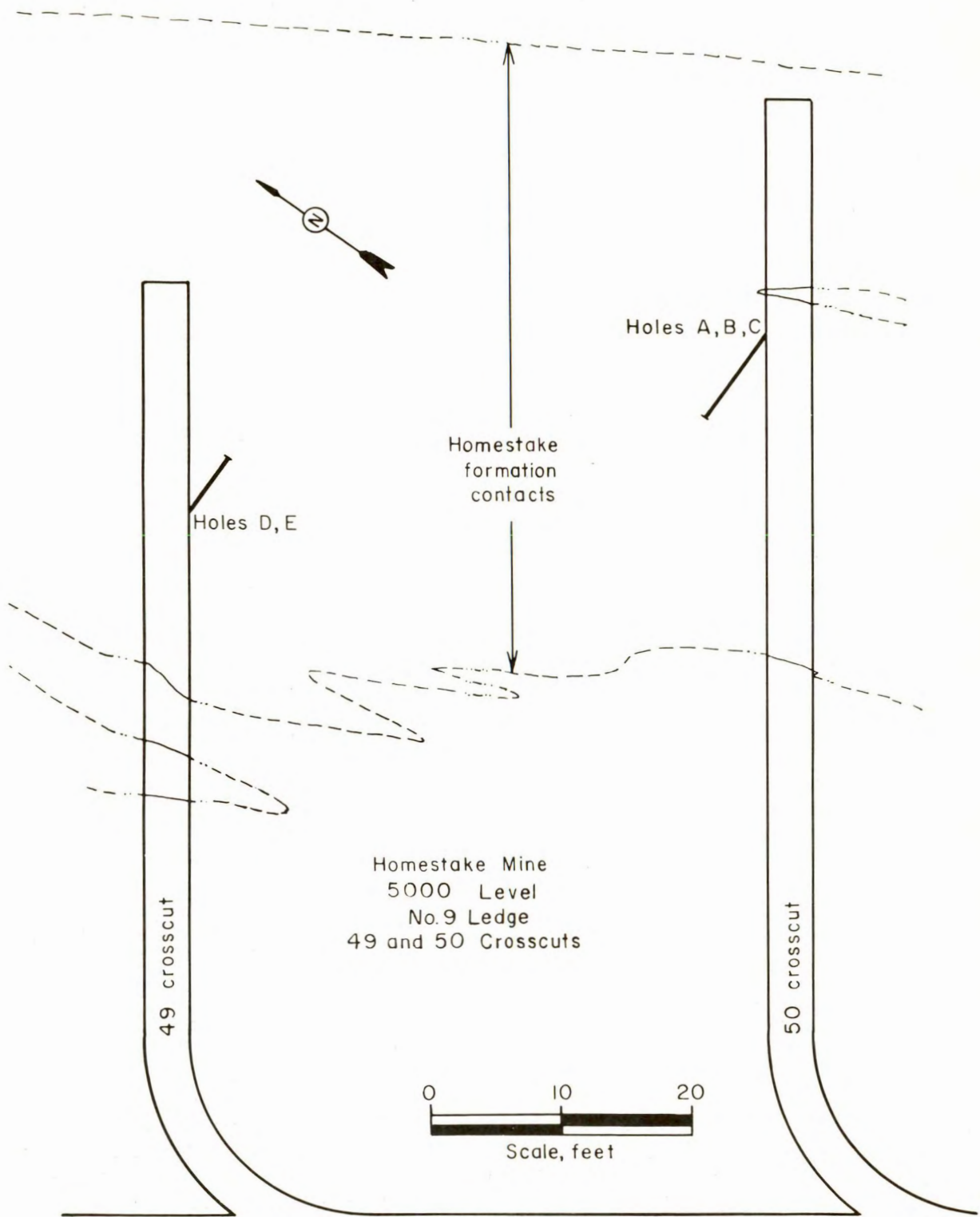


FIGURE 11. - Location of Diamond-Drill Holes Yielding 219 Assay Values for Experimental Sampling Study, Homestake Mine, Lead, S. Dak.

The drill core was cut transversely with a diamond saw into cylinders 1 inch long to obtain samples for assay weighing about 1 assay ton (29.1667 grams) each. Thus, no variation was introduced through splitting pulverized material or screening out coarse gold that could not be ground. Not all of the core was assayed, because sample preparation and assaying facilities were limited; instead, two random pairs of adjacent 1-inch pieces, a total of four potential samples out of a possible 12, were selected from each of the 56 feet of drill core. In this manner, 224, or 1/3 of the possible inches, were selected, and after losses in drilling and assaying, 219 assay values were obtained.

The second group of 900 assays were on diamond-drill core from EX core holes, drilled in the regular course of the company development of ore, at the locations shown in figure 12. These holes were also drilled in the Homestake Formation, and only core obtained at locations where ore was expected within the Homestake Formation was assayed.

The outstanding fact about both groups of data is that most of the gold is in only a few of the 1-inch cylinders, as is clearly displayed by ordering the assay values and plotting cumulative metal grade against number of observations. For the 219 observations, the resulting graph (fig. 13) shows that 50 percent of the observations with the lowest grade contain virtually no gold, whereas only 3 percent, 6 in number, of the highest grade observations contain 1/2 of the gold. For the 900 observations, the resulting graph (fig. 14) shows that, as before, 50 percent of the observations with the lowest grade contain virtually no gold, whereas 6.3 percent, 57 in number, of the highest grade observations contain 1/2 of the gold. As clearly shown by figure 15, the high-grade 1-inch cylinders of EX core are erratically distributed throughout the drill holes.

Using the computer program in appendix A, the summary data on table 18 were prepared and the confidence region for μ and σ were graphed in figure 16 according to Mood's method.

The means obtained from the two sets of data do not agree, and, moreover, the means estimated from the logarithms do not agree with those estimated from the raw data. It is interesting to consider briefly how these seemingly inconsistent results may be reconciled. The five diamond-drill holes that yielded the 219 observations were bored in an ore body selected as typical by the Homestake geological staff. The company assay data from this part of the mine indicate ore with a grade near that of the current mine average of about 0.33 ounce per ton (11.31 ppm); yet the average grade in each of the holes is higher. Based on the details of the company assay records and on visual examination of the crosscuts, some evidence holds that the holes were collared in higher than average grade ore. However, the confidence limits do not clearly indicate whether the holes in fact cut higher than average grade ore or whether their apparent highness in grade is merely due to statistical fluctuations and that the ore in the holes was in fact typical of that in the mine. Mined grade of ore is much more closely reflected by the set of 900 observations.

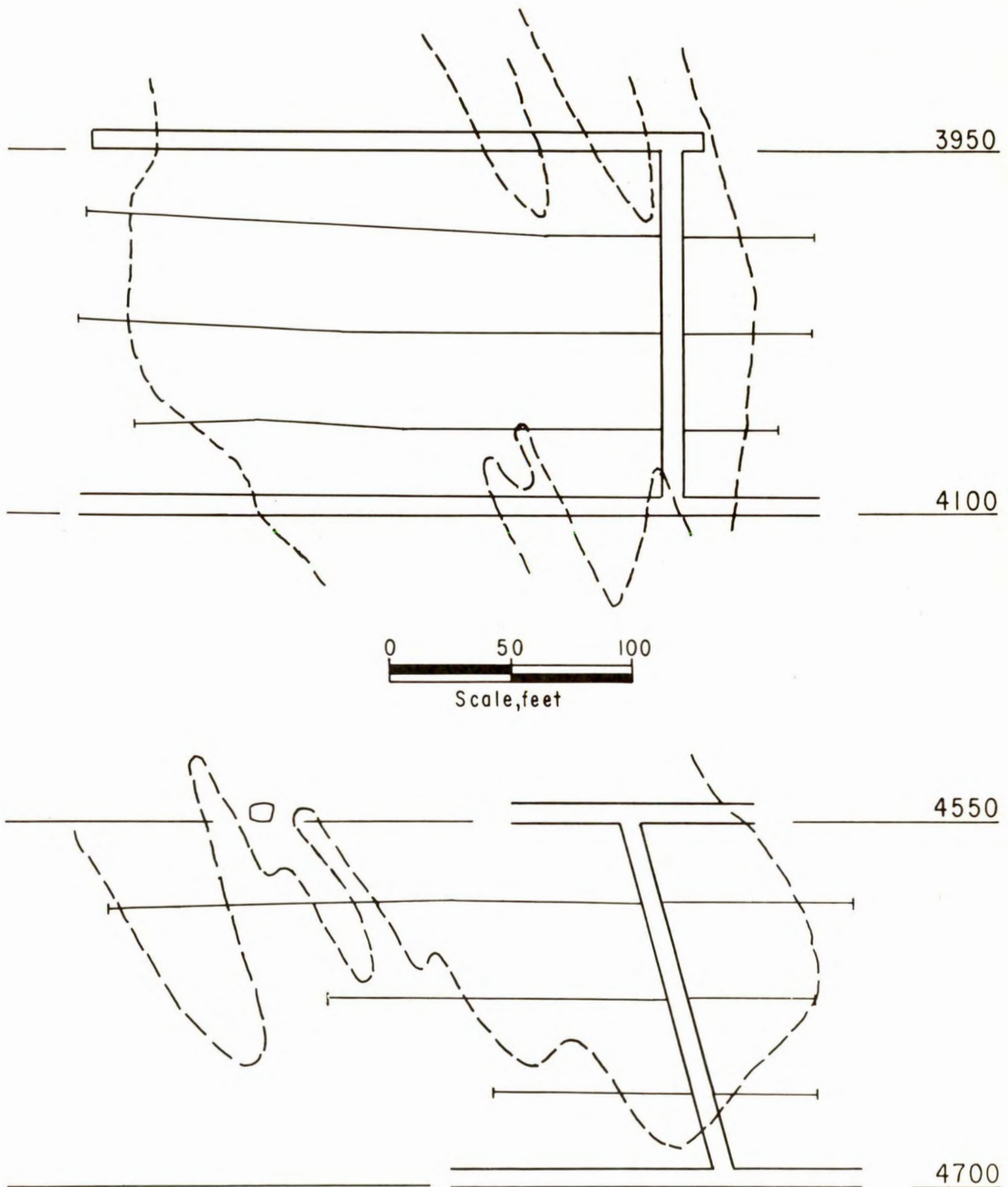


FIGURE 12. - Longitudinal Sections Showing Locations of Diamond-Drill Holes Yielding 900 Assay Values for Experimental Sampling Study, Homestake Mine, Lead, S. Dak.

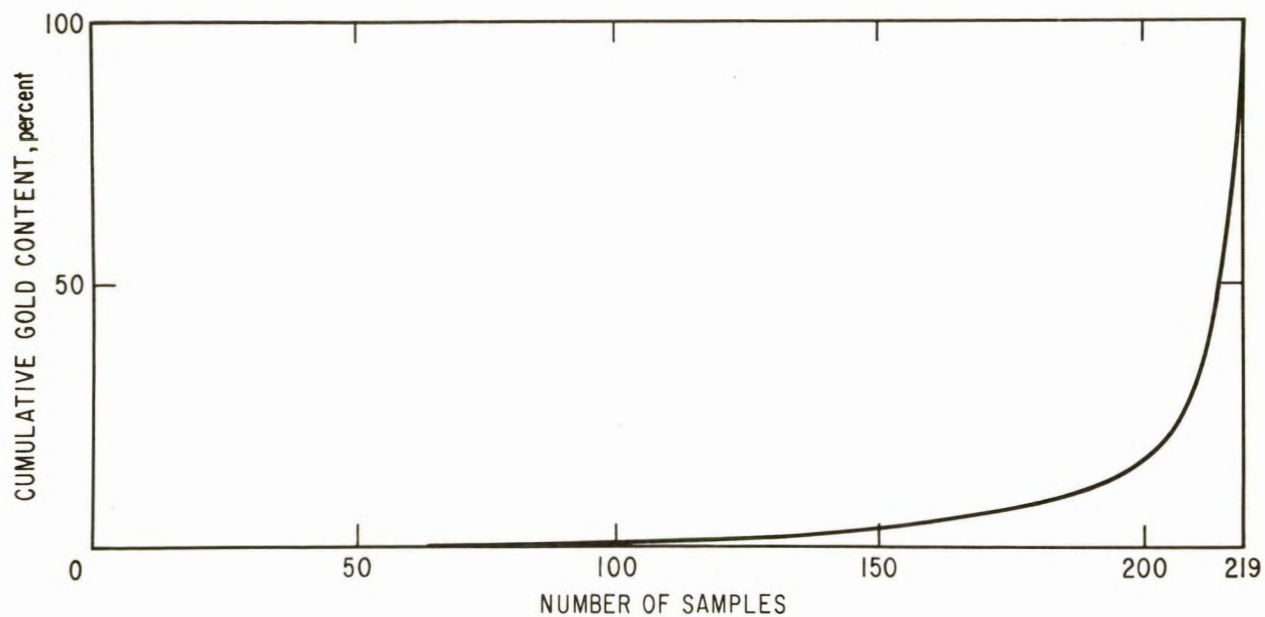


FIGURE 13. - Relation of Cumulative Gold Content to Number of Samples (Ordered by Assay Value) for Group of 219 Samples From Homestake Mine, Lead, S. Dak.

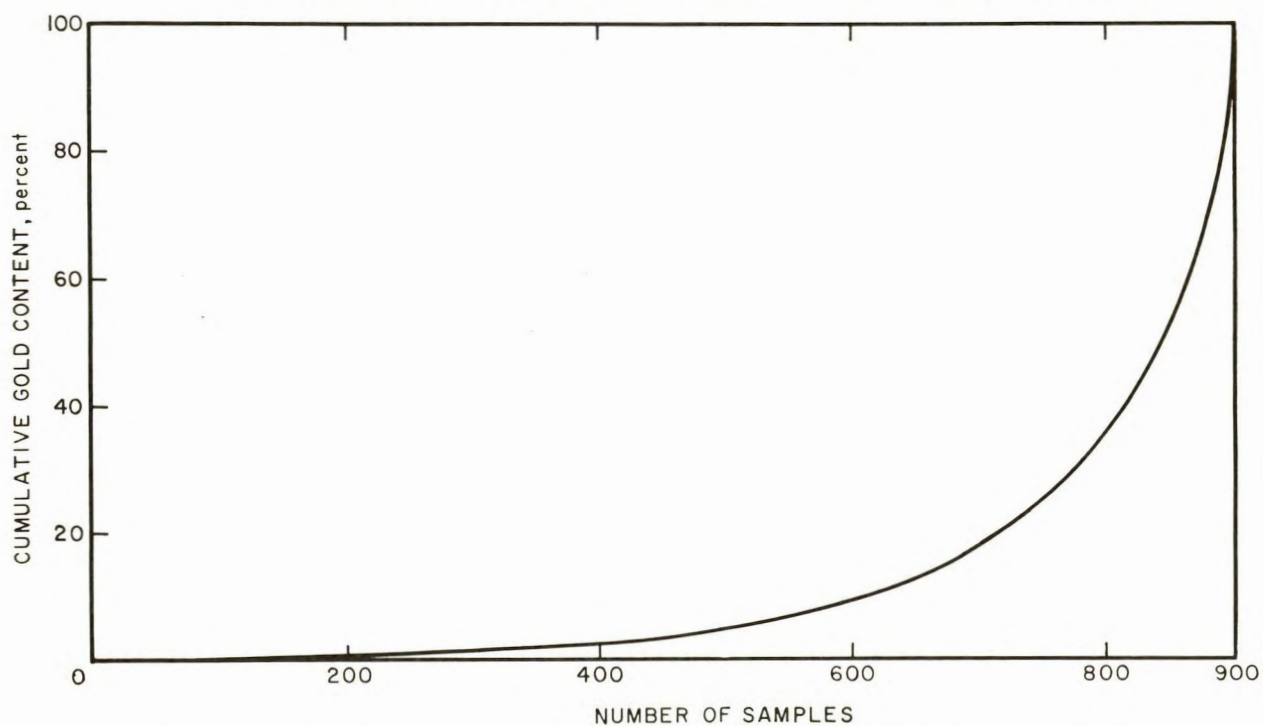


FIGURE 14. - Relation of Cumulative Gold Content to Number of Samples (Ordered by Assay Value) for Group of 900 Samples From the Homestake Mine, Lead, S. Dak.

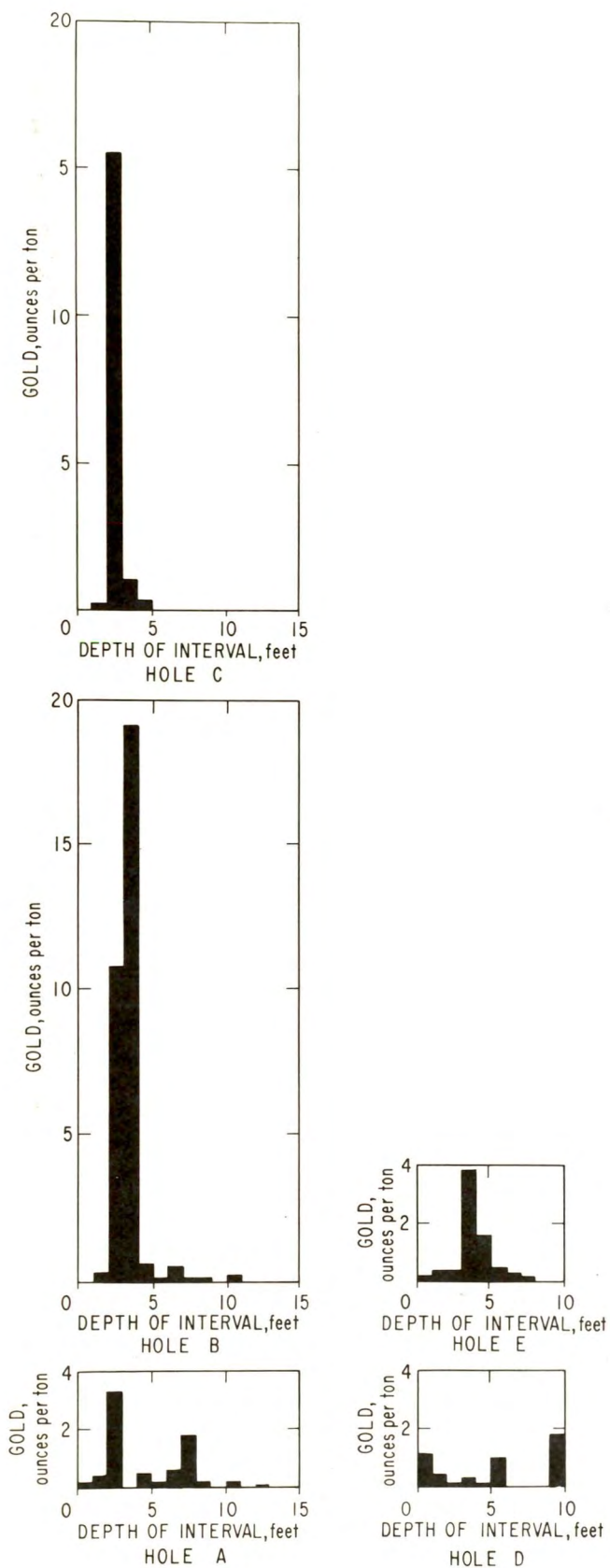


FIGURE 15. - Average Grade of Gold Assays at 1-Foot Intervals for Group of 219 Samples From the Homestake Mine, Lead, S. Dak.

TABLE 18. - Summary data calculated from two sets of gold assay data from the Homestake mine, Lead, S. Dak.

Item	Type of data			
	Gold, ppm		Gold, logarithms	
	Notation	Value	Notation	Value
219 OBSERVATIONS ON 1-INCH-LONG EX CORES				
Mean.....	\bar{w}	42.70	\bar{u}	1.036
Variance.....	s_w^2	28,609	s_u^2	5.6555
Standard deviation.....	s_w	169	s_u	2.378
Geometric mean.....	$e^{\bar{u}}$	2.817	-	-
Multiplying factor for geometric mean.	$\psi_n (\frac{1}{2}s_u^2)$	16.115	-	-
Estimate of mean from logarithms.....	m	45.4	-	-
900 OBSERVATIONS ON 1-FOOT-LONG EX CORES				
Mean.....	\bar{w}	7.59	\bar{u}	0.578
Variance.....	s_w^2	327	s_u^2	3.024
Standard deviation.....	s_w	18	s_u	1.739
Geometric mean.....	$e^{\bar{u}}$	1.783	-	-
Multiplying factor for geometric mean.	$\psi_n (\frac{1}{2}s_u^2)$	4.516	-	-
Estimate of mean from logarithms.....	m	8.1	-	-

A basic problem in estimating grade of gold ore is that as a consequence of the extremely skewed frequency distribution, there are always enough low-grade assays to obtain a close estimate of the mean of low-grade ore, but there are seldom, if ever, enough high-grade assays. In technical terms, the variance is not homoscedastic. This distinction is shown by table 19 which contrasts the statistics obtained by separating observations above and below 0.5 ounce. As pointed out before, the large number of observations permits a good estimate of range with narrow confidence limits for the low-grade material; but for the high-grade material, too few observations with a high standard deviation result in a poor estimate of grade with wide confidence limits.

The extreme variability in gold mineralization in the Homestake mine is revealed through the detailed sampling experiment from which the 219 1-inch cores were obtained for assay. This variability is that naturally present on the smallest scale that can be found by fire assay of one assay ton of ore. With cores of substantially larger volumes, as in the 1-foot increments, less variability is found.

TABLE 19. - Means, standard deviations, and confidence limits for samples above and below 17 ppm (0.5 ounce per ton) from set of 219 observations of gold assay data from the Homestake mine, Lead, S. Dak.

Item	Number of samples	Percent of samples	Mean gold assay, ounce per ton	Standard deviation	Confidence limits ¹	
					Lower	Upper
Samples <17 ppm...	166	76	0.0923	0.1204	0.0739	0.1107
Samples >17 ppm...	53	24	4.8583	9.1920	2.3204	7.3962
All samples.....	219	100	1.2457	4.9347	.5922	1.8993

¹These have been calculated as if random samples were involved in all cases.

This assumption is not strictly true for these data.

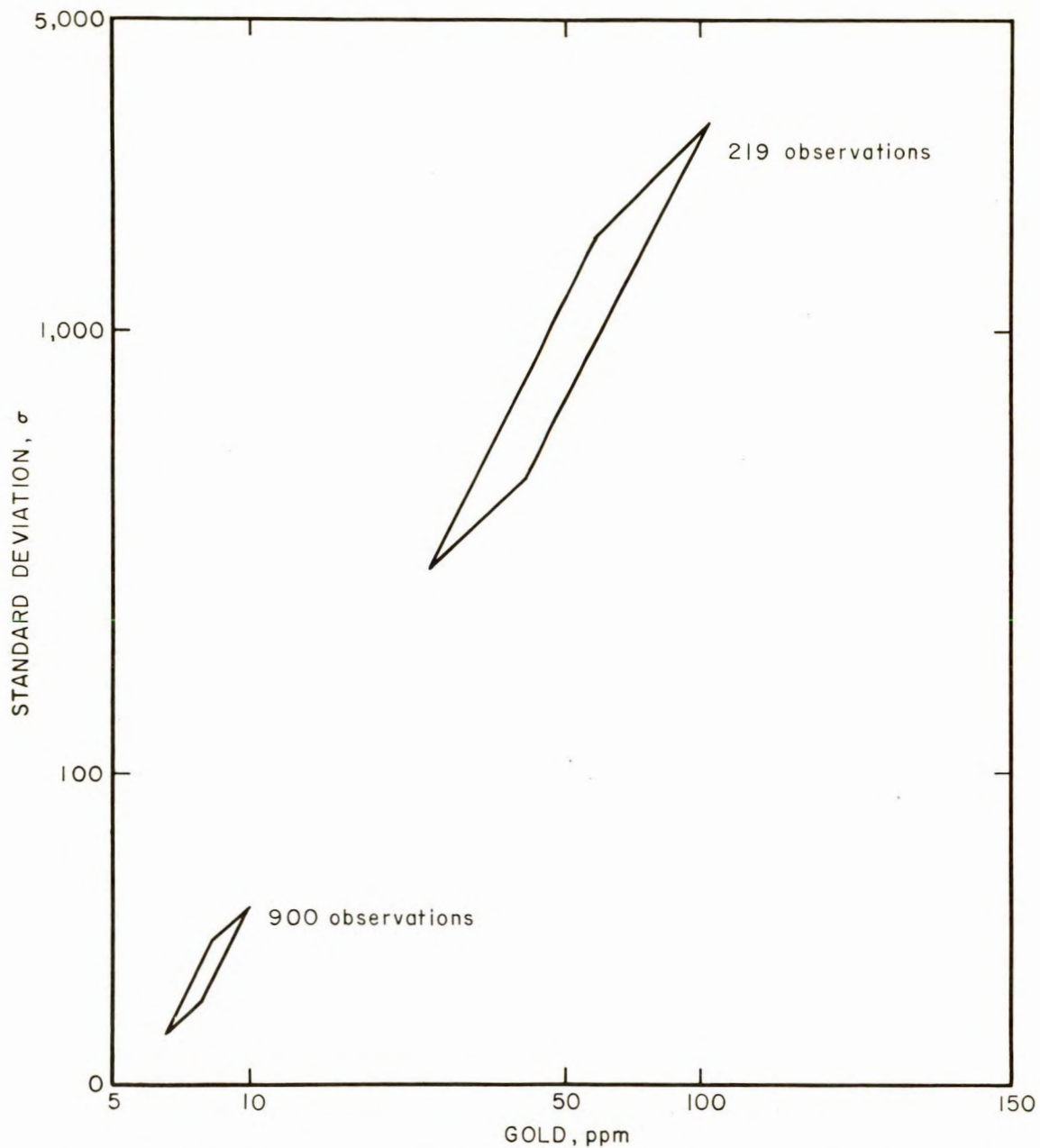


FIGURE 16. - Eighty-One-Percent Confidence Region in μ, σ Space for Groups of 219 and 900 Geological Samples From the Homestake Mine, Lead, S. Dak.

Example Data From the City Deep Mine, Central Witwatersrand,
Republic of South Africa

Another set of example data comprises gold assays from a block of ground in the City Deep mine, Central Witwatersrand, Republic of South Africa. The authors have published (7) an analysis of these data as they relate to guide stopping methods. In this report, only certain properties of the data connected with the lognormal distribution are summarized and interpreted.

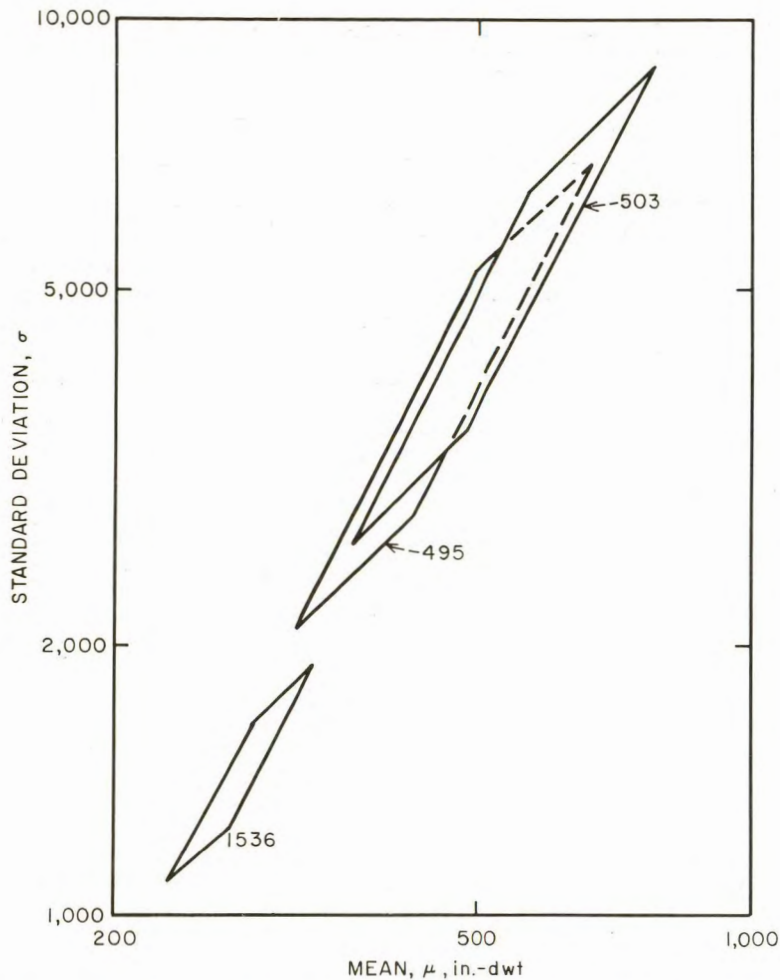


FIGURE 17. - Eighty-One-Percent Confidence Region in μ, σ Space for Groups of 1,536, 503, and 495 Observations From the City Deep Mine, Central Witwatersrand, Republic of South Africa.

these distributions are more or less lognormal. On the other hand, the distribution for inches is not straight, showing that these data are less skewed than they would be if lognormally distributed.

As in the Homestake mine, the outstanding fact about gold mineralization in the 1,000-foot-square block of the City Deep mine is that only a few of the observations account for most of the value. A plot of cumulative metal content (fig. 19) shows that fewer than 10 percent of the stope observations account for more than 50 percent of the total values. And when the 80 percent of sample points yielding the lowest in.-dwt values are averaged (table 21), these data, presumed to represent background mineralization, have a mean in.-dwt value of only 68.79 in the stopes. These results suggest that if the geographic locations of the best sample points could be predicted from the development assays, a mining strategy could be devised to extract most of the gold by stoping a relatively small part of the reef if other considerations,

Gold assay results were from a 1,000-foot-square block in the City Deep mine. The data comprised the results of 503 assays corresponding to the development samples (drift and raise samples) and 1,536 assays corresponding to the stope samples taken in the same part of the mine. For each sample, the company supplied the following information: north coordinate, east coordinate, width of the reef in inches, and gold in.-dwt value.

Table 20 presents calculations for 1,536 observations corresponding to stope data, for 503 observations from development data, and for 495 observations from development data selected for the purpose explained subsequently. The confidence regions calculated by Mood's method are graphed in figure 17.

Figure 18 shows that cumulative frequency distributions for in.-dwt and dwt values are essentially straight, indicating that

such as the necessity for longwall stoping were not predominant. Such a strategy is described in the original paper (7).

TABLE 20. - Summary assay data for a block of ground in the City Deep mine, Central Witwatersrand, Republic of South Africa

Item	Type of data			
	Gold, in.-dwt		Gold, logarithms	
	Notation	Value	Notation	Value
STOPE DATA, 1,536 OBSERVATIONS				
Mean.....	\bar{w}	192.40	\bar{u}	3.945
Variance.....	s_w^2	140,403	s_u^2	3.339
Standard deviation.....	s_w	375	s_u	1.827
Geometric mean.....	$e^{\bar{u}}$	51.675	-	-
Multiplying factor for geometric mean	$\psi_n (\frac{1}{2}s_u^2)$	5.294	-	-
Estimate of mean from logarithms.....	m	273.6	-	-
DEVELOPMENT DATA, 503 OBSERVATIONS				
Mean.....	\bar{w}	273.54	\bar{u}	4.056
Variance.....	s_w^2	369,381	s_u^2	4.387
Standard deviation.....	s_w	608	s_u	2.095
Geometric mean.....	$e^{\bar{u}}$	57.27	-	-
Multiplying factor for geometric mean	$\psi_n (\frac{1}{2}s_u^2)$	8.844	-	-
Estimate of mean from logarithms.....	m	510.6	-	-
DEVELOPMENT DATA, 495 OBSERVATIONS				
Mean.....	\bar{w}	227.38	\bar{u}	3.994
Variance.....	s_w^2	200,688	s_u^2	4.211
Standard deviation.....	s_w	488	s_u	2.052
Geometric mean.....	$e^{\bar{u}}$	54.262	-	-
Multiplying factor for geometric mean	$\psi_n (\frac{1}{2}s_u^2)$	8.103	-	-
Estimate of mean from logarithms.....	m	439.7	-	-

TABLE 21. - Summary assay data from 80 percent of sample points yielding lowest in.-dwt values (presumed to represent background data) in the City Deep mine, Central Witwatersrand, Republic of South Africa

Unit	Stope data			Development data		
	Mean	s	C	Mean	s	C
Dwt.....	3.93	5.82	1.48	6.48	9.99	1.54
In.	20.58	9.20	0.45	17.78	9.91	0.56
In.-dwt.....	68.79	71.12	1.03	91.27	106.32	1.16

Not only is the gold concentrated in only a few of the mine samples, but these samples were taken at points erratically located throughout the development workings, as shown in figure 20 which plots the locations of the 39 out of 503 development points accounting for 50 percent of the total in.-dwt gold content.

The data in table 20 indicate that the 274 in.-dwt estimate of grade from the development workings is a very poor measure of the 192 in.-dwt grade

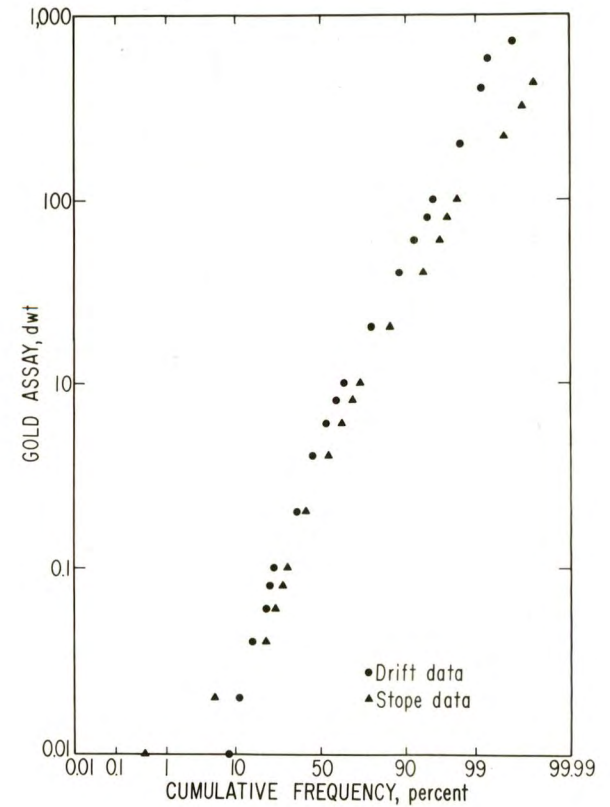
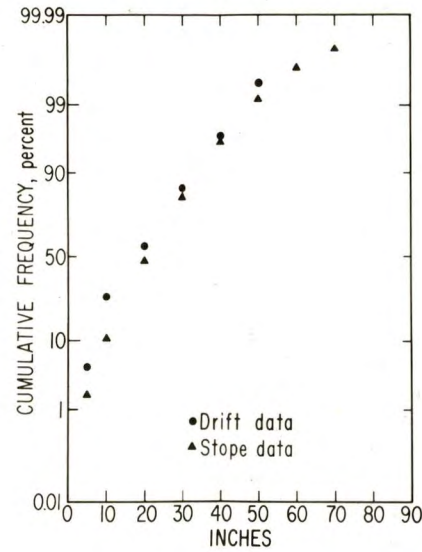
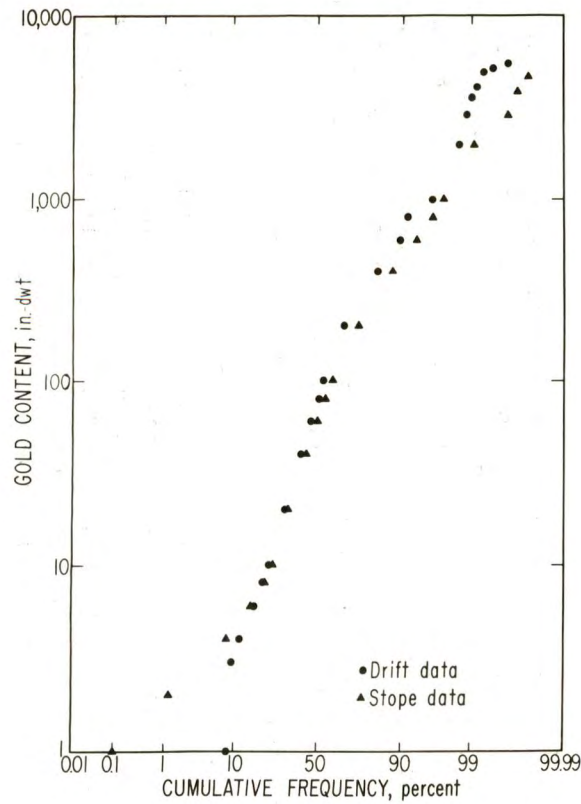


FIGURE 18. - Cumulative Frequency Distributions for In.-Dwt, Inch, and Dwt Data From the City Deep Mine, Central Witwatersrand, Republic of South Africa.

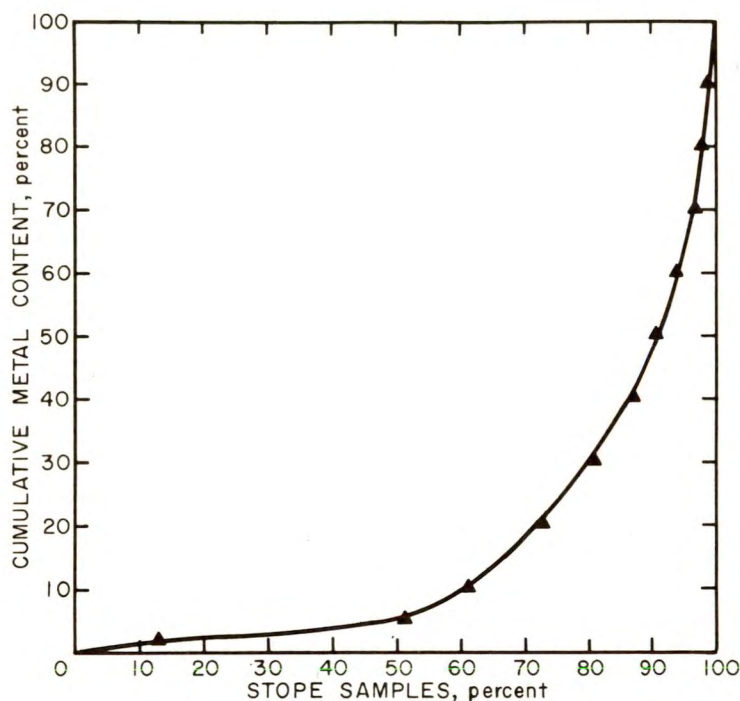


FIGURE 19. - Relation of Cumulative Metal Content to Percent of Stope Samples, City Deep Mine, Central Witwatersrand, Republic of South Africa.

vidual development observations are listed in order of increasing in.-dwt values, eight of the highest 15 are from observations in this cell. These eight observations were removed, and the means and standard deviations were recalculated; the results are presented in the last part of table 20. With 495 rather than 503 development observations, the new development mean of 227 in.-dwt is a much better estimate of the stope mean of 192 in.-dwt.

