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PREDICTION OF SODIUM
CONCENTRATION IN LIGNITE



UNITED STATES DEPARTMENT OF THE INTERIOR

BUREAU OF MINES

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By Manuel Gomez, Kathleen Hazen,
and Everett A. Sondreal

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PREDICTION OF SODIUM CONCENTRATION IN LIGNITE

by

Manuel Gomez,¹ Kathleen Hazen,² and Everett A. Sondreal³

ABSTRACT

Prediction models were constructed for seam elevation, overburden, and sodium concentration for a lignite deposit in Ward County, N. Dak. These models were developed by mathematical-statistical techniques using surface coordinates as independent variables to permit interpolation between drill holes. The results suggest that sodium concentration in lignite may be predicted in advance of mining. This information is helpful to mine planning, specifically, in scheduling production to deliver a uniform product to existing powerplants.

INTRODUCTION

Increase in power generation in the United States and particularly in the north-central region has focused attention on problems related to coal combustion. The apparent tendency for certain coal ash to be deposited on furnace walls and fireside surfaces of boiler tubes in coal burning powerplants has stimulated interest in the composition of the mineral matter. Since these fireside ash deposits decrease boiler efficiency and add to powerplant downtime, the quantitative identification of ash properties that contribute to boiler fouling will increase in importance as these reserves are exploited. Gronhovd and coworkers⁴ found that the increased sodium oxide content in lignite ash was related, in some instances, to an increase in the rate of ash deposition on boiler tubes. A later report⁵ concluded that the degree

¹Research chemist, Mine Systems Engineering Group, Bureau of Mines, Denver, Colo.

²Chemist, Mine Systems Engineering Group, Bureau of Mines, Denver, Colo.

³Chemical research engineer, Grand Forks Coal Research Laboratory, Bureau of Mines, Grand Forks, N. Dak.

⁴Gronhovd, G. H., R. J. Wagner, and A. J. Wittmaier. Comparison of Ash Fouling Tendencies of High- and Low-Sodium From a North Dakota Mine. Proc. Am. Power Conf., Chicago, Ill., Apr. 26-28, 1966, v. 28, 1966, pp. 632-644.

⁵Gronhovd, G. H., R. J. Wagner, and A. J. Wittmaier. A Study of the Ash Fouling Tendencies of North Dakota Lignite as Related to Its Sodium Content. Trans. AIME, v. 238, 1967, pp. 313-322.

of boiler fouling encountered in pulverized coal combustion was directly related to the sodium content of the fuel. Probe deposit rates in the superheater-reheater area showed an almost linear relationship with sodium concentration up to a sodium oxide content of about 6 percent in the ash. Above this level of sodium, the probe deposit rate appeared to level off. This report also noted the great variability of sodium concentration in North Dakota lignite seams and observed that, when sampling for sodium, 75-foot rather than 150-foot mine sample hole spacings might be advantageous.

The variability of sodium within the same mine was reported by Sondreal and coworkers.⁶ Values equal to twice the estimated standard deviation for sodium were given for groups of samples representative of the entire mine, clusters of samples taken along the bed at 20-foot intervals, and samples taken through two or more seams. Twice the estimated standard deviation is the approximate interval on either side of the mean within which 95 percent of the individual values would be expected to lie. Examples of sodium variability for mines operating in high, intermediate, and low sodium lignite are summarized as follows:

Sodium level	Range, percent		Two estimated standard deviations		
	of dry ash		Entire mine	20-ft groups	Two or more beds
	High	Low			
High.....	27.8	2.7	8.54	2.16	7.13
Intermediate.....	12.6	3.1	5.44	1.70	2.56
Low.....	.4	.2	.12	.06	.06

These data suggest that there is a rather high regional variability in contrast to the rather low local variability. If this situation is true, it is quite probable that high sodium concentrations may be found as islands along the seam. This type of sodium distribution may reflect the geologic history of the seam, especially the effect of water over geologic time.

This report presents results of a mathematical-statistical study applied to the prediction of sodium concentration in lignite seams. The prime objective of this report is to develop an acceptable prediction model for sodium in terms of variables that can be measured in advance of mining. The findings from this study dovetail with the larger Bureau of Mines goals to establish criteria for identifying lignites which have excessive boiler-fouling tendencies and to develop procedures that will help the power generating industry increase utilization of this low-cost fuel.

ACKNOWLEDGMENT

The authors are grateful to James L. Elder, Chief, Grand Forks Coal Research Laboratory, Bureau of Mines, Grand Forks, N. Dak., for providing the sodium data and for assistance in locating each of the mine samples.

⁶Sondreal, Everett A., Wayne R. Kube, and James L. Elder. Analysis of the Northern Great Plains Province Lignites and Their Ash: A Study of Variability. BuMines Rept. of Inv. 7158, 1968, 94 pp.

DISCUSSION AND RESULTS

Data Analyzed

Data used in the study were taken from a lignite deposit in Ward County, N. Dak. The data represent 101 core-drilled and auger-drilled samples taken by both company and Bureau samplers. Seam elevation and overburden values are estimates made from mine maps. Sodium values are Bureau results determined by flame photometry.⁷ Ranges, means, and standard deviations for these variables are given in table 1.

TABLE 1. - Basic statistics for variables used in model construction

Variable	Range		Mean	Standard deviation
	From	To		
East coordinate, arbitrary units....	1.64	6.39	3.812	1.2562
North coordinate, arbitrary units...	2.92	10.61	7.254	1.8753
Seam elevation, feet.....	1,830	1,880	1,858	14.00
Overburden, feet.....	19	58	39.7	10.59
Sodium content of ash, percent as Na ₂ O.....	0.3	12.1	5.69	3.13

Sample locations are presented in figure 1. These samples generally follow the mining pattern for this deposit and were not taken specifically for the present study.

To permit later evaluation of prediction models developed, 11 of the 101 samples, selected by a table of random numbers, were set aside for testing. All models were constructed using 90 data points.

Development of Prediction Models

The statistical response surface fitting method⁸ was used to develop the prediction equations or models for seam elevation, overburden, and sodium concentration. The procedure involves the statistical fitting of an undulating curved surface which can be expressed as a mathematical equation to a dependent variable. In this study all models (prediction equations for surfaces) were developed by multiple stepwise linear regression analysis using the least squares technique. The multivariable models are of the general form, as follows:

$$Y = B_0 + B_1 X_1 + B_2 X_2 + \dots + B_n X_n + E,$$

⁷Gibson, F. H., and W. H. Ode. Application of Rapid Methods for Analyzing Coal Ash and Related Materials. BuMines Rept. of Inv. 6036, 1962, 23 pp.

⁸Box, G. E. P. The Exploration and Exploitation of Response Surfaces. Biometrics, v. 10, 1954, pp. 16-60.

Hewlett, Richard F. Polynomial Surface Fitting Using Sample Data From an Underground Copper Deposit. BuMines Rept. of Inv. 6522, 1964, 27 pp.

