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TUNNEL BORING TECHNOLOGY

Disk Cutter Experiments in Sedimentary and Metamorphic Rocks

UNITED STATES DEPARTMENT OF THE INTERIOR

BUREAU OF MINES

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TUNNEL BORING TECHNOLOGY

Disk Cutter Experiments in Sedimentary and Metamorphic Rocks

by

Roger J. Morrell,[1] William E. Bruce,[2] and David A. Larson[3]

ABSTRACT

Disk-cutter experiments were performed on five rock types ranging in compressive strength from 9,000 psi to 27,000 psi. A specially constructed testing machine called a linear-cutter apparatus (LCA) was designed to load and traverse a free-rolling disk cutter across a sawed rock surface. The LCA was instrumented to measure the vertical and horizontal forces acting on the cutter during the run.

The ability of disk cutters to fragment rock was determined for both 60-degree and 90-degree cutting-edge angles, and relationships and regression equations were developed to predict cutter performance based on rock physical properties and applied forces.

INTRODUCTION

The greatly increased demand for faster and cheaper methods of constructing underground openings, such as tunnels and shafts, has strained present-day excavation technology. The National Academy of Sciences, Committee on Rapid Excavation, reports[4] that to a considerable extent the growing resource and urban development demands would be satisfied "if greatly improved underground-excavation technology were available, i.e., if the real cost of underground excavation were reduced 30 to 50 percent, and if the sustained rate of advance were increased 200 to 300 percent in both soft-medium and hard rock."

The committee also stated that, to a considerable extent, rock disintegration is the most important aspect in the process of excavation. It

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[4]National Academy of Sciences. Rapid Excavation: Significance, Needs, Opportunities. Washington, D.C., 1968, 48 pp.

represents a major cost factor, involving one-third to one-half of total excavation costs, and the rate of excavation basically is the factor that ultimately limits the rate of advance. Recognizing this fact, the Bureau of Mines has initiated a program to study this critical area of rapid excavation. This is the first in a proposed series of reports which will deal with the rock disintegration process in rapid excavation.

The first objective was to study the rock disintegration characteristics of disk cutters. This part of the investigation was limited to a study of two independent variables: the vertical force on the disk cutter, and the cutting edge angle of the disk cutter. The rock disintegration characteristics of disk cutters were determined by defining the relationships between the independent variables and such dependent variables as the horizontal force on the cutter and the dimensions of the craters produced.

The second objective was to predict the performance of a disk cutter in a particular rock, given the forces involved and physical properties of that rock. This objective was met by finding some unique relationship between a particular rock property or properties and the quantity to be found such as the depth of penetration of a disk cutter, the horizontal force acting on a disk cutter, or the volume of the craters produced. In a given rock the volume of crater produced was predicted on the basis of energy input.

These experiments were limited to a study of disk cutters, traversed in a linear path across the surface of a rock. The craters were formed by a single pass of the disk with no indexing effects between craters.

The experiments were performed on five rocks: three varieties of limestone, one dolomite, and one marble. The physical properties of these rocks are tabulated later in this report. All the rocks tested were fine-grained, well-cemented, and with no visible fractures or partings along the bedding. In all cases except Indiana limestone, type 1, the disk cutter was loaded onto the rock surface, perpendicular to the bedding.

Although the literature on previous experimental work on disk cutters was limited, some related work on milled-tooth or gear-type cutters was available. Peterson[5] used a milling table apparatus to roll a gear cutter in a straight line over a rock surface while measuring the three components of force acting on the cutter. In this work the depth of cut was predetermined and the three components of force acting on the cutter were measured as dependent variables. The cutter was indexed after each run so that several complete layers of the rock surface were removed during the experiments. The principal results of Peterson's work include (1) determination of the ratio of the three component forces acting on the cutter, (2) the conclusion that dulling of the cutter caused little variation in the force requirement, and (3) the conclusion that the normal force requirement is reduced when the cutter is skewed.

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[5] Peterson, Carl R. Rolling-Cutter Forces. Proc. 4th Conf., Drilling and Rock Mechanics, Austin, Tex., AIME Paper No. SPE 2393, Jan. 14-15, 1969, 10 pp.