The calculations outlined in the previous paragraph were made to demonstrate that a few geographically contiguous observations can have a profound influence upon the grade estimates, although the writers certainly do not suggest that in practice these values should be removed. In fact, exclusion of fewer than 2 percent of the development observations concentrated in one working place changed the estimated grade some 46 in.-dwt, or 20 percent. But in actual mining practice, there would be no basis for deciding which, if any, of the sample values should be excluded, and in many mines, it is precisely the occurrence of such high values that makes the mine payable.

Through the use of confidence intervals, the stope and development data may be compared. The essential assumption of this method is that the sampling is random. Actually, the development observations and stope observations comprise nearly systematic samples with a random start rather than random samples. However, it has been found (6) that samples of this type taken elsewhere appear to behave as if they were random so that the assumption of randomness is reasonable. Table 22 shows that the confidence interval for the in.-dwt

actually mined in the stopes. Indeed, the difference between these two values is so large that the authors first speculated that a bias must have been introduced by the sampling or assaying or a mistake made in transcribing the data, but examination of the computer output revealed a simple explanation. When the 1,000-foot-square block was divided into 100 cells by a 10 by 10 grid, the summarized assay data indicated a single anomalously high summary gold value in one of these cells. Because this value, 1,267 in.-dwt, is the average of 24 original observations from samples cut at the end of the northeasternmost development drift, it must be accounted for and cannot be supposed to reflect merely one or two erratically high or wrongly recorded assays. When all 503 indi-

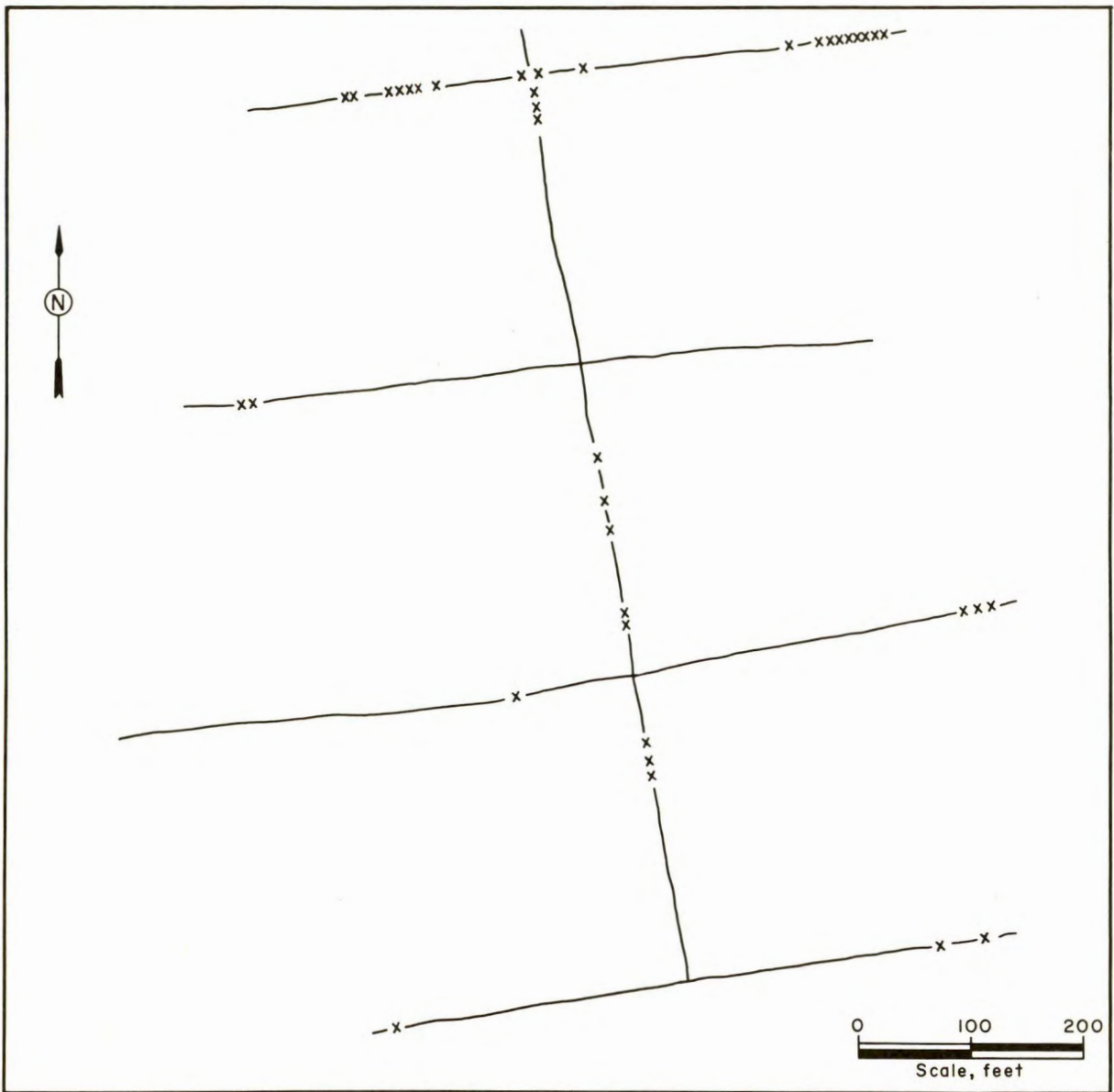


FIGURE 20. - Locations of the 39 out of 503 Development Points Accounting for 50 Percent of the Total In.-Dwt Gold Content, City Deep Mine, Central Witwatersrand, Republic of South Africa.

mean difference does not contain zero for the complete sample of 503 observations, but that it does contain zero for the sample with the eight observations removed.

TABLE 22. - Interval estimates comparing data from stopes and development workings in the City Deep mine, Central Witwatersrand, Republic of South Africa

Source of development data	Unit	Observed average difference between stope and development data	Confidence interval based on raw data	
			Lower limit	Upper limit
All data.....	Dwt....	10.6	4.2	17.0
	In. ...	-2.64	-3.66	-1.62
	In.-dwt	81	25	137
Data with eight high points removed.	Dwt....	3.6	.3	6.9
	In. ...	-2.47	-3.49	-1.45
	In.-dwt	35	-9	79

SOME PROBLEMS IN THE APPLICATION OF THE LOGNORMAL DISTRIBUTION TO GOLD ASSAY AND OTHER DATA

This section reviews four problems that arise in the application of the lognormal distribution to gold assay and other data: bias, departures of gold observations from the lognormal model, diamond-drill hole deflections, and the application of Sichel's tables. All of these problems are related in that the use of lognormal theory may increase efficiency of estimates, but the risk of bias must be recognized, and the probable amount of bias must be appraised.

Much of this section is a commentary on the work of H. S. Sichel (16-18) and D. G. Krige (9-13), pioneer investigators for many years in the interpretation through lognormal theory of gold assay data from South African mines. Their principal works are cited in the list of references, and these works contain references to their other publications pertinent to gold assay data. Some of the data discussed in this section are published data; others were supplied through the courtesy of Dr. Krige. Despite some criticisms the authors believe that Sichel's and Krige's work is the basis for present-day evaluation of lognormally distributed data in mining and geology and that their papers are indispensable to those interested in this subject.

Bias

In this subsection, bias is explained in more detail than in the introduction, and the illustrations demonstrate the problems that may be encountered with biased observations that are not exactly lognormally distributed. While for some data, lognormal methods may provide more precise statistics than the ordinary arithmetic mean, but if the distribution is not exactly lognormal, lognormal procedures may lead to biased results. Thus, unless there is a good theoretical basis or a historical record of empirical success with the method, applying the lognormal methods explained in this report must be made with care. Unfortunately, there is usually no way to decide if the lognormal distribution should be applied to a given problem without obtaining a great deal of data, sometimes more than would be normally gathered during the life of a modest sampling program. In this subsection, some of the

dangers of applying these procedures when they are inappropriate are illustrated. The effect of adding a constant to lognormal data when it is inappropriate to do so is examined, and then the effect of applying lognormal procedures when the distribution is inappropriate is briefly discussed.

Three sampling experiments were conducted to investigate the consequences of adding a constant to observations before applying the methods of estimation appropriate for lognormal data, when it was in fact inappropriate to add a constant. In the first two experiments, the observations follow a lognormal or modified lognormal distribution, and in the third, the observations follow the Chi-square distribution with 4 degrees of freedom which is skewed but not lognormal.

In the first sampling experiment, 1,000 samples of size 10 were taken from a modified lognormal population which was constructed by taking a lognormal population whose mean was 245 and whose variance of logarithms was 1 and subtracting 50 from all observations larger than 50 and replacing all observations smaller than 50 with the observation 1. The modified population with an arithmetic mean of 196 is of the kind postulated by Krige for gold observations that were originally lognormal but from which a constant has been removed with the smallest observations that the assayer would record as trace or 0 being replaced by an arbitrary small number, taken to be 1.

Table 23 gives the results of this experiment. The third column lists the estimated biases obtained by subtracting the means in column 2 from the theoretical mean of 196. The confidence intervals calculated for each sample of size 10 by Mood's method are nominal 90-percent intervals, so that about 10 percent are expected to fail to contain the population mean. The table shows that whether or not a constant is needed, using too large a constant does not on the average lead to much bias; but if one is needed, using too small a constant leads to intolerable bias. Similarly, the confidence intervals are more or less correct unless the constants are either too small or extremely large.

TABLE 23. - Statistics calculated in a sampling experiment to test the effect of adding a constant to a modified lognormal population

Constant	Average of 1,000 estimated means, assuming lognormality	Estimated bias	Percent of 90-percent confidence intervals that do not contain the population mean		Variance of logarithms
			Lower	Upper	
0	346	150	10.9	0.1	4.28
5	229	33	5.4	.2	2.33
10	210	14	5.2	.8	1.81
25	193	-3	5.2	2.7	1.18
50	186	-10	5.5	5.1	0.78
100	185	-11	5.7	8.6	.47
500	189	-7	8.0	15.1	.09
1,000	191	-5	8.5	16.3	.04

In the second sampling experiment, the effect of adding a constant when it was inappropriate to do so was investigated for a lognormal distribution with a mean of about 9.5 and with a mean and variance of logarithms both equal to 1.5. For different sample sizes and constants, table 24 gives the biases of estimates of means based on lognormal theory when constants are added erroneously to an originally lognormal distribution. The estimated biases, each based on 100 samples of the indicated sizes, are added to the means estimated with the specified constants to obtain the true means estimated without adding constants. These biases are not exact, because they depend on the particular random numbers chosen, but they do indicate the magnitude of the biases obtained. The table shows that biases of 10 percent can easily be introduced, and moreover, when the sample sizes are increased, the biases do not disappear as might be expected, but rather they are smallest for the samples of smallest size. For samples of sizes one and two, the biases are 0 regardless of the sizes of the constant added, because the arithmetic mean and the mean based on logarithms must be identical for those samples sizes.

TABLE 24. - Biases (for different sample sizes and constants)
of estimates of means based on lognormal theory
when constants are erroneously added to an
originally lognormal distribution with
 $\alpha = 1.5, \beta^2 = 1.5$

Sample size, n	Constants				
	0	1	2	5	10
3	0.00	0.17	0.21	0.19	0.12
4	.00	.30	.37	.36	.27
5	.00	.68	.84	.89	.81
6	.00	.69	.84	.90	.81
7	.00	.65	.78	.81	.70
8	.00	.63	.76	.78	.67
9	.00	.63	.75	.76	.64
10	.00	.62	.73	.72	.59
25	.00	.70	.79	.75	.59
50	.00	.79	.90	.85	.68
100	.00	.80	.91	.87	.71

In the third sampling experiment, some estimates were made based on logarithms when the observations did not follow a lognormal distribution. In order to make these estimates, samples were drawn from a Chi-square distribution with 4 degrees of freedom, and lognormal theory was applied. Interestingly, the estimates based on lognormal theory exhibited little bias. Thus, at least for this particular distribution, estimates based on the lognormal distribution are evidently rather robust, rather than requiring the observations to follow the lognormal distribution closely. Further work in this field is beyond the scope of this report but would be useful and could be based on Chi-square distributions with other than 4 degrees of freedom on the Weibull distribution, etc.

Departures of Gold Observations From the Lognormal Model

For some gold assay data, Krige postulates that the observations are too small to be lognormally distributed by an amount that is constant, or nearly constant, regardless of the sizes of individual observations. He suggests that a constant amount of gold might have been leached from the ore after its formation, but as he points out, a physical explanation is unnecessary. If gold observations do in fact depart from the lognormal model, lognormal theory is not entirely satisfactory, because the estimates of the mean are too large unless adjustments are made. Figure 21 which is plotted on logarithmic probability paper and table 25 portray a lognormal distribution with two different constants added; they illustrate that adding either a positive or a negative

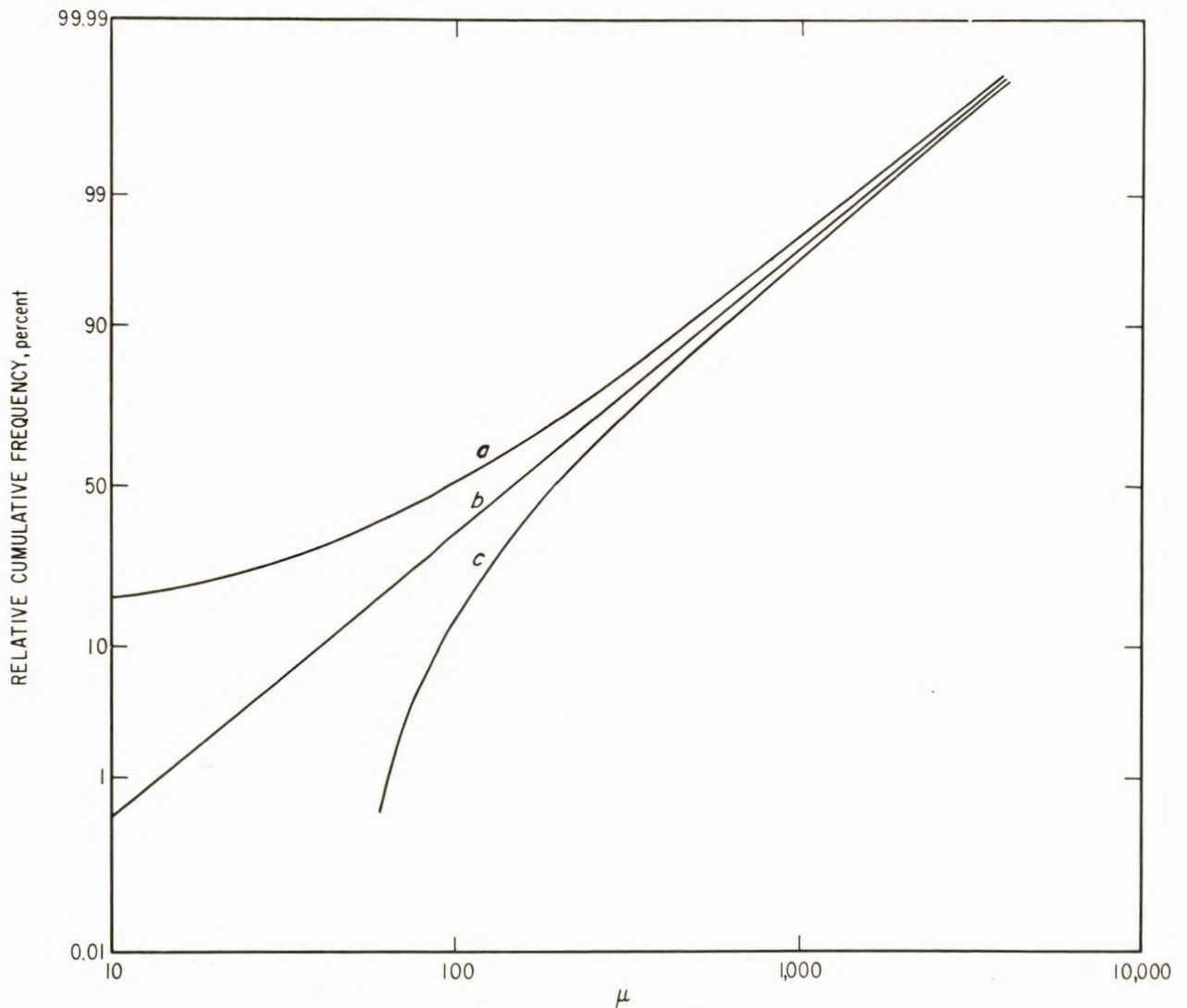


FIGURE 21. - Cumulative Frequency Distributions of Three Distributions: *a*, a Lognormal Distribution With a Constant of 50 Added; *b*, a Lognormal Distribution; *c*, a Lognormal Distribution With Constant of 50 Subtracted.

constant to observations that are originally lognormal makes the resulting distributions nonlognormal. On the other hand, if all of the original observations are too small by the same amount, adding the proper constant would make their distribution lognormal, as in changing from curve a to curve b in figure 21.

TABLE 25. - Frequency table for three distributions obtained by adding constants of -50, 0, and +50 to a lognormal distribution with $\alpha = 5$, $\beta^2 = 1$

Class interval	Relative cumulative frequency, percent		
	A(-50)	B(0)	C(+50)
0- 10	18.77	0.43	0
10- 20	23.20	2.07	0
20- 30	27.01	5.26	0
30- 40	31.15	9.55	0
40- 50	35.32	14.38	0
50- 60	38.76	18.77	0.43
60- 70	42.03	23.20	2.07
70- 80	45.10	27.01	5.26
80- 90	47.80	31.15	9.55
90- 100	50.75	35.32	14.38
100- 200	70.30	62.02	50.75
200- 300	80.71	76.46	70.30
300- 400	86.60	83.93	80.71
400- 500	90.47	88.75	86.60
500- 600	93.11	91.88	90.47
600- 700	95.00	94.09	93.11
700- 800	96.14	95.70	95.00
800- 900	96.98	96.69	96.14
900-1,000	97.68	97.34	96.98
1,000-2,000	99.55	99.51	99.49
2,000-3,000	99.89	99.88	99.87
3,000-4,000	99.94	99.94	99.93
4,000-5,000	99.99	99.98	99.98
5,000-6,000	100.00	100.00	100.00

Krige (11) discusses various ways to compute the constant and concludes that the preferred method is as follows:

Using logarithmic-probability paper, the basic distribution of z [the logarithm of the original observation] is first plotted, and then the distribution of $(z + \alpha)$ with the constant α [not the α of this report] being increased in stages until the resultant plot yields the most satisfactory straight line fit. For all practical purposes this procedure will be found adequate, as the fit is evidently not very sensitive to small variations in the value of α either side of the optimum value.

Table 26 summarizes data on 28 distributions of gold assay data from South African mines, originally published by Krige (11) whose paper gives the names of the mines and reefs, reef areas, frequency distributions, and other data. Four kinds of means are tabulated in table 26; the first three are the arithmetic mean, the lognormal mean computed without a constant, and the lognormal mean computed with a constant. The fourth kind of mean is based on mine production data, which were kindly furnished by Dr. Krige (1967, personal communication), who made the following comments about them:

Production data reasonably representative of the specific mine section covered by the distributions were available only for the nine mines shown. In.-dwt of gold actually accounted for on surface in gold recovered and in slime residues dump [are tabulated] corresponding approximately to ore from these sections of the mine represented by the distribution shown. The difference between this value and the mean of the distribution is due to a.--Bias in normal cutting of underground samples (generally negative), b.--Gold actually lost in mining operations (negative), and c.--Improvements (positive) due to selectively mining only ore above the accepted pay limit (significant mainly for mines numbers 27 and 28).

The table shows that on the average, 90.3 percent of the predicted value is obtained in mining. Finally, the coefficients of variation in table 26 are approximations, because they are calculated from standard deviations obtained from classed data in Krige's frequency tables rather than from the original observations.

What can be learned from table 26? First of all, the means estimated by lognormal theory from observations formed by adding a constant to each original observation are nearly the same as the arithmetic means, if appropriate constants are chosen. Although a constant is in principle chosen, as explained by Krige (11), in order to make a distribution that is as nearly lognormal as possible, in practice, subjective judgment is required. Table 26 shows that the estimates of grade obtained by adding a constant are generally slightly smaller than the arithmetic means; the constants may have been chosen to obtain these slightly conservative values, because most of the mine production grades are still lower.

Figure 22 plots on logarithmic probability paper the development values of ore from 16 mines in table 27. Because they are estimated from many observations for each mine, these means should be reliable; and because they are distributed approximately lognormally, a perhaps reasonable hypothesis is that all of these mines work ore whose gold assay values belong to a common lognormal population. This view can be restated as follows: the distribution of gold in these mines working several reefs constitutes one enormous lognormal population, and the differences in average grade from mine to mine represent natural statistical fluctuation on a large scale. For the entire postulated population, the mean grade based on lognormal theory is 485 in.-dwt (arithmetic mean of 490 in.-dwt) which may approximate the average grade of the reefs although the data are not weighted by the sizes of the mines.

TABLE 26. - Comparison of four kinds of means for 28 distributions of gold assay data

Distribution No.	Number of observations	Arithmetic mean, in.-dwt	Lognormal mean, in.-dwt		Constant, in.-dwt	Mine production grade, in.-dwt	Percent of predicted grade ¹	Coefficient of variation
			Without a constant	With a constant				
1	216	308	406	304	40	-	-	1.29
2	3,600	477	478	472	5	-	-	1.92
3	4,200	90	91	91	10	-	-	1.03
4	1,648	373	396	366	10	-	-	1.52
5	9,599	301	366	304	80	-	-	1.25
6	2,477	307	667	299	90	-	-	1.35
7	1,003	240	293	236	30	-	-	1.07
8	964	259	320	320	0	-	-	2.93
9	2,840	² 126	² 129	125	2	-	-	1.40
10	28,334	680	681	681	0	630	93	1.15
11	25,474	341	344	339	7	283	83	1.20
12	22,500	441	467	446	20	396	89	.89
13	1,000	448	449	444	20	-	-	1.08
14	14,920	537	550	534	30	-	-	.98
15	25,112	477	497	478	25	-	-	1.02
16	568	142	147	143	30	-	-	.80
17	517	³ 210	242	204	4	-	-	2.81
18	158	299	310	296	60	-	-	.86
19	12,238	1,230	1,244	1,229	12	-	-	1.81
20	17,726	934	989	931	35	-	-	1.28
21	33,031	319	358	319	50	254	80	.99
22	14,590	1,103	1,283	1,113	95	981	92	1.19
23	740	1,026	1,234	1,019	60	-	-	1.58
24	19,687	499	505	499	4	415	83	.87
25	44	362	445	356	25	-	-	1.59
26	8,436	559	610	559	70	492	88	.95
27	197	219	244	225	20	248	110	.99
28	1,000	261	339	257	55	245	95	1.35

¹Ratio of mine production mean to lognormal mean with a constant.

²Dwt instead of in.-dwt for this mine only.

³Based on value of all development to December 1959.

TABLE 27. - Comparison of borehole and development values from
16 representative South African gold mines

Mine number	Number of bore- holes	Number of original inter- sections plus deflec- tions	Coefficient of variation		Mean gold, in.-dwt		Devel- opment value in.-dwt	Ratio of development value to mean, percent	
			Based on col. (2)	Based on col. (3)	Arith- metic	Log- normal		Arith- metic	Log- normal
			(4)	(5)	(6)	(7)		(8)	(9)
1	5	8	0.60	(¹)	219	231	163	74	71
2	3	6	.60	1.24	140	147	320	229	218
3	3	7	1.78	.70	185	197	237	128	120
4	5	12	1.82	1.96	5,459	2,914	1,229	23	42
5	7	21	1.92	2.19	1,175	832	864	74	104
6	13	19	1.47	1.44	457	451	320	70	71
7	11	8	1.11	.96	267	272	502	188	185
8	5	15	1.04	1.15	1,238	1,218	1,107	89	91
9	5	-	.60	.62	952	1,006	468	49	47
10	5	11	(¹)	(¹)	270	254	185	69	73
11	4	8	1.02	.90	175	181	250	143	138
12	5	3	.92	1.15	606	583	332	55	57
13	2	8	.42	.32	206	237	545	265	230
14	5	16	.92	.92	234	240	475	203	198
15	5	-	.85	.64	184	230	432	235	188
16	5	-	(¹)	(¹)	176	186	404	230	217

¹Coefficient of variation not available.

Diamond-Drill Hole Deflections

Because an important use of lognormal theory has been to obtain estimates of grades based on diamond-drill-hole data, a more detailed examination of these data is pertinent to this report. Previously in this report, one drill hole was considered to have only one observation associated with it. But, actually for many drill holes more than one observation was originally available, because of the practice of deflecting holes. A deflection is obtained when the drill bit is forced out of the original borehole by placing a wedge-shaped piece of metal in the hole which causes the bit to cut a new hole nearby. Thus, another geological sample is obtained, and according to the number of deflections made, two or more original observations are obtained for a single hole. This subsection considers ways to improve estimates of means by making use of these original data rather than by averaging them together to obtain a single observation.

The deflection data discussed are from South African gold mines. Some of the data are from papers by Krige, of which two (9, 13) are particularly interesting. Other data are from a large set of unpublished borehole data that Dr. Krige kindly sent to the authors.

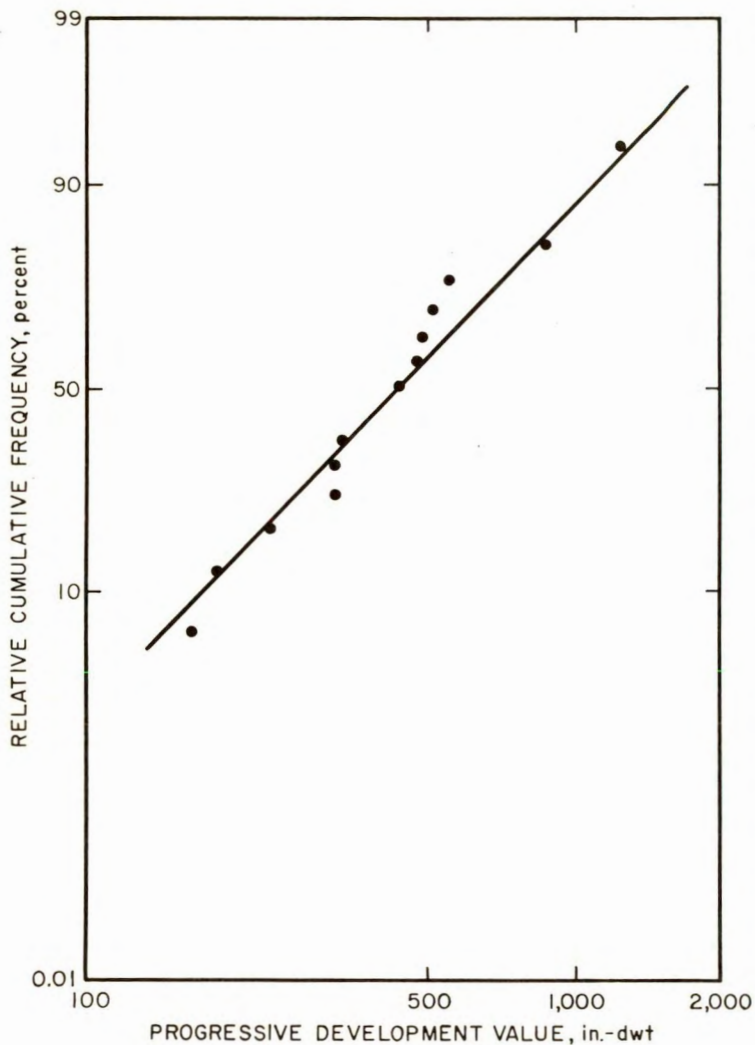


FIGURE 22. - Cumulative Frequency Distribution of Progressive Development Values From 16 Mines in Table 27.

while those for other holes are based on one or more deflections and hence should have smaller variances. Although the means calculated by method number 2 are biased (because a bias is introduced through the decision that is made to deflect some holes but not others), this bias is also present, although concealed, if one observation is set equal to one hole and also if the information on variability is ignored. However, the lower confidence limits calculated by the four different methods fortunately do not seriously disagree, because the effects of lowering the mean and reducing the variance of logarithms tend to compensate for each other. Further research on more data sets is needed to evaluate these tentative conclusions.

Further calculations are made on the data from table 28 and for drill holes in another South African mine in order to compare results with those of Krige and Sichel. The data averaged to obtain one observation per borehole are listed in table 29. The first four sets of data were discussed by

For borehole deflection data, there are four obvious methods to estimate means: (1) averaging the deflection values within each hole to obtain one observation for each hole and adding no constant, (2) retaining the deflection values within each hole to obtain one observation for each deflection and adding no constant, (3) like method (1) but with a constant added, and (4) like method (2) but with a constant added. In table 28, results of estimation for boreholes in four South African mines are compared for the four methods of calculation. Except for the Welkom data, treating each deflection value as an observation and using no constant, yields lower arithmetic estimates of the mean and also smaller lower confidence limits. Therefore, the authors suggest that method (2) is preferable, because the results are the most conservative. Treating the data as one observation per borehole averages a mixture of observations with different variances for the different holes, because the observations for some holes are based on no deflections

Krige (11) and the last set by Sichel (18); results of calculations of means and lower confidence limits by several methods are given in table 30.

TABLE 28. - Comparison of statistics calculated in different ways from boreholes in four South African gold mines

Type of data	Number of observations, n	Constant	Arithmetic mean	Mean based on lognormal distribution	Estimate of variance of logarithms	Lower confidence limit ¹
FREE STATE GEDULD MINE						
1 observation/hole.....	5	0	5,459	4,297	2.791	1,327
1 observation/deflection	7	0	4,243	3,147	1.932	1,288
1 observation/hole.....	5	60	5,459	4,246	2.553	1,349
1 observation/deflection	7	60	4,243	3,123	1.774	1,302
WELKOM MINE						
1 observation/hole.....	13	0	457	507	2.016	232
1 observation/deflection	21	0	477	717	3.065	291
1 observation/hole.....	13	60	457	451	1.049	243
1 observation/deflection	21	60	477	448	1.060	263
WESTERN HOLDINGS MINE						
1 observation/hole.....	7	0	1,175	809	1.439	382
1 observation/deflection	12	0	806	560	1.077	325
1 observation/hole.....	7	60	1,175	832	1.257	387
1 observation/deflection	12	60	806	572	.884	331
HARTEBEETSFontein MINE						
1 observation/hole.....	5	0	184	184	0.052	140
1 observation/deflection	16	0	176	187	.674	129
1 observation/hole.....	5	40	184	184	.035	136
1 observation/deflection	16	40	176	179	.326	133

¹Computed by Mood's method.

TABLE 29. - Averaged data, one observation per borehole, for five South African gold mines

Harmony mine, group No. 637 ¹	Free State Geduld mine ²	Welkom mine ²	Western Holdings mine ²	Hartebeetsfontein mine ²
329	1,084	154	6,399	242
277	1,747	1,252	237	211
111	1,202	308	418	136
189	23,037	15	280	160
5,071	227	525	182	171
-	-	377	383	-
-	-	109	327	-
-	-	48	-	-
-	-	1,560	-	-
-	-	70	-	-
-	-	1,221	-	-
-	-	237	-	-
-	-	68	-	-

¹Sichel, H. S. The Estimation of Means and Associated Confidence Limits for Small Samples From Lognormal Populations. Symp. on Mathematical Statistics and Computer Applications in Ore Valuation. Johannesburg, S. Afr. Inst. Min. and Met., 1966, pp. 106-122.

²Krige, D. G. Developments in the Valuation of Gold Mining Properties From Borehole Results. Seventh Commonwealth Mining and Metallurgical Congress, Johannesburg, Republic of South Africa, preprint, 1961, 23 pp.

TABLE 30. - Comparison of means and lower confidence limits for lognormally distributed data by applying four methods to data from five South African gold mines

Source of data	Number of observations	Con-stant	Arith-metic mean	Mean based on lognormal distribution	Lower confidence limit			
					Student's t applied to lognormal	Mood's method	Sichel's method (1966)	Sichel's method (1962)
Harmony....	5	0	1,195	864	210	309	320	190
	5	80	1,195	894	198	320	320	176
Free State. Geduld.....	5	0	5,459	4,297	875	1,327	1,444	788
	5	60	5,459	4,246	875	1,349	1,424	789
Welkom.....	13	0	457	507	253	232	260	220
	13	60	457	451	248	243	266	230
Western.... Holdings...	7	0	1,175	809	336	382	385	314
	7	60	1,175	832	332	387	367	311
Hartebeets- fontein...	5	0	184	184	147	140	158	140
	5	40	184	184	148	136	158	135

As shown in table 30, basing the mean on the lognormal distribution yields various results; sometimes the mean increases, sometimes it decreases, and sometimes it stays the same. For three of the mines, the means estimated by the lognormal distribution are lower than the arithmetic means, but on the average both kinds of estimates must yield the same mean provided that the observations are lognormally distributed.

There are four ways of calculating the lower confidence limit (with a nominal confidence coefficient of 95 percent): first, is to assume that the observations, m , that are estimated from the logarithms are normally distributed and to use the t -statistic as outlined in table 14; second, to use Mood's method; third, to use Sichel's 1966 tables; and fourth, to use Sichel's 1962 tables. The most conservative estimate of the lower confidence limit is that closest to 0, and for these data, this estimate is given by Sichel's 1962 tables, followed by the t -statistic method, Mood's method, and finally by Sichel's 1966 tables. Whether or not the constant is added makes little difference. As previously stated, the authors prefer Mood's method and therefore, regard Sichel's 1962 method and the method based on the t -statistic as too conservative while Sichel's 1966 method as not conservative enough. In summary, for these data which Sichel and Krige evidently believe are typical, Mood's method and Sichel's 1966 tables yield similar results and addition of the constant is unimportant.

Further Comments on Sichel's Tables

The numerical values in Sichel's 1966 tables (18) have been verified by Nicholas Thireous (1968, personal communication) of the Kennecott Copper

Corporation, Salt Lake City, Utah. For those confidence levels calculated by Sichel but not for others, Thireous has also developed formulas to interpolate in the tables for different numbers of observations and different values of SS_u/n (Sichel's variable V). Thireous's method of fitting polynomial equations to Sichel's percentage point T_p solves the problem of interpolation although his results are unpublished.

Several problems with Sichel's tables lead to preference for Mood's method when the choice is between these two. The first problem stems from Sichel's assumption of a population variance of logarithms β^2 of 0.7 which corresponds to a coefficient of variation δ of 1.0. Not only is this value of 0.7 lower than most of the published values (tables 26 through 28), but also for data with so low a coefficient of variation, lognormal methods of estimation offer little or no advantage. Although Sichel's tables could be expanded to a family of tables for different values of the population variance of logarithms β^2 , the problem of deciding which table to use would remain, because the small sample sizes provide values of V (in Sichel's notation) which are not reliable estimates of β^2 . [Sichel states (personal communication, 1969) that his approximation works well for variances of logarithms as large as 1.5.]

The second problem arises from Sichel's assumption that his t -statistic (not the variable t in the notation of this report) is approximately normally distributed. This assumption appears to fail for large values of the population variance of logarithms β^2 even if the assumption of a lognormal distribution for the observations is fulfilled.

A third problem stems from Sichel's apparent assumption of the independence of the distributions of his t -statistic and his V -statistic (not the variable V in the notation of this report) which is an estimate of the variance of the logarithms. This assumption probably is not true.

Because of these problems, the writers conclude that although Sichel has made a step forward, Mood's method yields results that are more conservative in setting the lower confidence limit, more reliable, and require fewer assumptions. The main drawback to Mood's method--the tedious calculations that are required--can be eliminated by using the computer program in appendix A of this report.

SUMMARY AND CONCLUSIONS

In this report, a detailed study of the lognormal distribution in relation to gold assay data has been presented, and different computations on several sets of gold assay data are illustrated. The computer program, appendix A, permits investigators with access to a computer to obtain several kinds of pertinent numerical results that they can interpret depending on their decisions about which of the assumptions are satisfied. The functions tabulated in appendix B should permit investigators without access to a computer to work more readily with the lognormal distribution than was previously possible.

The following conclusions are developed from this investigation:

(1) The logarithmic transformation should be used with care. If the lognormal distribution is inappropriate and especially if the data are somewhat truncated, a large bias can be introduced. In theory, this bias can be removed by adding a constant, but in practice the constant is difficult to estimate unless many data are available or unless the data in question are like those from another situation where many data are available.

(2) If the coefficient of variation is less than 1.2, the mean ordinarily should not be estimated from lognormal theory because: (a) the possible gain in efficiency is less than the risk of introducing bias if the lognormal distribution is inappropriate and (b) because the size of the constant is difficult to find.

(3) Even if the coefficient of variation is less than 1.2, it may be desirable to use Mood's method to obtain the lower confidence limit, especially if the number of observations is small, because use of a Student's t may give too small a lower limit. Whether or not a constant is added, lower confidence limits calculated by Mood's method are about the same.

(4) If a constant is added, it is better to make it somewhat too large rather than too small, because this will make the estimated mean less biased on the average. However, if the constant is much too large, the confidence limits become too narrow, because the variance of the logarithms has been made too small.

(5) Especially for gold assay data, the real difficulty is that a few high assays tend to dominate the statistical analyses, whichever particular ones are made. Extreme care must be taken in this situation, especially when dealing with ore near the cutoff grade. Overestimation is not desirable as it could result in development of a mine that can only be operated at a loss. Neither is it desirable to underestimate and thereby miss a mine on which a profit could be made.