Hill, William J., and William G. Hunter. A Review of Response Surface Methodology. A Literature Survey. Technometrics, v. 8, No. 4, November 1966, pp. 571-590.

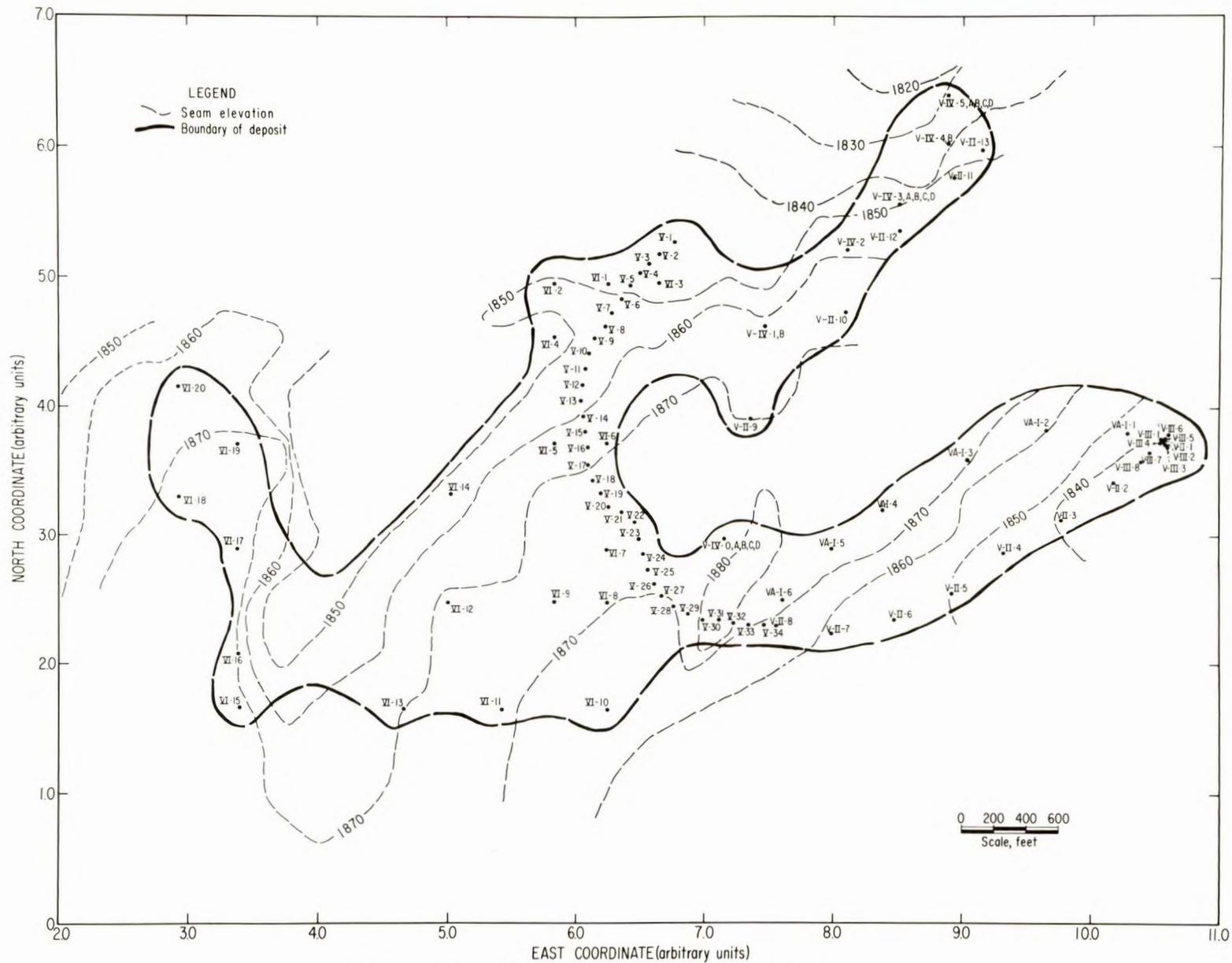


FIGURE 1. - Sample Locations in Lignite Deposit.

where

Y = the parameter of interest,

B_0 = constant term,

$B_{1,n}$ = partial regression coefficient for each independent variable,

$X_{1,n}$ = independent variables,

E = error.

Prior to regression analysis, the independent variables used were scaled and transformed into a population having the same predetermined mean according to the following relationship:

$$X = \left[\frac{(X_1 - \bar{X}_1)}{S_1} \right] + u,$$

where

X = transformed value of independent variable,

X_1 = original value of independent variable,

\bar{X}_1 = mean of independent variable,

S_1 = estimated standard deviation of independent variable,

u = desired mean of independent variable.

The partial regression coefficients for the independent variables of the model were determined by stepwise regression, using the standard matrix normally calculated for the solution of the coefficients. The stepwise regression approach was used because at each intermediate step a model is formed where the resulting solution of coefficients and relationships between the independent and dependent variables can be easily followed. The intermediate models are obtained by adding one variable at a time, where the variable added produces the greatest reduction in variance of the parameter of interest with respect to the variables not presently in the model. At each intermediate step the remaining unexplained standard error (variance) of the dependent variable, the R^2 or multiple correlation coefficient squared (fraction of the total variance in the dependent variable explained by the independent variables included in the model), and the standard error of each partial regression coefficient are computed.

The first step in the construction of an empirical model is to decide on various mathematical forms (that is, trigonometric, exponential, logarithmic, etc.) of the independent variables that can be tried as input to the model. The necessary transformations on the independent variables are then made, and the linear correlation coefficients are computed. This procedure is necessary to establish the intercorrelations between the independent variables and to

determine the correlations between those independent variables that would quite likely contribute most to the reduction in variance of the dependent variable. Based on this information, a complex model was constructed and the coefficients computed by stepwise regression. The process was repeated with nonsignificant variables eliminated and other variables introduced until a minimum standard error of the predicted value, or a maximum R^2 was obtained. After each successive model was formed, variables having coefficients with a computed t-test of less than 0.5 were considered statistically insignificant and were eliminated from the model. Although a t-test with 25 or more degrees of freedom would be approximately 2.0, based on a 95-percent confidence level, the lower limits were used because experience has shown such variables do permit the construction of useful models. Other variables were then decided upon and another complex model was constructed. This process was continued until the fraction of the variance in the dependent variable explained by the model was greater than 0.9 ($R^2 \geq 0.9$ or 90 percent). It was felt that the experimental error in the data was something less than 10 percent, and therefore a model which explained 90 percent or more of the variance was adequate for prediction. The final selection of the desired model was made after an examination of the individual residuals (residual = actual value minus predicted value) in which the model giving the minimum residuals was chosen.

The prediction models were constructed in the following order: seam elevation, overburden, and sodium concentration. The seam elevation model, given in the appendix, was developed using only the surface coordinates and transformations of the surface coordinates. This model had a multiple correlation coefficient squared of 0.985 and a standard error of estimate of 2.034. Observed and predicted seam elevations are shown in figure 2.