Despite substantial differences between the Bureau's work and Peterson's, the results show some similarity. In both investigations crater depth was found to vary approximately as a linear function of the vertical force on the cutter. The horizontal-vertical force ratio (F_h/F_v) increased as the vertical force increased and specific energies were found to be approximately 60 percent of the compressive strength of the rocks tested.

Teale[6] used an experimental machine similar to the Bureau's in that the vertical and horizontal forces and motion were induced by hydraulic cylinders. The chief difference was that Teale's device moved the rock while the cutter remained stationary. Much of Teale's work was concerned with the effect of gear-cutter-tooth geometry on rock fracture. Some of Teale's results that are comparable to the Bureau's work show general agreement. Up to a limit defined by the geometry of the disk, the depth of penetration is directly proportional to the normal thrust and the volume of crater produced is approximately proportional to the square of the depth of penetration.

Both Peterson and Roxborough[7] found that the simple compressive strength of the rock is not fundamentally related to the rock-breaking mechanism and therefore cannot be used as a reliable method of predicting "machinability." Although Roxborough proposes a special test to determine "machinability," the Bureau work, so far as we know, is the only research which has attempted to use singly or in combination various force parameters and the familiar physical properties of rock to predict disk-cutter performance. The results,[8] which are promising, show that two properties, Shore hardness and density, are good predictors. Although these two properties were most often used, tensile strength is also used in a prediction equation.

EQUIPMENT

Disk Cutters

The cutters used in these experiments were specially designed disks constructed of tool steel, which was heat treated and oil quenched to a hardness of from 58 to 62 Rockwell C. The 7-in-diameter disks were 1-in-thick, and had an included cutting-edge angle of either 60 or 90 degrees around the circumference. In addition, the cutting edge was finished off with a 1/32-in radius to minimize wear. A typical disk cutter is shown in figure 1.

Linear-Cutter Apparatus

A single disk cutter such as described above was mounted on a Bureau designed testing machine called a linear-cutter apparatus (LCA). This

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[6] Teale, R. The Mechanical Excavation of Rock-Experiments with Roller Cutters. Internat. J. Rock Mech. Mining Sci., v. 1, No. 1, January 1964, pp. 63-78.

[7] Roxborough, F. F. Rock Cutting Research for the Design and Operation of Tunnelling Machines. Tunnels and Tunnelling, v. 1, No. 3, September 1969, pp. 125-128.

[8] Bruce, William E., and Roger J. Morrell. Principles of Rock Cutting Applied to Mechanical Boring Machines. Proc. 2d Symp. on Rapid Excavation, Sacramento State College, Sacramento, Calif., Oct. 16-17, 1969, pp. 3-1 to 3-43.