(6) When high assay values are obtained, large numbers of observations are especially desirable if their cost is not excessive. Deflections appear to be highly worthwhile, especially in areas where additional mine samples are expensive, as in ore bodies far from drill sites or at great depth.

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APPENDIX A.--COMPUTER PROGRAM TO CALCULATE THE MEAN AND CONFIDENCE LIMITS
FOR THE LOGNORMAL DISTRIBUTION

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C   LOGNORMAL PROGRAM, ESTIMATE MEAN, CALCULATE CONFIDENCE REGIONS      00000100
C   VIA MOODS METHOD.                                                    00000200
C                                                                           00000300
C   BY J. SCHUENEMEYER, USBM, DENVER, COLO.                             00000400
C                                                                           00000500
C   CARD SEQUENCE                                                         COL *FORMAT* 00000600
C   1 - JOB DESCRIPTION           1-78 26A3 00000700
C   2 - PARAMETER                 00000800
C   OBN - TOTAL NUMBER OF OBSERVATIONS PER SET           1-10 F10.0 00000900
C   CHILO- LOWER CHI SQUARE PERCENTAGE POINT,           11-20 F10.0 00001000
C   OBN-1 DEGREES OF FREEDOM, TABLED VALUE             00001100
C   CHIH- UPPER CHI SQUARE PERCENTAGE POINT             21-30 F10.0 00001200
C   Z - PERCENT POINT STANDARD NORMAL DEVIATE,         31-40 F10.0 00001300
C   TABLED VALUE                                         00001400
C   C - CONSTANT                                         41-50 F10.0 00001500
C   3 - DATA (OBSERVATIONS), ONE/CARD                  1-10 F10.0 00001600
C   4 - 2 BLANKS FOR END OR PARAMETER AND ADDITIONAL SET OF DATA,    00001700
C                                                                           00001800
C   SYMBOL TABLE                                                         00001900
C   X - NATURAL LOGRATHIM OF W(INPUT DATA)              00002000
C   WBAR - ARITHMETIC MEAN OF RAW DATA                  00002100
C   XBAR - ARITHMETIC MEAN OF LNS (NATURAL LOGS)         00002200
C   GMEAN - GEOMETRIC MEAN                               00002300
C   V - MULTIPLYING FACTOR FOR GEOMETRIC MEAN           00002400
C   OGXBR - ESTIMATE OF MEAN (GMEAN * V)                00002500
C   XMEAN - MEAN OF OBSERVATIONS FOR CONFIDENCE REGION (CR) 00002600
C   XSD - STANDARD DEVIATION OF OBSERVATIONS FOR CR     00002700
C   SQRLD - UPPER LIMIT BETA SQ(VARIANCE OF LOGS)       00002800
C                                                                           00002900
C   1 FORMAT (26A3)                                           00003000
C   2 FORMAT (F10.0)                                          00003100
C   5 FORMAT (1H1,4X,26A3/)                                  00003200
C   6 FORMAT (5F10.0)                                        00003300
C   8 FORMAT (30X,21HPARAMETER INFORMATION//8X,6HSAMPLE,4X, 00003400
C   1 25HCHI SQUARE PERCENT POINTS,3X,16HPERCENT STANDARD/9X,4HSIZE, 00003500
C   2 9X,5HLLOWER,9X,5HUPPER,6X,14HNORMAL DEVIATE,6X,8HCONSTANT/ 00003600
C   3 I13,1X,2F14.3,F16.4,F16.2//37X,12HUBSERVATIONS//30X,3HRAW,14X, 00003700
C   4 18HNATURAL LOGARITHMS/)                                00003800
C   11 FORMAT (19X,F16.3,9X,F16.4)                          00003900
C   12 FORMAT(/39X,7HRESULTS//54X,3HRAW,10X,3HLNS//6X,4HMEAN,33X,F16.3, 00004000
C   1  F13.3/6X,8HVARIANCE,29X,F16.3,F13.3/6X,18HSTANDARD DEVIATION, 00004100
C   2  19X,F16.3,F13.3)                                     00004200
C   15 FORMAT (6X,14HGEOMETRIC MEAN,23X,F16.3/6X,          00004300
C   1  37HMULTIPLYING FACTOR FOR GEOMETRIC MEAN,F16.3/6X, 00004400
C   2  26HESTIMATE OF MEANS FROM LNS,11X,F16.3/)          00004500
C   18 FORMAT (28X,28HPOINTS FOR CONFIDENCE REGION//29X,3HRAW,33X,3HLNS/ 00004600
C   1  17X,4HMEAN,20X,4HS.D.,11X,4HMEAN,11X,4HS.D./)    00004700
C   19 FORMAT (5X,F18.3,6X,F18.3,2F15.3)                  00004800
C                                                                           00004900
C   DIMENSION XMEAN(30), XSD(30), XLN(30), XLNSD(30)      00005000

```

	DIMENSION P(90), Q(90), S(90), T(90)	00005100
	DIMENSION IDAAA(26)	00005200
C		00005300
C	READ PARAMETERS, INITIALIZE	00005400
C		00005500
	READ 1, IDAAA	00005600
22	PRINT 5, IDAAA	00005700
	SUMX = 0.0	00005800
	SUMSQX = 0.0	00005900
	SUMW = 0.0	00006000
	SUMSQW = 0.0	00006100
	I=0	00006200
	READ 6, OBN, CHILO, CHIMI, Z, C	00006300
	NOB = OBN	00006400
	IF (NOB) 23,23,24	00006500
23	STOP 8	00006600
24	SQOBN = SQRT (OBN)	00006700
	PRINT 8, NOB, CHILO, CHIMI, Z, C	00006800
C		00006900
C	READ DATA, OBTAIN LOGS, SUMS AND SUM OF SQUARES	00007000
C		00007100
	DO 25 I=1, NOB	00007200
	READ 2, W	00007300
	X=ALOG(W+C)	00007400
	PRINT 11, W, X	00007500
	SUMX =SUMX + X	00007600
	SUMW =SUMW + W	00007700
	SUMSQX= SUMSQX + X*X	00007800
25	SUMSQW = SUMSQW +W*W	00007900
	PRINT 5, IDAAA	00008000
	XBAR =SUMX/OBN	00008100
	WBAR =SUMW/OBN	00008200
	SQR = (SUMSQX - SUMX*SUMX/OBN)/(OBN - 1.)	00008300
	SQRW = (SUMSQW - SUMW*SUMW/OBN)/(OBN - 1.)	00008400
	SDX = SQRT (SQR)	00008500
	SDW = SQRT (SQRW)	00008600
	PRINT 12, WBAR, XBAR, SQRW, SQR, SDW, SDX	00008700
	GMEAN=EXP (XBAR)	00008800
C		00008900
C	CALCULATE MULTIPLYING FACTOR FOR GEOMETRIC MEAN	00009000
C		00009100
	ST = SQR/2.	00009200
	Q(1) = 1.	00009300
	T(1) = ST	00009400
	P(1) = (OBN - 1.)/OBN	00009500
	S(1) = P(1)*Q(1)*T(1)	00009600
	DO 30 K=2, 90	00009700
	R = K	00009800
	P(K) = P(K-1)* P(1)	00009900
	Q(K) = Q(K-1) * (OBN - 1.)/(OBN + 2.*R - 3.)	00010000