The overburden model was constructed using the surface coordinates, the predicted seam elevation, and transformations of these variables. The model for overburden reported in the appendix has a multiple correlation coefficient squared of 0.955 and a standard error of estimate of 2.702. Observed and predicted overburdens are shown in figure 3.

The model for sodium concentration was developed using the surface coordinates, the predicted seam elevation, and the predicted overburden, as well as transformations of these variables. The model for sodium had a multiple correlation coefficient squared of 0.925 and a standard error of estimate of 1.094. The sodium model is shown in the appendix. Observed and predicted values for sodium are reported in table 2.

It is observed that 92.5 to 98.5 percent of the variance was explained with these models, and that both seam elevation and overburden contributed to the reduction in variance for sodium. It appears likely, however, that neither seam elevation nor overburden are variables directly affecting sodium concentration, but rather, elevation and overburden only reflect other variables, not included in the present model, that do influence the quantity of sodium in the seam. Since elevation and overburden are related to the depositional history of the coal seam, it is reasonable to believe that there are geologic factors that, when measured and used as input to the model, may be better predictors of sodium than elevation or overburden.

TABLE 2. - Observed and predicted seam elevations, overburdens, and sodium concentrations in lignite

SAMPLE NUMBER	SURFACE COORDINATES (ARBITRARY UNITS)		SEAM ELEVATION, FEET		OVERBURDEN, FEET		SODIUM IN LIGNITE ASH, PERCENT		
	NORTH	EAST	OBSERVED	PRE-DICTED	OBSERVED	PRE-DICTED	OBSERVED	PRE-DICTED	RESIDUAL
VA-I-1	3,80	10,29	1847	1845	40	42	5,5	5,4	0,1
VA-I-2	3,83	9,66	1860	1861	30	29	2,6	3,2	-0,6
VA-I-3	3,60	9,05	1870	1870	20	20	3,6	3,6	0,0
VA-I-4	3,21	8,38	1874	1874	19	19	0,7	0,0	0,7
VA-I-5	2,90	7,99	1873	1875	23	21	0,5	1,1	-0,6
V-II-1	3,74	10,56	1841	1841	43	42	7,2	6,3	0,9
V-II-2	3,42	10,18	1838	1838	58	56	8,4	8,3	0,1
V-II-3	3,13	9,78	1840	1839	56	56	8,6	8,7	-0,1
V-II-4	2,86	9,33	1845	1845	57	57	8,3	8,0	0,3
V-II-5	2,55	8,93	1850	1850	55	55	5,1	6,2	-1,1
V-II-6	2,34	8,48	1855	1856	52	52	3,3	2,0	1,3
V-II-8	2,29	7,56	1869	1869	28	35	1,1	0,8	0,3
V-II-9	3,92	7,36	1870	1874	28	28	2,5	2,3	0,2
V-II-10	4,73	8,10	1867	1863	20	20	3,4	2,8	0,6
V-II-11	5,77	8,93	1851	1849	22	27	8,4	6,8	1,6
V-II-12	5,36	8,52	1856	1857	24	23	4,6	5,2	-0,6
V-II-13	5,98	9,15	1847	1847	30	27	3,2	4,3	-1,1
V-III-1	3,76	10,55	1841	1842	43	41	6,1	6,5	-0,4
V-III-2	3,72	10,58	1841	1841	43	44	6,9	6,0	0,9
V-III-3	3,69	10,60	1841	1840	43	46	6,8	6,0	0,8
V-III-4	3,73	10,54	1841	1841	43	43	5,6	6,3	-0,7
V-III-5	3,76	10,58	1841	1842	43	41	6,3	6,4	-0,1
V-III-6	3,78	10,61	1841	1842	43	40	5,7	6,4	-0,7
V-III-8	3,58	10,39	1838	1839	48	51	6,4	7,1	-0,7
V-IV-0A	2,98	7,15	1878	1876	20	21	0,5	0,7	-0,2
V-IV-0	2,98	7,15	1878	1876	20	21	0,4	0,7	-0,3
V-IV-0C	2,98	7,15	1878	1876	20	21	0,3	0,7	-0,4
V-IV-0D	2,98	7,15	1878	1876	20	21	0,5	0,7	-0,2
V-IV-1	4,63	7,48	1862	1863	25	25	3,6	3,7	-0,1
V-IV-1B	4,63	7,48	1862	1863	25	25	3,7	3,7	-0,0
V-IV-2	5,22	8,12	1858	1856	20	22	7,6	8,3	-0,7
V-IV-3B	5,56	8,51	1849	1850	30	29	8,9	8,2	0,7
V-IV-3C	5,56	8,51	1849	1850	30	29	7,2	8,2	-1,0
V-IV-3D	5,56	8,51	1849	1850	30	29	9,8	8,2	1,6
V-IV-4B	6,03	8,89	1840	1840	40	41	9,0	9,7	-0,7
V-IV-5A	6,39	8,88	1830	1830	52	52	9,9	9,6	0,3
V-IV-5	6,39	8,88	1830	1830	52	52	9,9	9,6	0,3
V-IV-5B	6,39	8,88	1830	1830	52	52	9,1	9,6	-0,5
V-IV-5C	6,39	8,88	1830	1830	52	52	9,7	9,6	0,1
V-IV-5D	6,39	8,88	1830	1830	52	52	9,8	9,6	0,2
V-1	5,28	6,77	1846	1845	47	47	9,2	9,4	-0,2
V-2	5,18	6,60	1847	1847	46	46	8,7	9,1	-0,4
V-3	5,11	6,57	1848	1848	46	45	8,6	9,0	-0,4
V-4	5,04	6,50	1848	1849	45	44	9,6	8,9	0,7
V-5	4,94	6,42	1849	1850	45	44	10,1	8,9	1,2
V-6	4,83	6,35	1851	1851	43	44	9,5	9,1	0,4

TABLE 2. - Observed and predicted seam elevations, overburdens, and sodium concentrations in lignite--Continued