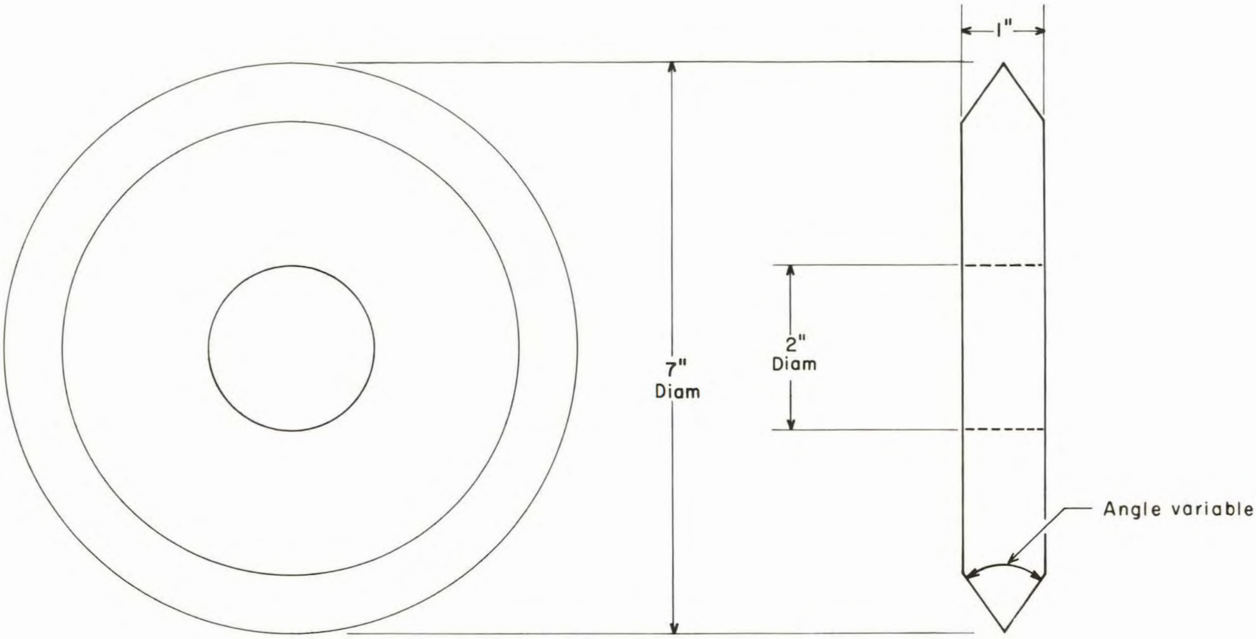

FIGURE 1. - Simple Disk Cutter.

apparatus was designed to load and traverse a rolling disk across the surface of a rock and to measure the forces acting on the cutter as a function of the distance traveled during the run. The vertical load and horizontal motion of the disk were provided by the pistons of two hydraulic cylinders while the forces acting on the disk were measured with strain-gage load transducers. The LCA is shown in figure 2.

The cutter under test was mounted on a shaft supported by needle bearings which allowed the cutter to revolve freely as it moved across the rock surface. The shaft was mounted in a yoke assembly attached to the end of the vertical piston rod. This vertically oriented hydraulic cylinder controlled the up-down movement of the cutter and provided the vertical force on the cutter. The vertical cylinder and yoke assembly were, in turn, attached by a trunnion mount to a carriage with eight wheels (fig. 2) which moved horizontally on rails above the rock sample. The carriage was driven along the rails by the piston of a horizontally mounted hydraulic cylinder, whose movement caused the disk to traverse the surface of the rock.

The vertical and horizontal pistons were controlled by four-way manual control valves. In addition, the hydraulic circuit for the vertical cylinder was equipped with a pressure accumulator and check valve to damp out large pressure fluctuations during the tests. The horizontal cylinder was equipped with a flow-control valve which regulated the speed of the carriage.

Although the circuit pressures in the LCA were monitored visually during operation, the actual forces acting on the cutter were measured electrically with strain-gage load transducers and simultaneously recorded on two X-Y recorders. The vertical force on the cutter was measured by strain gages