```

T(K) = ST * T(K-1)/R                                00010100
S(K) = S(K-1) + P(K)*Q(K)*T(K)                      00010200
IF (ABS (S(K) - S(K-1))) = 5,E-10) 28, 30, 30       00010300
28 KS = K                                             00010400
GD TD 32                                             00010500
30 CONTINUE                                          00010600
PAUSE 1                                              00010700
KS = 90                                              00010800
32 V = S(KS) + 1,                                    00010900
OGXBR = GMEAN*V - C                                  00011000
PRINT 15, GMEAN, V, OGXBR                           00011100
C                                                     00011200
C MEANS, SD, MEANS OF LOGS, AND SD OF LOGS FOR CONFIDENCE REGION 00011300
C                                                     00011400
SQRLO =SQR*(OBN-1)/CHIHI                             00011500
SQRHI =SQR*(OBN-1)/CHILO                             00011600
RGSQR = SQRHI - SQRLO                                00011700
SSQRLO = SQRT (SQRLO)                                00011800
SSQRHI = SQRT (SQRHI)                                00011900
XLOR = 2. * Z * SSQRLO /SQOBN                         00012000
XHR = 2. * Z * SSQRHI/SQOBN                          00012100
DO 40 I = 1, 11                                       00012200
K = I + 11                                             00012300
XIM1 = I - 1                                           00012400
ASQR =SQRLO +0.1*XIM1 *RGSQR                          00012500
ASQS = SQRT (ASQR)                                    00012600
XLNSD(I) = ASQS                                       00012700
XLNSD(K) = XLNSD(I)                                   00012800
XLN(K) = XBAR + Z * ASQS/SQOBN                         00012900
XLN(I) = XBAR - Z * ASQS/SQOBN                         00013000
XMEAN(K) = EXP (XLN(K) + 0.5 * ASQR) - C              00013100
XMEAN(I) = EXP (XLN(I) + 0.5 * ASQR) - C              00013200
TEMP = EXP (ASQR) - 1.                                 00013300
IF (TEMP) 40, 40, 35                                  00013400
35 XSD(I) = SQRT (TEMP * XMEAN(I) * XMEAN(I))         00013500
XSD(K) = SQRT (TEMP * XMEAN(K) * XMEAN(K))           00013600
40 CONTINUE                                          00013700
DO 50 I=2, 5                                          00013800
K = I + 21                                             00013900
XIM1 = I - 1                                           00014000
XLN(K) = XBAR + (-0.5 + 0.2 * XIM1) * XLOR           00014100
XMEAN(K) = EXP (XLN(K) + 0.5 * SQRLO) - C            00014200
XLNSD(K) = SSQRLO                                     00014300
VAR = (EXP (SQRLO) - 1.) * XMEAN(K)* XMEAN(K)        00014400
50 XSD(K) = SQRT (VAR)                                 00014500
DO 60 I = 2, 5                                       00014600
K = I + 25                                             00014700
XIM1 = I - 1                                           00014800
XLN(K) = XBAR + (-0.5 + 0.2 * XIM1) * XHR           00014900
XLNSD(K) = SSQRHI                                     00015000

```


SAMPLE INPUT CARD DATA

CARD COLUMNS

1		2		3		4		5		6		7		8	
1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890

WELKOM MINE, 13 OBSERVATIONS

13, 5,23 21,03 1,645 0,0

154,
525,
1560,
1252,
377,
70,
308,
109,
1221,
15,
48,
237,
68,

1		2		3		4		5		6		7		8	
1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890

WELKOM MINE, 13 OBSERVATIONS

PARAMETER INFORMATION

SAMPLE SIZE	CHI SQUARE LOWER	PERCENT POINTS UPPER	PERCENT STANDARD NORMAL DEVIATE	CONSTANT
13	5.230	21.030	1.6450	0.00

OBSERVATIONS

RAW	NATURAL LOGARITHMS
154.000	5.0370
525.000	6.2634
1560.000	7.3524
1252.000	7.1325
377.000	5.9322
70.000	4.2485
308.000	5.7301
109.000	4.6913
1221.000	7.1074
15.000	2.7081
48.000	3.8712
237.000	5.4681
68.000	4.2195

WELKOM MINE, 13 OBSERVATIONS

RESULTS

	RAW	LNS
MEAN	457.231	5.366
VARIANCE	282550.192	1.983
STANDARD DEVIATION	531.555	1.408
GEOMETRIC MEAN	214.066	
MULTIPLYING FACTOR FOR GEOMETRIC MEAN	2.370	
ESTIMATE OF MEANS FROM LNS	507.382	

WELKOM MINE, 13 OBSERVATIONS

POINTS FOR CONFIDENCE REGION

MEAN	RAW	S.D.	MEAN	LNS	S.D.
231.992		336.192	4.881		1.064
257.014		471.345	4.813		1.214
281.693		408.218	5.075		1.064
286.902		650.532	4.752		1.347
322.036		890.343	4.696		1.469
342.043		495.675	5.269		1.064
363.008		1213.010	4.645		1.581
410.584		1648.659	4.597		1.685
415.323		601.868	5.463		1.064
465.695		2238.297	4.552		1.784
504.301		730.812	5.657		1.064
529.450		3037.861	4.510		1.877
603.151		4123.749	4.469		1.966
612.343		987.381	5.852		1.064
688.320		5600.403	4.430		2.051
777.955		1426.714	5.920		1.214
786.731		7610.708	4.393		2.133
980.894		2224.117	5.981		1.347
1161.122		11232.508	4.782		2.133
1229.975		3400.555	6.036		1.469
1535.779		5131.889	6.087		1.581
1713.679		16577.856	5.172		2.133
1911.117		7673.905	6.135		1.685
2371.545		11398.488	6.180		1.784
2529.188		24466.958	5.561		2.133
2935.974		16845.925	6.223		1.877
3627.403		24800.581	6.263		1.966
3732.784		36110.343	5.950		2.133
4473.795		36400.311	6.302		2.051
5509.148		53294.605	6.339		2.133

APPENDIX B.--MULTIPLYING FACTORS FOR THE GEOMETRIC MEAN AND VARIANCE,
EXPONENTIAL FUNCTION, AND COEFFICIENT OF VARIATION

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
0.01	1.005	1.007	1.008	1.008	1.008	1.009	1.009	1.009
0.02	1.010	1.013	1.015	1.016	1.017	1.017	1.018	1.018
0.03	1.015	1.020	1.023	1.024	1.025	1.026	1.027	1.027
0.04	1.020	1.027	1.030	1.032	1.034	1.035	1.035	1.036
0.05	1.025	1.034	1.038	1.041	1.042	1.044	1.045	1.045
0.06	1.030	1.040	1.046	1.049	1.051	1.052	1.054	1.054
0.07	1.035	1.047	1.053	1.057	1.060	1.061	1.063	1.064
0.08	1.040	1.054	1.061	1.065	1.068	1.070	1.072	1.073
0.09	1.045	1.061	1.069	1.074	1.077	1.079	1.081	1.083
0.10	1.050	1.068	1.077	1.082	1.086	1.089	1.091	1.092
0.11	1.056	1.075	1.085	1.091	1.095	1.098	1.100	1.102
0.12	1.061	1.082	1.092	1.099	1.104	1.107	1.109	1.111
0.13	1.066	1.089	1.100	1.108	1.113	1.116	1.119	1.121
0.14	1.071	1.096	1.108	1.116	1.122	1.126	1.128	1.131
0.15	1.076	1.103	1.116	1.125	1.131	1.135	1.138	1.141
0.16	1.081	1.110	1.124	1.134	1.140	1.144	1.148	1.151
0.17	1.086	1.117	1.132	1.142	1.149	1.154	1.158	1.161
0.18	1.091	1.124	1.141	1.151	1.158	1.163	1.167	1.171
0.19	1.097	1.131	1.149	1.160	1.168	1.173	1.177	1.181
0.20	1.102	1.138	1.157	1.169	1.177	1.183	1.187	1.191
0.21	1.107	1.145	1.165	1.178	1.186	1.193	1.197	1.201
0.22	1.112	1.152	1.173	1.187	1.196	1.202	1.208	1.212
0.23	1.117	1.159	1.182	1.196	1.205	1.212	1.218	1.222
0.24	1.122	1.167	1.190	1.205	1.215	1.222	1.228	1.232
0.25	1.128	1.174	1.198	1.214	1.224	1.232	1.238	1.243
0.26	1.133	1.181	1.207	1.223	1.234	1.242	1.249	1.254
0.27	1.138	1.188	1.215	1.232	1.244	1.252	1.259	1.264
0.28	1.143	1.196	1.224	1.241	1.254	1.263	1.270	1.275
0.29	1.149	1.203	1.232	1.251	1.263	1.273	1.280	1.286
0.30	1.154	1.210	1.241	1.260	1.273	1.283	1.291	1.297
0.31	1.159	1.218	1.249	1.269	1.283	1.294	1.302	1.308
0.32	1.164	1.225	1.258	1.279	1.293	1.304	1.312	1.319
0.33	1.170	1.232	1.267	1.288	1.303	1.315	1.323	1.330
0.34	1.175	1.240	1.275	1.298	1.314	1.325	1.334	1.341
0.35	1.180	1.247	1.284	1.307	1.324	1.336	1.345	1.353
0.36	1.185	1.255	1.293	1.317	1.334	1.347	1.356	1.364
0.37	1.191	1.262	1.302	1.327	1.344	1.357	1.367	1.375
0.38	1.196	1.270	1.310	1.336	1.355	1.368	1.379	1.387
0.39	1.201	1.277	1.319	1.346	1.365	1.379	1.390	1.399
0.40	1.207	1.285	1.328	1.356	1.376	1.390	1.401	1.410

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	16	17
0.01	1.009	1.009	1.009	1.009	1.009	1.009	1.009	1.009
0.02	1.018	1.018	1.018	1.019	1.019	1.019	1.019	1.019
0.03	1.027	1.028	1.028	1.028	1.028	1.028	1.028	1.029
0.04	1.037	1.037	1.037	1.038	1.038	1.038	1.038	1.038
0.05	1.046	1.046	1.047	1.047	1.047	1.048	1.048	1.048
0.06	1.055	1.056	1.056	1.057	1.057	1.057	1.058	1.058
0.07	1.065	1.065	1.066	1.066	1.067	1.067	1.068	1.068
0.08	1.074	1.075	1.076	1.076	1.077	1.077	1.078	1.078
0.09	1.084	1.085	1.085	1.086	1.087	1.087	1.088	1.088
0.10	1.093	1.094	1.095	1.096	1.097	1.097	1.098	1.098
0.11	1.103	1.104	1.105	1.106	1.107	1.107	1.108	1.108
0.12	1.113	1.114	1.115	1.116	1.117	1.118	1.118	1.119
0.13	1.123	1.124	1.125	1.126	1.127	1.128	1.129	1.129
0.14	1.133	1.134	1.136	1.137	1.138	1.138	1.139	1.140
0.15	1.143	1.144	1.146	1.147	1.148	1.149	1.150	1.150
0.16	1.153	1.155	1.156	1.157	1.158	1.159	1.160	1.161
0.17	1.163	1.165	1.166	1.168	1.169	1.170	1.171	1.172
0.18	1.173	1.175	1.177	1.178	1.180	1.181	1.182	1.183
0.19	1.183	1.186	1.188	1.189	1.191	1.192	1.193	1.194
0.20	1.194	1.196	1.198	1.200	1.201	1.203	1.204	1.205
0.21	1.204	1.207	1.209	1.211	1.212	1.214	1.215	1.216
0.22	1.215	1.217	1.220	1.222	1.223	1.225	1.226	1.227
0.23	1.225	1.228	1.231	1.233	1.234	1.236	1.237	1.239
0.24	1.236	1.239	1.242	1.244	1.246	1.247	1.249	1.250
0.25	1.247	1.250	1.253	1.255	1.257	1.259	1.260	1.262
0.26	1.258	1.261	1.264	1.266	1.268	1.270	1.272	1.273
0.27	1.269	1.272	1.275	1.278	1.280	1.282	1.283	1.285
0.28	1.280	1.283	1.286	1.289	1.291	1.293	1.295	1.297
0.29	1.291	1.294	1.298	1.301	1.303	1.305	1.307	1.309
0.30	1.302	1.306	1.309	1.312	1.315	1.317	1.319	1.321
0.31	1.313	1.317	1.321	1.324	1.327	1.329	1.331	1.333
0.32	1.324	1.329	1.332	1.336	1.338	1.341	1.343	1.345
0.33	1.336	1.340	1.344	1.348	1.350	1.353	1.355	1.357
0.34	1.347	1.352	1.356	1.360	1.363	1.365	1.368	1.370
0.35	1.359	1.364	1.368	1.372	1.375	1.378	1.380	1.382
0.36	1.370	1.376	1.380	1.384	1.387	1.390	1.392	1.395
0.37	1.382	1.387	1.392	1.396	1.399	1.402	1.405	1.407
0.38	1.394	1.399	1.404	1.408	1.412	1.415	1.418	1.420
0.39	1.406	1.412	1.417	1.421	1.425	1.428	1.431	1.433
0.40	1.418	1.424	1.429	1.433	1.437	1.441	1.444	1.446

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	18	19	20	25	30	35	40	45
0.01	1.009	1.010	1.010	1.010	1.010	1.010	1.010	1.010
0.02	1.019	1.019	1.019	1.019	1.020	1.020	1.020	1.020
0.03	1.029	1.029	1.029	1.029	1.029	1.030	1.030	1.030
0.04	1.038	1.039	1.039	1.039	1.039	1.040	1.040	1.040
0.05	1.048	1.048	1.049	1.049	1.049	1.050	1.050	1.050
0.06	1.058	1.058	1.058	1.059	1.060	1.060	1.060	1.060
0.07	1.068	1.068	1.069	1.069	1.070	1.070	1.071	1.071
0.08	1.078	1.078	1.079	1.080	1.080	1.081	1.081	1.081
0.09	1.088	1.089	1.089	1.090	1.091	1.091	1.092	1.092
0.10	1.099	1.099	1.099	1.100	1.101	1.102	1.102	1.102
0.11	1.109	1.109	1.110	1.111	1.112	1.112	1.113	1.113
0.12	1.119	1.120	1.120	1.122	1.123	1.123	1.124	1.124
0.13	1.130	1.130	1.131	1.132	1.133	1.134	1.135	1.135
0.14	1.140	1.141	1.141	1.143	1.144	1.145	1.146	1.146
0.15	1.151	1.152	1.152	1.154	1.155	1.156	1.157	1.157
0.16	1.162	1.162	1.163	1.165	1.166	1.167	1.168	1.169
0.17	1.173	1.173	1.174	1.176	1.178	1.179	1.179	1.180
0.18	1.184	1.184	1.185	1.187	1.189	1.190	1.191	1.192
0.19	1.195	1.195	1.196	1.199	1.200	1.202	1.203	1.203
0.20	1.206	1.206	1.207	1.210	1.212	1.213	1.214	1.215
0.21	1.217	1.218	1.219	1.221	1.223	1.225	1.226	1.227
0.22	1.228	1.229	1.230	1.233	1.235	1.237	1.238	1.239
0.23	1.240	1.241	1.241	1.245	1.247	1.249	1.250	1.251
0.24	1.251	1.252	1.253	1.257	1.259	1.261	1.262	1.263
0.25	1.263	1.264	1.265	1.268	1.271	1.273	1.274	1.275
0.26	1.274	1.276	1.277	1.281	1.283	1.285	1.287	1.288
0.27	1.286	1.287	1.288	1.293	1.295	1.297	1.299	1.300
0.28	1.298	1.299	1.300	1.305	1.308	1.310	1.312	1.313
0.29	1.310	1.311	1.313	1.317	1.320	1.323	1.324	1.326
0.30	1.322	1.324	1.325	1.330	1.333	1.335	1.337	1.338
0.31	1.334	1.336	1.337	1.342	1.346	1.348	1.350	1.351
0.32	1.347	1.348	1.350	1.355	1.358	1.361	1.363	1.364
0.33	1.359	1.361	1.362	1.368	1.371	1.374	1.376	1.378
0.34	1.371	1.373	1.375	1.380	1.384	1.387	1.389	1.391
0.35	1.384	1.386	1.387	1.393	1.398	1.401	1.403	1.405
0.36	1.397	1.399	1.400	1.407	1.411	1.414	1.416	1.418
0.37	1.410	1.411	1.413	1.420	1.424	1.427	1.430	1.432
0.38	1.422	1.424	1.426	1.433	1.438	1.441	1.444	1.446
0.39	1.435	1.438	1.439	1.447	1.451	1.455	1.458	1.460
0.40	1.449	1.451	1.453	1.460	1.465	1.469	1.472	1.474

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	50	75	100	125	150	175	200
0.01	1.010	1.010	1.010	1.010	1.010	1.010	1.010
0.02	1.020	1.020	1.020	1.020	1.020	1.020	1.020
0.03	1.030	1.030	1.030	1.030	1.030	1.030	1.030
0.04	1.040	1.040	1.040	1.040	1.041	1.041	1.041
0.05	1.050	1.051	1.051	1.051	1.051	1.051	1.051
0.06	1.060	1.061	1.061	1.061	1.061	1.061	1.061
0.07	1.071	1.071	1.072	1.072	1.072	1.072	1.072
0.08	1.081	1.082	1.082	1.083	1.083	1.083	1.083
0.09	1.092	1.093	1.093	1.093	1.093	1.094	1.094
0.10	1.103	1.104	1.104	1.104	1.104	1.104	1.105
0.11	1.114	1.114	1.115	1.115	1.115	1.115	1.116
0.12	1.124	1.125	1.126	1.126	1.126	1.127	1.127
0.13	1.136	1.137	1.137	1.137	1.138	1.138	1.138
0.14	1.147	1.148	1.148	1.149	1.149	1.149	1.149
0.15	1.158	1.159	1.160	1.160	1.161	1.161	1.161
0.16	1.169	1.171	1.171	1.172	1.172	1.172	1.172
0.17	1.181	1.182	1.183	1.183	1.184	1.184	1.184
0.18	1.192	1.194	1.195	1.195	1.196	1.196	1.196
0.19	1.204	1.206	1.207	1.207	1.207	1.208	1.208
0.20	1.216	1.218	1.218	1.219	1.219	1.220	1.220
0.21	1.227	1.230	1.231	1.231	1.232	1.232	1.232
0.22	1.239	1.242	1.243	1.243	1.244	1.244	1.244
0.23	1.252	1.254	1.255	1.256	1.256	1.257	1.257
0.24	1.264	1.266	1.267	1.268	1.269	1.269	1.269
0.25	1.276	1.279	1.280	1.281	1.281	1.282	1.282
0.26	1.289	1.291	1.293	1.294	1.294	1.295	1.295
0.27	1.301	1.304	1.306	1.306	1.307	1.307	1.308
0.28	1.314	1.317	1.318	1.319	1.320	1.320	1.321
0.29	1.327	1.330	1.331	1.332	1.333	1.334	1.334
0.30	1.340	1.343	1.345	1.346	1.346	1.347	1.347
0.31	1.353	1.356	1.358	1.359	1.360	1.360	1.361
0.32	1.366	1.369	1.371	1.372	1.373	1.374	1.374
0.33	1.379	1.383	1.385	1.386	1.387	1.387	1.388
0.34	1.392	1.397	1.399	1.400	1.401	1.401	1.402
0.35	1.406	1.410	1.412	1.414	1.415	1.415	1.416
0.36	1.420	1.424	1.426	1.428	1.429	1.429	1.430
0.37	1.433	1.438	1.440	1.442	1.443	1.444	1.444
0.38	1.447	1.452	1.455	1.456	1.457	1.458	1.458
0.39	1.461	1.466	1.469	1.471	1.472	1.472	1.473
0.40	1.476	1.481	1.484	1.485	1.486	1.487	1.488

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	300	400	500	750	1000	5000	
0.01	1.010	1.010	1.010	1.010	1.010	1.010	1.010
0.02	1.020	1.020	1.020	1.020	1.020	1.020	1.020
0.03	1.030	1.030	1.030	1.030	1.030	1.030	1.030
0.04	1.041	1.041	1.041	1.041	1.041	1.041	1.041
0.05	1.051	1.051	1.051	1.051	1.051	1.051	1.051
0.06	1.062	1.062	1.062	1.062	1.062	1.062	1.062
0.07	1.072	1.072	1.072	1.072	1.072	1.072	1.073
0.08	1.083	1.083	1.083	1.083	1.083	1.083	1.083
0.09	1.094	1.094	1.094	1.094	1.094	1.094	1.094
0.10	1.105	1.105	1.105	1.105	1.105	1.105	1.105
0.11	1.116	1.116	1.116	1.116	1.116	1.116	1.116
0.12	1.127	1.127	1.127	1.127	1.127	1.127	1.127
0.13	1.138	1.138	1.138	1.139	1.139	1.139	1.139
0.14	1.150	1.150	1.150	1.150	1.150	1.150	1.150
0.15	1.161	1.161	1.161	1.162	1.162	1.162	1.162
0.16	1.173	1.173	1.173	1.173	1.173	1.173	1.174
0.17	1.185	1.185	1.185	1.185	1.185	1.185	1.185
0.18	1.196	1.197	1.197	1.197	1.197	1.197	1.197
0.19	1.208	1.209	1.209	1.209	1.209	1.209	1.209
0.20	1.220	1.221	1.221	1.221	1.221	1.221	1.221
0.21	1.233	1.233	1.233	1.233	1.233	1.234	1.234
0.22	1.245	1.245	1.245	1.246	1.246	1.246	1.246
0.23	1.257	1.258	1.258	1.258	1.258	1.259	1.259
0.24	1.270	1.270	1.270	1.271	1.271	1.271	1.271
0.25	1.283	1.283	1.283	1.283	1.284	1.284	1.284
0.26	1.296	1.296	1.296	1.296	1.296	1.297	1.297
0.27	1.308	1.309	1.309	1.309	1.310	1.310	1.310
0.28	1.322	1.322	1.322	1.322	1.323	1.323	1.323
0.29	1.335	1.335	1.335	1.336	1.336	1.336	1.336
0.30	1.348	1.349	1.349	1.349	1.349	1.350	1.350
0.31	1.362	1.362	1.362	1.363	1.363	1.363	1.363
0.32	1.375	1.376	1.376	1.376	1.377	1.377	1.377
0.33	1.389	1.389	1.390	1.390	1.390	1.391	1.391
0.34	1.403	1.403	1.404	1.404	1.404	1.405	1.405
0.35	1.417	1.417	1.418	1.418	1.418	1.419	1.419
0.36	1.431	1.432	1.432	1.432	1.433	1.433	1.433
0.37	1.445	1.446	1.446	1.447	1.447	1.448	1.448
0.38	1.460	1.460	1.461	1.461	1.462	1.462	1.462
0.39	1.474	1.475	1.475	1.476	1.476	1.477	1.477
0.40	1.489	1.490	1.490	1.491	1.491	1.492	1.492

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
0.41	1.212	1.293	1.337	1.366	1.386	1.401	1.413	1.422
0.42	1.217	1.300	1.346	1.376	1.397	1.412	1.424	1.434
0.43	1.223	1.308	1.355	1.386	1.407	1.423	1.436	1.446
0.44	1.228	1.316	1.364	1.396	1.418	1.435	1.448	1.458
0.45	1.234	1.323	1.373	1.406	1.429	1.446	1.459	1.470
0.46	1.239	1.331	1.383	1.416	1.440	1.457	1.471	1.482
0.47	1.244	1.339	1.392	1.426	1.451	1.469	1.483	1.494
0.48	1.250	1.347	1.401	1.436	1.462	1.480	1.495	1.507
0.49	1.255	1.354	1.410	1.447	1.473	1.492	1.507	1.519
0.50	1.261	1.362	1.420	1.457	1.484	1.504	1.519	1.532
0.51	1.266	1.370	1.429	1.467	1.495	1.515	1.531	1.544
0.52	1.271	1.378	1.438	1.478	1.506	1.527	1.544	1.557
0.53	1.277	1.386	1.448	1.488	1.517	1.539	1.556	1.570
0.54	1.282	1.394	1.457	1.499	1.529	1.551	1.569	1.583
0.55	1.288	1.402	1.467	1.509	1.540	1.563	1.581	1.596
0.56	1.293	1.410	1.476	1.520	1.552	1.575	1.594	1.609
0.57	1.299	1.418	1.486	1.531	1.563	1.587	1.606	1.622
0.58	1.304	1.426	1.495	1.542	1.575	1.600	1.619	1.635
0.59	1.310	1.434	1.505	1.552	1.586	1.612	1.632	1.648
0.60	1.315	1.442	1.515	1.563	1.598	1.624	1.645	1.662
0.61	1.321	1.450	1.525	1.574	1.610	1.637	1.658	1.675
0.62	1.326	1.458	1.534	1.585	1.622	1.649	1.671	1.689
0.63	1.332	1.466	1.544	1.596	1.634	1.662	1.684	1.702
0.64	1.337	1.474	1.554	1.607	1.646	1.675	1.698	1.716
0.65	1.343	1.483	1.564	1.618	1.658	1.687	1.711	1.730
0.66	1.349	1.491	1.574	1.630	1.670	1.700	1.724	1.744
0.67	1.354	1.499	1.584	1.641	1.682	1.713	1.738	1.758
0.68	1.360	1.507	1.594	1.652	1.694	1.726	1.752	1.772
0.69	1.365	1.516	1.604	1.663	1.707	1.739	1.765	1.786
0.70	1.371	1.524	1.614	1.675	1.719	1.752	1.779	1.801
0.71	1.377	1.532	1.624	1.686	1.731	1.766	1.793	1.815
0.72	1.382	1.541	1.635	1.698	1.744	1.779	1.807	1.829
0.73	1.388	1.549	1.645	1.709	1.756	1.792	1.821	1.844
0.74	1.393	1.558	1.655	1.721	1.769	1.806	1.835	1.859
0.75	1.399	1.566	1.665	1.733	1.782	1.819	1.849	1.873
0.76	1.405	1.575	1.676	1.744	1.795	1.833	1.864	1.888
0.77	1.410	1.583	1.686	1.756	1.808	1.847	1.878	1.903
0.78	1.416	1.592	1.697	1.768	1.820	1.861	1.892	1.918
0.79	1.422	1.600	1.707	1.780	1.833	1.874	1.907	1.934
0.80	1.427	1.609	1.718	1.792	1.847	1.888	1.922	1.949

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	16	17
0.41	1.430	1.436	1.441	1.446	1.450	1.454	1.457	1.459
0.42	1.442	1.448	1.454	1.459	1.463	1.467	1.470	1.473
0.43	1.454	1.461	1.467	1.472	1.476	1.480	1.483	1.486
0.44	1.466	1.473	1.479	1.485	1.489	1.493	1.496	1.499
0.45	1.479	1.486	1.492	1.498	1.502	1.506	1.510	1.513
0.46	1.491	1.499	1.505	1.511	1.515	1.520	1.523	1.527
0.47	1.504	1.512	1.518	1.524	1.529	1.533	1.537	1.540
0.48	1.516	1.525	1.531	1.537	1.542	1.547	1.551	1.554
0.49	1.529	1.538	1.545	1.551	1.556	1.561	1.565	1.568
0.50	1.542	1.551	1.558	1.564	1.570	1.574	1.579	1.582
0.51	1.555	1.564	1.571	1.578	1.583	1.588	1.593	1.597
0.52	1.568	1.577	1.585	1.592	1.597	1.603	1.607	1.611
0.53	1.581	1.591	1.599	1.605	1.611	1.617	1.621	1.626
0.54	1.594	1.604	1.612	1.619	1.626	1.631	1.636	1.640
0.55	1.608	1.618	1.626	1.633	1.640	1.645	1.650	1.655
0.56	1.621	1.631	1.640	1.648	1.654	1.660	1.665	1.670
0.57	1.634	1.645	1.654	1.662	1.669	1.675	1.680	1.685
0.58	1.648	1.659	1.668	1.676	1.683	1.689	1.695	1.700
0.59	1.662	1.673	1.682	1.691	1.698	1.704	1.710	1.715
0.60	1.675	1.687	1.697	1.705	1.713	1.719	1.725	1.730
0.61	1.689	1.701	1.711	1.720	1.728	1.734	1.740	1.746
0.62	1.703	1.715	1.726	1.735	1.743	1.750	1.756	1.761
0.63	1.717	1.730	1.741	1.750	1.758	1.765	1.771	1.777
0.64	1.731	1.744	1.755	1.765	1.773	1.780	1.787	1.793
0.65	1.746	1.759	1.770	1.780	1.788	1.796	1.803	1.809
0.66	1.760	1.774	1.785	1.795	1.804	1.812	1.818	1.825
0.67	1.774	1.788	1.800	1.811	1.820	1.827	1.834	1.841
0.68	1.789	1.803	1.815	1.826	1.835	1.843	1.851	1.857
0.69	1.804	1.818	1.831	1.842	1.851	1.859	1.867	1.873
0.70	1.818	1.833	1.846	1.857	1.867	1.876	1.883	1.890
0.71	1.833	1.849	1.862	1.873	1.883	1.892	1.900	1.907
0.72	1.848	1.864	1.877	1.889	1.899	1.908	1.916	1.924
0.73	1.863	1.879	1.893	1.905	1.916	1.925	1.933	1.941
0.74	1.878	1.895	1.909	1.921	1.932	1.942	1.950	1.958
0.75	1.894	1.911	1.925	1.938	1.949	1.958	1.967	1.975
0.76	1.909	1.926	1.941	1.954	1.965	1.975	1.984	1.992
0.77	1.924	1.942	1.957	1.971	1.982	1.993	2.002	2.010
0.78	1.940	1.958	1.974	1.987	1.999	2.010	2.019	2.027
0.79	1.956	1.974	1.990	2.004	2.016	2.027	2.037	2.045
0.80	1.971	1.991	2.007	2.021	2.034	2.045	2.054	2.063

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	18	19	20	25	30	35	40	45
0.41	1.462	1.464	1.466	1.474	1.479	1.483	1.486	1.488
0.42	1.475	1.477	1.480	1.488	1.493	1.497	1.500	1.502
0.43	1.489	1.491	1.493	1.502	1.507	1.511	1.514	1.517
0.44	1.502	1.505	1.507	1.516	1.521	1.526	1.529	1.532
0.45	1.516	1.518	1.521	1.530	1.536	1.540	1.544	1.546
0.46	1.530	1.532	1.535	1.544	1.550	1.555	1.558	1.561
0.47	1.544	1.546	1.549	1.558	1.565	1.570	1.573	1.576
0.48	1.558	1.560	1.563	1.573	1.580	1.585	1.589	1.591
0.49	1.572	1.575	1.577	1.588	1.595	1.600	1.604	1.607
0.50	1.586	1.589	1.592	1.602	1.610	1.615	1.619	1.622
0.51	1.600	1.603	1.606	1.617	1.625	1.630	1.635	1.638
0.52	1.615	1.618	1.621	1.632	1.640	1.646	1.650	1.654
0.53	1.629	1.633	1.636	1.648	1.656	1.662	1.666	1.670
0.54	1.644	1.647	1.651	1.663	1.671	1.677	1.682	1.686
0.55	1.659	1.662	1.666	1.678	1.687	1.693	1.698	1.702
0.56	1.674	1.677	1.681	1.694	1.703	1.709	1.714	1.718
0.57	1.689	1.693	1.696	1.710	1.719	1.725	1.731	1.735
0.58	1.704	1.708	1.712	1.725	1.735	1.742	1.747	1.751
0.59	1.719	1.723	1.727	1.741	1.751	1.758	1.764	1.768
0.60	1.735	1.739	1.743	1.757	1.768	1.775	1.781	1.785
0.61	1.750	1.755	1.759	1.774	1.784	1.792	1.798	1.802
0.62	1.766	1.770	1.774	1.790	1.801	1.809	1.815	1.819
0.63	1.782	1.786	1.791	1.807	1.818	1.826	1.832	1.837
0.64	1.798	1.802	1.807	1.823	1.835	1.843	1.849	1.854
0.65	1.814	1.819	1.823	1.840	1.852	1.860	1.867	1.872
0.66	1.830	1.835	1.840	1.857	1.869	1.878	1.885	1.890
0.67	1.846	1.851	1.856	1.874	1.887	1.896	1.903	1.908
0.68	1.863	1.868	1.873	1.891	1.904	1.914	1.921	1.926
0.69	1.879	1.885	1.890	1.909	1.922	1.932	1.939	1.945
0.70	1.896	1.902	1.907	1.926	1.940	1.950	1.957	1.963
0.71	1.913	1.919	1.924	1.944	1.958	1.968	1.976	1.982
0.72	1.930	1.936	1.941	1.962	1.976	1.987	1.995	2.001
0.73	1.947	1.953	1.959	1.980	1.995	2.005	2.014	2.020
0.74	1.964	1.971	1.976	1.998	2.013	2.024	2.033	2.039
0.75	1.982	1.988	1.994	2.016	2.032	2.043	2.052	2.059
0.76	1.999	2.006	2.012	2.035	2.051	2.062	2.071	2.078
0.77	2.017	2.024	2.030	2.053	2.070	2.082	2.091	2.098
0.78	2.035	2.042	2.048	2.072	2.089	2.101	2.111	2.118
0.79	2.053	2.060	2.066	2.091	2.108	2.121	2.131	2.138
0.80	2.071	2.078	2.085	2.110	2.128	2.141	2.151	2.159

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	50	75	100	125	150	175	200
0.41	1.490	1.495	1.498	1.500	1.501	1.502	1.502
0.42	1.504	1.510	1.513	1.515	1.516	1.517	1.517
0.43	1.519	1.525	1.528	1.530	1.531	1.532	1.533
0.44	1.534	1.540	1.543	1.545	1.546	1.547	1.548
0.45	1.548	1.555	1.558	1.560	1.562	1.563	1.563
0.46	1.563	1.570	1.574	1.576	1.577	1.578	1.579
0.47	1.579	1.586	1.589	1.591	1.593	1.594	1.594
0.48	1.594	1.601	1.605	1.607	1.608	1.610	1.610
0.49	1.609	1.617	1.621	1.623	1.624	1.626	1.626
0.50	1.625	1.633	1.637	1.639	1.641	1.642	1.643
0.51	1.641	1.649	1.653	1.655	1.657	1.658	1.659
0.52	1.656	1.665	1.669	1.672	1.673	1.674	1.675
0.53	1.672	1.681	1.685	1.688	1.690	1.691	1.692
0.54	1.688	1.697	1.702	1.705	1.707	1.708	1.709
0.55	1.705	1.714	1.719	1.722	1.723	1.725	1.726
0.56	1.721	1.731	1.736	1.739	1.741	1.742	1.743
0.57	1.738	1.748	1.753	1.756	1.758	1.759	1.760
0.58	1.755	1.765	1.770	1.773	1.775	1.777	1.778
0.59	1.771	1.782	1.787	1.791	1.793	1.794	1.796
0.60	1.789	1.799	1.805	1.808	1.811	1.812	1.813
0.61	1.806	1.817	1.823	1.826	1.829	1.830	1.831
0.62	1.823	1.835	1.841	1.844	1.847	1.848	1.850
0.63	1.841	1.853	1.859	1.862	1.865	1.867	1.868
0.64	1.858	1.871	1.877	1.881	1.883	1.885	1.887
0.65	1.876	1.889	1.895	1.899	1.902	1.904	1.905
0.66	1.894	1.907	1.914	1.918	1.921	1.923	1.924
0.67	1.912	1.926	1.933	1.937	1.940	1.942	1.943
0.68	1.931	1.945	1.952	1.956	1.959	1.961	1.963
0.69	1.949	1.964	1.971	1.975	1.978	1.981	1.982
0.70	1.968	1.983	1.990	1.995	1.998	2.000	2.002
0.71	1.987	2.002	2.010	2.015	2.018	2.020	2.022
0.72	2.006	2.022	2.030	2.034	2.038	2.040	2.042
0.73	2.025	2.041	2.050	2.055	2.058	2.060	2.062
0.74	2.045	2.061	2.070	2.075	2.078	2.081	2.083
0.75	2.064	2.081	2.090	2.095	2.099	2.101	2.103
0.76	2.084	2.101	2.110	2.116	2.120	2.122	2.124
0.77	2.104	2.122	2.131	2.137	2.140	2.143	2.145
0.78	2.124	2.143	2.152	2.158	2.162	2.164	2.167
0.79	2.144	2.163	2.173	2.179	2.183	2.186	2.188
0.80	2.165	2.184	2.194	2.200	2.205	2.207	2.210

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	300	400	500	750	1000	5000	
0.41	1.504	1.505	1.505	1.506	1.506	1.507	1.507
0.42	1.519	1.520	1.520	1.521	1.521	1.522	1.522
0.43	1.534	1.535	1.535	1.536	1.536	1.537	1.537
0.44	1.549	1.550	1.551	1.551	1.552	1.552	1.553
0.45	1.565	1.566	1.566	1.567	1.567	1.568	1.568
0.46	1.581	1.581	1.582	1.583	1.583	1.584	1.584
0.47	1.596	1.597	1.598	1.599	1.599	1.600	1.600
0.48	1.612	1.613	1.614	1.615	1.615	1.616	1.616
0.49	1.628	1.629	1.630	1.631	1.631	1.632	1.632
0.50	1.645	1.646	1.646	1.647	1.647	1.648	1.649
0.51	1.661	1.662	1.663	1.664	1.664	1.665	1.665
0.52	1.678	1.679	1.679	1.680	1.681	1.682	1.682
0.53	1.694	1.695	1.696	1.697	1.698	1.699	1.699
0.54	1.711	1.712	1.713	1.714	1.715	1.716	1.716
0.55	1.728	1.730	1.730	1.731	1.732	1.733	1.733
0.56	1.746	1.747	1.748	1.749	1.749	1.750	1.751
0.57	1.763	1.764	1.765	1.766	1.767	1.768	1.768
0.58	1.781	1.782	1.783	1.784	1.784	1.786	1.786
0.59	1.798	1.800	1.801	1.802	1.802	1.804	1.804
0.60	1.816	1.818	1.819	1.820	1.820	1.822	1.822
0.61	1.834	1.836	1.837	1.838	1.839	1.840	1.840
0.62	1.853	1.854	1.855	1.856	1.857	1.859	1.859
0.63	1.871	1.873	1.874	1.875	1.876	1.877	1.878
0.64	1.890	1.892	1.893	1.894	1.894	1.896	1.896
0.65	1.909	1.910	1.911	1.913	1.913	1.915	1.916
0.66	1.928	1.930	1.931	1.932	1.933	1.934	1.935
0.67	1.947	1.949	1.950	1.951	1.952	1.954	1.954
0.68	1.966	1.968	1.969	1.971	1.972	1.973	1.974
0.69	1.986	1.988	1.989	1.991	1.991	1.993	1.994
0.70	2.006	2.008	2.009	2.011	2.011	2.013	2.014
0.71	2.026	2.028	2.029	2.031	2.032	2.033	2.034
0.72	2.046	2.048	2.049	2.051	2.052	2.054	2.054
0.73	2.066	2.069	2.070	2.072	2.072	2.075	2.075
0.74	2.087	2.089	2.091	2.092	2.093	2.095	2.096
0.75	2.108	2.110	2.111	2.113	2.114	2.116	2.117
0.76	2.129	2.131	2.133	2.134	2.135	2.138	2.138
0.77	2.150	2.152	2.154	2.156	2.157	2.159	2.160
0.78	2.171	2.174	2.175	2.177	2.178	2.181	2.181
0.79	2.193	2.196	2.197	2.199	2.200	2.203	2.203
0.80	2.215	2.218	2.219	2.221	2.222	2.225	2.225

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
0.81	1.433	1.617	1.728	1.804	1.860	1.902	1.936	1.964
0.82	1.439	1.626	1.739	1.816	1.873	1.917	1.951	1.980
0.83	1.445	1.635	1.750	1.828	1.886	1.931	1.966	1.995
0.84	1.450	1.643	1.760	1.841	1.900	1.945	1.981	2.011
0.85	1.456	1.652	1.771	1.853	1.913	1.959	1.996	2.027
0.86	1.462	1.661	1.782	1.865	1.927	1.974	2.012	2.043
0.87	1.467	1.670	1.793	1.878	1.940	1.988	2.027	2.059
0.88	1.473	1.679	1.804	1.890	1.954	2.003	2.042	2.075
0.89	1.479	1.687	1.815	1.903	1.968	2.018	2.058	2.091
0.90	1.485	1.696	1.826	1.915	1.981	2.033	2.074	2.107
0.91	1.491	1.705	1.837	1.928	1.995	2.047	2.089	2.124
0.92	1.496	1.714	1.848	1.940	2.009	2.062	2.105	2.140
0.93	1.502	1.723	1.859	1.953	2.023	2.077	2.121	2.157
0.94	1.508	1.732	1.870	1.966	2.037	2.093	2.137	2.173
0.95	1.514	1.741	1.881	1.979	2.051	2.108	2.153	2.190
0.96	1.520	1.750	1.893	1.992	2.066	2.123	2.169	2.207
0.97	1.525	1.759	1.904	2.005	2.080	2.139	2.186	2.224
0.98	1.531	1.768	1.915	2.018	2.094	2.154	2.202	2.241
0.99	1.537	1.777	1.927	2.031	2.109	2.170	2.218	2.259
1.00	1.543	1.786	1.938	2.044	2.123	2.185	2.235	2.276
1.01	1.549	1.796	1.949	2.057	2.138	2.201	2.252	2.294
1.02	1.555	1.805	1.961	2.071	2.153	2.217	2.269	2.311
1.03	1.561	1.814	1.973	2.084	2.168	2.233	2.285	2.329
1.04	1.567	1.823	1.984	2.098	2.182	2.249	2.303	2.347
1.05	1.573	1.832	1.996	2.111	2.197	2.265	2.320	2.365
1.06	1.579	1.842	2.008	2.125	2.212	2.281	2.337	2.383
1.07	1.584	1.851	2.019	2.138	2.228	2.298	2.354	2.401
1.08	1.590	1.860	2.031	2.152	2.243	2.314	2.372	2.419
1.09	1.596	1.870	2.043	2.166	2.258	2.331	2.389	2.438
1.10	1.602	1.879	2.055	2.180	2.273	2.347	2.407	2.456
1.11	1.608	1.889	2.067	2.193	2.289	2.364	2.425	2.475
1.12	1.614	1.898	2.079	2.207	2.304	2.381	2.442	2.494
1.13	1.620	1.908	2.091	2.221	2.320	2.397	2.460	2.512
1.14	1.626	1.917	2.103	2.235	2.336	2.414	2.478	2.531
1.15	1.632	1.927	2.115	2.250	2.351	2.432	2.497	2.551
1.16	1.638	1.936	2.127	2.264	2.367	2.449	2.515	2.570
1.17	1.644	1.946	2.140	2.278	2.383	2.466	2.533	2.589
1.18	1.650	1.956	2.152	2.292	2.399	2.483	2.552	2.609
1.19	1.656	1.965	2.164	2.307	2.415	2.501	2.570	2.628
1.20	1.662	1.975	2.177	2.321	2.431	2.518	2.589	2.648

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	16	17
0.81	1.987	2.007	2.024	2.038	2.051	2.062	2.072	2.081
0.82	2.003	2.023	2.040	2.055	2.068	2.080	2.090	2.099
0.83	2.019	2.040	2.057	2.073	2.086	2.098	2.108	2.118
0.84	2.036	2.057	2.074	2.090	2.104	2.116	2.127	2.136
0.85	2.052	2.073	2.092	2.108	2.122	2.134	2.145	2.155
0.86	2.068	2.090	2.109	2.125	2.140	2.152	2.164	2.174
0.87	2.085	2.107	2.126	2.143	2.158	2.171	2.182	2.193
0.88	2.102	2.124	2.144	2.161	2.176	2.189	2.201	2.212
0.89	2.118	2.142	2.162	2.179	2.195	2.208	2.220	2.231
0.90	2.135	2.159	2.180	2.197	2.213	2.227	2.239	2.250
0.91	2.152	2.177	2.198	2.216	2.232	2.246	2.259	2.270
0.92	2.169	2.194	2.216	2.234	2.251	2.265	2.278	2.290
0.93	2.187	2.212	2.234	2.253	2.270	2.284	2.298	2.310
0.94	2.204	2.230	2.252	2.272	2.289	2.304	2.317	2.330