SAMPLE NUMBER	SURFACE COORDINATES (ARBITRARY UNITS)		SEAM ELEVATION, FEET		OVERBURDEN, FEET		SODIUM IN LIGNITE ASH, PERCENT		
	NORTH	EAST	OBSERVED	PRE-DICTED	OBSERVED	PRE-DICTED	OBSERVED	PRE-DICTED	RESIDUAL
V- 7	4,73	6,28	1853	1852	44	44	10,2	9,1	1,1
V- 8	4,62	6,23	1853	1852	43	45	9,0	9,3	-0,3
V- 9	4,53	6,14	1853	1853	46	45	9,3	9,2	0,1
V-10	4,42	6,10	1853	1853	46	46	8,2	9,0	-0,8
V-11	4,29	6,07	1855	1854	47	47	8,2	8,3	-0,1
V-12	4,17	6,05	1856	1856	47	47	5,9	6,8	-0,9
V-14	3,93	6,06	1861	1860	46	46	4,2	4,0	0,2
V-15	3,81	6,07	1864	1863	42	44	4,0	3,6	0,4
V-16	3,69	6,09	1866	1866	41	42	3,5	3,4	0,1
V-17	3,55	6,09	1869	1869	40	40	2,5	3,8	-1,3
V-18	3,43	6,13	1870	1870	39	39	3,8	3,9	-0,1
V-19	3,33	6,19	1870	1870	40	39	5,4	3,7	1,7
V-20	3,23	6,26	1870	1870	41	40	3,0	3,8	-0,8
V-21	3,18	6,35	1870	1869	42	40	3,2	3,8	-0,6
V-22	3,11	6,45	1870	1869	42	41	6,0	4,0	2,0
V-23	2,97	6,49	1870	1869	41	41	5,3	3,9	1,4
V-24	2,86	6,52	1870	1870	41	40	0,7	3,0	-2,3
V-25	2,73	6,56	1870	1872	40	38	0,5	2,3	-1,8
V-26	2,62	6,58	1870	1873	30	36	0,6	2,0	-1,4
V-28	2,45	6,76	1869	1877	40	31	2,2	1,5	0,7
V-29	2,36	6,87	1875	1877	30	30	2,2	1,9	0,3
V-30	2,34	6,98	1880	1876	25	29	2,8	2,7	0,1
V-31	2,34	7,12	1880	1876	25	28	3,0	3,4	-0,4
V-32	2,32	7,23	1880	1874	25	29	5,4	3,9	1,5
V-33	2,30	7,34	1872	1873	32	30	3,3	3,6	-0,3
V-34	2,30	7,46	1869	1871	42	32	1,8	2,4	-0,6
VI- 1	4,95	6,25	1849	1851	45	43	6,7	8,2	-1,5
VI- 2	4,95	5,83	1850	1848	44	45	12,1	11,9	0,2
VI- 3	4,96	6,65	1849	1849	45	45	9,8	9,8	0,0
VI- 4	4,54	5,83	1850	1853	44	44	10,1	10,1	-0,0
VI- 6	3,72	6,23	1868	1867	40	39	2,1	2,5	-0,4
VI- 8	2,48	6,24	1870	1873	40	37	4,3	2,7	1,6
VI- 9	2,48	5,83	1870	1870	40	42	6,7	4,5	2,2
VI-10	1,64	6,24	1865	1865	56	55	4,7	4,8	-0,1
VI-11	1,64	5,42	1870	1870	50	51	7,4	8,8	-1,4
VI-12	2,47	5,00	1870	1868	47	49	5,4	5,9	-0,5
VI-13	1,64	4,66	1870	1870	46	45	8,4	9,0	-0,6
VI-14	3,33	5,02	1854	1855	57	56	8,1	7,9	0,2
VI-15	1,66	3,39	1870	1870	54	54	9,4	8,5	0,9
VI-16	2,08	3,38	1870	1871	50	49	2,1	1,8	0,3
VI-17	2,89	3,37	1880	1880	42	42	5,7	5,7	-0,0
VI-18	3,30	2,92	1870	1871	46	46	2,1	3,0	-0,9
VI-19	3,71	3,37	1870	1869	40	39	7,3	6,8	0,5
VI-20	4,15	2,92	1864	1864	46	46	9,1	8,9	0,2

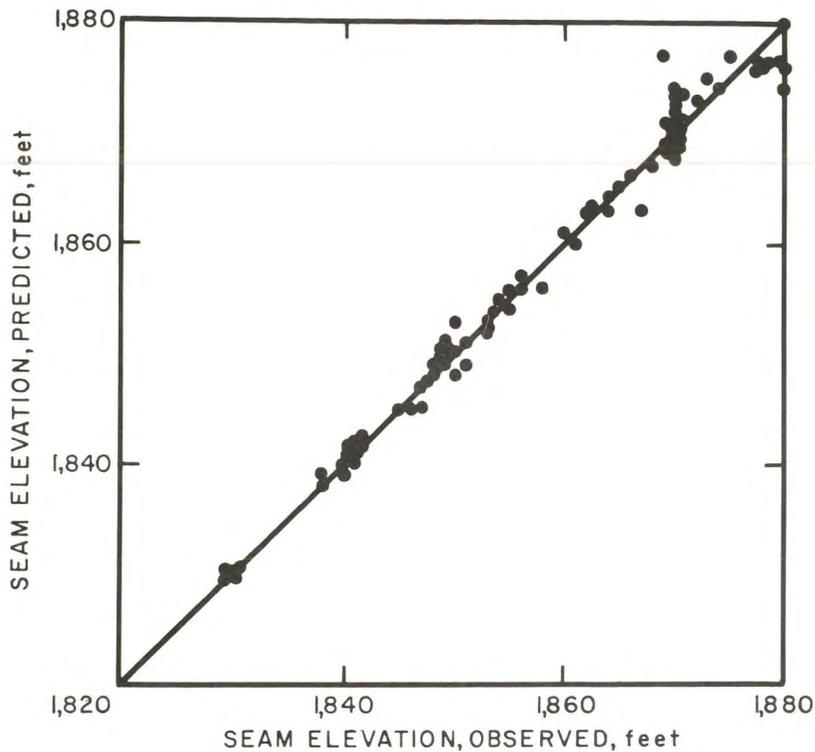


FIGURE 2. - Predicted and Observed Seam Elevation.

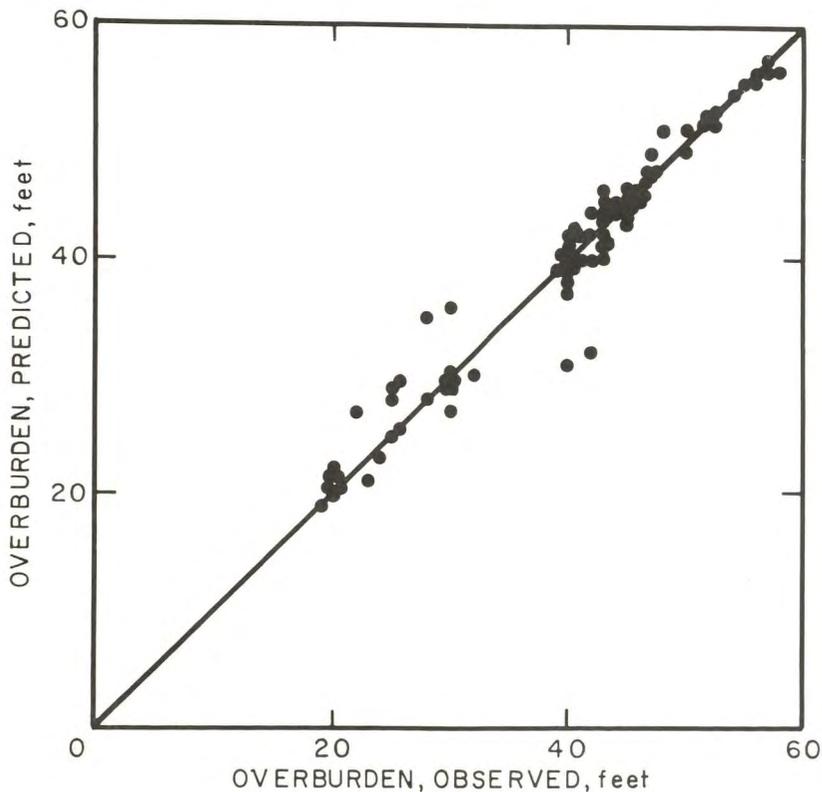


FIGURE 3. - Predicted and Observed Overburden.