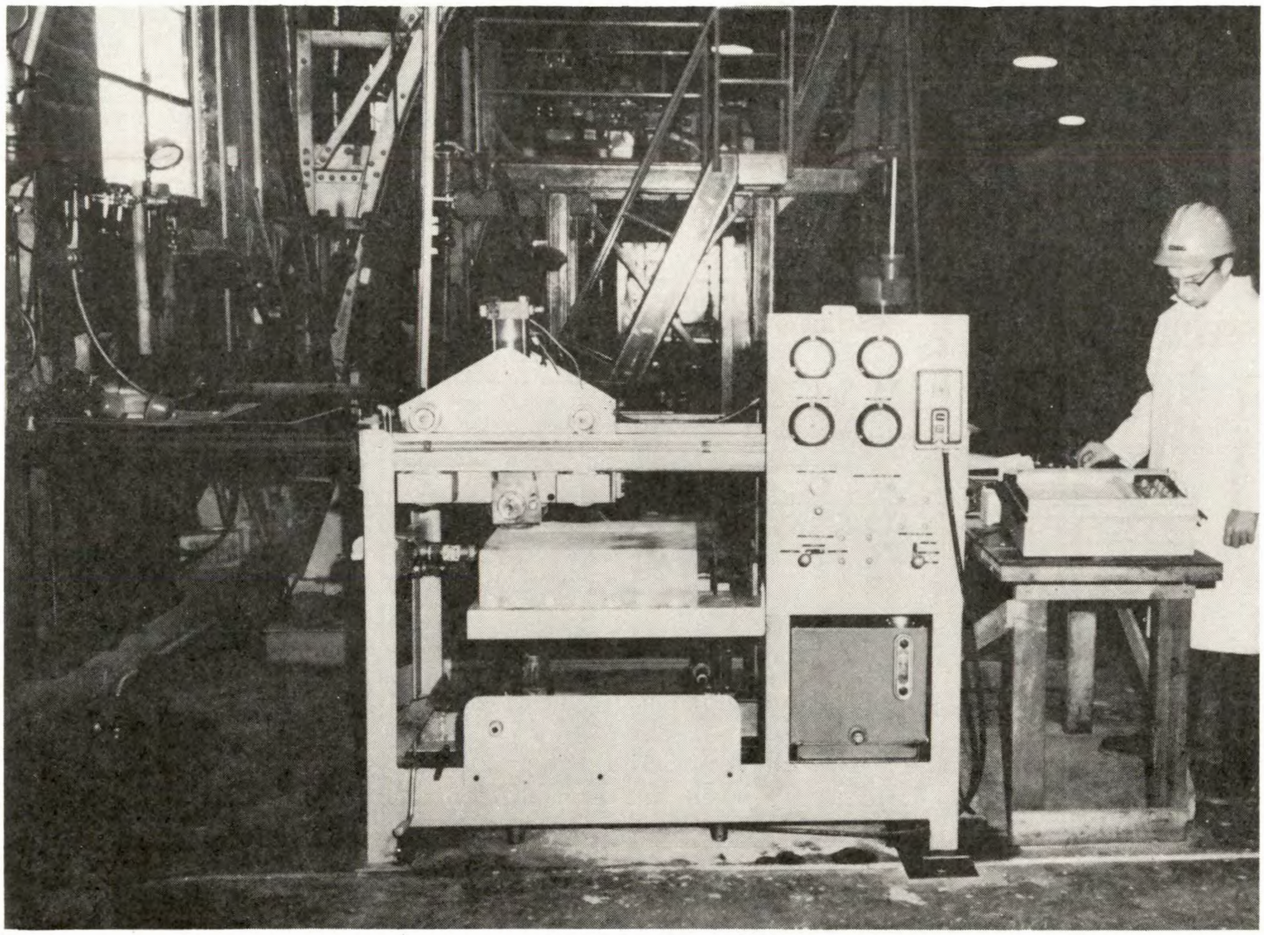

FIGURE 2. - Linear-Cutter Apparatus.

mounted on the piston rod of the vertically oriented cylinder, and the horizontal force by strain gages mounted on a horizontally oriented yoke. The load-transducer instrumentation was statically calibrated for both horizontal and vertical transducers. Since the yoke and the trunnions connect the cutter-shaft assembly to the carriage, the net horizontal force on the cutter could be measured independently of the gross force delivered by the horizontal piston to the entire carriage. The X-input to both recorders was obtained from a 10-turn potentiometer which measured the horizontal distance traveled by the cutter. A schematic of the instrumentation used in these experiments is shown in figure 3.

PROCEDURE

Operation

The rock specimen under test was placed on the rock platform located directly under the disk cutter. The height of the platform was adjusted so that when the disk was loaded down and penetrated into the rock, the yoke attached to the cutter shaft assembly would be level. This procedure insured that the load transducer mounted on the yoke would measure only the horizontal