0.95	2.221	2.248	2.271	2.291	2.308	2.323	2.337	2.350
0.96	2.239	2.266	2.289	2.310	2.327	2.343	2.357	2.370
0.97	2.257	2.284	2.308	2.329	2.347	2.363	2.378	2.391
0.98	2.274	2.303	2.327	2.348	2.367	2.383	2.398	2.411
0.99	2.292	2.321	2.346	2.367	2.386	2.403	2.418	2.432
1.00	2.310	2.340	2.365	2.387	2.406	2.424	2.439	2.453
1.01	2.329	2.358	2.384	2.407	2.427	2.444	2.460	2.474
1.02	2.347	2.377	2.404	2.427	2.447	2.465	2.481	2.495
1.03	2.365	2.396	2.423	2.447	2.467	2.486	2.502	2.517
1.04	2.384	2.416	2.443	2.467	2.488	2.507	2.523	2.538
1.05	2.403	2.435	2.463	2.487	2.509	2.528	2.545	2.560
1.06	2.421	2.454	2.483	2.508	2.530	2.549	2.566	2.582
1.07	2.440	2.474	2.503	2.528	2.551	2.570	2.588	2.604
1.08	2.459	2.493	2.523	2.549	2.572	2.592	2.610	2.627
1.09	2.478	2.513	2.543	2.570	2.593	2.614	2.632	2.649
1.10	2.498	2.533	2.564	2.591	2.615	2.636	2.655	2.672
1.11	2.517	2.553	2.585	2.612	2.636	2.658	2.677	2.695
1.12	2.537	2.574	2.606	2.633	2.658	2.680	2.700	2.718
1.13	2.556	2.594	2.626	2.655	2.680	2.703	2.723	2.741
1.14	2.576	2.614	2.648	2.677	2.702	2.725	2.746	2.764
1.15	2.596	2.635	2.669	2.698	2.725	2.748	2.769	2.788
1.16	2.616	2.656	2.690	2.720	2.747	2.771	2.792	2.811
1.17	2.636	2.677	2.712	2.743	2.770	2.794	2.816	2.835
1.18	2.657	2.698	2.734	2.765	2.793	2.817	2.839	2.859
1.19	2.677	2.719	2.755	2.787	2.816	2.841	2.863	2.884
1.20	2.698	2.740	2.777	2.810	2.839	2.864	2.887	2.908

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	18	19	20	25	30	35	40	45
0.81	2.089	2.097	2.103	2.130	2.148	2.161	2.171	2.179
0.82	2.108	2.115	2.122	2.149	2.168	2.181	2.192	2.200
0.83	2.126	2.134	2.141	2.169	2.188	2.202	2.212	2.221
0.84	2.145	2.153	2.160	2.188	2.208	2.222	2.233	2.242
0.85	2.164	2.172	2.179	2.208	2.228	2.243	2.254	2.263
0.86	2.183	2.191	2.199	2.228	2.249	2.264	2.275	2.285
0.87	2.202	2.211	2.218	2.249	2.270	2.285	2.297	2.306
0.88	2.221	2.230	2.238	2.269	2.291	2.306	2.319	2.328
0.89	2.241	2.250	2.258	2.290	2.312	2.328	2.340	2.350
0.90	2.260	2.270	2.278	2.310	2.333	2.350	2.362	2.372
0.91	2.280	2.290	2.298	2.331	2.354	2.372	2.385	2.395
0.92	2.300	2.310	2.318	2.352	2.376	2.394	2.407	2.418
0.93	2.320	2.330	2.339	2.374	2.398	2.416	2.430	2.440
0.94	2.341	2.350	2.360	2.395	2.420	2.438	2.452	2.464
0.95	2.361	2.371	2.380	2.417	2.442	2.461	2.475	2.487
0.96	2.382	2.392	2.401	2.439	2.465	2.484	2.499	2.510
0.97	2.402	2.413	2.423	2.461	2.487	2.507	2.522	2.534
0.98	2.423	2.434	2.444	2.483	2.510	2.530	2.546	2.558
0.99	2.444	2.455	2.466	2.505	2.533	2.554	2.570	2.582
1.00	2.465	2.477	2.487	2.528	2.556	2.578	2.594	2.607
1.01	2.487	2.498	2.509	2.551	2.580	2.601	2.618	2.631
1.02	2.508	2.520	2.531	2.574	2.604	2.626	2.643	2.656
1.03	2.530	2.542	2.553	2.597	2.627	2.650	2.667	2.681
1.04	2.552	2.564	2.576	2.620	2.651	2.675	2.692	2.706
1.05	2.574	2.587	2.598	2.644	2.676	2.699	2.717	2.732
1.06	2.596	2.609	2.621	2.668	2.700	2.724	2.743	2.758
1.07	2.619	2.632	2.644	2.692	2.725	2.749	2.768	2.783
1.08	2.641	2.655	2.667	2.716	2.750	2.775	2.794	2.810
1.09	2.664	2.678	2.691	2.740	2.775	2.801	2.820	2.836
1.10	2.687	2.701	2.714	2.765	2.800	2.826	2.847	2.863
1.11	2.710	2.725	2.738	2.789	2.826	2.853	2.873	2.890
1.12	2.734	2.748	2.762	2.814	2.851	2.879	2.900	2.917
1.13	2.757	2.772	2.786	2.840	2.877	2.905	2.927	2.944
1.14	2.781	2.796	2.810	2.865	2.904	2.932	2.954	2.972
1.15	2.805	2.820	2.834	2.891	2.930	2.959	2.982	3.000
1.16	2.829	2.845	2.859	2.916	2.957	2.987	3.010	3.028
1.17	2.853	2.869	2.884	2.942	2.984	3.014	3.038	3.056
1.18	2.877	2.894	2.909	2.969	3.011	3.042	3.066	3.085
1.19	2.902	2.919	2.934	2.995	3.038	3.070	3.094	3.114
1.20	2.927	2.944	2.960	3.022	3.065	3.098	3.123	3.143

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	50	75	100	125	150	175	200
0.81	2.186	2.206	2.216	2.222	2.226	2.229	2.232
0.82	2.207	2.227	2.238	2.244	2.248	2.251	2.254
0.83	2.228	2.249	2.259	2.266	2.271	2.274	2.276
0.84	2.249	2.270	2.282	2.288	2.293	2.296	2.299
0.85	2.270	2.293	2.304	2.311	2.316	2.319	2.321
0.86	2.292	2.315	2.326	2.334	2.338	2.342	2.344
0.87	2.314	2.337	2.349	2.357	2.362	2.365	2.368
0.88	2.336	2.360	2.372	2.380	2.385	2.389	2.391
0.89	2.358	2.383	2.395	2.403	2.408	2.412	2.415
0.90	2.381	2.406	2.419	2.427	2.432	2.436	2.439
0.91	2.403	2.429	2.443	2.451	2.456	2.460	2.463
0.92	2.426	2.453	2.466	2.475	2.480	2.484	2.487
0.93	2.449	2.477	2.491	2.499	2.505	2.509	2.512
0.94	2.473	2.500	2.515	2.524	2.530	2.534	2.537
0.95	2.496	2.525	2.539	2.548	2.555	2.559	2.562
0.96	2.520	2.549	2.564	2.573	2.580	2.584	2.588
0.97	2.544	2.574	2.589	2.599	2.605	2.610	2.613
0.98	2.568	2.599	2.615	2.624	2.631	2.636	2.639
0.99	2.592	2.624	2.640	2.650	2.657	2.662	2.665
1.00	2.617	2.649	2.666	2.676	2.683	2.688	2.692
1.01	2.642	2.675	2.692	2.702	2.709	2.714	2.718
1.02	2.667	2.701	2.718	2.729	2.736	2.741	2.745
1.03	2.692	2.727	2.745	2.756	2.763	2.768	2.772
1.04	2.718	2.753	2.771	2.783	2.790	2.796	2.800
1.05	2.744	2.780	2.798	2.810	2.818	2.823	2.827
1.06	2.770	2.807	2.826	2.837	2.845	2.851	2.855
1.07	2.796	2.834	2.853	2.865	2.873	2.879	2.884
1.08	2.822	2.861	2.881	2.893	2.902	2.908	2.912
1.09	2.849	2.889	2.909	2.922	2.930	2.936	2.941
1.10	2.876	2.916	2.938	2.950	2.959	2.965	2.970
1.11	2.903	2.945	2.966	2.979	2.988	2.995	3.000
1.12	2.931	2.973	2.995	3.009	3.018	3.024	3.029
1.13	2.958	3.002	3.024	3.038	3.047	3.054	3.059
1.14	2.986	3.031	3.054	3.068	3.077	3.084	3.089
1.15	3.014	3.060	3.083	3.098	3.108	3.115	3.120
1.16	3.043	3.089	3.113	3.128	3.138	3.145	3.151
1.17	3.072	3.119	3.144	3.159	3.169	3.176	3.182
1.18	3.101	3.149	3.174	3.190	3.200	3.208	3.213
1.19	3.130	3.179	3.205	3.221	3.232	3.239	3.245
1.20	3.159	3.210	3.236	3.252	3.263	3.271	3.277

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	300	400	500	750	1000	5000	
0.81	2.237	2.240	2.241	2.243	2.245	2.247	2.248
0.82	2.259	2.262	2.264	2.266	2.267	2.270	2.270
0.83	2.282	2.285	2.286	2.289	2.290	2.293	2.293
0.84	2.305	2.307	2.309	2.312	2.313	2.316	2.316
0.85	2.327	2.330	2.332	2.335	2.336	2.339	2.340
0.86	2.351	2.354	2.356	2.358	2.359	2.362	2.363
0.87	2.374	2.377	2.379	2.382	2.383	2.386	2.387
0.88	2.398	2.401	2.403	2.406	2.407	2.410	2.411
0.89	2.422	2.425	2.427	2.430	2.431	2.434	2.435
0.90	2.446	2.449	2.451	2.454	2.455	2.459	2.460
0.91	2.470	2.474	2.476	2.479	2.480	2.483	2.484
0.92	2.495	2.498	2.500	2.503	2.505	2.508	2.509
0.93	2.520	2.523	2.525	2.528	2.530	2.534	2.534
0.94	2.545	2.548	2.551	2.554	2.555	2.559	2.560
0.95	2.570	2.574	2.576	2.579	2.581	2.585	2.586
0.96	2.595	2.600	2.602	2.605	2.607	2.611	2.612
0.97	2.621	2.625	2.628	2.631	2.633	2.637	2.638
0.98	2.647	2.652	2.654	2.658	2.659	2.663	2.664
0.99	2.674	2.678	2.681	2.684	2.686	2.690	2.691
1.00	2.700	2.705	2.707	2.711	2.713	2.717	2.718
1.01	2.727	2.732	2.735	2.738	2.740	2.744	2.746
1.02	2.754	2.759	2.762	2.766	2.767	2.772	2.773
1.03	2.782	2.787	2.789	2.793	2.795	2.800	2.801
1.04	2.809	2.814	2.817	2.821	2.823	2.828	2.829
1.05	2.837	2.842	2.845	2.849	2.851	2.856	2.858
1.06	2.866	2.871	2.874	2.878	2.880	2.885	2.886
1.07	2.894	2.899	2.903	2.907	2.909	2.914	2.915
1.08	2.923	2.928	2.932	2.936	2.938	2.943	2.945
1.09	2.952	2.957	2.961	2.965	2.967	2.973	2.974
1.10	2.981	2.987	2.990	2.995	2.997	3.003	3.004
1.11	3.011	3.017	3.020	3.025	3.027	3.033	3.034
1.12	3.041	3.047	3.050	3.055	3.058	3.063	3.065
1.13	3.071	3.077	3.081	3.086	3.088	3.094	3.096
1.14	3.102	3.108	3.112	3.117	3.119	3.125	3.127
1.15	3.132	3.139	3.143	3.148	3.150	3.157	3.158
1.16	3.164	3.170	3.174	3.179	3.182	3.188	3.190
1.17	3.195	3.202	3.206	3.211	3.214	3.220	3.222
1.18	3.227	3.234	3.238	3.243	3.246	3.253	3.254
1.19	3.259	3.266	3.270	3.276	3.279	3.285	3.287
1.20	3.291	3.298	3.303	3.308	3.311	3.318	3.320

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
1.21	1.669	1.985	2.189	2.336	2.448	2.536	2.608	2.668
1.22	1.675	1.994	2.202	2.351	2.464	2.554	2.627	2.688
1.23	1.681	2.004	2.214	2.365	2.480	2.572	2.646	2.708
1.24	1.687	2.014	2.227	2.380	2.497	2.590	2.665	2.728
1.25	1.693	2.024	2.239	2.395	2.514	2.608	2.685	2.749
1.26	1.699	2.034	2.252	2.410	2.530	2.626	2.704	2.769
1.27	1.705	2.044	2.265	2.425	2.547	2.644	2.724	2.790
1.28	1.711	2.054	2.278	2.440	2.564	2.663	2.743	2.810
1.29	1.717	2.064	2.290	2.455	2.581	2.681	2.763	2.831
1.30	1.724	2.074	2.303	2.470	2.598	2.700	2.783	2.852
1.31	1.730	2.084	2.316	2.485	2.615	2.718	2.803	2.873
1.32	1.736	2.094	2.329	2.501	2.632	2.737	2.823	2.895
1.33	1.742	2.104	2.342	2.516	2.649	2.756	2.843	2.916
1.34	1.748	2.114	2.355	2.531	2.667	2.775	2.864	2.938
1.35	1.754	2.124	2.368	2.547	2.684	2.794	2.884	2.959
1.36	1.761	2.134	2.382	2.563	2.702	2.813	2.905	2.981
1.37	1.767	2.144	2.395	2.578	2.720	2.833	2.925	3.003
1.38	1.773	2.155	2.408	2.594	2.737	2.852	2.946	3.025
1.39	1.779	2.165	2.422	2.610	2.755	2.872	2.967	3.047
1.40	1.786	2.175	2.435	2.626	2.773	2.891	2.988	3.070
1.41	1.792	2.185	2.448	2.642	2.791	2.911	3.010	3.092
1.42	1.798	2.196	2.462	2.658	2.809	2.931	3.031	3.115
1.43	1.804	2.206	2.475	2.674	2.827	2.951	3.052	3.138
1.44	1.811	2.217	2.489	2.690	2.846	2.971	3.074	3.161
1.45	1.817	2.227	2.503	2.706	2.864	2.991	3.096	3.184
1.46	1.823	2.237	2.516	2.722	2.883	3.011	3.118	3.207
1.47	1.830	2.248	2.530	2.739	2.901	3.032	3.140	3.230
1.48	1.836	2.258	2.544	2.755	2.920	3.052	3.162	3.254
1.49	1.842	2.269	2.558	2.772	2.939	3.073	3.184	3.277
1.50	1.849	2.280	2.572	2.788	2.957	3.094	3.206	3.301
1.51	1.855	2.290	2.586	2.805	2.976	3.115	3.229	3.325
1.52	1.861	2.301	2.600	2.822	2.995	3.136	3.251	3.349
1.53	1.868	2.312	2.614	2.839	3.015	3.157	3.274	3.373
1.54	1.874	2.322	2.628	2.856	3.034	3.178	3.297	3.398
1.55	1.880	2.333	2.642	2.873	3.053	3.199	3.320	3.422
1.56	1.887	2.344	2.656	2.890	3.073	3.221	3.343	3.447
1.57	1.893	2.355	2.671	2.907	3.092	3.242	3.367	3.472
1.58	1.900	2.365	2.685	2.924	3.112	3.264	3.390	3.497
1.59	1.906	2.376	2.699	2.942	3.132	3.286	3.414	3.522
1.60	1.913	2.387	2.714	2.959	3.151	3.308	3.437	3.547

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	16	17
1.21	2.718	2.762	2.800	2.833	2.862	2.888	2.912	2.933
1.22	2.739	2.784	2.822	2.856	2.886	2.912	2.936	2.958
1.23	2.760	2.805	2.844	2.879	2.909	2.936	2.961	2.983
1.24	2.782	2.827	2.867	2.902	2.933	2.961	2.985	3.008
1.25	2.803	2.849	2.890	2.926	2.957	2.985	3.010	3.033
1.26	2.824	2.872	2.913	2.949	2.981	3.010	3.036	3.059
1.27	2.846	2.894	2.936	2.973	3.006	3.035	3.061	3.085
1.28	2.867	2.917	2.959	2.997	3.030	3.060	3.087	3.111
1.29	2.889	2.939	2.983	3.021	3.055	3.085	3.112	3.137
1.30	2.911	2.962	3.006	3.045	3.080	3.111	3.138	3.163
1.31	2.933	2.985	3.030	3.070	3.105	3.136	3.164	3.190
1.32	2.956	3.008	3.054	3.094	3.130	3.162	3.191	3.217
1.33	2.978	3.032	3.078	3.119	3.156	3.188	3.217	3.244
1.34	3.001	3.055	3.102	3.144	3.181	3.214	3.244	3.271
1.35	3.023	3.079	3.127	3.169	3.207	3.241	3.271	3.298
1.36	3.046	3.103	3.152	3.195	3.233	3.267	3.298	3.326
1.37	3.069	3.126	3.176	3.220	3.259	3.294	3.325	3.354
1.38	3.092	3.151	3.201	3.246	3.286	3.321	3.353	3.382
1.39	3.116	3.175	3.226	3.272	3.312	3.348	3.381	3.410
1.40	3.139	3.199	3.252	3.298	3.339	3.376	3.409	3.438
1.41	3.163	3.224	3.277	3.324	3.366	3.403	3.437	3.467
1.42	3.187	3.249	3.303	3.350	3.393	3.431	3.465	3.496
1.43	3.211	3.274	3.329	3.377	3.420	3.459	3.494	3.525
1.44	3.235	3.299	3.355	3.404	3.448	3.487	3.522	3.555
1.45	3.259	3.324	3.381	3.431	3.476	3.515	3.552	3.584
1.46	3.283	3.349	3.407	3.458	3.503	3.544	3.581	3.614
1.47	3.308	3.375	3.434	3.485	3.532	3.573	3.610	3.644
1.48	3.333	3.401	3.460	3.513	3.560	3.602	3.640	3.674
1.49	3.357	3.427	3.487	3.541	3.588	3.631	3.670	3.705
1.50	3.382	3.453	3.514	3.569	3.617	3.661	3.700	3.735
1.51	3.408	3.479	3.542	3.597	3.646	3.690	3.730	3.766
1.52	3.433	3.505	3.569	3.625	3.675	3.720	3.761	3.797
1.53	3.458	3.532	3.597	3.654	3.705	3.750	3.791	3.829
1.54	3.484	3.559	3.624	3.682	3.734	3.781	3.822	3.861
1.55	3.510	3.586	3.652	3.711	3.764	3.811	3.854	3.892
1.56	3.536	3.613	3.681	3.741	3.794	3.842	3.885	3.925
1.57	3.562	3.640	3.709	3.770	3.824	3.873	3.917	3.957
1.58	3.588	3.668	3.738	3.799	3.855	3.904	3.949	3.989
1.59	3.615	3.696	3.766	3.829	3.885	3.936	3.981	4.022
1.60	3.642	3.723	3.795	3.859	3.916	3.967	4.013	4.055

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	18	19	20	25	30	35	40	45
1.21	2.952	2.969	2.985	3.049	3.093	3.126	3.152	3.172
1.22	2.977	2.995	3.011	3.076	3.121	3.155	3.181	3.202
1.23	3.003	3.021	3.037	3.103	3.150	3.184	3.211	3.232
1.24	3.028	3.047	3.064	3.131	3.178	3.213	3.241	3.262
1.25	3.054	3.073	3.090	3.159	3.207	3.243	3.271	3.293
1.26	3.080	3.099	3.117	3.187	3.236	3.273	3.301	3.324
1.27	3.106	3.126	3.144	3.215	3.265	3.303	3.332	3.355
1.28	3.133	3.152	3.171	3.243	3.295	3.333	3.362	3.386
1.29	3.159	3.179	3.198	3.272	3.324	3.363	3.394	3.418
1.30	3.186	3.207	3.226	3.301	3.354	3.394	3.425	3.450
1.31	3.213	3.234	3.253	3.330	3.385	3.425	3.457	3.482
1.32	3.240	3.262	3.281	3.360	3.415	3.457	3.489	3.514
1.33	3.268	3.290	3.310	3.389	3.446	3.488	3.521	3.547
1.34	3.295	3.318	3.338	3.419	3.477	3.520	3.553	3.580
1.35	3.323	3.346	3.367	3.450	3.508	3.552	3.586	3.613
1.36	3.351	3.374	3.396	3.480	3.540	3.585	3.619	3.647
1.37	3.380	3.403	3.425	3.511	3.572	3.617	3.653	3.681
1.38	3.408	3.432	3.454	3.542	3.604	3.650	3.686	3.715
1.39	3.437	3.461	3.484	3.573	3.636	3.684	3.720	3.750
1.40	3.466	3.491	3.513	3.604	3.669	3.717	3.755	3.785
1.41	3.495	3.520	3.543	3.636	3.702	3.751	3.789	3.820
1.42	3.524	3.550	3.574	3.668	3.735	3.785	3.824	3.855
1.43	3.554	3.580	3.604	3.700	3.769	3.820	3.859	3.891
1.44	3.584	3.610	3.635	3.733	3.802	3.854	3.895	3.927
1.45	3.614	3.641	3.666	3.766	3.836	3.889	3.931	3.964
1.46	3.644	3.672	3.697	3.799	3.871	3.925	3.967	4.001
1.47	3.675	3.703	3.729	3.832	3.905	3.960	4.003	4.038
1.48	3.706	3.734	3.760	3.865	3.940	3.996	4.040	4.075
1.49	3.737	3.766	3.792	3.899	3.976	4.033	4.077	4.113
1.50	3.768	3.797	3.825	3.933	4.011	4.069	4.115	4.151
1.51	3.799	3.829	3.857	3.968	4.047	4.106	4.152	4.189
1.52	3.831	3.862	3.890	4.003	4.083	4.143	4.190	4.228
1.53	3.863	3.894	3.923	4.038	4.119	4.181	4.229	4.267
1.54	3.895	3.927	3.956	4.073	4.156	4.219	4.268	4.307
1.55	3.928	3.960	3.990	4.108	4.193	4.257	4.307	4.347
1.56	3.960	3.993	4.023	4.144	4.231	4.296	4.346	4.387
1.57	3.993	4.027	4.057	4.180	4.268	4.334	4.386	4.427
1.58	4.027	4.060	4.092	4.217	4.306	4.374	4.426	4.468
1.59	4.060	4.095	4.126	4.253	4.345	4.413	4.467	4.509
1.60	4.094	4.129	4.161	4.291	4.383	4.453	4.507	4.551

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	50	75	100	125	150	175	200
1.21	3.189	3.241	3.268	3.284	3.295	3.304	3.310
1.22	3.219	3.272	3.299	3.316	3.328	3.336	3.342
1.23	3.249	3.303	3.332	3.349	3.361	3.369	3.375
1.24	3.280	3.335	3.364	3.382	3.394	3.402	3.409
1.25	3.311	3.367	3.397	3.415	3.427	3.436	3.442
1.26	3.342	3.400	3.430	3.448	3.461	3.470	3.476
1.27	3.374	3.432	3.463	3.482	3.495	3.504	3.511
1.28	3.405	3.465	3.497	3.516	3.529	3.538	3.545
1.29	3.437	3.499	3.531	3.550	3.564	3.573	3.580
1.30	3.470	3.532	3.565	3.585	3.599	3.608	3.616
1.31	3.502	3.566	3.599	3.620	3.634	3.644	3.651
1.32	3.535	3.600	3.634	3.655	3.670	3.680	3.688
1.33	3.568	3.635	3.670	3.691	3.706	3.716	3.724
1.34	3.602	3.670	3.705	3.727	3.742	3.753	3.761
1.35	3.636	3.705	3.741	3.763	3.779	3.790	3.798
1.36	3.670	3.740	3.777	3.800	3.816	3.827	3.835
1.37	3.704	3.776	3.814	3.837	3.853	3.864	3.873
1.38	3.739	3.812	3.851	3.875	3.891	3.903	3.911
1.39	3.774	3.849	3.888	3.913	3.929	3.941	3.950
1.40	3.809	3.886	3.926	3.951	3.968	3.980	3.989
1.41	3.845	3.923	3.964	3.989	4.006	4.019	4.028
1.42	3.881	3.961	4.002	4.028	4.046	4.058	4.068
1.43	3.917	3.998	4.041	4.068	4.085	4.098	4.108
1.44	3.954	4.037	4.080	4.107	4.125	4.139	4.149
1.45	3.991	4.075	4.120	4.147	4.166	4.179	4.189
1.46	4.028	4.114	4.160	4.188	4.207	4.220	4.231
1.47	4.066	4.154	4.200	4.228	4.248	4.262	4.272
1.48	4.104	4.193	4.241	4.270	4.289	4.304	4.315
1.49	4.142	4.233	4.282	4.311	4.331	4.346	4.357
1.50	4.181	4.274	4.323	4.353	4.374	4.389	4.400
1.51	4.220	4.315	4.365	4.396	4.417	4.432	4.443
1.52	4.259	4.356	4.407	4.439	4.460	4.475	4.487
1.53	4.299	4.398	4.450	4.482	4.504	4.519	4.531
1.54	4.339	4.440	4.493	4.525	4.548	4.564	4.576
1.55	4.379	4.482	4.536	4.570	4.592	4.609	4.621
1.56	4.420	4.525	4.580	4.614	4.637	4.654	4.667
1.57	4.461	4.568	4.624	4.659	4.683	4.700	4.713
1.58	4.503	4.611	4.669	4.704	4.728	4.746	4.759
1.59	4.545	4.655	4.714	4.750	4.775	4.792	4.806
1.60	4.587	4.700	4.759	4.796	4.821	4.839	4.853

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	300	400	500	750	1000	5000	
1.21	3.324	3.331	3.336	3.342	3.345	3.352	3.353
1.22	3.357	3.365	3.369	3.375	3.378	3.385	3.387
1.23	3.390	3.398	3.403	3.409	3.412	3.419	3.421
1.24	3.424	3.432	3.437	3.443	3.446	3.454	3.456
1.25	3.458	3.466	3.471	3.477	3.481	3.488	3.490
1.26	3.492	3.501	3.506	3.512	3.515	3.523	3.525
1.27	3.527	3.535	3.541	3.547	3.551	3.559	3.561
1.28	3.562	3.571	3.576	3.583	3.586	3.595	3.597
1.29	3.598	3.606	3.612	3.619	3.622	3.631	3.633
1.30	3.633	3.642	3.648	3.655	3.658	3.667	3.669
1.31	3.669	3.678	3.684	3.691	3.695	3.704	3.706
1.32	3.706	3.715	3.721	3.728	3.732	3.741	3.743
1.33	3.743	3.752	3.758	3.765	3.769	3.779	3.781
1.34	3.780	3.789	3.795	3.803	3.807	3.817	3.819
1.35	3.817	3.827	3.833	3.841	3.845	3.855	3.857
1.36	3.855	3.865	3.871	3.880	3.884	3.894	3.896
1.37	3.893	3.904	3.910	3.918	3.923	3.933	3.935
1.38	3.932	3.943	3.949	3.958	3.962	3.972	3.975
1.39	3.971	3.982	3.988	3.997	4.002	4.012	4.015
1.40	4.011	4.022	4.028	4.037	4.042	4.052	4.055
1.41	4.050	4.062	4.068	4.077	4.082	4.093	4.096
1.42	4.091	4.102	4.109	4.118	4.123	4.134	4.137
1.43	4.131	4.143	4.150	4.159	4.164	4.176	4.179
1.44	4.172	4.184	4.191	4.201	4.206	4.218	4.221
1.45	4.214	4.226	4.233	4.243	4.248	4.260	4.263
1.46	4.255	4.268	4.275	4.285	4.290	4.303	4.306
1.47	4.298	4.310	4.318	4.328	4.334	4.346	4.349
1.48	4.340	4.353	4.361	4.372	4.377	4.390	4.393
1.49	4.383	4.397	4.405	4.415	4.421	4.434	4.437
1.50	4.427	4.440	4.448	4.459	4.465	4.478	4.482
1.51	4.471	4.484	4.493	4.504	4.510	4.523	4.527
1.52	4.515	4.529	4.538	4.549	4.555	4.569	4.572
1.53	4.560	4.574	4.583	4.595	4.600	4.615	4.618
1.54	4.605	4.620	4.629	4.640	4.646	4.661	4.665
1.55	4.651	4.666	4.675	4.687	4.693	4.708	4.711
1.56	4.697	4.712	4.721	4.734	4.740	4.755	4.759
1.57	4.743	4.759	4.768	4.781	4.787	4.803	4.807
1.58	4.790	4.806	4.816	4.829	4.835	4.851	4.855
1.59	4.838	4.854	4.864	4.877	4.884	4.900	4.904
1.60	4.886	4.902	4.912	4.926	4.933	4.949	4.953

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
1.61	1.919	2.398	2.728	2.976	3.171	3.330	3.461	3.573
1.62	1.925	2.409	2.743	2.994	3.191	3.352	3.485	3.598
1.63	1.932	2.420	2.758	3.012	3.212	3.374	3.509	3.624
1.64	1.938	2.431	2.772	3.029	3.232	3.396	3.534	3.650
1.65	1.945	2.442	2.787	3.047	3.252	3.419	3.558	3.676
1.66	1.951	2.453	2.802	3.065	3.273	3.442	3.583	3.702
1.67	1.958	2.464	2.817	3.083	3.293	3.464	3.607	3.729
1.68	1.964	2.475	2.832	3.101	3.314	3.487	3.632	3.755
1.69	1.971	2.487	2.847	3.119	3.334	3.510	3.657	3.782
1.70	1.977	2.498	2.862	3.137	3.355	3.533	3.682	3.809
1.71	1.984	2.509	2.877	3.155	3.376	3.557	3.707	3.836
1.72	1.991	2.520	2.892	3.174	3.397	3.580	3.733	3.863
1.73	1.997	2.532	2.907	3.192	3.418	3.604	3.758	3.890
1.74	2.004	2.543	2.922	3.211	3.440	3.627	3.784	3.918
1.75	2.010	2.554	2.938	3.229	3.461	3.651	3.810	3.945
1.76	2.017	2.566	2.953	3.248	3.483	3.675	3.836	3.973
1.77	2.023	2.577	2.968	3.267	3.504	3.699	3.862	4.001
1.78	2.030	2.589	2.984	3.286	3.526	3.723	3.888	4.029
1.79	2.037	2.600	2.999	3.304	3.548	3.747	3.915	4.058
1.80	2.043	2.612	3.015	3.323	3.570	3.772	3.942	4.086
1.81	2.050	2.623	3.031	3.343	3.592	3.796	3.968	4.115
1.82	2.057	2.635	3.046	3.362	3.614	3.821	3.995	4.144
1.83	2.063	2.647	3.062	3.381	3.636	3.846	4.022	4.173
1.84	2.070	2.658	3.078	3.400	3.658	3.871	4.049	4.202
1.85	2.077	2.670	3.094	3.420	3.681	3.896	4.077	4.232
1.86	2.083	2.682	3.110	3.439	3.703	3.921	4.104	4.261
1.87	2.090	2.693	3.126	3.459	3.726	3.946	4.132	4.291
1.88	2.097	2.705	3.142	3.478	3.749	3.972	4.160	4.321
1.89	2.104	2.717	3.158	3.498	3.772	3.998	4.188	4.351
1.90	2.110	2.729	3.174	3.518	3.795	4.023	4.216	4.381
1.91	2.117	2.741	3.190	3.538	3.818	4.049	4.244	4.412
1.92	2.124	2.753	3.207	3.558	3.841	4.075	4.273	4.442
1.93	2.131	2.765	3.223	3.578	3.864	4.101	4.301	4.473
1.94	2.137	2.777	3.239	3.598	3.888	4.128	4.330	4.504
1.95	2.144	2.789	3.256	3.619	3.911	4.154	4.359	4.535
1.96	2.151	2.801	3.272	3.639	3.935	4.181	4.388	4.567
1.97	2.158	2.813	3.289	3.660	3.959	4.207	4.418	4.598
1.98	2.165	2.825	3.306	3.680	3.983	4.234	4.447	4.630
1.99	2.171	2.837	3.322	3.701	4.007	4.261	4.477	4.662
2.00	2.178	2.849	3.339	3.721	4.031	4.288	4.506	4.694

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	16	17
1.61	3.668	3.752	3.825	3.889	3.947	3.999	4.046	4.089
1.62	3.695	3.780	3.854	3.920	3.978	4.031	4.079	4.122
1.63	3.723	3.808	3.884	3.950	4.010	4.064	4.112	4.156
1.64	3.750	3.837	3.913	3.981	4.042	4.096	4.145	4.190
1.65	3.777	3.866	3.943	4.012	4.074	4.129	4.179	4.225
1.66	3.805	3.895	3.973	4.043	4.106	4.162	4.213	4.259
1.67	3.833	3.924	4.004	4.075	4.138	4.195	4.247	4.294
1.68	3.861	3.953	4.034	4.107	4.171	4.229	4.281	4.329
1.69	3.889	3.983	4.065	4.138	4.204	4.263	4.316	4.365
1.70	3.918	4.013	4.096	4.171	4.237	4.297	4.351	4.400
1.71	3.946	4.043	4.127	4.203	4.270	4.331	4.386	4.436
1.72	3.975	4.073	4.159	4.235	4.304	4.366	4.421	4.472
1.73	4.004	4.103	4.190	4.268	4.338	4.400	4.457	4.509
1.74	4.033	4.134	4.222	4.301	4.372	4.435	4.493	4.545
1.75	4.062	4.164	4.254	4.334	4.406	4.471	4.529	4.582
1.76	4.092	4.195	4.287	4.368	4.441	4.506	4.566	4.620
1.77	4.121	4.226	4.319	4.402	4.475	4.542	4.602	4.657
1.78	4.151	4.258	4.352	4.435	4.510	4.578	4.639	4.695
1.79	4.181	4.289	4.385	4.470	4.546	4.614	4.677	4.733
1.80	4.212	4.321	4.418	4.504	4.581	4.651	4.714	4.772
1.81	4.242	4.353	4.451	4.539	4.617	4.688	4.752	4.810
1.82	4.273	4.385	4.485	4.574	4.653	4.725	4.790	4.849
1.83	4.303	4.418	4.519	4.609	4.689	4.762	4.828	4.888
1.84	4.334	4.450	4.553	4.644	4.726	4.800	4.867	4.928
1.85	4.366	4.483	4.587	4.680	4.763	4.838	4.906	4.968
1.86	4.397	4.516	4.621	4.715	4.800	4.876	4.945	5.008
1.87	4.429	4.549	4.656	4.751	4.837	4.914	4.984	5.048
1.88	4.460	4.583	4.691	4.788	4.874	4.953	5.024	5.089
1.89	4.492	4.616	4.726	4.824	4.912	4.992	5.064	5.130
1.90	4.525	4.650	4.762	4.861	4.950	5.031	5.104	5.171
1.91	4.557	4.684	4.797	4.898	4.989	5.071	5.145	5.213
1.92	4.589	4.719	4.833	4.935	5.027	5.110	5.186	5.255
1.93	4.622	4.753	4.869	4.973	5.066	5.150	5.227	5.297
1.94	4.655	4.788	4.906	5.011	5.105	5.191	5.269	5.340
1.95	4.688	4.823	4.942	5.049	5.145	5.232	5.310	5.382
1.96	4.722	4.858	4.979	5.087	5.185	5.272	5.352	5.426
1.97	4.755	4.894	5.016	5.126	5.224	5.314	5.395	5.469
1.98	4.789	4.929	5.054	5.165	5.265	5.355	5.438	5.513
1.99	4.823	4.965	5.091	5.204	5.305	5.397	5.481	5.557
2.00	4.857	5.001	5.129	5.243	5.346	5.439	5.524	5.601

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	18	19	20	25	30	35	40	45
1.61	4.128	4.163	4.196	4.328	4.422	4.493	4.549	4.593
1.62	4.162	4.198	4.232	4.365	4.461	4.534	4.590	4.635
1.63	4.196	4.233	4.267	4.403	4.501	4.575	4.632	4.678
1.64	4.231	4.269	4.303	4.442	4.541	4.616	4.674	4.721
1.65	4.266	4.304	4.339	4.480	4.581	4.657	4.717	4.765
1.66	4.301	4.340	4.376	4.519	4.622	4.699	4.760	4.809
1.67	4.337	4.376	4.413	4.558	4.663	4.742	4.803	4.853
1.68	4.373	4.413	4.450	4.598	4.704	4.784	4.847	4.898
1.69	4.409	4.450	4.487	4.638	4.746	4.827	4.891	4.943
1.70	4.445	4.487	4.525	4.678	4.788	4.871	4.936	4.988
1.71	4.482	4.524	4.563	4.718	4.830	4.915	4.981	5.034
1.72	4.519	4.562	4.601	4.759	4.873	4.959	5.026	5.080
1.73	4.556	4.599	4.639	4.800	4.916	5.003	5.072	5.127
1.74	4.593	4.638	4.678	4.842	4.959	5.048	5.118	5.174
1.75	4.631	4.676	4.717	4.883	5.003	5.094	5.165	5.222
1.76	4.669	4.715	4.757	4.925	5.047	5.139	5.212	5.270
1.77	4.708	4.754	4.796	4.968	5.092	5.185	5.259	5.318
1.78	4.746	4.793	4.836	5.011	5.137	5.232	5.307	5.367
1.79	4.785	4.833	4.877	5.054	5.182	5.279	5.355	5.416
1.80	4.824	4.873	4.917	5.097	5.227	5.326	5.403	5.466
1.81	4.864	4.913	4.958	5.141	5.273	5.374	5.452	5.516
1.82	4.904	4.953	5.000	5.185	5.320	5.422	5.502	5.566
1.83	4.944	4.994	5.041	5.230	5.367	5.470	5.552	5.617
1.84	4.984	5.036	5.083	5.275	5.414	5.519	5.602	5.669
1.85	5.025	5.077	5.125	5.320	5.461	5.569	5.653	5.721
1.86	5.066	5.119	5.168	5.366	5.509	5.618	5.704	5.773
1.87	5.107	5.161	5.211	5.412	5.558	5.668	5.755	5.826
1.88	5.149	5.203	5.254	5.458	5.606	5.719	5.807	5.879
1.89	5.190	5.246	5.297	5.505	5.656	5.770	5.860	5.933
1.90	5.233	5.289	5.341	5.552	5.705	5.821	5.913	5.987
1.91	5.275	5.333	5.385	5.600	5.755	5.873	5.966	6.041
1.92	5.318	5.376	5.430	5.647	5.806	5.926	6.020	6.097
1.93	5.361	5.420	5.475	5.696	5.856	5.978	6.075	6.152
1.94	5.405	5.465	5.520	5.744	5.908	6.032	6.129	6.208
1.95	5.449	5.509	5.566	5.794	5.959	6.085	6.185	6.265
1.96	5.493	5.554	5.612	5.843	6.011	6.139	6.240	6.322
1.97	5.537	5.600	5.658	5.893	6.064	6.194	6.297	6.380
1.98	5.582	5.646	5.704	5.943	6.117	6.249	6.353	6.438
1.99	5.627	5.692	5.751	5.994	6.170	6.305	6.411	6.496
2.00	5.673	5.738	5.799	6.045	6.224	6.361	6.468	6.555

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	50	75	100	125	150	175	200
1.61	4.630	4.745	4.805	4.843	4.868	4.887	4.901
1.62	4.673	4.790	4.852	4.890	4.916	4.935	4.949
1.63	4.716	4.835	4.899	4.938	4.964	4.983	4.998
1.64	4.760	4.881	4.946	4.986	5.013	5.032	5.047
1.65	4.804	4.928	4.993	5.034	5.062	5.082	5.097
1.66	4.849	4.975	5.042	5.083	5.111	5.132	5.147
1.67	4.894	5.022	5.090	5.132	5.161	5.182	5.198
1.68	4.939	5.070	5.139	5.182	5.211	5.233	5.249
1.69	4.985	5.118	5.189	5.233	5.262	5.284	5.300
1.70	5.031	5.167	5.239	5.283	5.314	5.336	5.353
1.71	5.078	5.216	5.289	5.335	5.366	5.388	5.405
1.72	5.125	5.266	5.340	5.387	5.418	5.441	5.458
1.73	5.172	5.316	5.392	5.439	5.471	5.494	5.512
1.74	5.220	5.366	5.444	5.492	5.524	5.548	5.566
1.75	5.269	5.417	5.496	5.545	5.578	5.603	5.621
1.76	5.317	5.469	5.549	5.599	5.633	5.657	5.676
1.77	5.367	5.521	5.602	5.653	5.688	5.713	5.732
1.78	5.416	5.573	5.656	5.708	5.743	5.769	5.788
1.79	5.466	5.626	5.711	5.763	5.799	5.825	5.845
1.80	5.517	5.679	5.766	5.819	5.856	5.882	5.903
1.81	5.568	5.733	5.821	5.876	5.913	5.940	5.961
1.82	5.619	5.788	5.877	5.933	5.971	5.998	6.019
1.83	5.671	5.842	5.934	5.990	6.029	6.057	6.078
1.84	5.724	5.898	5.991	6.048	6.088	6.116	6.138
1.85	5.776	5.954	6.048	6.107	6.147	6.176	6.198
1.86	5.830	6.010	6.106	6.166	6.207	6.237	6.259
1.87	5.883	6.067	6.165	6.226	6.267	6.298	6.321
1.88	5.938	6.124	6.224	6.286	6.329	6.359	6.383
1.89	5.992	6.182	6.284	6.347	6.390	6.422	6.445
1.90	6.048	6.241	6.344	6.409	6.453	6.485	6.509
1.91	6.103	6.300	6.405	6.471	6.515	6.548	6.573
1.92	6.160	6.360	6.467	6.533	6.579	6.612	6.637
1.93	6.216	6.420	6.529	6.597	6.643	6.677	6.702
1.94	6.273	6.481	6.591	6.661	6.708	6.742	6.768
1.95	6.331	6.542	6.655	6.725	6.773	6.808	6.834
1.96	6.389	6.604	6.719	6.790	6.839	6.875	6.902
1.97	6.448	6.666	6.783	6.856	6.906	6.942	6.969
1.98	6.507	6.729	6.848	6.922	6.973	7.010	7.038
1.99	6.567	6.793	6.914	6.989	7.041	7.078	7.107
2.00	6.628	6.857	6.980	7.057	7.109	7.147	7.176

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	300	400	500	750	1000	5000	
1.61	4.934	4.951	4.961	4.975	4.982	4.999	5.003
1.62	4.983	5.000	5.011	5.025	5.032	5.049	5.053
1.63	5.032	5.050	5.061	5.075	5.082	5.099	5.104
1.64	5.082	5.100	5.111	5.126	5.133	5.151	5.155
1.65	5.133	5.151	5.162	5.177	5.184	5.202	5.207
1.66	5.184	5.202	5.213	5.229	5.236	5.255	5.259
1.67	5.235	5.254	5.265	5.281	5.289	5.307	5.312
1.68	5.287	5.306	5.318	5.334	5.342	5.361	5.365
1.69	5.339	5.359	5.371	5.387	5.395	5.414	5.419
1.70	5.392	5.412	5.424	5.441	5.449	5.469	5.474
1.71	5.445	5.466	5.478	5.495	5.503	5.524	5.529
1.72	5.499	5.520	5.533	5.550	5.559	5.579	5.584
1.73	5.554	5.575	5.588	5.605	5.614	5.635	5.641
1.74	5.609	5.631	5.644	5.661	5.670	5.692	5.697
1.75	5.664	5.687	5.700	5.718	5.727	5.749	5.755
1.76	5.720	5.743	5.757	5.775	5.784	5.807	5.812
1.77	5.777	5.800	5.814	5.833	5.842	5.865	5.871
1.78	5.834	5.858	5.872	5.891	5.901	5.924	5.930
1.79	5.892	5.916	5.930	5.950	5.960	5.983	5.989
1.80	5.950	5.975	5.989	6.009	6.019	6.044	6.050
1.81	6.009	6.034	6.049	6.069	6.080	6.104	6.110
1.82	6.069	6.094	6.109	6.130	6.140	6.166	6.172
1.83	6.129	6.155	6.170	6.191	6.202	6.227	6.234
1.84	6.190	6.216	6.232	6.253	6.264	6.290	6.297
1.85	6.251	6.278	6.294	6.316	6.327	6.353	6.360
1.86	6.313	6.340	6.356	6.379	6.390	6.417	6.424
1.87	6.375	6.403	6.420	6.442	6.454	6.481	6.488
1.88	6.438	6.466	6.484	6.507	6.518	6.546	6.553
1.89	6.502	6.531	6.548	6.572	6.583	6.612	6.619
1.90	6.566	6.596	6.613	6.637	6.649	6.679	6.686
1.91	6.631	6.661	6.679	6.704	6.716	6.746	6.753
1.92	6.697	6.727	6.746	6.770	6.783	6.813	6.821
1.93	6.763	6.794	6.813	6.838	6.851	6.882	6.889
1.94	6.830	6.861	6.881	6.906	6.919	6.951	6.959
1.95	6.897	6.930	6.949	6.975	6.989	7.021	7.029
1.96	6.966	6.998	7.018	7.045	7.058	7.091	7.099
1.97	7.035	7.068	7.088	7.115	7.129	7.162	7.171
1.98	7.104	7.138	7.159	7.186	7.200	7.234	7.243
1.99	7.174	7.209	7.230	7.258	7.272	7.307	7.315
2.00	7.245	7.280	7.302	7.331	7.345	7.380	7.389

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
2.01	2.185	2.862	3.356	3.742	4.055	4.316	4.536	4.726
2.02	2.192	2.874	3.373	3.763	4.080	4.343	4.567	4.759
2.03	2.199	2.886	3.390	3.784	4.104	4.371	4.597	4.792
2.04	2.206	2.899	3.407	3.805	4.129	4.398	4.627	4.824
2.05	2.213	2.911	3.424	3.827	4.154	4.426	4.658	4.858
2.06	2.219	2.923	3.441	3.848	4.179	4.454	4.689	4.891
2.07	2.226	2.936	3.459	3.869	4.203	4.482	4.720	4.924
2.08	2.233	2.948	3.476	3.891	4.229	4.511	4.751	4.958
2.09	2.240	2.961	3.493	3.912	4.254	4.539	4.782	4.992
2.10	2.247	2.973	3.511	3.934	4.279	4.568	4.814	5.026
2.11	2.254	2.986	3.528	3.956	4.305	4.597	4.845	5.060
2.12	2.261	2.998	3.546	3.977	4.330	4.626	4.877	5.095