Evaluation of Prediction Models

The locations of the 11 samples used for model evaluation are shown in figure 4. Observed and predicted values for seam elevation, overburden, and sodium concentration are presented in table 3.

Predicted values for seam elevation and overburden are in excellent agreement with corresponding observed values. This is the expected situation since the models explain 98.5 percent of the variance for elevation and 95.5 percent of the variance for overburden.

Predicted values for sodium are in good agreement with observed values with the exception of sample V-II-7. This sample point lies on the periphery of the cored area (see fig. 4) and, quite likely, is outside the range of the prediction model.

Prediction of Sodium Between Drill Holes

The model developed for sodium concentration represents a surface that is valid within the cored area. This surface can be utilized to predict within the range of data used to construct the model and to establish a regular pattern of predicted values over the

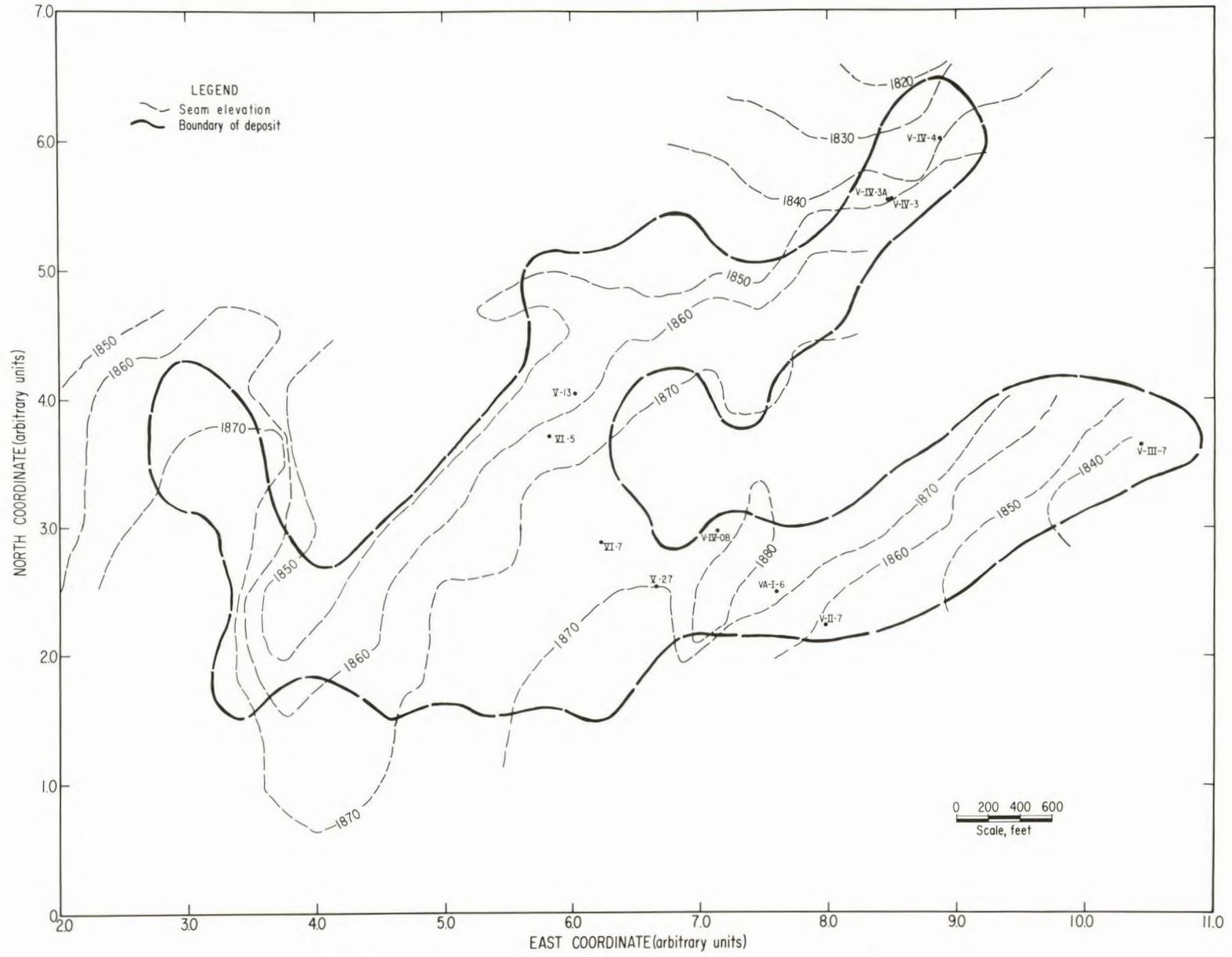


FIGURE 4. - Location of Samples Used for Model Evaluation.

cored area. The regular grid pattern can then be used to construct a contour map representative of sodium concentration. Data in figure 5 show the distribution of sodium in the present area of interest. The predicted values at regular grid points and the contours shown are computer output.

TABLE 3. - Evaluation of prediction models for elevation, overburden, and sodium concentration

Sample number	Surface coordinates (arbitrary units)		Seam elevation, feet		Overburden, feet		Sodium in lignite ash, percent		
	North	East	Observed	Pre-dicted	Observed	Pre-dicted	Observed	Pre-dicted	Residual
VA-I-6	2.49	7.61	1873	1874	22	28	0.5	2.9	-2.4
V-II-7	2.23	7.99	1859	1861	48	49	4.3	0.0	4.3
V-III-7	3.65	10.47	1839	1840	45	48	6.4	6.1	0.3
V-IV-0B	2.98	7.15	1878	1876	20	21	0.4	0.7	-0.3
V-IV-3A	5.56	8.51	1849	1850	30	29	8.6	8.2	0.4
V-IV-3	5.56	8.51	1849	1850	30	29	9.2	8.2	1.0
V-IV-4	6.03	8.89	1840	1840	40	41	10.2	9.7	0.5
V-13	4.05	6.03	1858	1858	47	47	4.4	5.2	-0.8
V-27	2.53	6.67	1870	1875	40	33	0.4	1.6	-1.2
VI-5	3.72	5.83	1864	1863	44	47	3.0	3.9	-0.9
VI-7	2.89	6.23	1870	1869	40	43	6.2	5.2	1.0

The results indicate that areas of high sodium concentration are scattered throughout the deposit. A more extensive study is necessary to determine whether these high sodium areas are randomly distributed in the deposit or whether they exhibit a predictable pattern. The properties of the overburden, such as permeability, and seam characteristics, such as thickness and number of clay or rock partings, need to be observed and measured to establish the effect of geologic factors on sodium distribution.