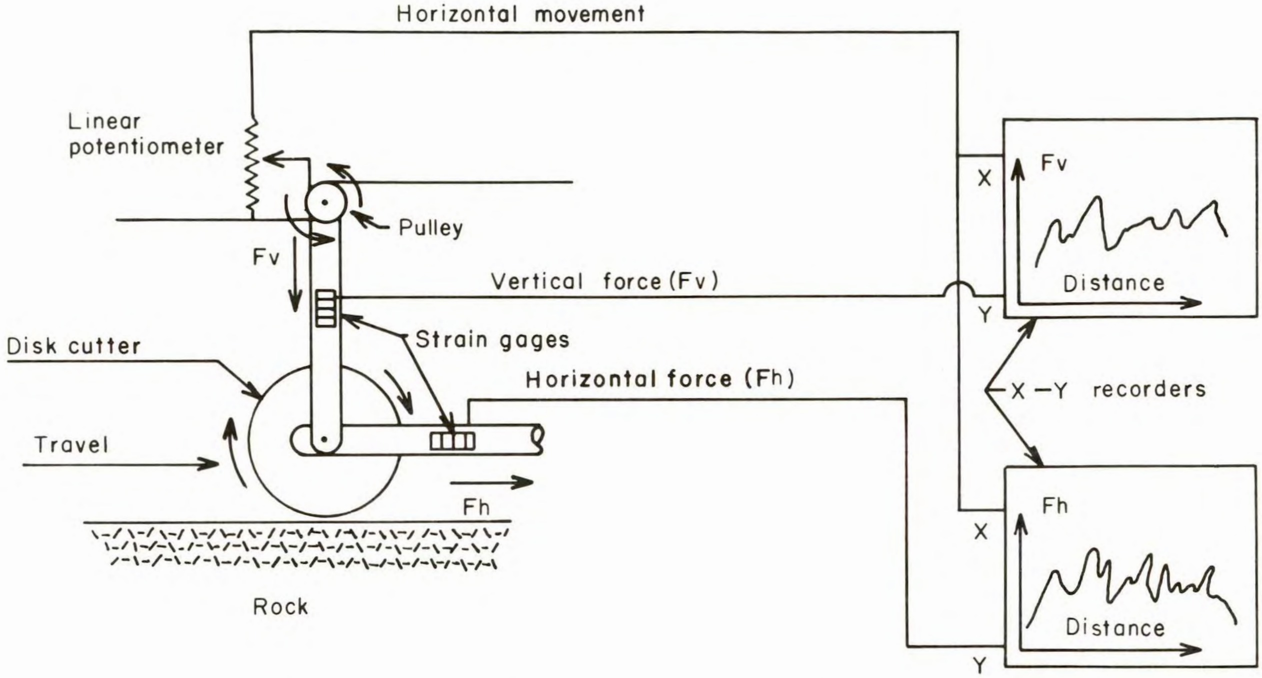

FIGURE 3. - Instrumentation Schematic for Linear-Cutter Apparatus.

force component acting on the cutter. The rock was then laterally positioned to avoid interference (indexing) with craters from previous runs, and locked in place.

The cutter was loaded near one end of the rock and the vertical force adjusted to the desired level. With the vertical load preset, the horizontal cylinder was activated and the cutter traversed the rock. The horizontal velocity was held constant at 3 ips during all of these experiments. At the end of the run, usually about 20 in, the disk was unloaded.

Collection of Data

The traces of the vertical and horizontal forces recorded during the run were later analyzed to obtain the average forces acting on the cutter. This was accomplished by measuring the area under the force curves with a planimeter and dividing this area by the distance traveled by the cutter. The balance of the raw data was obtained from crater measurements. The width and depth of the crater, measured with a scale and micrometer probe, respectively, were taken at the same 2-in intervals along the length of the crater. The volume of the crater was calculated by dividing the weight of the chips created during the run by the density of the rock. Physical properties of all rocks utilized are presented in table 1.