2.13	2.268	3.011	3.563	3.999	4.356	4.655	4.909	5.129
2.14	2.275	3.024	3.581	4.021	4.382	4.684	4.941	5.164
2.15	2.282	3.036	3.599	4.043	4.408	4.713	4.974	5.199
2.16	2.289	3.049	3.616	4.066	4.434	4.743	5.006	5.235
2.17	2.296	3.062	3.634	4.088	4.460	4.772	5.039	5.270
2.18	2.303	3.075	3.652	4.110	4.486	4.802	5.072	5.306
2.19	2.310	3.088	3.670	4.133	4.513	4.832	5.105	5.342
2.20	2.317	3.101	3.688	4.156	4.539	4.862	5.138	5.378
2.21	2.324	3.114	3.706	4.178	4.566	4.893	5.172	5.414
2.22	2.331	3.126	3.725	4.201	4.593	4.923	5.206	5.451
2.23	2.338	3.139	3.743	4.224	4.620	4.954	5.239	5.488
2.24	2.345	3.153	3.761	4.247	4.647	4.984	5.273	5.525
2.25	2.352	3.166	3.780	4.270	4.674	5.015	5.308	5.562
2.26	2.360	3.179	3.798	4.293	4.702	5.046	5.342	5.599
2.27	2.367	3.192	3.817	4.316	4.729	5.078	5.377	5.637
2.28	2.374	3.205	3.835	4.340	4.757	5.109	5.411	5.675
2.29	2.381	3.218	3.854	4.363	4.785	5.141	5.446	5.713
2.30	2.388	3.231	3.872	4.387	4.812	5.172	5.482	5.751
2.31	2.395	3.245	3.891	4.411	4.840	5.204	5.517	5.789
2.32	2.402	3.258	3.910	4.434	4.869	5.236	5.553	5.828
2.33	2.410	3.271	3.929	4.458	4.897	5.268	5.588	5.867
2.34	2.417	3.285	3.948	4.482	4.925	5.301	5.624	5.906
2.35	2.424	3.298	3.967	4.506	4.954	5.333	5.660	5.946
2.36	2.431	3.312	3.986	4.530	4.983	5.366	5.697	5.985
2.37	2.438	3.325	4.005	4.555	5.011	5.399	5.733	6.025
2.38	2.446	3.339	4.025	4.579	5.040	5.432	5.770	6.065
2.39	2.453	3.352	4.044	4.604	5.069	5.465	5.807	6.105
2.40	2.460	3.366	4.063	4.628	5.099	5.499	5.844	6.146

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	16	17
2.01	4.892	5.038	5.167	5.283	5.387	5.482	5.568	5.646
2.02	4.927	5.074	5.205	5.323	5.429	5.524	5.612	5.691
2.03	4.961	5.111	5.244	5.363	5.470	5.567	5.656	5.737
2.04	4.997	5.148	5.283	5.403	5.512	5.611	5.700	5.782
2.05	5.032	5.185	5.322	5.444	5.554	5.654	5.745	5.829
2.06	5.067	5.223	5.361	5.485	5.597	5.698	5.790	5.875
2.07	5.103	5.261	5.401	5.526	5.640	5.742	5.836	5.922
2.08	5.139	5.299	5.441	5.568	5.683	5.787	5.882	5.969
2.09	5.175	5.337	5.481	5.610	5.726	5.832	5.928	6.016
2.10	5.212	5.375	5.521	5.652	5.770	5.877	5.975	6.064
2.11	5.248	5.414	5.562	5.694	5.814	5.923	6.021	6.112
2.12	5.285	5.453	5.603	5.737	5.858	5.968	6.069	6.161
2.13	5.322	5.492	5.644	5.780	5.903	6.014	6.116	6.209
2.14	5.359	5.532	5.685	5.823	5.948	6.061	6.164	6.259
2.15	5.397	5.572	5.727	5.867	5.993	6.108	6.212	6.308
2.16	5.435	5.612	5.769	5.911	6.039	6.155	6.261	6.358
2.17	5.473	5.652	5.811	5.955	6.085	6.202	6.310	6.408
2.18	5.511	5.692	5.854	5.999	6.131	6.250	6.359	6.459
2.19	5.549	5.733	5.897	6.044	6.177	6.298	6.409	6.510
2.20	5.588	5.774	5.940	6.089	6.224	6.347	6.459	6.561
2.21	5.627	5.815	5.983	6.134	6.271	6.395	6.509	6.613
2.22	5.666	5.857	6.027	6.180	6.319	6.445	6.560	6.665
2.23	5.705	5.898	6.071	6.226	6.366	6.494	6.611	6.718
2.24	5.745	5.941	6.115	6.272	6.414	6.544	6.662	6.771
2.25	5.785	5.983	6.160	6.319	6.463	6.594	6.714	6.824
2.26	5.825	6.025	6.204	6.366	6.512	6.644	6.766	6.877
2.27	5.865	6.068	6.250	6.413	6.561	6.695	6.818	6.931
2.28	5.906	6.111	6.295	6.460	6.610	6.747	6.871	6.986
2.29	5.947	6.155	6.341	6.508	6.660	6.798	6.925	7.041
2.30	5.988	6.198	6.387	6.556	6.710	6.850	6.978	7.096
2.31	6.029	6.242	6.433	6.605	6.760	6.902	7.032	7.151
2.32	6.071	6.286	6.479	6.653	6.811	6.955	7.087	7.207
2.33	6.113	6.331	6.526	6.703	6.862	7.008	7.141	7.264
2.34	6.155	6.376	6.574	6.752	6.914	7.061	7.196	7.321
2.35	6.197	6.421	6.621	6.802	6.966	7.115	7.252	7.378
2.36	6.240	6.466	6.669	6.852	7.018	7.169	7.308	7.435
2.37	6.282	6.512	6.717	6.902	7.070	7.224	7.364	7.493
2.38	6.326	6.557	6.765	6.953	7.123	7.279	7.421	7.552
2.39	6.369	6.604	6.814	7.004	7.176	7.334	7.478	7.611
2.40	6.413	6.650	6.863	7.055	7.230	7.389	7.536	7.670

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	18	19	20	25	30	35	40	45
2.01	5.718	5.785	5.846	6.096	6.278	6.417	6.527	6.615
2.02	5.765	5.832	5.895	6.148	6.333	6.474	6.585	6.675
2.03	5.811	5.880	5.943	6.200	6.388	6.531	6.644	6.736
2.04	5.858	5.927	5.992	6.253	6.444	6.589	6.704	6.797
2.05	5.905	5.976	6.041	6.306	6.500	6.648	6.764	6.859
2.06	5.953	6.024	6.090	6.360	6.556	6.707	6.825	6.921
2.07	6.000	6.073	6.140	6.414	6.613	6.766	6.886	6.984
2.08	6.049	6.122	6.191	6.468	6.671	6.826	6.948	7.048
2.09	6.097	6.172	6.241	6.523	6.729	6.886	7.011	7.112
2.10	6.146	6.222	6.292	6.578	6.787	6.947	7.074	7.176
2.11	6.196	6.273	6.344	6.634	6.846	7.008	7.137	7.241
2.12	6.245	6.323	6.396	6.690	6.905	7.070	7.201	7.307
2.13	6.295	6.374	6.448	6.746	6.965	7.133	7.265	7.373
2.14	6.346	6.426	6.500	6.803	7.026	7.196	7.330	7.440
2.15	6.396	6.478	6.553	6.861	7.086	7.259	7.396	7.507
2.16	6.448	6.530	6.607	6.919	7.148	7.323	7.462	7.575
2.17	6.499	6.583	6.661	6.977	7.210	7.388	7.529	7.644
2.18	6.551	6.636	6.715	7.036	7.272	7.453	7.596	7.713
2.19	6.603	6.690	6.769	7.095	7.335	7.519	7.664	7.783
2.20	6.656	6.743	6.825	7.155	7.398	7.585	7.733	7.853
2.21	6.709	6.798	6.880	7.215	7.462	7.652	7.802	7.924
2.22	6.763	6.852	6.936	7.276	7.527	7.719	7.872	7.996
2.23	6.816	6.908	6.992	7.337	7.592	7.787	7.942	8.068
2.24	6.871	6.963	7.049	7.399	7.657	7.855	8.013	8.141
2.25	6.925	7.019	7.106	7.461	7.723	7.924	8.084	8.214
2.26	6.980	7.075	7.164	7.524	7.790	7.994	8.156	8.288
2.27	7.036	7.132	7.221	7.587	7.857	8.064	8.229	8.363
2.28	7.092	7.189	7.280	7.651	7.924	8.135	8.302	8.438
2.29	7.148	7.247	7.339	7.715	7.993	8.206	8.376	8.514
2.30	7.204	7.305	7.398	7.780	8.061	8.278	8.451	8.591
2.31	7.261	7.363	7.458	7.845	8.131	8.351	8.526	8.668
2.32	7.319	7.422	7.518	7.911	8.201	8.424	8.602	8.746
2.33	7.377	7.482	7.579	7.977	8.271	8.498	8.678	8.825
2.34	7.435	7.541	7.640	8.043	8.342	8.572	8.755	8.905
2.35	7.494	7.601	7.701	8.111	8.414	8.647	8.833	8.985
2.36	7.553	7.662	7.763	8.179	8.486	8.723	8.912	9.066
2.37	7.613	7.723	7.826	8.247	8.559	8.799	8.991	9.147
2.38	7.673	7.785	7.889	8.316	8.632	8.876	9.071	9.229
2.39	7.733	7.847	7.952	8.385	8.706	8.954	9.151	9.312
2.40	7.794	7.909	8.016	8.455	8.781	9.032	9.232	9.396

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	50	75	100	125	150	175	200
2.01	6.688	6.922	7.047	7.125	7.178	7.217	7.247
2.02	6.750	6.987	7.114	7.194	7.248	7.288	7.318
2.03	6.812	7.053	7.183	7.263	7.319	7.359	7.390
2.04	6.874	7.120	7.251	7.334	7.390	7.431	7.462
2.05	6.937	7.187	7.321	7.405	7.462	7.503	7.535
2.06	7.001	7.255	7.391	7.476	7.534	7.577	7.609
2.07	7.065	7.323	7.462	7.548	7.608	7.651	7.684
2.08	7.130	7.392	7.533	7.621	7.682	7.726	7.759
2.09	7.195	7.462	7.605	7.695	7.756	7.801	7.835
2.10	7.261	7.532	7.678	7.769	7.832	7.877	7.912
2.11	7.327	7.603	7.751	7.844	7.908	7.954	7.989
2.12	7.394	7.675	7.826	7.920	7.985	8.032	8.068
2.13	7.462	7.747	7.900	7.996	8.062	8.110	8.147
2.14	7.530	7.820	7.976	8.074	8.141	8.189	8.226
2.15	7.599	7.893	8.052	8.152	8.220	8.269	8.307
2.16	7.669	7.968	8.129	8.230	8.299	8.350	8.388
2.17	7.739	8.043	8.207	8.310	8.380	8.431	8.470
2.18	7.809	8.118	8.285	8.390	8.461	8.514	8.553
2.19	7.881	8.195	8.364	8.471	8.544	8.597	8.637
2.20	7.953	8.272	8.444	8.552	8.627	8.681	8.722
2.21	8.025	8.350	8.525	8.635	8.710	8.765	8.807
2.22	8.098	8.428	8.606	8.718	8.795	8.851	8.893
2.23	8.172	8.507	8.688	8.802	8.880	8.937	8.980
2.24	8.247	8.587	8.771	8.887	8.966	9.024	9.068
2.25	8.322	8.668	8.855	8.973	9.053	9.112	9.157
2.26	8.398	8.749	8.940	9.059	9.141	9.201	9.247
2.27	8.474	8.831	9.025	9.146	9.230	9.291	9.337
2.28	8.551	8.914	9.111	9.234	9.319	9.381	9.428
2.29	8.629	8.998	9.198	9.324	9.410	9.473	9.521
2.30	8.708	9.082	9.286	9.413	9.501	9.565	9.614
2.31	8.787	9.167	9.374	9.504	9.593	9.658	9.708
2.32	8.867	9.253	9.463	9.595	9.686	9.752	9.803
2.33	8.947	9.340	9.554	9.688	9.780	9.847	9.899
2.34	9.029	9.428	9.645	9.781	9.875	9.943	9.995
2.35	9.111	9.516	9.737	9.875	9.971	10.040	10.093
2.36	9.193	9.605	9.829	9.970	10.067	10.138	10.192
2.37	9.277	9.695	9.923	10.066	10.165	10.237	10.291
2.38	9.361	9.786	10.017	10.163	10.263	10.336	10.392
2.39	9.446	9.878	10.113	10.261	10.363	10.437	10.493
2.40	9.532	9.970	10.209	10.359	10.463	10.538	10.596

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	300	400	500	750	1000	5000	
2.01	7.317	7.353	7.374	7.404	7.418	7.454	7.463
2.02	7.389	7.426	7.448	7.478	7.493	7.529	7.538
2.03	7.462	7.499	7.522	7.552	7.568	7.605	7.614
2.04	7.536	7.574	7.597	7.628	7.643	7.681	7.691
2.05	7.611	7.649	7.672	7.704	7.720	7.758	7.768
2.06	7.686	7.725	7.749	7.781	7.797	7.836	7.846
2.07	7.762	7.802	7.826	7.858	7.875	7.915	7.925
2.08	7.838	7.879	7.904	7.937	7.954	7.994	8.004
2.09	7.916	7.957	7.982	8.016	8.033	8.074	8.085
2.10	7.994	8.036	8.062	8.096	8.113	8.155	8.166
2.11	8.073	8.116	8.142	8.177	8.195	8.237	8.248
2.12	8.153	8.196	8.223	8.258	8.276	8.320	8.331
2.13	8.233	8.278	8.305	8.341	8.359	8.404	8.415
2.14	8.315	8.360	8.387	8.424	8.443	8.488	8.499
2.15	8.397	8.443	8.471	8.508	8.527	8.573	8.585
2.16	8.480	8.526	8.555	8.593	8.612	8.659	8.671
2.17	8.563	8.611	8.640	8.679	8.699	8.746	8.758
2.18	8.648	8.697	8.726	8.766	8.786	8.834	8.846
2.19	8.733	8.783	8.813	8.853	8.873	8.923	8.935
2.20	8.820	8.870	8.900	8.941	8.962	9.012	9.025
2.21	8.907	8.958	8.989	9.031	9.052	9.103	9.116
2.22	8.995	9.047	9.078	9.121	9.142	9.194	9.207
2.23	9.084	9.136	9.168	9.212	9.233	9.286	9.300
2.24	9.173	9.227	9.260	9.304	9.326	9.380	9.393
2.25	9.264	9.318	9.352	9.396	9.419	9.474	9.488
2.26	9.355	9.411	9.445	9.490	9.513	9.569	9.583
2.27	9.448	9.504	9.538	9.585	9.608	9.665	9.679
2.28	9.541	9.598	9.633	9.680	9.704	9.762	9.777
2.29	9.635	9.693	9.729	9.777	9.801	9.860	9.875
2.30	9.730	9.790	9.826	9.875	9.899	9.959	9.974
2.31	9.826	9.887	9.923	9.973	9.998	10.059	10.074
2.32	9.923	9.985	10.022	10.073	10.098	10.160	10.176
2.33	10.021	10.084	10.122	10.173	10.199	10.262	10.278
2.34	10.120	10.183	10.222	10.274	10.301	10.365	10.381
2.35	10.220	10.284	10.324	10.377	10.404	10.469	10.485
2.36	10.320	10.386	10.426	10.480	10.508	10.574	10.591
2.37	10.422	10.489	10.530	10.585	10.613	10.680	10.697
2.38	10.525	10.593	10.635	10.691	10.719	10.787	10.805
2.39	10.629	10.698	10.740	10.797	10.826	10.896	10.913
2.40	10.734	10.804	10.847	10.905	10.934	11.005	11.023

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
2.41	2.467	3.380	4.083	4.653	5.128	5.532	5.881	6.187
2.42	2.475	3.393	4.102	4.678	5.158	5.566	5.919	6.228
2.43	2.482	3.407	4.122	4.703	5.187	5.600	5.957	6.269
2.44	2.489	3.421	4.142	4.728	5.217	5.634	5.995	6.310
2.45	2.496	3.435	4.161	4.753	5.247	5.668	6.033	6.352
2.46	2.504	3.448	4.181	4.778	5.277	5.703	6.071	6.394
2.47	2.511	3.462	4.201	4.803	5.307	5.737	6.110	6.436
2.48	2.518	3.476	4.221	4.829	5.337	5.772	6.149	6.479
2.49	2.526	3.490	4.241	4.854	5.368	5.807	6.188	6.521
2.50	2.533	3.504	4.261	4.880	5.399	5.842	6.227	6.564
2.51	2.540	3.518	4.281	4.905	5.429	5.877	6.266	6.608
2.52	2.548	3.532	4.302	4.931	5.460	5.913	6.306	6.651
2.53	2.555	3.546	4.322	4.957	5.491	5.949	6.346	6.695
2.54	2.563	3.560	4.342	4.983	5.523	5.984	6.386	6.739
2.55	2.570	3.575	4.363	5.010	5.554	6.021	6.426	6.783
2.56	2.577	3.589	4.383	5.036	5.585	6.057	6.467	6.827
2.57	2.585	3.603	4.404	5.062	5.617	6.093	6.507	6.872
2.58	2.592	3.617	4.425	5.089	5.649	6.130	6.548	6.917
2.59	2.600	3.632	4.445	5.115	5.681	6.166	6.589	6.962
2.60	2.607	3.646	4.466	5.142	5.713	6.203	6.631	7.007
2.61	2.615	3.660	4.487	5.169	5.745	6.241	6.672	7.053
2.62	2.622	3.675	4.508	5.196	5.777	6.278	6.714	7.099
2.63	2.630	3.689	4.529	5.223	5.810	6.315	6.756	7.145
2.64	2.637	3.704	4.550	5.250	5.843	6.353	6.799	7.192
2.65	2.645	3.718	4.571	5.277	5.875	6.391	6.841	7.238
2.66	2.652	3.733	4.593	5.305	5.908	6.429	6.884	7.285
2.67	2.660	3.747	4.614	5.332	5.942	6.467	6.927	7.333
2.68	2.667	3.762	4.635	5.360	5.975	6.506	6.970	7.380
2.69	2.675	3.777	4.657	5.388	6.008	6.544	7.013	7.428
2.70	2.682	3.792	4.678	5.415	6.042	6.583	7.057	7.476
2.71	2.690	3.806	4.700	5.443	6.076	6.622	7.101	7.524
2.72	2.698	3.821	4.722	5.471	6.110	6.662	7.145	7.573
2.73	2.705	3.836	4.743	5.500	6.144	6.701	7.189	7.622
2.74	2.713	3.851	4.765	5.528	6.178	6.741	7.234	7.671
2.75	2.720	3.866	4.787	5.556	6.212	6.780	7.279	7.720
2.76	2.728	3.881	4.809	5.585	6.247	6.821	7.324	7.770
2.77	2.736	3.896	4.831	5.613	6.281	6.861	7.369	7.820
2.78	2.743	3.911	4.853	5.642	6.316	6.901	7.415	7.870
2.79	2.751	3.926	4.876	5.671	6.351	6.942	7.461	7.921
2.80	2.759	3.941	4.898	5.700	6.386	6.983	7.507	7.972

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	16	17
2.41	6.456	6.697	6.912	7.107	7.284	7.445	7.593	7.730
2.42	6.501	6.744	6.962	7.159	7.338	7.502	7.652	7.790
2.43	6.545	6.791	7.012	7.212	7.393	7.559	7.711	7.850
2.44	6.590	6.839	7.062	7.264	7.448	7.616	7.770	7.911
2.45	6.635	6.887	7.113	7.318	7.504	7.673	7.829	7.973
2.46	6.680	6.935	7.164	7.371	7.559	7.731	7.889	8.035
2.47	6.725	6.983	7.215	7.425	7.616	7.790	7.950	8.097
2.48	6.771	7.032	7.267	7.479	7.672	7.848	8.010	8.160
2.49	6.817	7.081	7.319	7.534	7.729	7.908	8.072	8.223
2.50	6.863	7.131	7.371	7.588	7.786	7.967	8.133	8.286
2.51	6.910	7.180	7.423	7.644	7.844	8.027	8.195	8.351
2.52	6.957	7.230	7.476	7.699	7.902	8.088	8.258	8.415
2.53	7.004	7.281	7.530	7.755	7.961	8.148	8.321	8.480
2.54	7.051	7.331	7.583	7.812	8.019	8.210	8.384	8.546
2.55	7.099	7.382	7.637	7.868	8.079	8.271	8.448	8.611
2.56	7.147	7.433	7.692	7.925	8.138	8.333	8.513	8.678
2.57	7.195	7.485	7.746	7.983	8.198	8.396	8.577	8.745
2.58	7.244	7.537	7.801	8.041	8.259	8.459	8.642	8.812
2.59	7.293	7.589	7.856	8.099	8.320	8.522	8.708	8.880
2.60	7.342	7.642	7.912	8.157	8.381	8.586	8.774	8.948
2.61	7.391	7.695	7.968	8.216	8.443	8.650	8.841	9.017
2.62	7.441	7.748	8.025	8.276	8.505	8.715	8.908	9.086
2.63	7.491	7.801	8.081	8.335	8.567	8.780	8.975	9.156
2.64	7.541	7.855	8.138	8.396	8.630	8.845	9.043	9.226
2.65	7.592	7.909	8.196	8.456	8.694	8.911	9.112	9.297
2.66	7.643	7.964	8.254	8.517	8.757	8.978	9.181	9.368
2.67	7.694	8.019	8.312	8.578	8.821	9.045	9.250	9.440
2.68	7.746	8.074	8.370	8.640	8.886	9.112	9.320	9.512
2.69	7.797	8.129	8.429	8.702	8.951	9.180	9.390	9.585
2.70	7.849	8.185	8.489	8.765	9.017	9.248	9.461	9.658
2.71	7.902	8.241	8.548	8.827	9.083	9.317	9.532	9.732
2.72	7.955	8.298	8.608	8.891	9.149	9.386	9.604	9.806
2.73	8.008	8.355	8.669	8.954	9.216	9.455	9.677	9.881
2.74	8.061	8.412	8.730	9.019	9.283	9.526	9.749	9.956
2.75	8.115	8.470	8.791	9.083	9.351	9.596	9.823	10.032
2.76	8.169	8.527	8.852	9.148	9.419	9.667	9.896	10.109
2.77	8.223	8.586	8.914	9.213	9.487	9.739	9.971	10.186
2.78	8.278	8.644	8.977	9.279	9.556	9.811	10.046	10.263
2.79	8.332	8.703	9.039	9.345	9.626	9.883	10.121	10.341
2.80	8.388	8.763	9.102	9.412	9.696	9.956	10.197	10.420

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	18	19	20	25	30	35	40	45
2.41	7.855	7.972	8.080	8.525	8.856	9.111	9.314	9.480
2.42	7.917	8.035	8.145	8.597	8.932	9.190	9.397	9.565
2.43	7.980	8.099	8.211	8.668	9.008	9.271	9.480	9.651
2.44	8.042	8.164	8.276	8.740	9.085	9.351	9.564	9.737
2.45	8.105	8.228	8.343	8.813	9.163	9.433	9.649	9.825
2.46	8.169	8.294	8.410	8.886	9.241	9.515	9.734	9.913
2.47	8.233	8.359	8.477	8.960	9.320	9.598	9.820	10.002
2.48	8.298	8.426	8.545	9.035	9.399	9.682	9.907	10.091
2.49	8.363	8.492	8.613	9.110	9.479	9.766	9.995	10.182
2.50	8.428	8.559	8.682	9.185	9.560	9.851	10.083	10.273
2.51	8.494	8.627	8.751	9.261	9.642	9.937	10.172	10.365
2.52	8.560	8.695	8.821	9.338	9.724	10.023	10.262	10.457
2.53	8.627	8.764	8.891	9.416	9.807	10.110	10.352	10.551
2.54	8.695	8.833	8.962	9.494	9.890	10.198	10.444	10.645
2.55	8.763	8.903	9.033	9.572	9.974	10.286	10.536	10.740
2.56	8.831	8.973	9.105	9.651	10.059	10.376	10.629	10.836
2.57	8.900	9.044	9.178	9.731	10.145	10.466	10.723	10.933
2.58	8.969	9.115	9.251	9.812	10.231	10.556	10.817	11.031
2.59	9.039	9.187	9.324	9.893	10.318	10.648	10.912	11.129
2.60	9.109	9.259	9.398	9.974	10.405	10.740	11.009	11.228
2.61	9.180	9.332	9.473	10.057	10.494	10.833	11.105	11.328
2.62	9.251	9.405	9.548	10.140	10.583	10.927	11.203	11.429
2.63	9.323	9.479	9.624	10.223	10.672	11.022	11.302	11.531
2.64	9.396	9.553	9.700	10.308	10.763	11.117	11.401	11.634
2.65	9.469	9.628	9.777	10.393	10.854	11.213	11.501	11.738
2.66	9.542	9.704	9.854	10.478	10.946	11.310	11.602	11.842
2.67	9.616	9.780	9.932	10.564	11.038	11.408	11.704	11.948
2.68	9.690	9.856	10.011	10.651	11.132	11.507	11.807	12.054
2.69	9.765	9.933	10.090	10.739	11.226	11.606	11.911	12.161
2.70	9.841	10.011	10.170	10.827	11.321	11.706	12.015	12.269
2.71	9.917	10.089	10.250	10.916	11.416	11.807	12.121	12.378
2.72	9.994	10.168	10.331	11.006	11.513	11.909	12.227	12.488
2.73	10.071	10.248	10.412	11.096	11.610	12.012	12.334	12.599
2.74	10.149	10.327	10.494	11.187	11.708	12.115	12.442	12.711
2.75	10.227	10.408	10.577	11.279	11.807	12.220	12.551	12.824
2.76	10.306	10.489	10.660	11.371	11.906	12.325	12.661	12.938
2.77	10.385	10.571	10.744	11.464	12.007	12.431	12.772	13.053
2.78	10.465	10.653	10.829	11.558	12.108	12.538	12.884	13.168
2.79	10.546	10.736	10.914	11.653	12.210	12.646	12.997	13.285
2.80	10.627	10.819	11.000	11.748	12.313	12.755	13.110	13.403

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	50	75	100	125	150	175	200
2.41	9.618	10.064	10.306	10.459	10.564	10.641	10.700
2.42	9.705	10.158	10.404	10.560	10.667	10.745	10.804
2.43	9.793	10.253	10.503	10.661	10.770	10.849	10.910
2.44	9.882	10.349	10.603	10.764	10.874	10.955	11.016
2.45	9.971	10.445	10.704	10.867	10.979	11.061	11.124
2.46	10.062	10.543	10.806	10.971	11.086	11.169	11.232
2.47	10.153	10.642	10.909	11.077	11.193	11.278	11.342
2.48	10.245	10.741	11.012	11.183	11.301	11.387	11.453
2.49	10.337	10.841	11.117	11.291	11.410	11.498	11.565
2.50	10.431	10.943	11.223	11.399	11.521	11.610	11.678
2.51	10.525	11.045	11.329	11.509	11.632	11.723	11.792
2.52	10.620	11.148	11.437	11.619	11.745	11.837	11.907
2.53	10.716	11.252	11.545	11.731	11.858	11.952	12.023
2.54	10.813	11.357	11.655	11.843	11.973	12.068	12.140
2.55	10.911	11.463	11.766	11.957	12.089	12.185	12.259
2.56	11.009	11.570	11.877	12.072	12.206	12.304	12.378
2.57	11.108	11.678	11.990	12.187	12.324	12.423	12.499
2.58	11.209	11.787	12.104	12.304	12.443	12.544	12.621
2.59	11.310	11.897	12.219	12.422	12.563	12.666	12.744
2.60	11.412	12.007	12.335	12.541	12.684	12.789	12.869
2.61	11.515	12.119	12.452	12.662	12.807	12.913	12.994
2.62	11.618	12.232	12.570	12.783	12.930	13.038	13.121
2.63	11.723	12.346	12.689	12.906	13.055	13.165	13.249
2.64	11.829	12.461	12.809	13.029	13.181	13.293	13.378
2.65	11.935	12.577	12.930	13.154	13.309	13.422	13.508
2.66	12.042	12.694	13.053	13.280	13.437	13.552	13.640
2.67	12.151	12.812	13.177	13.407	13.567	13.684	13.773
2.68	12.260	12.932	13.301	13.536	13.698	13.817	13.907
2.69	12.370	13.052	13.427	13.666	13.830	13.951	14.043
2.70	12.481	13.173	13.555	13.796	13.964	14.086	14.180
2.71	12.594	13.296	13.683	13.929	14.098	14.223	14.318
2.72	12.707	13.419	13.812	14.062	14.234	14.361	14.457
2.73	12.821	13.544	13.943	14.197	14.372	14.500	14.598
2.74	12.936	13.670	14.075	14.332	14.510	14.641	14.740
2.75	13.052	13.797	14.208	14.470	14.650	14.783	14.884
2.76	13.169	13.925	14.343	14.608	14.791	14.926	15.029
2.77	13.287	14.054	14.478	14.748	14.934	15.071	15.175
2.78	13.406	14.185	14.615	14.889	15.078	15.217	15.323
2.79	13.527	14.316	14.754	15.031	15.223	15.364	15.472
2.80	13.648	14.449	14.893	15.175	15.370	15.513	15.623

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	300	400	500	750	1000	5000	
2.41	10.839	10.911	10.955	11.014	11.043	11.116	11.134
2.42	10.946	11.019	11.064	11.123	11.154	11.227	11.246
2.43	11.054	11.128	11.173	11.234	11.265	11.340	11.359
2.44	11.163	11.239	11.284	11.346	11.378	11.454	11.473
2.45	11.273	11.350	11.397	11.460	11.491	11.569	11.588
2.46	11.384	11.462	11.510	11.574	11.606	11.685	11.705
2.47	11.497	11.576	11.624	11.689	11.722	11.802	11.822
2.48	11.610	11.690	11.740	11.806	11.839	11.921	11.941
2.49	11.724	11.806	11.856	11.923	11.958	12.040	12.061
2.50	11.840	11.923	11.974	12.042	12.077	12.161	12.182
2.51	11.957	12.041	12.093	12.163	12.198	12.283	12.305
2.52	12.074	12.160	12.213	12.284	12.320	12.407	12.429
2.53	12.193	12.281	12.334	12.406	12.443	12.531	12.553
2.54	12.313	12.403	12.457	12.530	12.567	12.657	12.680
2.55	12.435	12.525	12.580	12.655	12.692	12.784	12.807
2.56	12.557	12.649	12.705	12.781	12.819	12.912	12.936
2.57	12.681	12.775	12.831	12.908	12.947	13.042	13.066
2.58	12.806	12.901	12.959	13.037	13.077	13.173	13.197
2.59	12.932	13.029	13.088	13.167	13.207	13.305	13.330
2.60	13.059	13.158	13.217	13.298	13.339	13.439	13.464
2.61	13.188	13.288	13.349	13.431	13.472	13.573	13.599
2.62	13.318	13.419	13.481	13.565	13.607	13.710	13.736
2.63	13.449	13.552	13.615	13.700	13.743	13.847	13.874
2.64	13.582	13.686	13.750	13.836	13.880	13.986	14.013
2.65	13.715	13.822	13.887	13.974	14.019	14.127	14.154
2.66	13.850	13.959	14.024	14.114	14.159	14.268	14.296
2.67	13.987	14.097	14.164	14.254	14.300	14.412	14.440
2.68	14.124	14.236	14.304	14.396	14.443	14.556	14.585
2.69	14.263	14.377	14.446	14.540	14.587	14.702	14.732
2.70	14.404	14.519	14.590	14.685	14.733	14.850	14.880
2.71	14.546	14.663	14.734	14.831	14.880	14.999	15.029
2.72	14.689	14.808	14.881	14.979	15.029	15.150	15.180
2.73	14.833	14.954	15.028	15.128	15.179	15.302	15.333
2.74	14.979	15.102	15.177	15.279	15.330	15.455	15.487
2.75	15.127	15.252	15.328	15.431	15.483	15.610	15.643
2.76	15.275	15.402	15.480	15.585	15.638	15.767	15.800
2.77	15.426	15.555	15.634	15.740	15.794	15.925	15.959
2.78	15.577	15.709	15.789	15.897	15.952	16.085	16.119
2.79	15.731	15.864	15.945	16.055	16.111	16.247	16.281
2.80	15.885	16.021	16.103	16.215	16.272	16.410	16.445

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
2.81	2.766	3.956	4.920	5.729	6.422	7.024	7.553	8.023
2.82	2.774	3.972	4.943	5.758	6.457	7.065	7.599	8.074
2.83	2.782	3.987	4.965	5.788	6.493	7.106	7.646	8.126
2.84	2.790	4.002	4.988	5.817	6.529	7.148	7.693	8.178
2.85	2.797	4.017	5.011	5.847	6.565	7.190	7.740	8.230
2.86	2.805	4.033	5.033	5.877	6.601	7.232	7.788	8.283
2.87	2.813	4.048	5.056	5.906	6.637	7.274	7.836	8.335
2.88	2.821	4.064	5.079	5.936	6.674	7.317	7.884	8.389
2.89	2.828	4.079	5.102	5.966	6.710	7.359	7.932	8.442
2.90	2.836	4.095	5.125	5.997	6.747	7.402	7.981	8.496
2.91	2.844	4.110	5.149	6.027	6.784	7.445	8.029	8.550
2.92	2.852	4.126	5.172	6.057	6.821	7.489	8.078	8.604
2.93	2.860	4.142	5.195	6.088	6.858	7.532	8.128	8.659
2.94	2.867	4.157	5.219	6.119	6.896	7.576	8.177	8.714
2.95	2.875	4.173	5.242	6.150	6.934	7.620	8.227	8.769
2.96	2.883	4.189	5.266	6.180	6.971	7.664	8.277	8.824
2.97	2.891	4.205	5.289	6.212	7.009	7.708	8.327	8.880
2.98	2.899	4.220	5.313	6.243	7.047	7.753	8.378	8.936
2.99	2.907	4.236	5.337	6.274	7.086	7.798	8.429	8.993
3.00	2.915	4.252	5.361	6.305	7.124	7.843	8.480	9.050
T	SAMPLE SIZE							
	10	11	12	13	14	15	16	17
2.81	8.443	8.822	9.166	9.479	9.766	10.030	10.273	10.499
2.82	8.499	8.882	9.230	9.547	9.837	10.104	10.350	10.578
2.83	8.555	8.943	9.294	9.615	9.908	10.178	10.428	10.659
2.84	8.612	9.004	9.359	9.683	9.980	10.253	10.506	10.739
2.85	8.669	9.065	9.424	9.752	10.052	10.329	10.584	10.821
2.86	8.726	9.126	9.490	9.821	10.125	10.405	10.663	10.903
2.87	8.784	9.188	9.556	9.891	10.198	10.481	10.743	10.985
2.88	8.841	9.250	9.622	9.961	10.272	10.558	10.823	11.068
2.89	8.900	9.313	9.689	10.032	10.346	10.636	10.904	11.152
2.90	8.958	9.376	9.756	10.103	10.421	10.714	10.985	11.236
2.91	9.017	9.439	9.823	10.174	10.496	10.792	11.067	11.321
2.92	9.076	9.503	9.891	10.246	10.572	10.872	11.149	11.406
2.93	9.136	9.567	9.960	10.319	10.648	10.951	11.232	11.492
2.94	9.196	9.632	10.029	10.391	10.724	11.031	11.315	11.579
2.95	9.256	9.697	10.098	10.465	10.802	11.112	11.399	11.666
2.96	9.317	9.762	10.168	10.539	10.879	11.193	11.484	11.754
2.97	9.378	9.828	10.238	10.613	10.957	11.275	11.569	11.842
2.98	9.439	9.894	10.308	10.688	11.036	11.357	11.655	11.931
2.99	9.501	9.961	10.380	10.763	11.115	11.440	11.741	12.021
3.00	9.563	10.028	10.451	10.839	11.195	11.524	11.828	12.111

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE							
	18	19	20	25	30	35	40	45
2.81	10.708	10.904	11.086	11.844	12.416	12.864	13.225	13.522
2.82	10.791	10.988	11.173	11.941	12.521	12.975	13.340	13.641
2.83	10.873	11.074	11.261	12.038	12.626	13.086	13.457	13.762
2.84	10.957	11.159	11.349	12.136	12.732	13.199	13.574	13.884
2.85	11.041	11.246	11.438	12.235	12.839	13.312	13.693	14.007
2.86	11.125	11.333	11.527	12.335	12.947	13.426	13.812	14.131
2.87	11.211	11.421	11.617	12.436	13.055	13.541	13.933	14.256
2.88	11.296	11.509	11.708	12.537	13.165	13.657	14.054	14.382
2.89	11.383	11.598	11.800	12.639	13.275	13.774	14.177	14.509
2.90	11.470	11.688	11.892	12.742	13.386	13.892	14.300	14.637
2.91	11.558	11.778	11.985	12.846	13.499	14.011	14.425	14.766
2.92	11.646	11.869	12.078	12.950	13.612	14.131	14.551	14.897
2.93	11.735	11.961	12.173	13.055	13.726	14.252	14.677	15.028
2.94	11.824	12.053	12.267	13.162	13.840	14.374	14.805	15.161
2.95	11.914	12.146	12.363	13.268	13.956	14.497	14.934	15.294
2.96	12.005	12.240	12.459	13.376	14.073	14.621	15.064	15.429
2.97	12.097	12.334	12.556	13.485	14.190	14.746	15.195	15.565
2.98	12.189	12.429	12.654	13.594	14.309	14.872	15.327	15.703
2.99	12.281	12.525	12.752	13.704	14.428	14.999	15.460	15.841
3.00	12.375	12.621	12.851	13.815	14.549	15.127	15.594	15.980
T	SAMPLE SIZE							
	50	75	100	125	150	175	200	
2.81	13.770	14.583	15.034	15.320	15.518	15.664	15.775	
2.82	13.893	14.719	15.176	15.467	15.668	15.815	15.928	
2.83	14.018	14.855	15.319	15.614	15.819	15.969	16.083	
2.84	14.143	14.993	15.464	15.764	15.971	16.123	16.240	
2.85	14.270	15.132	15.610	15.914	16.125	16.280	16.398	
2.86	14.398	15.272	15.757	16.066	16.280	16.437	16.557	
2.87	14.526	15.414	15.906	16.220	16.437	16.597	16.719	
2.88	14.656	15.556	16.056	16.375	16.595	16.757	16.881	
2.89	14.787	15.701	16.208	16.531	16.755	16.920	17.045	
2.90	14.919	15.846	16.361	16.689	16.916	17.083	17.211	
2.91	15.053	15.993	16.515	16.848	17.079	17.249	17.379	
2.92	15.187	16.141	16.671	17.009	17.244	17.416	17.548	
2.93	15.323	16.290	16.828	17.172	17.410	17.584	17.718	
2.94	15.459	16.441	16.987	17.335	17.577	17.755	17.891	
2.95	15.597	16.593	17.147	17.501	17.746	17.927	18.065	
2.96	15.736	16.746	17.309	17.668	17.917	18.100	18.240	
2.97	15.877	16.901	17.472	17.836	18.089	18.275	18.417	
2.98	16.018	17.057	17.637	18.007	18.263	18.452	18.597	
2.99	16.161	17.215	17.803	18.178	18.439	18.630	18.777	
3.00	16.305	17.374	17.971	18.352	18.616	18.811	18.960	

TABLE B-1. - MULTIPLYING FACTOR FOR THE GEOMETRIC MEAN--CONTINUED

T	SAMPLE SIZE						
	300	400	500	750	1000	5000	
2.81	16.042	16.179	16.263	16.377	16.434	16.574	16.610
2.82	16.199	16.339	16.425	16.540	16.598	16.741	16.777
2.83	16.359	16.501	16.587	16.705	16.764	16.909	16.945
2.84	16.520	16.664	16.752	16.871	16.931	17.079	17.116
2.85	16.682	16.829	16.918	17.039	17.101	17.250	17.288
2.86	16.846	16.995	17.086	17.209	17.271	17.423	17.461
2.87	17.012	17.163	17.255	17.380	17.444	17.598	17.637
2.88	17.179	17.333	17.427	17.554	17.618	17.775	17.814
2.89	17.348	17.504	17.599	17.728	17.794	17.953	17.993
2.90	17.518	17.677	17.774	17.905	17.971	18.133	18.174
2.91	17.691	17.852	17.950	18.083	18.151	18.315	18.357
2.92	17.865	18.028	18.128	18.263	18.332	18.499	18.541
2.93	18.040	18.206	18.308	18.445	18.515	18.685	18.728
2.94	18.218	18.386	18.489	18.629	18.700	18.872	18.916
2.95	18.397	18.568	18.673	18.814	18.886	19.061	19.106
2.96	18.577	18.751	18.858	19.002	19.075	19.253	19.298
2.97	18.760	18.937	19.045	19.191	19.265	19.446	19.492
2.98	18.944	19.124	19.234	19.382	19.458	19.641	19.688
2.99	19.130	19.313	19.424	19.575	19.652	19.838	19.886
3.00	19.318	19.504	19.617	19.770	19.848	20.037	20.085

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
0.02	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
0.04	0.040	0.041	0.041	0.041	0.041	0.041	0.042	0.042
0.06	0.061	0.062	0.062	0.063	0.063	0.063	0.064	0.064
0.08	0.081	0.083	0.084	0.085	0.085	0.086	0.086	0.087
0.10	0.102	0.104	0.106	0.108	0.109	0.109	0.110	0.111
0.12	0.122	0.126	0.129	0.131	0.133	0.134	0.135	0.136
0.14	0.143	0.148	0.152	0.155	0.157	0.159	0.160	0.161
0.16	0.164	0.171	0.176	0.180	0.183	0.185	0.187	0.188
0.18	0.185	0.194	0.200	0.205	0.209	0.212	0.214	0.216
0.20	0.207	0.217	0.225	0.231	0.236	0.239	0.242	0.245
0.22	0.228	0.241	0.251	0.258	0.264	0.268	0.272	0.275
0.24	0.250	0.265	0.277	0.285	0.292	0.298	0.302	0.306
0.26	0.271	0.289	0.303	0.314	0.322	0.328	0.333	0.338
0.28	0.293	0.314	0.330	0.342	0.352	0.360	0.366	0.371
0.30	0.315	0.339	0.358	0.372	0.383	0.392	0.399	0.405
0.32	0.337	0.365	0.386	0.403	0.415	0.426	0.434	0.441
0.34	0.360	0.391	0.415	0.434	0.448	0.460	0.470	0.478
0.36	0.382	0.417	0.445	0.466	0.482	0.496	0.507	0.516
0.38	0.405	0.444	0.475	0.498	0.517	0.532	0.545	0.555
0.40	0.427	0.471	0.505	0.532	0.553	0.570	0.584	0.596
0.42	0.450	0.498	0.537	0.566	0.590	0.609	0.625	0.638
0.44	0.473	0.526	0.569	0.602	0.628	0.649	0.667	0.682
0.46	0.496	0.555	0.601	0.638	0.667	0.691	0.710	0.727
0.48	0.520	0.583	0.635	0.675	0.707	0.733	0.755	0.773
0.50	0.543	0.613	0.669	0.713	0.748	0.777	0.801	0.821
0.52	0.567	0.642	0.703	0.751	0.790	0.822	0.848	0.871
0.54	0.590	0.672	0.738	0.791	0.833	0.868	0.897	0.922
0.56	0.614	0.703	0.774	0.832	0.878	0.916	0.947	0.974
0.58	0.638	0.733	0.811	0.873	0.923	0.964	0.999	1.029
0.60	0.662	0.765	0.849	0.915	0.970	1.015	1.052	1.085
0.62	0.687	0.796	0.887	0.959	1.018	1.066	1.107	1.142
0.64	0.711	0.829	0.925	1.003	1.067	1.119	1.164	1.202
0.66	0.736	0.861	0.965	1.049	1.117	1.174	1.222	1.263
0.68	0.761	0.894	1.005	1.095	1.169	1.230	1.282	1.326
0.70	0.786	0.928	1.046	1.143	1.222	1.287	1.343	1.391
0.72	0.811	0.962	1.088	1.191	1.276	1.347	1.407	1.458
0.74	0.836	0.996	1.131	1.241	1.331	1.407	1.472	1.527
0.76	0.861	1.031	1.174	1.291	1.388	1.469	1.539	1.598
0.78	0.887	1.066	1.218	1.343	1.446	1.533	1.607	1.671