CONCLUSIONS

The following trends and conclusions are supported by the data:

1. Statistical-mathematical models for seam elevation, overburden, and sodium concentration were developed using surface coordinates and transformations of surface coordinates as input to the models.
2. The models developed permit interpolation of sodium values between drill holes for data from the one mine tested.
3. In the deposit examined, the modeling technique described may be used to forecast sodium concentration along the seam in advance of mining.
4. The data presented suggest that sodium concentration is dependent on the depositional history of the lignite.

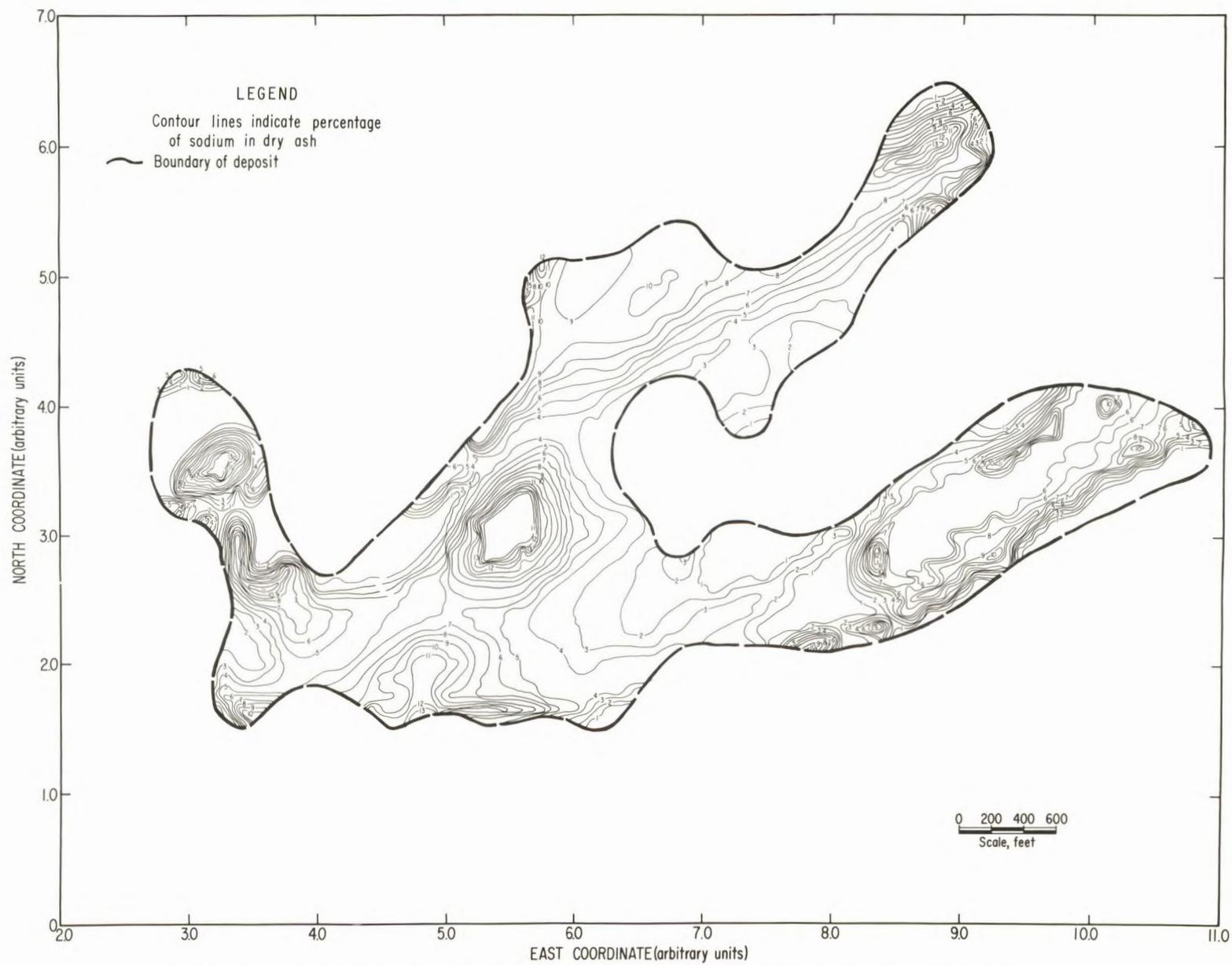


FIGURE 5. - Isogram of Sodium Concentration Developed From Response Surface Regression Model.

RECOMMENDATIONS

Use of this method by the lignite mining industry will be motivated by their interest in obtaining a maximum amount of information from a minimum number of core holes drilled to survey coal quality ahead of stripping and mining. While present results based on data from one mine are encouraging, adoption of the method by the industry will depend on a further successful demonstration of its practicality at a mine site where blending of coal is practiced to average out high sodium concentrations. Therefore, it is recommended that a further study be performed at a mine where the method can most feasibly be incorporated into mining practice.

APPENDIX.--PREDICTION MODELS¹

$$\begin{aligned}
\text{Seam elevation} = & 1770.0262 + 80.280350 (\text{NO}) + 367.57998 \left(e^{-\text{NO}} \times \sin(\text{EA}) \right) \\
& 1.7677966 \left(e^{\text{NO}} - e^{-\text{NO}} \right) + 23.866313 \left(\sin^3(\text{NO}) \times \cos^2(\text{EA}) \right) \\
& - 10.205978 \left(\sin^3(\text{EA}) \times \cos^2(\text{NO}) \right) + 112.78484 \left(\sin(\sin(\text{NO})) \times \right. \\
& \left. \cos(\cos(\text{EA})) \right) - 41.348262 \left(e^{\left(\sin^3(\text{NO} - \text{EA}) \times \cos^2(\text{EA} - \text{NO}) \right)} \right) \\
& + .50556539 \left(\cos(\sin(\text{NO})) \times \sin(\sin(\text{EA})) \times (\text{NO} + \text{EA})^3 \right) \\
& - 2.8713928 \left(\sin(\cos(\text{NO})) \times \cos(\sin(\text{EA})) \times \text{EA}^3 / \text{NO}^2 \right) \\
& - 4.790246 \left((\cos(\cos(\text{NO})) \times \text{EA}^2) + (\sin(\sin(\text{EA})) \times \text{NO}^2) \right) \\
& + 1.3667624 \left(\left(e^{\text{NO}} - e^{-\text{NO}} \right) \times \sin^3(\text{EA}) \times \cos^2(\text{EA}) \right) \\
& - 5.0384723 \left(\sin(\sin(\text{NO} - \text{EA})) \times \text{EA}^3 / \text{NO}^2 \right) - 105.28178 \\
& \left(e^{\cos(\cos(\text{EA}))} \times \text{NO} / e^{\text{NO}} \right) + .021484841 \left(\text{NO}^2 \times \left(e^{\text{EA}} - e^{-\text{EA}} \right) \right) \\
& - 1.7664006 \left(\left(e^{\text{EA}} - e^{-\text{EA}} \right) \times \sin^3(\text{NO}) \times \cos^2(\text{NO}) \right) \\
& - 1.0925511 \left(\left(e^{\text{NO}} - e^{-\text{NO}} + e^{\text{EA}} - e^{-\text{EA}} \right) \times \sin(\sin(\text{NO})) \times \right. \\
& \left. \cos(\cos(\text{EA})) \right) + 32177153 \left(e^{\sin(\text{NO})} \times \left(e^{\text{EA}} - e^{-\text{EA}} \right) \right) \\
& - 4.7467927 \left(\text{NO}^2 \times \sin(\text{EA}) \right) - 139.60904 \left((\text{EA} - \text{NO})^2 \times \right. \\
& \left. \text{EA} / e^{\text{EA}} \times \text{NO} / e^{\text{NO}} \right) - 14.163283 \left(\sin^4(\text{EA}) \times \cos^5(\text{NO}) \right) \\
& + 5.5634048 \left(\sin^8(\text{EA}) \times \cos^9(\text{NO}) \right) - .26657367 \left(e^{\sin(\text{EA})} \times \right. \\
& \left. (\text{EA} - \text{NO})^4 \right) - 92350512 \left(e^{\sin(\text{EA})} \times (\text{NO} + (\text{EA})^2) \right) - 93024008 \\
& \left((\text{EA} - \text{NO})^3 \times \sin(\sin(\text{NO} + \text{EA})) \right) + 2.2670771 \left((\text{EA} - \text{NO})^2 \times \right. \\
& \left. (\text{NO} + \text{EA})^2 \times \text{EA} / e^{\text{EA}} \right)
\end{aligned}$$