TABLE 1. - Physical properties of rocks tested

Geologic name	Salem Limestone	Salem Limestone	Oneota Member, Prairie du Chien Formation	Holston Limestone	Cordell Dolomite Member, Manistique Formation
Commercial name	Indiana limestone, type 2	Indiana limestone, type 1	Kasota stone	Tennessee marble	Valders white rock
Locality	Bedford, Ind.	Bedford, Ind.	Kasota, Minn.	Knoxville, Tenn.	Valders, Wis.
Compressive strength.......psi	9,126	9,991	13,184	16,809	27,230
Tensile strength...........psi	679	502	792	1,219	793
Shore hardness scleroscope units	27	32	37	55	68
Apparent density.....slugs/ft^3	4.455	4.635	4.818	5.186	5.056
Apparent density.......g/cu cm	2.302	2.395	2.487	2.681	2.613
Static Young's modulus..10^6 psi	3.5	4.4	5.7	9.0	5.7
Longitudinal velocity......fps	14,570	14,610	17,119	20,058	12,815
Bar velocity...............fps	13,062	12,007	14,708	16,845	12,118
Shear velocity.............fps	11,482	8,489	9,360	10,590	8,513
Dynamic Young's modulus 10^6 psi	5.29	4.65	7.42	10.29	5.17
Poisson's ratio...............	0.27	0.33	0.28	0.32	0.20
Shear modulus..........10^6 psi	2.09	2.32	2.90	4.07	2.55

RESULTS OF LINEAR CUTTER EXPERIMENTS WITH DISK CUTTERS

Typical Craters

Figure 4 shows a number of test craters typical of those created during the disk-cutter experiments. These craters were formed by a disk cutter with a 90-degree cutting edge in a test block of limestone. The illustrated craters, approximately 21-in long, 0.3- to 2-in wide, and 0.1- to 0.5-in deep, were formed with vertical loads of from 3,600 to 9,600 lb.

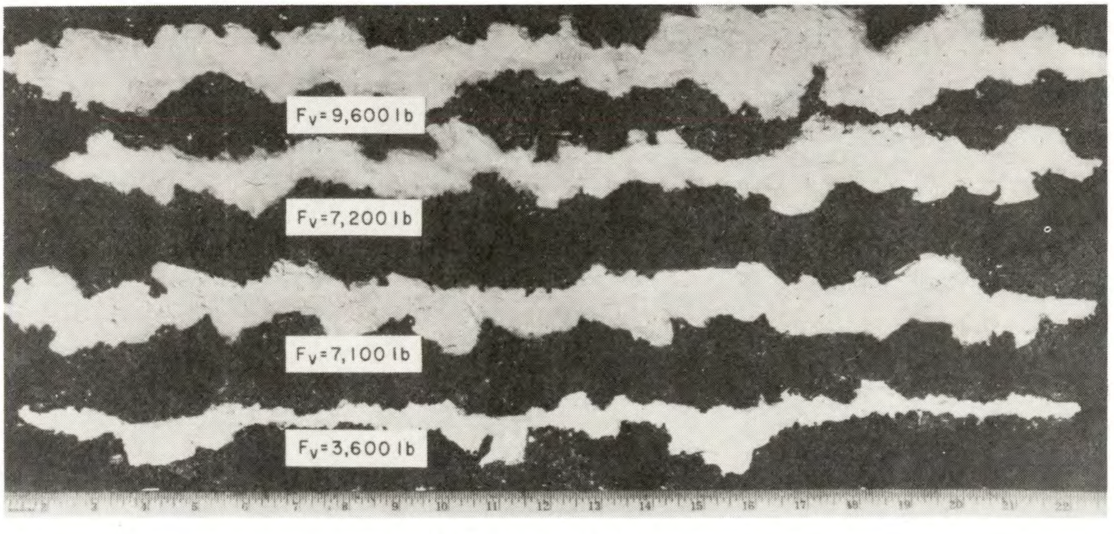

FIGURE 4. - Typical Craters.

Crater Width as a Function of Crater Depth

Crater width was defined as the average width of the crater as calculated from measurements taken at 2-in intervals along its entire length and crater depth as the average depth of the crater as calculated from measurements made at the same 2-in intervals along its entire length.

Figure 5 shows the crater width as a function of crater depth for both the 60-degree and 90-degree disk cutters in Tennessee marble. The crater depth is plotted as the independent variable and crater width as the dependent variable. These curves are typical of those produced for the other four rocks tested in these experiments. The crater depth-crater width relation for the majority of rocks was found to be linear as shown in equation 1:

$$W = W_o + KD, \tag{1}$$

where W = average crater width, in,

W_o = intercept, in,

K = the slope of straight line, in/in,

and D = average crater depth, in, and any subscript refers to the included angle of the cutting edge.