0.80	0.913	1.102	1.263	1.396	1.506	1.599	1.678	1.747

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	20	25
0.02	0.020	0.020	0.020	0.020	0.020	0.020	0.021	0.021
0.04	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042
0.06	0.064	0.064	0.064	0.064	0.064	0.064	0.065	0.065
0.08	0.087	0.087	0.088	0.088	0.088	0.088	0.089	0.089
0.10	0.111	0.112	0.112	0.112	0.112	0.113	0.113	0.114
0.12	0.136	0.137	0.137	0.138	0.138	0.138	0.140	0.140
0.14	0.162	0.163	0.164	0.164	0.165	0.165	0.167	0.168
0.16	0.189	0.190	0.191	0.192	0.193	0.193	0.196	0.197
0.18	0.218	0.219	0.220	0.221	0.222	0.223	0.226	0.228
0.20	0.247	0.249	0.250	0.251	0.253	0.254	0.257	0.260
0.22	0.277	0.279	0.281	0.283	0.284	0.286	0.290	0.293
0.24	0.309	0.311	0.314	0.316	0.317	0.319	0.325	0.328
0.26	0.341	0.345	0.347	0.350	0.352	0.354	0.360	0.365
0.28	0.375	0.379	0.382	0.385	0.388	0.390	0.398	0.403
0.30	0.411	0.415	0.419	0.422	0.425	0.428	0.437	0.443
0.32	0.447	0.452	0.456	0.460	0.464	0.467	0.478	0.485
0.34	0.485	0.491	0.496	0.500	0.504	0.508	0.521	0.529
0.36	0.524	0.531	0.536	0.542	0.546	0.550	0.565	0.575
0.38	0.564	0.572	0.579	0.585	0.590	0.594	0.612	0.623
0.40	0.606	0.615	0.623	0.629	0.635	0.640	0.660	0.673
0.42	0.650	0.659	0.668	0.675	0.682	0.688	0.710	0.725
0.44	0.695	0.706	0.715	0.724	0.731	0.738	0.763	0.779
0.46	0.741	0.753	0.764	0.773	0.782	0.789	0.817	0.836
0.48	0.789	0.803	0.814	0.825	0.834	0.843	0.874	0.895
0.50	0.839	0.854	0.867	0.879	0.889	0.898	0.933	0.956
0.52	0.890	0.906	0.921	0.934	0.946	0.956	0.995	1.020
0.54	0.943	0.961	0.977	0.992	1.004	1.016	1.059	1.087
0.56	0.997	1.018	1.035	1.051	1.065	1.078	1.125	1.157
0.58	1.054	1.076	1.096	1.113	1.128	1.142	1.194	1.229
0.60	1.112	1.136	1.158	1.177	1.193	1.209	1.266	1.304
0.62	1.172	1.199	1.222	1.243	1.261	1.278	1.341	1.383
0.64	1.235	1.263	1.289	1.311	1.331	1.349	1.418	1.464
0.66	1.299	1.330	1.357	1.382	1.404	1.423	1.499	1.549
0.68	1.365	1.399	1.428	1.455	1.479	1.500	1.582	1.637
0.70	1.433	1.469	1.502	1.531	1.556	1.580	1.669	1.729
0.72	1.503	1.543	1.578	1.609	1.637	1.662	1.759	1.824
0.74	1.576	1.618	1.656	1.690	1.720	1.747	1.852	1.923
0.76	1.650	1.696	1.737	1.773	1.806	1.835	1.949	2.026
0.78	1.727	1.777	1.820	1.859	1.895	1.926	2.049	2.132
0.80	1.807	1.860	1.907	1.949	1.986	2.021	2.153	2.243

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	30	40	50	100	200	500	1000	
0.02	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021
0.04	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042
0.06	0.065	0.065	0.065	0.065	0.066	0.066	0.066	0.066
0.08	0.089	0.089	0.090	0.090	0.090	0.090	0.090	0.090
0.10	0.114	0.115	0.115	0.116	0.116	0.116	0.116	0.116
0.12	0.141	0.142	0.142	0.143	0.143	0.144	0.144	0.144
0.14	0.169	0.170	0.170	0.172	0.172	0.173	0.173	0.173
0.16	0.198	0.199	0.200	0.202	0.203	0.203	0.203	0.204
0.18	0.229	0.231	0.232	0.234	0.235	0.236	0.236	0.236
0.20	0.261	0.263	0.265	0.268	0.269	0.270	0.270	0.270
0.22	0.295	0.298	0.300	0.303	0.305	0.306	0.306	0.307
0.24	0.331	0.334	0.336	0.340	0.343	0.344	0.344	0.345
0.26	0.368	0.372	0.374	0.380	0.382	0.384	0.385	0.385
0.28	0.407	0.412	0.415	0.421	0.424	0.426	0.427	0.428
0.30	0.448	0.453	0.457	0.464	0.468	0.471	0.471	0.472
0.32	0.490	0.497	0.501	0.510	0.515	0.517	0.518	0.519
0.34	0.535	0.543	0.548	0.558	0.563	0.567	0.568	0.569
0.36	0.582	0.591	0.596	0.608	0.615	0.618	0.620	0.621
0.38	0.631	0.641	0.648	0.661	0.668	0.673	0.674	0.676
0.40	0.682	0.694	0.701	0.717	0.725	0.730	0.732	0.734
0.42	0.735	0.748	0.757	0.775	0.784	0.790	0.792	0.794
0.44	0.791	0.806	0.816	0.836	0.847	0.854	0.856	0.858
0.46	0.849	0.866	0.877	0.900	0.912	0.920	0.923	0.925
0.48	0.909	0.929	0.941	0.967	0.981	0.990	0.993	0.996
0.50	0.973	0.994	1.008	1.037	1.053	1.063	1.066	1.070
0.52	1.039	1.063	1.078	1.111	1.129	1.140	1.143	1.147
0.54	1.107	1.134	1.151	1.188	1.208	1.220	1.224	1.229
0.56	1.179	1.209	1.228	1.269	1.291	1.305	1.309	1.314
0.58	1.254	1.287	1.308	1.354	1.378	1.393	1.399	1.404
0.60	1.332	1.368	1.392	1.442	1.469	1.486	1.492	1.498
0.62	1.413	1.453	1.479	1.535	1.565	1.584	1.590	1.597
0.64	1.497	1.542	1.570	1.631	1.665	1.686	1.693	1.700
0.66	1.585	1.634	1.665	1.733	1.770	1.793	1.801	1.809
0.68	1.677	1.730	1.764	1.839	1.879	1.905	1.913	1.922
0.70	1.772	1.830	1.868	1.949	1.994	2.022	2.032	2.041
0.72	1.871	1.935	1.976	2.065	2.114	2.145	2.156	2.166
0.74	1.974	2.044	2.088	2.186	2.240	2.274	2.285	2.297
0.76	2.081	2.157	2.206	2.312	2.371	2.408	2.421	2.434
0.78	2.193	2.275	2.328	2.444	2.509	2.549	2.563	2.577
0.80	2.309	2.398	2.455	2.582	2.652	2.697	2.712	2.727

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
0.82	0.938	1.138	1.309	1.450	1.567	1.666	1.751	1.824
0.84	0.964	1.175	1.355	1.505	1.629	1.735	1.825	1.904
0.86	0.991	1.213	1.403	1.561	1.693	1.806	1.902	1.986
0.88	1.017	1.250	1.451	1.618	1.759	1.878	1.981	2.070
0.90	1.043	1.289	1.500	1.677	1.826	1.952	2.062	2.157
0.92	1.070	1.327	1.550	1.737	1.894	2.029	2.145	2.246
0.94	1.097	1.366	1.601	1.798	1.964	2.107	2.230	2.337
0.96	1.124	1.406	1.653	1.860	2.036	2.187	2.317	2.431
0.98	1.151	1.446	1.705	1.924	2.109	2.269	2.407	2.528
1.00	1.178	1.487	1.759	1.989	2.185	2.353	2.499	2.627
1.02	1.206	1.529	1.813	2.055	2.261	2.439	2.594	2.730
1.04	1.233	1.570	1.868	2.122	2.340	2.527	2.691	2.834
1.06	1.261	1.613	1.925	2.191	2.420	2.618	2.790	2.942
1.08	1.289	1.656	1.982	2.262	2.502	2.710	2.892	3.053
1.10	1.317	1.699	2.040	2.333	2.586	2.805	2.997	3.166
1.12	1.345	1.743	2.099	2.406	2.671	2.902	3.104	3.283
1.14	1.374	1.787	2.159	2.481	2.759	3.001	3.214	3.403
1.16	1.402	1.832	2.220	2.557	2.848	3.103	3.327	3.526
1.18	1.431	1.878	2.283	2.634	2.940	3.207	3.443	3.652
1.20	1.460	1.924	2.346	2.713	3.033	3.313	3.561	3.781
1.22	1.489	1.971	2.410	2.793	3.128	3.422	3.682	3.914
1.24	1.518	2.018	2.475	2.875	3.226	3.534	3.807	4.050
1.26	1.548	2.066	2.541	2.959	3.325	3.648	3.934	4.190
1.28	1.577	2.114	2.609	3.044	3.427	3.765	4.065	4.334
1.30	1.607	2.163	2.677	3.131	3.530	3.884	4.199	4.481
1.32	1.637	2.213	2.746	3.219	3.636	4.006	4.336	4.631
1.34	1.667	2.263	2.817	3.309	3.744	4.131	4.476	4.786
1.36	1.698	2.314	2.889	3.401	3.855	4.258	4.620	4.945
1.38	1.728	2.365	2.961	3.494	3.967	4.389	4.767	5.107
1.40	1.759	2.417	3.035	3.589	4.082	4.522	4.917	5.274
1.42	1.790	2.470	3.111	3.686	4.199	4.659	5.072	5.445
1.44	1.821	2.523	3.187	3.784	4.319	4.798	5.230	5.620
1.46	1.852	2.577	3.264	3.885	4.441	4.941	5.391	5.799
1.48	1.883	2.631	3.343	3.987	4.566	5.086	5.556	5.983
1.50	1.915	2.686	3.423	4.091	4.693	5.235	5.726	6.171
1.52	1.946	2.742	3.504	4.197	4.823	5.387	5.899	6.364
1.54	1.978	2.798	3.586	4.305	4.955	5.543	6.076	6.561
1.56	2.010	2.855	3.670	4.415	5.090	5.701	6.257	6.764
1.58	2.042	2.913	3.754	4.526	5.227	5.864	6.443	6.971
1.60	2.075	2.971	3.841	4.640	5.368	6.029	6.632	7.183

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	20	25
0.82	1.888	1.945	1.996	2.041	2.081	2.118	2.261	2.358
0.84	1.972	2.033	2.087	2.136	2.180	2.219	2.373	2.478
0.86	2.059	2.124	2.182	2.234	2.281	2.324	2.489	2.602
0.88	2.148	2.218	2.280	2.335	2.386	2.431	2.609	2.731
0.90	2.240	2.314	2.381	2.440	2.494	2.543	2.733	2.864
0.92	2.335	2.414	2.484	2.548	2.606	2.658	2.862	3.003
0.94	2.432	2.516	2.592	2.659	2.721	2.777	2.995	3.146
0.96	2.532	2.622	2.702	2.774	2.840	2.900	3.133	3.295
0.98	2.635	2.730	2.816	2.893	2.963	3.027	3.276	3.449
1.00	2.741	2.842	2.933	3.015	3.090	3.158	3.424	3.610
1.02	2.850	2.957	3.054	3.141	3.220	3.293	3.577	3.775
1.04	2.962	3.076	3.179	3.271	3.356	3.432	3.735	3.947
1.06	3.077	3.198	3.307	3.405	3.495	3.577	3.899	4.126
1.08	3.196	3.324	3.439	3.543	3.638	3.725	4.069	4.310
1.10	3.317	3.453	3.575	3.686	3.787	3.879	4.244	4.501
1.12	3.443	3.586	3.715	3.832	3.939	4.037	4.425	4.699
1.14	3.571	3.723	3.859	3.983	4.097	4.201	4.613	4.905
1.16	3.703	3.863	4.008	4.139	4.259	4.369	4.806	5.117
1.18	3.839	4.008	4.160	4.299	4.426	4.543	5.007	5.337
1.20	3.979	4.157	4.318	4.464	4.599	4.722	5.214	5.564
1.22	4.122	4.309	4.480	4.635	4.776	4.907	5.428	5.800
1.24	4.269	4.467	4.646	4.810	4.959	5.097	5.649	6.044
1.26	4.420	4.628	4.817	4.990	5.148	5.294	5.877	6.296
1.28	4.575	4.794	4.993	5.175	5.342	5.496	6.113	6.557
1.30	4.735	4.965	5.175	5.366	5.542	5.705	6.357	6.827
1.32	4.898	5.140	5.361	5.563	5.748	5.919	6.609	7.107
1.34	5.066	5.320	5.552	5.765	5.960	6.141	6.869	7.396
1.36	5.238	5.506	5.749	5.973	6.179	6.369	7.137	7.695
1.38	5.415	5.696	5.952	6.187	6.404	6.604	7.414	8.004
1.40	5.597	5.891	6.160	6.407	6.635	6.846	7.700	8.323
1.42	5.783	6.092	6.374	6.634	6.873	7.095	7.996	8.654
1.44	5.974	6.297	6.594	6.867	7.118	7.352	8.300	8.995
1.46	6.170	6.509	6.820	7.106	7.371	7.616	8.615	9.349
1.48	6.371	6.726	7.052	7.352	7.630	7.888	8.940	9.713
1.50	6.577	6.949	7.290	7.606	7.897	8.168	9.275	10.091
1.52	6.788	7.178	7.536	7.866	8.172	8.456	9.620	10.480
1.54	7.005	7.412	7.787	8.134	8.454	8.753	9.976	10.883
1.56	7.227	7.653	8.046	8.409	8.745	9.058	10.344	11.299
1.58	7.455	7.901	8.311	8.691	9.044	9.372	10.723	11.729
1.60	7.689	8.154	8.584	8.982	9.351	9.695	11.114	12.173

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	30	40	50	100	200	500	1000	
0.82	2.429	2.526	2.588	2.726	2.803	2.851	2.868	2.885
0.84	2.554	2.659	2.727	2.876	2.960	3.013	3.031	3.049
0.86	2.685	2.797	2.871	3.033	3.124	3.181	3.201	3.221
0.88	2.820	2.942	3.021	3.197	3.295	3.358	3.380	3.402
0.90	2.960	3.091	3.177	3.368	3.475	3.543	3.566	3.590
0.92	3.106	3.247	3.340	3.546	3.662	3.736	3.761	3.787
0.94	3.257	3.410	3.510	3.732	3.858	3.938	3.965	3.994
0.96	3.414	3.578	3.686	3.927	4.062	4.149	4.179	4.209
0.98	3.577	3.754	3.870	4.129	4.275	4.369	4.402	4.435
1.00	3.747	3.936	4.061	4.340	4.498	4.600	4.635	4.671
1.02	3.922	4.125	4.259	4.560	4.731	4.841	4.879	4.917
1.04	4.105	4.322	4.466	4.790	4.974	5.092	5.133	5.175
1.06	4.294	4.527	4.681	5.029	5.227	5.355	5.399	5.445
1.08	4.490	4.739	4.904	5.278	5.492	5.630	5.678	5.726
1.10	4.693	4.960	5.137	5.538	5.768	5.916	5.968	6.021
1.12	4.904	5.189	5.378	5.809	6.056	6.216	6.272	6.328
1.14	5.123	5.427	5.630	6.091	6.356	6.529	6.589	6.650
1.16	5.349	5.674	5.891	6.385	6.670	6.855	6.920	6.986
1.18	5.584	5.931	6.162	6.691	6.997	7.196	7.266	7.337
1.20	5.828	6.197	6.444	7.010	7.338	7.552	7.627	7.703
1.22	6.080	6.473	6.737	7.343	7.694	7.924	8.004	8.086
1.24	6.341	6.760	7.041	7.689	8.065	8.311	8.398	8.486
1.26	6.612	7.058	7.358	8.049	8.452	8.716	8.809	8.903
1.28	6.893	7.367	7.686	8.424	8.855	9.139	9.238	9.339
1.30	7.183	7.687	8.027	8.815	9.276	9.579	9.686	9.794
1.32	7.484	8.020	8.381	9.221	9.714	10.039	10.153	10.270
1.34	7.796	8.364	8.749	9.644	10.171	10.519	10.641	10.766
1.36	8.119	8.722	9.131	10.085	10.648	11.020	11.150	11.284
1.38	8.453	9.093	9.527	10.543	11.144	11.542	11.681	11.825
1.40	8.799	9.477	9.939	11.020	11.661	12.087	12.236	12.389
1.42	9.157	9.876	10.366	11.516	12.200	12.655	12.814	12.979
1.44	9.527	10.289	10.809	12.033	12.762	13.247	13.418	13.594
1.46	9.911	10.718	11.269	12.570	13.347	13.865	14.048	14.235
1.48	10.308	11.162	11.746	13.129	13.957	14.510	14.704	14.905
1.50	10.718	11.622	12.242	13.710	14.592	15.182	15.390	15.604
1.52	11.143	12.099	12.755	14.315	15.254	15.882	16.104	16.333
1.54	11.582	12.593	13.288	14.944	15.943	16.613	16.850	17.094
1.56	12.037	13.105	13.841	15.598	16.660	17.374	17.627	17.888
1.58	12.507	13.635	14.414	16.278	17.408	18.168	18.438	18.716
1.60	12.993	14.185	15.009	16.985	18.186	18.996	19.283	19.579

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
1.62	2.107	3.030	3.928	4.756	5.511	6.199	6.826	7.401
1.64	2.140	3.090	4.017	4.873	5.657	6.371	7.025	7.623
1.66	2.173	3.150	4.107	4.993	5.805	6.548	7.228	7.851
1.68	2.206	3.211	4.198	5.115	5.957	6.728	7.435	8.085
1.70	2.240	3.273	4.291	5.239	6.112	6.912	7.647	8.324
1.72	2.273	3.335	4.385	5.365	6.269	7.100	7.864	8.568
1.74	2.307	3.398	4.480	5.493	6.430	7.292	8.086	8.818
1.76	2.341	3.462	4.577	5.624	6.593	7.488	8.312	9.074
1.78	2.375	3.526	4.675	5.757	6.760	7.687	8.544	9.336
1.80	2.409	3.591	4.775	5.892	6.930	7.891	8.781	9.605
1.82	2.444	3.657	4.876	6.029	7.103	8.100	9.023	9.879
1.84	2.478	3.724	4.978	6.169	7.280	8.312	9.270	10.160
1.86	2.513	3.791	5.082	6.311	7.460	8.529	9.523	10.447
1.88	2.548	3.859	5.188	6.455	7.643	8.750	9.781	10.741
1.90	2.583	3.928	5.295	6.602	7.830	8.976	10.045	11.041
1.92	2.619	3.997	5.403	6.751	8.020	9.207	10.314	11.349
1.94	2.654	4.067	5.514	6.903	8.214	9.442	10.590	11.663
1.96	2.690	4.138	5.625	7.057	8.411	9.682	10.871	11.984
1.98	2.726	4.210	5.738	7.214	8.612	9.926	11.158	12.313
2.00	2.762	4.283	5.853	7.373	8.816	10.176	11.452	12.649
2.02	2.799	4.356	5.969	7.535	9.025	10.430	11.752	12.993
2.04	2.835	4.430	6.087	7.700	9.237	10.690	12.058	13.344
2.06	2.872	4.505	6.207	7.867	9.453	10.954	12.370	13.704
2.08	2.909	4.581	6.328	8.038	9.673	11.224	12.689	14.071
2.10	2.946	4.657	6.451	8.210	9.897	11.500	13.015	14.446
2.12	2.983	4.734	6.576	8.386	10.125	11.780	13.348	14.830
2.14	3.021	4.812	6.702	8.565	10.357	12.066	13.688	15.222
2.16	3.059	4.891	6.831	8.746	10.594	12.358	14.034	15.623
2.18	3.097	4.971	6.960	8.930	10.834	12.656	14.388	16.033
2.20	3.135	5.051	7.092	9.117	11.079	12.959	14.750	16.451
2.22	3.173	5.133	7.225	9.308	11.329	13.268	15.118	16.879
2.24	3.212	5.215	7.361	9.501	11.582	13.583	15.495	17.316
2.26	3.250	5.298	7.498	9.697	11.840	13.904	15.879	17.762
2.28	3.289	5.382	7.636	9.896	12.103	14.231	16.271	18.218
2.30	3.329	5.466	7.777	10.099	12.370	14.565	16.671	18.684
2.32	3.368	5.552	7.920	10.305	12.642	14.905	17.079	19.160
2.34	3.408	5.638	8.064	10.513	12.919	15.251	17.495	19.646
2.36	3.447	5.726	8.211	10.726	13.201	15.604	17.920	20.143
2.38	3.487	5.814	8.359	10.941	13.487	15.964	18.354	20.650
2.40	3.527	5.903	8.509	11.160	13.779	16.330	18.796	21.168

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	20	25
1.62	7.928	8.415	8.864	9.280	9.667	10.028	11.518	12.631
1.64	8.174	8.682	9.151	9.587	9.992	10.370	11.934	13.105
1.66	8.426	8.956	9.446	9.902	10.326	10.722	12.362	13.594
1.68	8.683	9.237	9.749	10.226	10.669	11.084	12.804	14.099
1.70	8.947	9.525	10.060	10.558	11.022	11.456	13.260	14.620
1.72	9.218	9.820	10.379	10.900	11.385	11.839	13.729	15.158
1.74	9.496	10.123	10.707	11.250	11.758	12.233	14.213	15.714
1.76	9.780	10.434	11.043	11.610	12.141	12.637	14.712	16.288
1.78	10.071	10.753	11.388	11.980	12.534	13.053	15.226	16.880
1.80	10.369	11.079	11.742	12.360	12.939	13.481	15.756	17.491
1.82	10.674	11.414	12.105	12.750	13.354	13.921	16.302	18.121
1.84	10.987	11.758	12.477	13.150	13.781	14.373	16.864	18.772
1.86	11.307	12.109	12.859	13.561	14.219	14.837	17.443	19.443
1.88	11.635	12.470	13.251	13.982	14.669	15.315	18.039	20.136
1.90	11.971	12.839	13.653	14.415	15.131	15.805	18.654	20.851
1.92	12.315	13.218	14.065	14.859	15.606	16.309	19.286	21.588
1.94	12.666	13.606	14.487	15.315	16.094	16.827	19.938	22.348
1.96	13.027	14.004	14.921	15.783	16.594	17.359	20.609	23.132
1.98	13.395	14.411	15.365	16.263	17.108	17.906	21.300	23.941
2.00	13.773	14.828	15.821	16.755	17.636	18.467	22.011	24.776
2.02	14.159	15.256	16.288	17.260	18.177	19.044	22.744	25.636
2.04	14.555	15.693	16.766	17.778	18.733	19.636	23.498	26.523
2.06	14.959	16.142	17.257	18.309	19.304	20.245	24.274	27.437
2.08	15.373	16.601	17.760	18.854	19.889	20.869	25.073	28.380
2.10	15.797	17.071	18.275	19.413	20.490	21.511	25.895	29.352
2.12	16.230	17.553	18.804	19.987	21.107	22.169	26.741	30.354
2.14	16.673	18.046	19.345	20.575	21.740	22.846	27.612	31.387
2.16	17.127	18.551	19.900	21.177	22.389	23.540	28.508	32.452
2.18	17.591	19.068	20.468	21.796	23.056	24.253	29.430	33.549
2.20	18.066	19.597	21.050	22.429	23.739	24.984	30.378	34.680
2.22	18.551	20.139	21.647	23.079	24.440	25.735	31.354	35.845
2.24	19.047	20.693	22.258	23.745	25.160	26.506	32.358	37.046
2.26	19.555	21.261	22.884	24.428	25.897	27.297	33.391	38.283
2.28	20.074	21.842	23.525	25.127	26.654	28.109	34.453	39.558
2.30	20.605	22.437	24.182	25.845	27.430	28.942	35.546	40.872
2.32	21.148	23.045	24.854	26.580	28.226	29.797	36.670	42.225
2.34	21.703	23.668	25.543	27.333	29.042	30.674	37.825	43.618
2.36	22.271	24.305	26.248	28.105	29.879	31.574	39.014	45.054
2.38	22.851	24.957	26.971	28.896	30.737	32.497	40.236	46.533
2.40	23.444	25.624	27.710	29.706	31.616	33.444	41.493	48.056

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	30	40	50	100	200	500	1000	
1.62	13.496	14.754	15.626	17.720	18.997	19.859	20.165	20.481
1.64	14.016	15.344	16.265	18.484	19.841	20.758	21.084	21.421
1.66	14.554	15.954	16.928	19.279	20.720	21.696	22.042	22.401
1.68	15.109	16.586	17.615	20.105	21.634	22.672	23.042	23.424
1.70	15.684	17.241	18.328	20.963	22.587	23.691	24.083	24.490
1.72	16.278	17.919	19.067	21.855	23.579	24.751	25.170	25.602
1.74	16.891	18.621	19.832	22.783	24.611	25.857	26.302	26.762
1.76	17.526	19.348	20.626	23.746	25.685	27.009	27.482	27.972
1.78	18.181	20.100	21.448	24.748	26.803	28.209	28.712	29.233
1.80	18.858	20.878	22.300	25.789	27.967	29.460	29.994	30.549
1.82	19.558	21.684	23.183	26.870	29.178	30.763	31.331	31.920
1.84	20.281	22.518	24.098	27.993	30.439	32.121	32.724	33.350
1.86	21.028	23.381	25.046	29.160	31.751	33.535	34.175	34.841
1.88	21.800	24.275	26.029	30.373	33.115	35.008	35.688	36.395
1.90	22.597	25.199	27.046	31.632	34.536	36.543	37.265	38.015
1.92	23.420	26.155	28.100	32.941	36.013	38.142	38.907	39.705
1.94	24.270	27.144	29.191	34.299	37.551	39.807	40.619	41.465
1.96	25.148	28.167	30.322	35.711	39.150	41.541	42.403	43.301
1.98	26.055	29.226	31.493	37.177	40.814	43.347	44.262	45.215
2.00	26.991	30.321	32.706	38.699	42.546	45.229	46.198	47.209
2.02	27.957	31.453	33.961	40.280	44.346	47.188	48.216	49.288
2.04	28.955	32.625	35.262	41.921	46.219	49.228	50.318	51.455
2.06	29.985	33.836	36.608	43.626	48.168	51.353	52.507	53.713
2.08	31.049	35.089	38.002	45.395	50.194	53.566	54.789	56.067
2.10	32.147	36.384	39.445	47.233	52.302	55.870	57.165	58.520
2.12	33.279	37.723	40.939	49.140	54.494	58.269	59.641	61.077
2.14	34.449	39.107	42.485	51.121	56.773	60.767	62.220	63.741
2.16	35.655	40.539	44.085	53.176	59.144	63.368	64.906	66.517
2.18	36.900	42.019	45.742	55.310	61.609	66.075	67.704	69.411
2.20	38.185	43.548	47.457	57.525	64.172	68.895	70.618	72.426
2.22	39.511	45.129	49.231	59.825	66.838	71.830	73.654	75.568
2.24	40.879	46.764	51.068	62.211	69.609	74.885	76.815	78.841
2.26	42.290	48.453	52.968	64.687	72.491	78.067	80.108	82.252
2.28	43.746	50.198	54.934	67.258	75.487	81.378	83.537	85.807
2.30	45.248	52.002	56.969	69.925	78.602	84.825	87.108	89.510
2.32	46.797	53.866	59.074	72.693	81.840	88.413	90.828	93.369
2.34	48.394	55.792	61.251	75.565	85.207	92.149	94.701	97.389
2.36	50.042	57.782	63.504	78.545	88.706	96.036	98.734	101.577
2.38	51.741	59.839	65.835	81.638	92.344	100.083	102.935	105.941
2.40	53.494	61.963	68.246	84.846	96.126	104.295	107.308	110.487

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
2.42	3.568	5.993	8.662	11.382	14.075	16.704	19.247	21.697
2.44	3.609	6.084	8.816	11.608	14.377	17.084	19.707	22.237
2.46	3.649	6.175	8.972	11.837	14.684	17.471	20.177	22.789
2.48	3.690	6.268	9.131	12.069	14.996	17.866	20.655	23.352
2.50	3.732	6.362	9.291	12.306	15.314	18.268	21.143	23.927
2.52	3.773	6.456	9.453	12.545	15.636	18.677	21.641	24.514
2.54	3.815	6.552	9.618	12.789	15.965	19.095	22.149	25.113
2.56	3.857	6.648	9.785	13.036	16.299	19.519	22.667	25.725
2.58	3.899	6.746	9.954	13.287	16.638	19.952	23.195	26.349
2.60	3.941	6.844	10.125	13.541	16.984	20.392	23.733	26.987
2.62	3.984	6.943	10.298	13.800	17.335	20.841	24.282	27.637
2.64	4.026	7.044	10.473	14.062	17.692	21.298	24.842	28.302
2.66	4.069	7.145	10.651	14.328	18.055	21.763	25.412	28.979
2.68	4.113	7.247	10.831	14.599	18.424	22.237	25.994	29.671
2.70	4.156	7.350	11.013	14.873	18.799	22.719	26.587	30.377
2.72	4.200	7.454	11.198	15.151	19.181	23.210	27.191	31.097
2.74	4.244	7.560	11.385	15.434	19.569	23.709	27.807	31.832
2.76	4.288	7.666	11.574	15.720	19.963	24.218	28.435	32.582
2.78	4.332	7.773	11.765	16.011	20.364	24.736	29.075	33.347
2.80	4.377	7.881	11.959	16.306	20.771	25.263	29.728	34.128
2.82	4.421	7.991	12.156	16.606	21.185	25.800	30.392	34.924
2.84	4.466	8.101	12.354	16.910	21.606	26.346	31.070	35.737
2.86	4.512	8.212	12.556	17.218	22.034	26.902	31.760	36.565
2.88	4.557	8.325	12.760	17.531	22.468	27.468	32.463	37.411
2.90	4.603	8.438	12.966	17.848	22.910	28.044	33.180	38.273
2.92	4.649	8.553	13.175	18.170	23.359	28.630	33.911	39.153
2.94	4.695	8.669	13.386	18.497	23.815	29.226	34.655	40.050
2.96	4.741	8.786	13.600	18.828	24.279	29.833	35.413	40.965
2.98	4.788	8.903	13.817	19.164	24.750	30.450	36.185	41.898
3.00	4.834	9.022	14.036	19.505	25.228	31.079	36.972	42.850
3.02	4.881	9.142	14.258	19.851	25.714	31.718	37.773	43.820
3.04	4.929	9.264	14.483	20.202	26.208	32.368	38.590	44.809
3.06	4.976	9.386	14.710	20.557	26.710	33.030	39.421	45.818
3.08	5.024	9.509	14.940	20.918	27.220	33.703	40.268	46.847
3.10	5.072	9.634	15.173	21.284	27.738	34.388	41.131	47.896
3.12	5.120	9.760	15.409	21.655	28.265	35.085	42.010	48.966
3.14	5.169	9.887	15.647	22.031	28.799	35.793	42.905	50.056
3.16	5.217	10.015	15.889	22.413	29.342	36.514	43.816	51.167
3.18	5.266	10.144	16.133	22.800	29.894	37.247	44.744	52.301
3.20	5.315	10.274	16.380	23.193	30.454	37.993	45.689	53.456

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	20	25
2.42	24.050	26.306	28.467	30.537	32.518	34.416	42.785	49.625
2.44	24.670	27.004	29.242	31.388	33.443	35.412	44.113	51.240
2.46	25.303	27.718	30.036	32.259	34.391	36.434	45.479	52.904
2.48	25.950	28.449	30.848	33.152	35.362	37.483	46.883	54.617
2.50	26.612	29.196	31.680	34.066	36.358	38.558	48.326	56.380
2.52	27.288	29.960	32.531	35.003	37.378	39.661	49.810	58.197
2.54	27.978	30.742	33.403	35.963	38.424	40.791	51.334	60.066
2.56	28.684	31.541	34.294	36.945	39.496	41.951	52.901	61.991
2.58	29.405	32.358	35.207	37.952	40.595	43.140	54.512	63.972
2.60	30.142	33.194	36.141	38.982	41.720	44.359	56.167	66.012
2.62	30.895	34.049	37.096	40.037	42.874	45.608	57.868	68.112
2.64	31.664	34.923	38.074	41.118	44.056	46.889	59.615	70.273
2.66	32.450	35.816	39.074	42.224	45.266	48.203	61.411	72.497
2.68	33.252	36.729	40.098	43.357	46.507	49.549	63.256	74.786
2.70	34.072	37.663	41.145	44.517	47.777	50.929	65.151	77.141
2.72	34.909	38.618	42.217	45.704	49.079	52.343	67.099	79.565
2.74	35.765	39.594	43.313	46.919	50.412	53.793	69.099	82.060
2.76	36.638	40.591	44.434	48.163	51.778	55.279	71.154	84.626
2.78	37.530	41.610	45.581	49.437	53.177	56.801	73.265	87.267
2.80	38.441	42.652	46.754	50.740	54.609	58.361	75.433	89.984
2.82	39.371	43.717	47.953	52.074	56.076	59.960	77.660	92.780
2.84	40.321	44.806	49.180	53.439	57.579	61.598	79.947	95.655
2.86	41.291	45.918	50.435	54.836	59.118	63.277	82.295	98.614
2.88	42.281	47.054	51.719	56.266	60.693	64.997	84.707	101.657
2.90	43.292	48.215	53.031	57.729	62.306	66.759	87.184	104.787
2.92	44.324	49.402	54.373	59.226	63.958	68.564	89.727	108.006
2.94	45.378	50.614	55.745	60.758	65.649	70.413	92.339	111.318
2.96	46.453	51.853	57.147	62.325	67.380	72.307	95.020	114.723
2.98	47.551	53.118	58.582	63.929	69.152	74.248	97.773	118.226
3.00	48.672	54.411	60.048	65.569	70.967	76.235	100.599	121.828
3.02	49.816	55.732	61.547	67.247	72.824	78.271	103.500	125.531
3.04	50.984	57.081	63.079	68.964	74.725	80.355	106.478	129.340
3.06	52.175	58.459	64.646	70.720	76.671	82.490	109.536	133.256
3.08	53.391	59.866	66.247	72.516	78.662	84.677	112.674	137.283
3.10	54.633	61.304	67.884	74.354	80.701	86.916	115.895	141.423
3.12	55.899	62.772	69.556	76.233	82.787	89.209	119.201	145.679
3.14	57.192	64.272	71.266	78.154	84.922	91.557	122.594	150.055
3.16	58.511	65.803	73.013	80.120	87.106	93.961	126.076	154.553
3.18	59.856	67.367	74.799	82.130	89.342	96.422	129.649	159.178
3.20	61.229	68.964	76.624	84.185	91.629	98.942	133.316	163.931

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	30	40	50	100	200	500	1000	
2.42	55.301	64.158	70.739	88.175	100.057	108.679	111.863	115.223
2.44	57.165	66.425	73.319	91.628	104.143	113.241	116.605	120.158
2.46	59.086	68.766	75.986	95.210	108.390	117.990	121.543	125.298
2.48	61.067	71.185	78.745	98.927	112.804	122.932	126.684	130.652
2.50	63.109	73.684	81.598	102.782	117.391	128.075	132.037	136.231
2.52	65.215	76.264	84.549	106.781	122.159	133.427	137.611	142.041
2.54	67.385	78.929	87.600	110.929	127.114	138.997	143.414	148.094
2.56	69.623	81.681	90.755	115.232	132.263	144.793	149.456	154.400
2.58	71.929	84.522	94.017	119.694	137.614	150.824	155.746	160.967
2.60	74.306	87.457	97.389	124.322	143.175	157.100	162.295	167.808
2.62	76.756	90.487	100.876	129.122	148.953	163.631	169.113	174.934
2.64	79.280	93.615	104.481	134.099	154.958	170.427	176.210	182.357
2.66	81.882	96.845	108.208	139.261	161.197	177.497	183.599	190.088
2.68	84.563	100.180	112.060	144.614	167.679	184.855	191.291	198.140
2.70	87.326	103.623	116.042	150.165	174.415	192.509	199.298	206.527
2.72	90.173	107.176	120.158	155.921	181.413	200.474	207.633	215.262
2.74	93.106	110.845	124.412	161.889	188.684	208.761	216.310	224.360
2.76	96.128	114.631	128.809	168.076	196.238	217.382	225.342	233.835
2.78	99.241	118.540	133.354	174.492	204.086	226.352	234.744	243.704
2.80	102.449	122.573	138.050	181.143	212.239	235.683	244.530	253.982
2.82	105.753	126.737	142.904	188.039	220.709	245.391	254.717	264.686
2.84	109.156	131.033	147.919	195.188	229.508	255.491	265.320	275.834
2.86	112.662	135.467	153.102	202.599	238.648	265.998	276.356	287.443
2.88	116.272	140.043	158.457	210.282	248.143	276.928	287.843	299.534
2.90	119.991	144.764	163.990	218.246	258.006	288.299	299.799	312.125
2.92	123.821	149.636	169.707	226.501	268.251	300.127	312.243	325.238
2.94	127.766	154.663	175.613	235.058	278.892	312.431	325.195	338.893
2.96	131.828	159.849	181.716	243.927	289.945	325.230	338.675	353.114
2.98	136.011	165.200	188.020	253.119	301.426	338.544	352.704	367.922
3.00	140.318	170.721	194.532	262.647	313.349	352.392	367.305	383.343
3.02	144.753	176.416	201.259	272.521	325.733	366.797	382.501	399.402
3.04	149.319	182.291	208.209	282.755	338.595	381.781	398.316	416.124
3.06	154.021	188.351	215.386	293.361	351.953	397.365	414.775	433.537
3.08	158.861	194.602	222.800	304.352	365.826	413.575	431.904	451.669
3.10	163.844	201.050	230.458	315.742	380.233	430.435	449.729	470.551
3.12	168.974	207.700	238.366	327.544	395.195	447.971	468.280	490.212
3.14	174.255	214.559	246.534	339.774	410.732	466.209	487.584	510.685
3.16	179.691	221.633	254.969	352.447	426.867	485.178	507.674	532.002
3.18	185.286	228.929	263.680	365.578	443.621	504.907	528.580	554.199
3.20	191.045	236.453	272.675	379.184	461.019	525.425	550.335	577.312

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
3.22	5.365	10.406	16.631	23.590	31.023	38.752	46.652	54.633
3.24	5.415	10.539	16.884	23.994	31.601	39.523	47.632	55.833
3.26	5.464	10.673	17.140	24.403	32.188	40.308	48.629	57.057
3.28	5.515	10.808	17.399	24.818	32.784	41.105	49.645	58.304
3.30	5.565	10.945	17.661	25.238	33.389	41.917	50.680	59.574
3.32	5.616	11.082	17.927	25.665	34.004	42.742	51.733	60.869
3.34	5.667	11.221	18.195	26.097	34.628	43.581	52.805	62.189
3.36	5.718	11.362	18.467	26.535	35.262	44.434	53.897	63.534
3.38	5.769	11.503	18.741	26.980	35.906	45.302	55.008	64.905
3.40	5.821	11.646	19.019	27.430	36.559	46.184	56.139	66.302
3.42	5.872	11.790	19.301	27.887	37.223	47.081	57.290	67.725
3.44	5.925	11.935	19.585	28.349	37.897	47.992	58.462	69.175
3.46	5.977	12.082	19.873	28.818	38.581	48.919	59.655	70.652
3.48	6.030	12.230	20.164	29.294	39.275	49.862	60.869	72.158
3.50	6.082	12.379	20.459	29.776	39.980	50.820	62.105	73.691
3.52	6.136	12.529	20.756	30.264	40.696	51.794	63.363	75.254
3.54	6.189	12.681	21.058	30.759	41.423	52.784	64.643	76.846
3.56	6.243	12.834	21.362	31.261	42.161	53.790	65.945	78.467
3.58	6.297	12.989	21.671	31.770	42.909	54.813	67.271	80.119
3.60	6.351	13.145	21.983	32.285	43.669	55.853	68.620	81.802
3.62	6.405	13.302	22.298	32.807	44.441	56.910	69.992	83.516
3.64	6.460	13.461	22.617	33.337	45.224	57.984	71.389	85.262
3.66	6.515	13.621	22.939	33.873	46.019	59.076	72.810	87.040
3.68	6.570	13.782	23.266	34.417	46.826	60.185	74.256	88.851
3.70	6.625	13.945	23.596	34.967	47.644	61.313	75.727	90.695
3.72	6.681	14.109	23.929	35.525	48.475	62.458	77.224	92.574
3.74	6.737	14.275	24.267	36.091	49.319	63.623	78.747	94.487
3.76	6.793	14.442	24.608	36.664	50.174	64.806	80.296	96.435
3.78	6.850	14.611	24.953	37.244	51.043	66.009	81.873	98.420
3.80	6.907	14.781	25.302	37.833	51.924	67.231	83.476	100.440
3.82	6.964	14.952	25.655	38.429	52.819	68.472	85.107	102.497
3.84	7.021	15.125	26.012	39.032	53.726	69.734	86.766	104.592
3.86	7.078	15.300	26.372	39.644	54.647	71.015	88.454	106.725
3.88	7.136	15.476	26.737	40.264	55.582	72.317	90.171	108.897
3.90	7.194	15.653	27.106	40.891	56.530	73.641	91.917	111.108
3.92	7.253	15.832	27.479	41.527	57.492	74.985	93.693	113.359
3.94	7.311	16.012	27.856	42.172	58.468	76.350	95.499	115.651
3.96	7.370	16.194	28.237	42.824	59.458	77.738	97.336	117.984
3.98	7.429	16.378	28.622	43.485	60.462	79.147	99.205	120.360
4.00	7.489	16.563	29.012	44.155	61.482	80.578	101.105	122.778

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	20	25
3.22	62.630	70.594	78.489	86.287	93.969	101.522	137.079	168.817
3.24	64.060	72.259	80.394	88.436	96.364	104.163	140.940	173.839
3.26	65.518	73.959	82.341	90.633	98.814	106.867	144.902	179.001
3.28	67.005	75.695	84.330	92.880	101.321	109.635	148.967	184.305
3.30	68.523	77.467	86.363	95.177	103.885	112.468	153.137	189.757
3.32	70.071	79.276	88.439	97.526	106.508	115.367	157.416	195.360
3.34	71.650	81.123	90.561	99.926	109.192	118.336	161.805	201.118
3.36	73.260	83.008	92.728	102.381	111.937	121.373	166.308	207.035