¹Variables identified at the end of the appendix.

$$\begin{aligned}
\text{Overburden} = & 70.397455 + .059286964 \left(\text{EA} \times \text{NO} / \text{PEL}^2 \right)^2 + 9.3019249 \times 10^{-8} \\
& \left(\text{PEL} \times \text{NO} \times \text{EA} \right)^5 + .028889831 \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \times \left(\text{EA} - \text{NO} \right)^5 \right) \\
& + 1.1872666 \left(\text{PEL} \times \text{EA} / \text{NO}^2 \right)^2 - .17125392 \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \right. \\
& \times e^{\sin(\text{NO})} \times \cos(\cos(\text{EA})) \left. \right) + 2.2796710 \left(\cos(\cos(\text{EA})) + \text{EA}^2 \right) \\
& \times \cos(\cos(\text{NO})) \times \text{PEL}^3 \left. \right) + .051547641 \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \times \right. \\
& \left. \left(e^{\text{NO}} - e^{-\text{NO}} \right) \right) + 836.37939 \left(\left(\text{PEL} / \left(e^{\text{PEL}} \times \text{EA}^2 \right) \right) + \left(\text{NO} / \left(e^{\text{NO}} \times \text{PEL}^2 \right) \right) \right) \\
& - 20.588829 \left(\cos(\text{PEL}) \times \cos(\text{EA}) \times \cos(\text{NO}) \times \cos(\text{EA} + \text{NO}) \right) \\
& - .46385918 \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \times \text{EA}^4 / \left(\text{NO} \times \text{PEL}^2 \right) \right) + 32.855872 \\
& \left(\sin(\text{PEL}) \times \sin(\text{EA}) \times \sin(\text{NO}) \times \sin(\text{EA} - \text{NO}) \right) - 1.3990489 \times 10^{-3} \\
& \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \times e^{\sin(\text{NO})} \left(\text{EA} - \text{NO} \right)^3 \right) + 2.0352658 \\
& \left(\sin(\sin(\text{PEL})) \times \left(\text{NO} / \text{EA} \right)^3 \right) - 1.7080433 \times 10^3 \left(\left(\text{PEL} / e^{\text{PEL}} \times \right. \right. \\
& \left. \left. \text{NO} / e^{\text{NO}} \right) + \left(\text{PEL} / e^{\text{PEL}} \times \text{EA} / e^{\text{EA}} \right) \right) + 28.960629 \left(\text{EA}^2 \times \text{PEL} / e^{\text{PEL}} \times \right. \\
& \left. \text{NO} / e^{\text{NO}} \right) - 58.667782 \left(\text{NO}^2 \times \text{PEL} / e^{\text{PEL}} \times \text{EA} / e^{\text{EA}} \right) + 53.572099 \\
& \left(\sin(\text{PEL}) \times \sin(\text{NO}) \times e^{\cos(\text{EA})} \right) + 5.2006331 \left(\cos(\text{PEL}) \times \right. \\
& \left. \cos(\text{EA}) \times e^{\sin(\text{NO})} \right) + 13.009927 \left(\left(\text{PEL}^2 + \text{NO}^2 \right) \times \text{EA} / e^{\text{EA}} \right) \\
& - .041866517 \left(\left(\text{PEL} + \text{PEL}^2 + \text{PEL}^3 \right) \times \left(\text{NO} + \text{NO}^2 + \text{NO}^3 \right) \right) \\
& + 2.6605247 \left(\sin(\sin(\text{PEL})) \times \sin(\sin(\text{EA} \times \text{NO})) \right) - 4.3070305 \\
& \times 10^{-3} \left(\left(\text{EA} + \text{EA}^2 + \text{EA}^3 \right) \times \left(\text{NO} + \text{NO}^2 + \text{NO}^3 \right) \right) + 6.2554103 \times 10^{-4} \\
& \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \times \left(\text{EA} + \text{NO} \right)^3 \right) - 1.4098875 \left(\cos(\cos(\text{EA})) \times \text{PEL}^3 \right) \\
& + 74.503737 \left(\sin(\sin(\text{PEL})) \times \cos(\cos(\text{NO})) \right) - 7.6204306 \\
& \left(\sin(\sin(\text{PEL})) \times \sin(\sin(\text{EA} - \text{NO})) \right) + 5.6729621 \left(\sin(\sin(\text{PEL} \times \text{NO})) \right) \\
& + \cos(\cos(\text{PEL} \times \text{EA})) \left. \right) + .055643694 \left(\left(e^{\text{EA}} - e^{-\text{EA}} \right) \times \sin(\sin(\text{PEL} \right. \\
& \left. \times \text{NO})) \right)
\end{aligned}$$