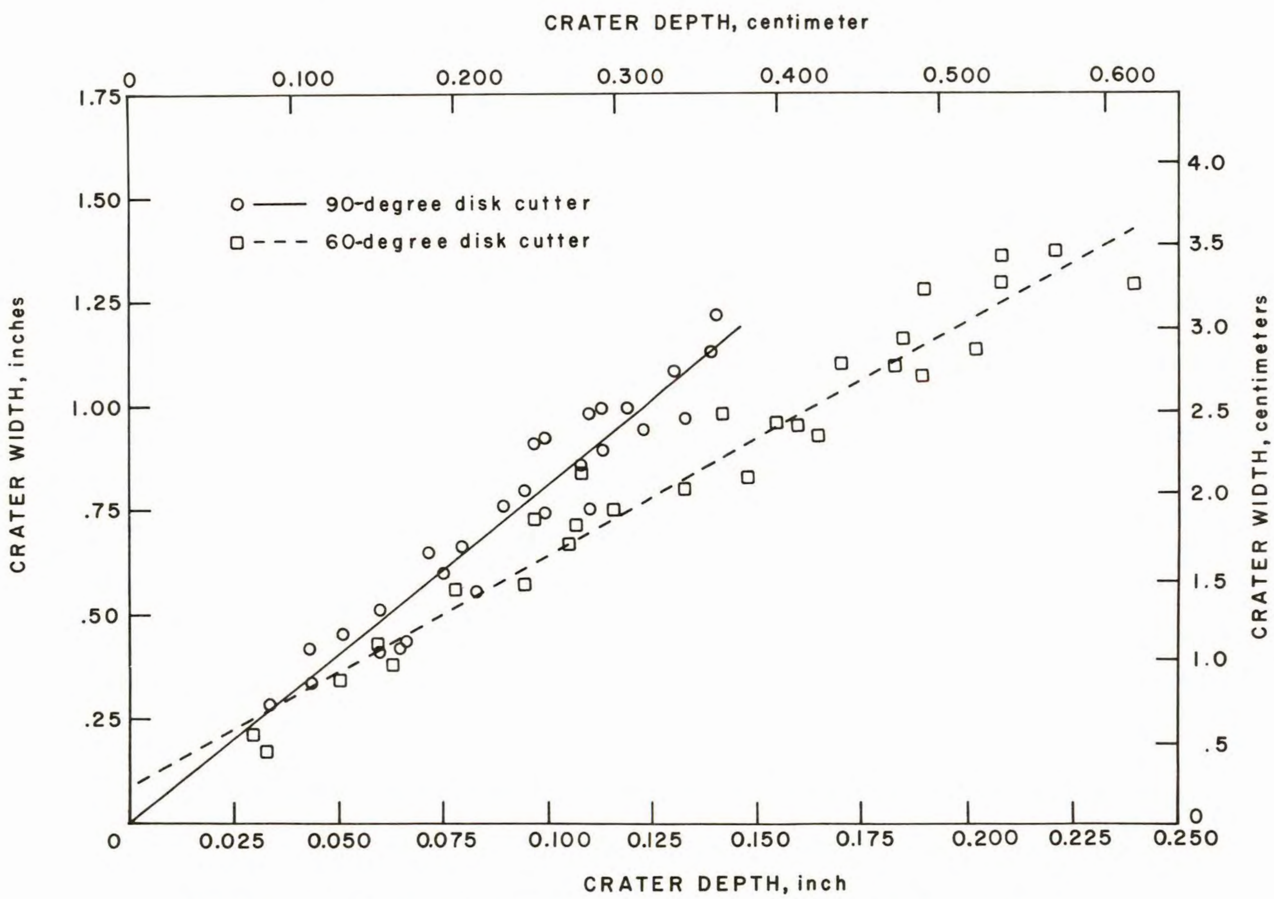

FIGURE 5. - Crater Width-Crater Depth Relation for Tennessee Marble.