3.38	74.903	84.933	94.942	104.890	114.744	124.482	170.927	213.114
3.40	76.578	86.897	97.204	107.454	117.616	127.664	175.665	219.361
3.42	78.287	88.903	99.514	110.076	120.553	130.920	180.525	225.780
3.44	80.030	90.949	101.873	112.755	123.557	134.252	185.510	232.374
3.46	81.807	93.038	104.284	115.493	126.630	137.662	190.623	239.150
3.48	83.619	95.170	106.745	118.292	129.771	141.151	195.867	246.110
3.50	85.468	97.346	109.259	121.153	132.984	144.721	201.244	253.261
3.52	87.352	99.567	111.827	124.076	136.270	148.374	206.759	260.607
3.54	89.274	101.833	114.449	127.063	139.630	152.111	212.415	268.153
3.56	91.233	104.145	117.127	130.116	143.065	155.934	218.215	275.904
3.58	93.231	106.505	119.861	133.236	146.577	159.846	224.161	283.866
3.60	95.268	108.912	122.653	136.423	150.169	163.848	230.259	292.044
3.62	97.344	111.369	125.504	139.680	153.841	167.941	236.512	300.443
3.64	99.461	113.876	128.415	143.008	157.594	172.129	242.922	309.070
3.66	101.619	116.433	131.388	146.408	161.432	176.412	249.495	317.929
3.68	103.820	119.042	134.422	149.881	165.355	180.794	256.233	327.028
3.70	106.062	121.704	137.521	153.430	169.366	185.275	263.141	336.371
3.72	108.349	124.420	140.684	157.055	173.466	189.859	270.223	345.966
3.74	110.679	127.190	143.913	160.759	177.656	194.547	277.482	355.819
3.76	113.054	130.016	147.209	164.542	181.940	199.341	284.924	365.936
3.78	115.475	132.899	150.574	168.406	186.318	204.244	292.552	376.325
3.80	117.943	135.839	154.009	172.354	190.793	209.259	300.371	386.991
3.82	120.458	138.838	157.515	176.386	195.366	214.386	308.385	397.942
3.84	123.021	141.897	161.094	180.504	200.040	219.630	316.598	409.186
3.86	125.633	145.017	164.746	184.710	204.817	224.992	325.017	420.729
3.88	128.295	148.200	168.474	189.005	209.698	230.474	333.644	432.580
3.90	131.007	151.445	172.279	193.392	214.687	236.080	342.486	444.745
3.92	133.772	154.755	176.162	197.873	219.784	241.811	351.547	457.234
3.94	136.588	158.130	180.125	202.448	224.993	247.671	360.833	470.054
3.96	139.458	161.572	184.169	207.120	230.315	253.662	370.348	483.213
3.98	142.383	165.082	188.296	211.891	235.753	259.787	380.098	496.720
4.00	145.363	168.661	192.508	216.763	241.310	266.048	390.088	510.584

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	30	40	50	100	200	500	1000	
3.22	196.973	244.211	281.964	393.281	479.085	546.765	572.974	601.378
3.24	203.074	252.211	291.555	407.886	497.844	568.957	596.531	626.437
3.26	209.352	260.461	301.458	423.018	517.322	592.037	621.045	652.529
3.28	215.813	268.967	311.684	438.696	537.546	616.040	646.554	679.696
3.30	222.462	277.737	322.241	454.937	558.545	641.001	673.097	707.982
3.32	229.304	286.779	333.141	471.762	580.348	666.960	700.716	737.434
3.34	236.345	296.102	344.393	489.192	602.985	693.954	729.456	768.100
3.36	243.589	305.713	356.010	507.248	626.488	722.027	759.360	800.028
3.38	251.043	315.621	368.003	525.952	650.888	751.219	790.476	833.271
3.40	258.712	325.835	380.382	545.326	676.221	781.575	822.853	867.883
3.42	266.602	336.364	393.161	565.395	702.520	813.142	856.540	903.919
3.44	274.720	347.218	406.352	586.181	729.824	845.967	891.592	941.439
3.46	283.070	358.405	419.967	607.712	758.168	880.100	928.062	980.502
3.48	291.661	369.936	434.020	630.011	787.593	915.593	966.009	1021.17
3.50	300.498	381.821	448.524	653.108	818.139	952.499	1005.49	1063.52
3.52	309.588	394.070	463.493	677.029	849.848	990.874	1046.57	1107.60
3.54	318.938	406.694	478.943	701.802	882.764	1030.78	1089.31	1153.50
3.56	328.556	419.705	494.886	727.459	916.932	1072.27	1133.78	1201.29
3.58	338.447	433.112	511.340	754.029	952.400	1115.41	1180.04	1251.04
3.60	348.621	446.929	528.320	781.545	989.215	1160.26	1228.18	1302.83
3.62	359.084	461.167	545.841	810.040	1027.43	1206.90	1278.26	1356.76
3.64	369.844	475.839	563.921	839.547	1067.09	1255.39	1330.36	1412.90
3.66	380.910	490.956	582.576	870.102	1108.26	1305.81	1384.57	1471.34
3.68	392.289	506.533	601.826	901.741	1150.99	1358.23	1440.96	1532.19
3.70	403.991	522.582	621.687	934.501	1195.35	1412.73	1499.64	1595.54
3.72	416.023	539.118	642.178	968.423	1241.38	1469.39	1560.68	1661.48
3.74	428.395	556.154	663.320	1003.55	1289.15	1528.31	1624.18	1730.14
3.76	441.117	573.705	685.132	1039.91	1338.74	1589.56	1690.24	1801.62
3.78	454.196	591.787	707.634	1077.56	1390.20	1653.23	1758.97	1876.03
3.80	467.644	610.414	730.848	1116.55	1443.61	1719.44	1830.48	1953.49
3.82	481.469	629.602	754.795	1156.90	1499.03	1788.26	1904.86	2034.14
3.84	495.682	649.368	779.498	1198.69	1556.56	1859.82	1982.24	2118.09
3.86	510.294	669.729	804.980	1241.94	1616.25	1934.21	2062.74	2205.49
3.88	525.315	690.701	831.264	1286.73	1678.20	2011.54	2146.49	2296.48
3.90	540.756	712.303	858.376	1333.08	1742.48	2091.94	2233.61	2391.20
3.92	556.628	734.552	886.340	1381.07	1809.20	2175.52	2324.24	2489.80
3.94	572.942	757.467	915.183	1430.75	1878.42	2262.41	2418.52	2592.45
3.96	589.712	781.067	944.931	1482.17	1950.26	2352.73	2516.59	2699.31
3.98	606.948	805.373	975.611	1535.40	2024.80	2446.63	2618.61	2810.55
4.00	624.663	830.405	1007.25	1590.50	2102.15	2544.24	2724.74	2926.36

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
4.02	7.549	16.750	29.406	44.833	62.516	82.033	103.037	125.240
4.04	7.609	16.938	29.804	45.520	63.565	83.510	105.002	127.745
4.06	7.669	17.128	30.207	46.216	64.629	85.010	107.000	130.296
4.08	7.730	17.319	30.614	46.921	65.708	86.535	109.032	132.892
4.10	7.790	17.512	31.025	47.635	66.803	88.083	111.099	135.535
4.12	7.852	17.707	31.441	48.358	67.914	89.655	113.200	138.224
4.14	7.913	17.903	31.862	49.090	69.040	91.252	115.336	140.962
4.16	7.975	18.101	32.287	49.832	70.183	92.874	117.508	143.748
4.18	8.037	18.300	32.716	50.583	71.342	94.521	119.716	146.583
4.20	8.099	18.502	33.151	51.343	72.518	96.194	121.961	149.469
4.22	8.162	18.705	33.590	52.114	73.710	97.893	124.244	152.406
4.24	8.224	18.909	34.033	52.894	74.919	99.618	126.565	155.394
4.26	8.287	19.115	34.482	53.683	76.146	101.370	128.924	158.435
4.28	8.351	19.323	34.935	54.483	77.389	103.149	131.322	161.530
4.30	8.415	19.533	35.393	55.293	78.651	104.955	133.761	164.679
4.32	8.479	19.744	35.856	56.113	79.930	106.789	136.239	167.883
4.34	8.543	19.958	36.324	56.944	81.227	108.652	138.759	171.143
4.36	8.607	20.172	36.797	57.784	82.543	110.543	141.319	174.460
4.38	8.672	20.389	37.275	58.636	83.876	112.463	143.923	177.836
4.40	8.737	20.607	37.758	59.498	85.229	114.413	146.568	181.269
4.42	8.803	20.828	38.246	60.371	86.601	116.392	149.258	184.763
4.44	8.869	21.050	38.739	61.254	87.991	118.402	151.991	188.317
4.46	8.935	21.273	39.238	62.149	89.402	120.442	154.769	191.933
4.48	9.001	21.499	39.742	63.054	90.831	122.514	157.593	195.612
4.50	9.068	21.727	40.251	63.971	92.281	124.616	160.462	199.354
4.52	9.135	21.956	40.765	64.899	93.751	126.751	163.378	203.161
4.54	9.202	22.187	41.285	65.839	95.241	128.918	166.342	207.033
4.56	9.269	22.420	41.810	66.790	96.752	131.118	169.354	210.972
4.58	9.337	22.655	42.341	67.753	98.284	133.351	172.414	214.979
4.60	9.405	22.892	42.878	68.728	99.837	135.618	175.524	219.055
4.62	9.474	23.130	43.419	69.715	101.411	137.918	178.685	223.200
4.64	9.543	23.371	43.967	70.714	103.007	140.254	181.896	227.416
4.66	9.612	23.613	44.520	71.725	104.625	142.624	185.159	231.704
4.68	9.681	23.858	45.079	72.749	106.265	145.030	188.474	236.066
4.70	9.751	24.104	45.644	73.785	107.927	147.472	191.843	240.502
4.72	9.821	24.353	46.215	74.833	109.612	149.950	195.266	245.013
4.74	9.891	24.603	46.791	75.894	111.320	152.466	198.744	249.601
4.76	9.962	24.855	47.374	76.969	113.052	155.018	202.277	254.266
4.78	10.032	25.110	47.962	78.056	114.807	157.609	205.866	259.011
4.80	10.104	25.366	48.557	79.156	116.586	160.238	209.513	263.836

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	10	11	12	13	14	15	20	25
4.02	148.398	172.310	196.805	221.738	246.987	272.449	400.324	524.814
4.04	151.491	176.032	201.190	226.817	252.787	278.993	410.812	539.419
4.06	154.643	179.826	205.664	232.003	258.712	285.681	421.558	554.408
4.08	157.853	183.694	210.229	237.298	264.766	292.518	432.567	569.791
4.10	161.123	187.638	214.886	242.703	270.950	299.507	443.846	585.578
4.12	164.455	191.659	219.638	248.222	277.267	306.649	455.400	601.778
4.14	167.849	195.758	224.486	253.856	283.720	313.950	467.237	618.403
4.16	171.306	199.938	229.431	259.608	290.312	321.411	479.363	635.463
4.18	174.828	204.198	234.477	265.479	297.045	329.037	491.784	652.968
4.20	178.415	208.541	239.624	271.472	303.922	336.831	504.507	670.930
4.22	182.069	212.968	244.874	277.590	310.946	344.795	517.539	689.360
4.24	185.790	217.480	250.230	283.835	318.120	352.934	530.887	708.269
4.26	189.580	222.080	255.693	290.209	325.448	361.252	544.559	727.670
4.28	193.441	226.768	261.265	296.715	332.931	369.751	558.562	747.575
4.30	197.372	231.547	266.948	303.355	340.573	378.135	572.903	767.995
4.32	201.376	236.417	272.745	310.132	348.377	387.309	587.591	788.945
4.34	205.454	241.381	278.658	317.048	356.347	396.376	602.632	810.437
4.36	209.606	246.440	284.688	324.107	364.486	405.640	618.036	832.484
4.38	213.834	251.596	290.837	331.310	372.796	415.106	633.810	855.099
4.40	218.140	256.850	297.109	338.661	381.282	424.776	649.963	878.298
4.42	222.525	262.205	303.505	346.162	389.947	434.656	666.503	902.094
4.44	226.990	267.661	310.027	353.817	398.794	444.749	683.440	926.503
4.46	231.535	273.221	316.678	361.628	407.827	455.060	700.782	951.538
4.48	236.164	278.887	323.460	369.598	417.050	465.594	718.538	977.215
4.50	240.877	284.660	330.376	377.730	426.466	476.354	736.718	1003.55
4.52	245.675	290.543	337.427	386.028	436.079	487.345	755.331	1030.56
4.54	250.560	296.537	344.617	394.494	445.894	498.573	774.387	1058.26
4.56	255.534	302.644	351.948	403.131	455.913	510.041	793.897	1086.67
4.58	260.597	308.866	359.422	411.943	466.140	521.755	813.870	1115.80
4.60	265.752	315.205	367.043	420.934	476.581	533.720	834.316	1145.67
4.62	271.000	321.663	374.812	430.106	487.239	545.940	855.247	1176.31
4.64	276.341	328.243	382.733	439.462	498.118	558.420	876.673	1207.72
4.66	281.779	334.946	390.807	449.007	509.223	571.167	898.606	1239.94
4.68	287.315	341.774	399.039	458.744	520.558	584.184	921.056	1272.96
4.70	292.949	348.730	407.431	468.676	532.127	597.478	944.036	1306.83
4.72	298.684	355.816	415.985	478.807	543.935	611.055	967.557	1341.55
4.74	304.522	363.034	424.705	489.141	555.986	624.918	991.631	1377.15
4.76	310.463	370.386	433.593	499.681	568.286	639.076	1016.27	1413.65
4.78	316.511	377.875	442.653	510.432	580.839	653.532	1041.49	1451.07
4.80	322.666	385.503	451.887	521.398	593.650	668.294	1067.30	1489.44

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	30	40	50	100	200	500	1000	
4.02	642.870	856.182	1039.88	1647.53	2182.41	2645.71	2835.15	3046.91
4.04	661.581	882.728	1073.53	1706.56	2265.69	2751.19	2949.99	3172.40
4.06	680.811	910.063	1108.24	1767.66	2352.11	2860.83	3069.45	3303.04
4.08	700.573	938.211	1144.02	1830.89	2441.77	2974.81	3193.72	3439.04
4.10	720.881	967.195	1180.92	1896.34	2534.80	3093.29	3322.99	3580.61
4.12	741.750	997.039	1218.97	1964.08	2631.33	3216.45	3457.45	3727.98
4.14	763.194	1027.77	1258.21	2034.19	2731.48	3344.47	3597.32	3881.39
4.16	785.228	1059.40	1298.66	2106.74	2835.39	3477.55	3742.81	4041.09
4.18	807.869	1091.98	1340.37	2181.82	2943.19	3615.87	3894.15	4207.33
4.20	831.131	1125.51	1383.37	2259.53	3055.04	3759.65	4051.57	4380.38
4.22	855.032	1160.04	1427.71	2339.94	3171.09	3909.10	4215.32	4560.52
4.24	879.588	1195.58	1473.42	2423.15	3291.48	4064.45	4385.64	4748.04
4.26	904.816	1232.17	1520.55	2509.26	3416.38	4225.92	4562.80	4943.24
4.28	930.734	1269.84	1569.13	2598.36	3545.96	4393.75	4747.08	5146.44
4.30	957.360	1308.61	1619.21	2690.56	3680.38	4568.20	4938.76	5357.96
4.32	984.713	1348.52	1670.84	2785.97	3819.84	4749.52	5138.14	5578.14
4.34	1012.81	1389.61	1724.06	2884.69	3964.51	4937.98	5345.51	5807.33
4.36	1041.68	1431.89	1778.92	2986.83	4114.59	5133.86	5561.21	6045.92
4.38	1071.32	1475.42	1835.47	3092.52	4270.27	5337.45	5785.57	6294.27
4.40	1101.78	1520.22	1893.76	3201.87	4431.77	5549.06	6018.93	6552.79
4.42	1133.06	1566.33	1953.84	3315.01	4599.30	5769.00	6261.65	6821.89
4.44	1165.19	1613.78	2015.76	3432.07	4773.09	5997.59	6514.10	7102.01
4.46	1198.19	1662.62	2079.58	3553.17	4953.35	6235.17	6776.69	7393.60
4.48	1232.08	1712.88	2145.36	3678.47	5140.34	6482.09	7049.80	7697.12
4.50	1266.88	1764.60	2213.16	3808.09	5334.30	6738.73	7333.86	8013.06
4.52	1302.63	1817.83	2283.02	3942.19	5535.49	7005.45	7629.31	8341.93
4.54	1339.34	1872.61	2355.02	4080.92	5744.17	7282.65	7936.60	8684.27
4.56	1377.04	1928.97	2429.23	4224.43	5960.63	7570.75	8256.21	9040.61
4.58	1415.75	1986.97	2505.69	4372.89	6185.13	7870.17	8588.62	9411.53
4.60	1455.51	2046.65	2584.49	4526.47	6418.00	8181.35	8934.36	9797.64
4.62	1496.33	2108.06	2665.69	4685.33	6659.52	8504.74	9293.95	10199.5
4.64	1538.24	2171.24	2749.35	4849.67	6910.03	8840.84	9667.94	10617.9
4.66	1581.28	2236.25	2835.57	5019.65	7169.84	9190.13	10056.9	11053.3
4.68	1625.47	2303.13	2924.40	5195.48	7439.31	9553.13	10461.5	11506.6
4.70	1670.84	2371.94	3015.93	5377.35	7718.78	9930.38	10882.2	11978.4
4.72	1717.43	2442.74	3110.24	5565.47	8008.63	10322.4	11319.8	12469.5
4.74	1765.25	2515.57	3207.40	5760.03	8309.24	10729.8	11774.9	12980.7
4.76	1814.36	2590.50	3307.51	5961.28	8621.00	11153.2	12248.2	13512.9
4.78	1864.76	2667.58	3410.65	6169.42	8944.32	11593.2	12740.5	14066.7
4.80	1916.51	2746.88	3516.90	6384.69	9279.63	12050.5	13252.5	14643.3

TABLE B-2. - MULTIPLYING FACTOR FOR THE VARIANCE--CONTINUED

T	SAMPLE SIZE							
	2	3	4	5	6	7	8	9
4.82	10.175	25.624	49.158	80.270	118.389	162.906	213.218	268.742
4.84	10.247	25.885	49.764	81.397	120.216	165.614	216.982	273.731
4.86	10.319	26.147	50.377	82.537	122.068	168.362	220.805	278.804
4.88	10.392	26.412	50.997	83.692	123.945	171.150	224.689	283.962
4.90	10.465	26.678	51.622	84.860	125.848	173.979	228.635	289.207
4.92	10.538	26.947	52.254	86.042	127.776	176.850	232.643	294.540
4.94	10.611	27.218	52.893	87.239	129.730	179.763	236.714	299.962
4.96	10.685	27.491	53.537	88.449	131.710	182.719	240.849	305.475
4.98	10.759	27.766	54.189	89.674	133.716	185.718	245.050	311.080
5.00	10.833	28.043	54.847	90.914	135.750	188.761	249.316	316.778

T	SAMPLE SIZE							
	10	11	12	13	14	15	20	25
4.82	328.930	393.272	461.300	532.581	606.723	683.367	1093.71	1528.77
4.84	335.305	401.186	470.893	543.987	620.065	698.757	1120.74	1569.09
4.86	341.793	409.245	480.671	555.620	633.679	714.471	1148.40	1610.42
4.88	348.396	417.453	490.636	567.483	647.572	730.515	1176.71	1652.78
4.90	355.115	425.813	500.792	579.581	661.748	746.895	1205.68	1696.21
4.92	361.953	434.326	511.142	591.919	676.213	763.619	1235.32	1740.73
4.94	368.911	442.997	521.690	604.500	690.973	780.692	1265.65	1786.36
4.96	375.992	451.826	532.439	617.329	706.033	798.123	1296.68	1833.13
4.98	383.197	460.817	543.392	630.411	721.399	815.917	1328.43	1881.07
5.00	390.528	469.973	554.555	643.751	737.076	834.081	1360.92	1930.20

T	SAMPLE SIZE							
	30	40	50	100	200	500	1000	
4.82	1969.64	2828.44	3626.37	6607.33	9627.36	12525.7	13784.9	15243.4
4.84	2024.17	2912.35	3739.14	6837.59	9987.97	13019.5	14338.7	15868.0
4.86	2080.15	2998.65	3855.31	7075.73	10361.9	13532.6	14914.6	16518.2
4.88	2137.62	3087.43	3974.98	7322.00	10749.8	14065.9	15513.6	17195.0
4.90	2196.60	3178.74	4098.25	7576.69	11151.9	14620.0	16136.5	17899.4
4.92	2257.15	3272.66	4225.24	7840.07	11568.9	15195.8	16784.3	18632.7
4.94	2319.29	3369.26	4356.04	8112.44	12001.4	15794.2	17458.0	19395.9
4.96	2383.07	3468.61	4490.77	8394.10	12449.9	16416.0	18158.6	20190.4
4.98	2448.54	3570.79	4629.54	8685.36	12914.9	17062.1	18887.3	21017.3
5.00	2515.72	3675.88	4772.48	8986.54	13397.1	17733.5	19645.1	21878.0

TABLE B-3. - EXPONENTIAL FUNCTION

x	e^x	x	e^x	x	e^x	x	e^x
0.01	1.0101	0.36	1.4333	0.71	2.0340	1.06	2.8864
0.02	1.0202	0.37	1.4477	0.72	2.0544	1.07	2.9154
0.03	1.0305	0.38	1.4623	0.73	2.0751	1.08	2.9447
0.04	1.0408	0.39	1.4770	0.74	2.0959	1.09	2.9743
0.05	1.0513	0.40	1.4918	0.75	2.1170	1.10	3.0042
0.06	1.0618	0.41	1.5068	0.76	2.1383	1.11	3.0344
0.07	1.0725	0.42	1.5220	0.77	2.1598	1.12	3.0649
0.08	1.0833	0.43	1.5373	0.78	2.1815	1.13	3.0957
0.09	1.0942	0.44	1.5527	0.79	2.2034	1.14	3.1268
0.10	1.1052	0.45	1.5683	0.80	2.2255	1.15	3.1582
0.11	1.1163	0.46	1.5841	0.81	2.2479	1.16	3.1899
0.12	1.1275	0.47	1.6000	0.82	2.2705	1.17	3.2220
0.13	1.1388	0.48	1.6161	0.83	2.2933	1.18	3.2544
0.14	1.1503	0.49	1.6323	0.84	2.3164	1.19	3.2871
0.15	1.1618	0.50	1.6487	0.85	2.3396	1.20	3.3201
0.16	1.1735	0.51	1.6653	0.86	2.3632	1.21	3.3535
0.17	1.1853	0.52	1.6820	0.87	2.3869	1.22	3.3872
0.18	1.1972	0.53	1.6989	0.88	2.4109	1.23	3.4212
0.19	1.2092	0.54	1.7160	0.89	2.4351	1.24	3.4556
0.20	1.2214	0.55	1.7333	0.90	2.4596	1.25	3.4903
0.21	1.2337	0.56	1.7507	0.91	2.4843	1.26	3.5254
0.22	1.2461	0.57	1.7683	0.92	2.5093	1.27	3.5609
0.23	1.2586	0.58	1.7860	0.93	2.5345	1.28	3.5966
0.24	1.2712	0.59	1.8040	0.94	2.5600	1.29	3.6328
0.25	1.2840	0.60	1.8221	0.95	2.5857	1.30	3.6693
0.26	1.2969	0.61	1.8404	0.96	2.6117	1.31	3.7062
0.27	1.3100	0.62	1.8589	0.97	2.6379	1.32	3.7434
0.28	1.3231	0.63	1.8776	0.98	2.6645	1.33	3.7810
0.29	1.3364	0.64	1.8965	0.99	2.6912	1.34	3.8190
0.30	1.3499	0.65	1.9155	1.00	2.7183	1.35	3.8574
0.31	1.3634	0.66	1.9348	1.01	2.7456	1.36	3.8962
0.32	1.3771	0.67	1.9542	1.02	2.7732	1.37	3.9354
0.33	1.3910	0.68	1.9739	1.03	2.8011	1.38	3.9749
0.34	1.4049	0.69	1.9937	1.04	2.8292	1.39	4.0149
0.35	1.4191	0.70	2.0138	1.05	2.8577	1.40	4.0552

TABLE B-3. - EXPONENTIAL FUNCTION--CONTINUED

x	e^x	x	e^x	x	e^x	x	e^x
1.41	4.0960	1.76	5.8124	2.11	8.2482	2.46	11.705
1.42	4.1371	1.77	5.8709	2.12	8.3311	2.47	11.822
1.43	4.1787	1.78	5.9299	2.13	8.4149	2.48	11.941
1.44	4.2207	1.79	5.9895	2.14	8.4994	2.49	12.061
1.45	4.2631	1.80	6.0496	2.15	8.5849	2.50	12.182
1.46	4.3060	1.81	6.1104	2.16	8.6711	2.51	12.305
1.47	4.3492	1.82	6.1719	2.17	8.7583	2.52	12.429
1.48	4.3929	1.83	6.2339	2.18	8.8463	2.53	12.554
1.49	4.4371	1.84	6.2965	2.19	8.9352	2.54	12.680
1.50	4.4817	1.85	6.3598	2.20	9.0250	2.55	12.807
1.51	4.5267	1.86	6.4237	2.21	9.1157	2.56	12.936
1.52	4.5722	1.87	6.4883	2.22	9.2073	2.57	13.066
1.53	4.6182	1.88	6.5535	2.23	9.2999	2.58	13.197
1.54	4.6646	1.89	6.6194	2.24	9.3933	2.59	13.330
1.55	4.7115	1.90	6.6859	2.25	9.4877	2.60	13.464
1.56	4.7588	1.91	6.7531	2.26	9.5831	2.61	13.599
1.57	4.8066	1.92	6.8210	2.27	9.6794	2.62	13.736
1.58	4.8550	1.93	6.8895	2.28	9.7767	2.63	13.874
1.59	4.9037	1.94	6.9588	2.29	9.8749	2.64	14.013
1.60	4.9530	1.95	7.0287	2.30	9.9742	2.65	14.154
1.61	5.0028	1.96	7.0993	2.31	10.074	2.66	14.296
1.62	5.0531	1.97	7.1707	2.32	10.176	2.67	14.440
1.63	5.1039	1.98	7.2427	2.33	10.278	2.68	14.585
1.64	5.1552	1.99	7.3155	2.34	10.381	2.69	14.732
1.65	5.2070	2.00	7.3891	2.35	10.486	2.70	14.880
1.66	5.2593	2.01	7.4633	2.36	10.591	2.71	15.029
1.67	5.3122	2.02	7.5383	2.37	10.697	2.72	15.180
1.68	5.3656	2.03	7.6141	2.38	10.805	2.73	15.333
1.69	5.4195	2.04	7.6906	2.39	10.913	2.74	15.487
1.70	5.4739	2.05	7.7679	2.40	11.023	2.75	15.643
1.71	5.5290	2.06	7.8460	2.41	11.134	2.76	15.800
1.72	5.5845	2.07	7.9248	2.42	11.246	2.77	15.959
1.73	5.6407	2.08	8.0045	2.43	11.359	2.78	16.119
1.74	5.6973	2.09	8.0849	2.44	11.473	2.79	16.281
1.75	5.7546	2.10	8.1662	2.45	11.588	2.80	16.445

TABLE B-3. - EXPONENTIAL FUNCTION--CONTINUED

x	e^x	x	e^x	x	e^x	x	e^x
2.81	16.610	3.16	23.571	3.51	33.448	3.86	47.465
2.82	16.777	3.17	23.807	3.52	33.784	3.87	47.942
2.83	16.945	3.18	24.047	3.53	34.124	3.88	48.424
2.84	17.116	3.19	24.288	3.54	34.467	3.89	48.911
2.85	17.288	3.20	24.533	3.55	34.813	3.90	49.402
2.86	17.462	3.21	24.779	3.56	35.163	3.91	49.899
2.87	17.637	3.22	25.028	3.57	35.517	3.92	50.400
2.88	17.814	3.23	25.280	3.58	35.874	3.93	50.907
2.89	17.993	3.24	25.534	3.59	36.234	3.94	51.419
2.90	18.174	3.25	25.790	3.60	36.598	3.95	51.935
2.91	18.357	3.26	26.050	3.61	36.966	3.96	52.457
2.92	18.541	3.27	26.311	3.62	37.338	3.97	52.985
2.93	18.728	3.28	26.576	3.63	37.713	3.98	53.517
2.94	18.916	3.29	26.843	3.64	38.092	3.99	54.055
2.95	19.106	3.30	27.113	3.65	38.475	4.00	54.598
2.96	19.298	3.31	27.385	3.66	38.861	4.01	55.147
2.97	19.492	3.32	27.660	3.67	39.252	4.02	55.701
2.98	19.688	3.33	27.938	3.68	39.646	4.03	56.261
2.99	19.886	3.34	28.219	3.69	40.045	4.04	56.826
3.00	20.086	3.35	28.503	3.70	40.447	4.05	57.397
3.01	20.287	3.36	28.789	3.71	40.854	4.06	57.974
3.02	20.491	3.37	29.079	3.72	41.264	4.07	58.557
3.03	20.697	3.38	29.371	3.73	41.679	4.08	59.145
3.04	20.905	3.39	29.666	3.74	42.098	4.09	59.740
3.05	21.115	3.40	29.964	3.75	42.521	4.10	60.340
3.06	21.328	3.41	30.265	3.76	42.948	4.11	60.947
3.07	21.542	3.42	30.569	3.77	43.380	4.12	61.559
3.08	21.758	3.43	30.877	3.78	43.816	4.13	62.178
3.09	21.977	3.44	31.187	3.79	44.256	4.14	62.803
3.10	22.198	3.45	31.500	3.80	44.701	4.15	63.434
3.11	22.421	3.46	31.817	3.81	45.150	4.16	64.072
3.12	22.646	3.47	32.137	3.82	45.604	4.17	64.715
3.13	22.874	3.48	32.460	3.83	46.063	4.18	65.366
3.14	23.104	3.49	32.786	3.84	46.525	4.19	66.023
3.15	23.336	3.50	33.115	3.85	46.993	4.20	66.686

TABLE B-3. - EXPONENTIAL FUNCTION--CONTINUED

x	e^x	x	e^x	x	e^x	x	e^x
4.21	67.357	4.56	95.583	4.91	135.64	5.26	192.48
4.22	68.033	4.57	96.544	4.92	137.00	5.27	194.42
4.23	68.717	4.58	97.514	4.93	138.38	5.28	196.37
4.24	69.408	4.59	98.494	4.94	139.77	5.29	198.34
4.25	70.105	4.60	99.484	4.95	141.17	5.30	200.34
4.26	70.810	4.61	100.48	4.96	142.59	5.31	202.35
4.27	71.522	4.62	101.49	4.97	144.03	5.32	204.38
4.28	72.240	4.63	102.51	4.98	145.47	5.33	206.44
4.29	72.966	4.64	103.54	4.99	146.94	5.34	208.51
4.30	73.700	4.65	104.58	5.00	148.41	5.35	210.61
4.31	74.440	4.66	105.64	5.01	149.90	5.36	212.72
4.32	75.189	4.67	106.70	5.02	151.41	5.37	214.86
4.33	75.944	4.68	107.77	5.03	152.93	5.38	217.02
4.34	76.708	4.69	108.85	5.04	154.47	5.39	219.20
4.35	77.478	4.70	109.95	5.05	156.02	5.40	221.41
4.36	78.257	4.71	111.05	5.06	157.59	5.41	223.63
4.37	79.044	4.72	112.17	5.07	159.17	5.42	225.88
4.38	79.838	4.73	113.30	5.08	160.77	5.43	228.15
4.39	80.640	4.74	114.43	5.09	162.39	5.44	230.44
4.40	81.451	4.75	115.58	5.10	164.02	5.45	232.76
4.41	82.269	4.76	116.75	5.11	165.67	5.46	235.10
4.42	83.096	4.77	117.92	5.12	167.34	5.47	237.46
4.43	83.931	4.78	119.10	5.13	169.02	5.48	239.85
4.44	84.775	4.79	120.30	5.14	170.72	5.49	242.26
4.45	85.627	4.80	121.51	5.15	172.43	5.50	244.69
4.46	86.488	4.81	122.73	5.16	174.16	5.51	247.15
4.47	87.357	4.82	123.97	5.17	175.91	5.52	249.64
4.48	88.235	4.83	125.21	5.18	177.68	5.53	252.14
4.49	89.121	4.84	126.47	5.19	179.47	5.54	254.68
4.50	90.017	4.85	127.74	5.20	181.27	5.55	257.24
4.51	90.922	4.86	129.02	5.21	183.09	5.56	259.82
4.52	91.836	4.87	130.32	5.22	184.93	5.57	262.43
4.53	92.759	4.88	131.63	5.23	186.79	5.58	265.07
4.54	93.691	4.89	132.95	5.24	188.67	5.59	267.74
4.55	94.632	4.90	134.29	5.25	190.57	5.60	270.43

TABLE B-3. - EXPONENTIAL FUNCTION--CONTINUED

x	e^x	x	e^x	x	e^x	x	e^x
5.61	273.14	5.96	387.61	6.31	550.04	6.66	780.55
5.62	275.89	5.97	391.51	6.32	555.57	6.67	788.40
5.63	278.66	5.98	395.44	6.33	561.16	6.68	796.32
5.64	281.46	5.99	399.41	6.34	566.80	6.69	804.32
5.65	284.29	6.00	403.43	6.35	572.49	6.70	812.41
5.66	287.15	6.01	407.48	6.36	578.25	6.71	820.57
5.67	290.03	6.02	411.58	6.37	584.06	6.72	828.82
5.68	292.95	6.03	415.72	6.38	589.93	6.73	837.15
5.69	295.89	6.04	419.89	6.39	595.86	6.74	845.56
5.70	298.87	6.05	424.11	6.40	601.85	6.75	854.06
5.71	301.87	6.06	428.38	6.41	607.89	6.76	862.64
5.72	304.90	6.07	432.68	6.42	614.00	6.77	871.31
5.73	307.97	6.08	437.03	6.43	620.17	6.78	880.07
5.74	311.06	6.09	441.42	6.44	626.41	6.79	888.91
5.75	314.19	6.10	445.86	6.45	632.70	6.80	897.85
5.76	317.35	6.11	450.34	6.46	639.06	6.81	906.87
5.77	320.54	6.12	454.86	6.47	645.48	6.82	915.99
5.78	323.76	6.13	459.44	6.48	651.97	6.83	925.19
5.79	327.01	6.14	464.05	6.49	658.52	6.84	934.49
5.80	330.30	6.15	468.72	6.50	665.14	6.85	943.88
5.81	333.62	6.16	473.43	6.51	671.83	6.86	953.37
5.82	336.97	6.17	478.19	6.52	678.58	6.87	962.95
5.83	340.36	6.18	482.99	6.53	685.40	6.88	972.63
5.84	343.78	6.19	487.85	6.54	692.29	6.89	982.40
5.85	347.23	6.20	492.75	6.55	699.24	6.90	992.27
5.86	350.72	6.21	497.70	6.56	706.27	6.91	1002.2
5.87	354.25	6.22	502.70	6.57	713.37	6.92	1012.3
5.88	357.81	6.23	507.76	6.58	720.54	6.93	1022.5
5.89	361.41	6.24	512.86	6.59	727.78	6.94	1032.8
5.90	365.04	6.25	518.01	6.60	735.10	6.95	1043.1
5.91	368.71	6.26	523.22	6.61	742.48	6.96	1053.6
5.92	372.41	6.27	528.48	6.62	749.95	6.97	1064.2
5.93	376.15	6.28	533.79	6.63	757.48	6.98	1074.9
5.94	379.93	6.29	539.15	6.64	765.09	6.99	1085.7
5.95	383.75	6.30	544.57	6.65	772.78	7.00	1096.6

TABLE B-3. - EXPONENTIAL FUNCTION--CONTINUED

x	e^x	x	e^x	x	e^x	x	e^x
7.01	1107.7	7.36	1571.8	7.71	2230.5	8.06	3165.3
7.02	1118.8	7.37	1587.6	7.72	2253.0	8.07	3197.1
7.03	1130.0	7.38	1603.6	7.73	2275.6	8.08	3229.2
7.04	1141.4	7.39	1619.7	7.74	2298.5	8.09	3261.7
7.05	1152.9	7.40	1636.0	7.75	2321.6	8.10	3294.5
7.06	1164.4	7.41	1652.4	7.76	2344.9	8.11	3327.6
7.07	1176.1	7.42	1669.0	7.77	2368.5	8.12	3361.0
7.08	1188.0	7.43	1685.8	7.78	2392.3	8.13	3394.8
7.09	1199.9	7.44	1702.8	7.79	2416.3	8.14	3428.9
7.10	1212.0	7.45	1719.9	7.80	2440.6	8.15	3463.4
7.11	1224.1	7.46	1737.1	7.81	2465.1	8.16	3498.2
7.12	1236.5	7.47	1754.6	7.82	2489.9	8.17	3533.3
7.13	1248.9	7.48	1772.2	7.83	2514.9	8.18	3568.9
7.14	1261.4	7.49	1790.1	7.84	2540.2	8.19	3604.7
7.15	1274.1	7.50	1808.0	7.85	2565.7	8.20	3641.0
7.16	1286.9	7.51	1826.2	7.86	2591.5	8.21	3677.5
7.17	1299.8	7.52	1844.6	7.87	2617.6	8.22	3714.5
7.18	1312.9	7.53	1863.1	7.88	2643.9	8.23	3751.8
7.19	1326.1	7.54	1881.8	7.89	2670.4	8.24	3789.5
7.20	1339.4	7.55	1900.7	7.90	2697.3	8.25	3827.6
7.21	1352.9	7.56	1919.8	7.91	2724.4	8.26	3866.1
7.22	1366.5	7.57	1939.1	7.92	2751.8	8.27	3904.9
7.23	1380.2	7.58	1958.6	7.93	2779.4	8.28	3944.2
7.24	1394.1	7.59	1978.3	7.94	2807.4	8.29	3983.8
7.25	1408.1	7.60	1998.2	7.95	2835.6	8.30	4023.9
7.26	1422.3	7.61	2018.3	7.96	2864.1	8.31	4064.3
7.27	1436.6	7.62	2038.6	7.97	2892.9	8.32	4105.2
7.28	1451.0	7.63	2059.1	7.98	2921.9	8.33	4146.4
7.29	1465.6	7.64	2079.7	7.99	2951.3	8.34	4188.1
7.30	1480.3	7.65	2100.6	8.00	2981.0	8.35	4230.2
7.31	1495.2	7.66	2121.8	8.01	3010.9	8.36	4272.7
7.32	1510.2	7.67	2143.1	8.02	3041.2	8.37	4315.6
7.33	1525.4	7.68	2164.6	8.03	3071.7	8.38	4359.0
7.34	1540.7	7.69	2186.4	8.04	3102.6	8.39	4402.8
7.35	1556.2	7.70	2208.3	8.05	3133.8	8.40	4447.1

TABLE B-3. - EXPONENTIAL FUNCTION--CONTINUED

x	e ^x	x	e ^x	x	e ^x	x	e ^x
8.41	4491.8	8.76	6374.1	9.11	9045.3	9.46	12836
8.42	4536.9	8.77	6438.2	9.12	9136.2	9.47	12965
8.43	4582.5	8.78	6502.9	9.13	9228.0	9.48	13095
8.44	4628.6	8.79	6568.2	9.14	9320.8	9.49	13227
8.45	4675.1	8.80	6634.2	9.15	9414.4	9.50	13360
8.46	4722.1	8.81	6700.9	9.16	9509.1	9.51	13494
8.47	4769.5	8.82	6768.3	9.17	9604.6	9.52	13630
8.48	4817.4	8.83	6836.3	9.18	9701.2	9.53	13767
8.49	4865.9	8.84	6905.0	9.19	9798.7	9.54	13905
8.50	4914.8	8.85	6974.4	9.20	9897.1	9.55	14045
8.51	4964.2	8.86	7044.5	9.21	9996.6	9.56	14186
8.52	5014.1	8.87	7115.3	9.22	10097	9.57	14328
8.53	5064.4	8.88	7186.8	9.23	10199	9.58	14472
8.54	5115.3	8.89	7259.0	9.24	10301	9.59	14618
8.55	5166.8	8.90	7332.0	9.25	10405	9.60	14765
8.56	5218.7	8.91	7405.7	9.26	10509	9.61	14913
8.57	5271.1	8.92	7480.1	9.27	10615	9.62	15063
8.58	5324.1	8.93	7555.3	9.28	10721	9.63	15214
8.59	5377.6	8.94	7631.2	9.29	10829	9.64	15367
8.60	5431.7	8.95	7707.9	9.30	10938	9.65	15522
8.61	5486.2	8.96	7785.4	9.31	11048	9.66	15678
8.62	5541.4	8.97	7863.6	9.32	11159	9.67	15835
8.63	5597.1	8.98	7942.6	9.33	11271	9.68	15994
8.64	5653.3	8.99	8022.5	9.34	11384	9.69	16155
8.65	5710.1	9.00	8103.1	9.35	11499	9.70	16318
8.66	5767.5	9.01	8184.5	9.36	11614	9.71	16482
8.67	5825.5	9.02	8266.8	9.37	11731	9.72	16647
8.68	5884.0	9.03	8349.9	9.38	11849	9.73	16815
8.69	5943.2	9.04	8433.8	9.39	11968	9.74	16984
8.70	6002.9	9.05	8518.5	9.40	12088	9.75	17154
8.71	6063.2	9.06	8604.2	9.41	12210	9.76	17327
8.72	6124.2	9.07	8690.6	9.42	12333	9.77	17501
8.73	6185.7	9.08	8778.0	9.43	12457	9.78	17677
8.74	6247.9	9.09	8866.2	9.44	12582	9.79	17854
8.75	6310.7	9.10	8955.3	9.45	12708	9.80	18034
9.81	18215	9.86	19149	9.91	20131	9.96	21163
9.82	18398	9.87	19341	9.92	20333	9.97	21375
9.83	18583	9.88	19536	9.93	20537	9.98	21590
9.84	18770	9.89	19732	9.94	20744	9.99	21807
9.85	18958	9.90	19930	9.95	20952	10.00	22026

TABLE B-4. - COEFFICIENT OF VARIATION

σ	γ	σ	γ	σ	γ
0.05	0.05	1.80	4.95	3.55	545.25
0.10	0.10	1.85	5.44	3.60	651.97
0.15	0.15	1.90	6.00	3.65	781.53
0.20	0.20	1.95	6.62	3.70	939.17
0.25	0.25	2.00	7.32	3.75	1131.44
0.30	0.31	2.05	8.12	3.80	1366.49
0.35	0.36	2.10	9.01	3.85	1654.49
0.40	0.42	2.15	10.04	3.90	2008.21
0.45	0.47	2.20	11.20	3.95	2443.65
0.50	0.53	2.25	12.53	4.00	2980.96
0.55	0.59	2.30	14.05	4.05	3645.50
0.60	0.66	2.35	15.79	4.10	4469.36
0.65	0.73	2.40	17.79	4.15	5493.11
0.70	0.80	2.45	20.09	4.20	6768.26
0.75	0.87	2.50	22.74	4.25	8360.30
0.80	0.95	2.55	25.80	4.30	10352.67
0.85	1.03	2.60	29.35	4.35	12851.94
0.90	1.12	2.65	33.48	4.40	15994.50
0.95	1.21	2.70	38.27	4.45	19955.30
1.00	1.31	2.75	43.86	4.50	24959.26
1.05	1.42	2.80	50.39	4.55	31296.14
1.10	1.53	2.85	58.04	4.60	39340.11
1.15	1.66	2.90	67.01	4.65	49575.40
1.20	1.79	2.95	77.57	4.70	62630.02
1.25	1.94	3.00	90.01	4.75	79320.35
1.30	2.10	3.05	104.71	4.80	100709.96
1.35	2.28	3.10	122.12	4.85	128187.59
1.40	2.47	3.15	142.77	4.90	163570.60
1.45	2.68	3.20	167.33	4.95	209242.68
1.50	2.91	3.25	196.61	5.00	268337.29
1.55	3.17	3.30	231.60	5.05	344982.85
1.60	3.45	3.35	273.48	5.10	444631.00
1.65	3.77	3.40	323.76	5.15	574496.91
1.70	4.12	3.45	384.23	5.20	744151.57
1.75	4.51	3.50	457.14	5.25	966319.69