$$\begin{aligned}
\text{Sodium} = & 33.878973 - .016302009 \left(e^{\cos(\cos(\text{POV}))} \times (\text{EA}/\text{NO})^4 \right. \\
& \left. \times \sin(\text{PEL})/e^{\sin(\text{PEL})} \right) + .029457627 \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \times \right. \\
& \left. \left(\text{POV}/e^{\text{POV}} + \text{EA}/e^{\text{EA}} + \text{NO}/e^{\text{NO}} + (\text{EA} - \text{NO})/e^{(\text{EA} - \text{NO})} \right) \right) \\
& - .43243501 \left(\sin(\sin(\text{PEL})) \times (\text{EA}^3 + \text{NO}^3 + \text{POV}^3) \right) - .51895499 \\
& \left(\cos(\cos(\text{POV})) \times (\text{EA}^2 + \text{NO}^2) \times \sin(\text{PEL})/e^{\sin(\text{PEL})} \right) + 8.5184382 \\
& \times 10^{-3} \left(\sin(\sin(\text{PEL}))/e^{\sin(\sin(\text{PEL}))} \times \left(e^{\text{POV}} - e^{-\text{POV}} \right) \times \right. \\
& \left. \left(e^{\text{EA}} - e^{-\text{EA}} \right) \times \sin(\sin(\text{EA} - \text{NO}))/e^{\sin(\sin(\text{EA} - \text{NO}))} \right) + .30933872 \\
& \left(\text{POV}^3 \times (\sin(\sin(\text{EA})) + \cos(\cos(\text{NO}))) \times \sin(\sin(\text{PEL}))/e^{\sin(\sin(\text{PEL}))} \right) \\
& - 2.3649368 \left(\sin(\sin(\text{PEL}))/e^{\sin(\sin(\text{PEL}))} \times \text{POV}/e^{\text{POV}} \times \right. \\
& \left. \cos(\sin(\text{EA}))/e^{\cos(\sin(\text{EA}))} \times \left(e^{\text{NO}} - e^{-\text{NO}} \right) \right) - 1.5394881 \\
& \left(\sin(\sin(\text{EA})) \times (\text{POV}^2 + \text{NO}^2) \times \sin(\sin(\text{PEL}))/e^{\sin(\sin(\text{PEL}))} \right) \\
& + 26.967714 \left(\text{EA} \times \text{PEL}/e^{\text{PEL}} \right) + 66.917269 \left((\cos(\cos(\text{POV}))) + \right. \\
& \left. \sin(\sin(\text{EA})) + \cos(\cos(\text{NO})) \right) \times \text{PEL}/e^{\text{PEL}} - 11.113891 \left(\left(e^{\sin(\text{EA} - \text{NO})} \right. \right. \\
& \left. \left. + e^{\cos(\text{NO} - \text{EA})} \right) \times \sin(\sin(\text{PEL})) \right) - 2.8731643 \left(\text{PEL}/e^{\text{PEL}} \times \right. \\
& \left. \left(e^{\text{POV}} - e^{-\text{POV}} \right) \times \cos(\sin(\text{EA} - \text{NO}))/e^{\cos(\sin(\text{EA} - \text{NO}))} \right) - 6.6038558 \\
& \times 10^{-3} \left(\sin(\sin(\text{NO})) \times \cos(\cos(\text{EA})) \times \text{POV}^3 \times \text{PEL}^3 \right) + .095916123 \\
& \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \times \cos(\cos(\text{POV})) \times \sin(\sin(\text{EA})) \times \cos(\cos(\text{NO})) \right) \\
& - .052215692 \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \times (\text{EA}^2 + \text{NO}^2 + \text{POV}^2) \right) - .073615777 \\
& \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \times (\cos(\sin(\text{EA})) + \sin(\cos(\text{NO})) + \sin(\cos(\text{POV}))) \right) \\
& + .081877397 \left(\left(e^{\text{PEL}} - e^{-\text{PEL}} \right) \times (\text{POV}^3 + (\text{EA} - \text{NO})^3 + (\text{NO} - \text{EA})^3) \right) \\
& - .27610256 \left((\text{POV}^3 + \text{EA}^3) \times \sin(\sin(\text{PEL}))/e^{\sin(\sin(\text{PEL}))} \right) \\
& + 2.8682649 \left(\cos^2(\text{EA} - \text{NO}) \times \sin^3(\text{PEL} + \text{POV}) \right) + 83717464
\end{aligned}$$

$$\begin{aligned}
& \left(\left(e^{\text{PEL} - \text{PEL}} \right) \times \text{POV} / e^{\text{POV}} \right) + 57.919337 \left(\sin(\sin(\text{POV})) \times \right. \\
& \left. \cos(\cos(\text{EA} - \text{NO})) \times \text{PEL} / e^{\text{PEL}} \right) - 20.927646 \left(\sin(\sin(\text{PEL})) \right. \\
& \left. \times \text{POV} / e^{\text{POV}} \times \left(e^{(\text{EA} - \text{NO})} - e^{-(\text{EA} - \text{NO})} \right) \times \sin(\sin(\text{EA} - \text{NO})) \right. \\
& \left. / e^{\sin(\sin(\text{EA} - \text{NO}))} \right) + 3.3791684 \times 10^{-3} \left(\sin(\sin(\text{PEL})) \times \sin^2(\sin(\text{EA})) \right. \\
& \left. \times \text{POV}^3 \times \text{NO}^3 \right) + 3432.8647 \left(\text{PEL} / e^{\text{PEL}} \times \text{EA} / e^{\text{EA}} \times \text{NO} / e^{\text{NO}} \right) + 3.6213802 \\
& \left(e^{\sin(\sin(\text{PEL}))} \times e^{\sin(\cos(\text{POV}))} \times e^{\cos(\sin(\text{EA} - \text{NO}))} \right) - .056103501 \\
& \left((\text{EA} \times \text{NO})^3 + (\text{PEL} \times \text{POV})^3 \right) + 71.452858 \left(\sin(\sin(\text{PEL})) \right) \\
& - .087937395 (\text{EA}^4) + 2.9125319 (\text{EA}^2) + .54460493 \left(\cos(\text{PEL}) \times \right. \\
& \left. \cos(\text{EA}) \times \text{NO}^2 \right) + .31147194 \left(\cos(\text{POV}) \times \cos(\text{NO}) \times \text{EA}^2 \right) \\
& + 0.55837163 (\text{EA} \times \text{NO})^3 - 248.82929 \left(\cos(\sin(\text{PEL})) / e^{\cos(\sin(\text{PEL}))} \right) \\
& \left. \times \text{POV} / e^{\text{POV}} \times \cos(\sin(\text{EA} - \text{NO})) / e^{\cos(\sin(\text{EA} - \text{NO}))} \right) - .69427436 \\
& \left(\cos(\text{POV}) \times \cos(\text{EA}) \times \text{NO}^2 \right) + 2.890383 \left(\cos(\text{POV}) \times \cos(\text{PEL}) \times \right. \\
& \left. (\text{NO} / \text{EA})^2 \right) + 5.2602596 \times 10^{-5} \left(\left(e^{\text{PEL} - \text{PEL}} \right) \times \left(e^{\text{POV} - \text{POV}} \right) \times \right. \\
& \left. \left(e^{\text{EA} - \text{EA}} \right) \times \left(e^{\text{NO} - \text{NO}} \right) \right)
\end{aligned}$$

Variables:

- EA = East coordinate, arbitrary.
- NO = North coordinate, arbitrary.
- PEL = Predicted seam elevation.
- POV = Predicted overburden.