The best-fit equations computed from the crater width and depth data for all the rocks tested is shown in table 2. The intercept, W_0, in equation 1, has no physical significance since it is not possible to produce a crater with a zero depth and a finite width or vice versa. Since in most cases the intercepts are small, they can be dropped from the equations without a significant loss of accuracy.

TABLE 2. - Crater width as a function of crater depth

Rock type	Crater width, in, as a function of crater depth, in	S_e[1]
Indiana limestone, type 1...	$W_{60} = 0.010 + 6.312\ D_{60}$ $W_{90} = 0.032 + 8.161\ D_{90}$	0.083 .075
Indiana limestone, type 2...	$W_{60} = 7.953\ D_{60}^{1.109}$ $W_{90} = -0.029 + 8.391\ D_{90}$	.113 .113
Kasota stone................	$W_{60} = 0.167 + 5.588\ D_{60}$ $W_{90} = 8.608\ D_{90}^{1.046}$	.101 .178
Tennessee marble............	$W_{60} = 0.084 + 5.645\ D_{60}$ $W_{90} = -0.017 + 8.262\ D_{90}$	.074 .072
Valders white rock..........	$W_{60} = 0.076 + 6.079\ D_{60}$ $W_{90} = 0.001 + 6.620\ D_{90}$	.143 .068

[1] Standard error of estimate.

The depth-width equations in table 2 show three important results. First, the crater width was, with two exceptions, a linear function of the crater depth for both cutting angles in the rocks tested. The exceptions were the 60-degree cutter in type 2 Indiana limestone and the 90-degree cutter in Kasota stone. These rock-cutter combinations showed the crater width as a function of crater depth raised to a power of 1.11 and 1.05, respectively. Because these power forms occurred in two different rocks with two different cutting edges, they were thought to be due to experimental error and not to a change in the fundamental width-depth relationship. Moreover, the statistical hypothesis that the exponent for Kasota stone was equal to 1.00 could not be rejected at the 95-percent confidence level.

The second observation made from the width-depth equations was that the 90-degree disk produced a wider crater than did the 60-degree disk for the same crater depth. To determine more precisely the effect of cutter-edge angle on crater width, the ratio of the slopes of the best-fit straight lines (table 2) was calculated for each rock type. The intercepts in the original equations were neglected for simplicity.

Then

$$\frac{W_{90}}{W_{60}} = \frac{K_{90} D_{90}}{K_{60} D_{60}}. \qquad (2)$$

For the same value of crater depth for both cutters equation 2 reduces to:

$$\frac{W_{90}}{W_{60}} = \frac{K_{90}}{K_{60}} = C, \tag{3}$$

where C = a constant comparing the effect of cutter angle on crater width.

Using equation 3, the ratios were calculated at 1.30, 1.13, 1.41, 1.44, and 1.08 for types 1 and 2 Indiana limestone, Kasota stone, Tennessee marble, and Valders white rock, respectively. These ratios show that for the same depth of cut, the 90-degree cutter produces a somewhat wider crater than does the 60-degree cutter.

The last observation made from the equations in table 2 is that the crater width also appears to be a function of the rock tested. To give some idea of the change in the craters for the different rocks tested, the crater width varied from a high of 8.3 to a low of 6.6 times the crater depth for the 90-degree cutter; while for the 60-degree cutter the crater width varied from a high of 7.4 to a low of 5.7 times the depth of the crater. These ratios can easily be obtained from the equations in table 2 since the width-to-depth ratio is in general equal to the constant of proportionality given in the linear-form equations. No attempt was made to relate rock physical properties to crater width. However, since crater width and depth are related by the equations in table 2, and knowing crater depth from prediction equation 23 (later in the report), one can calculate crater width for those rocks tested.

At this point the effect of vertical force on the crater width should also be considered. Although it has been noted that a 90-degree cutter would create a wider crater than a 60-degree cutter where both have the same depth of penetration, it will be shown that on the average, a 90-degree cutter requires 1.67 times the vertical load to produce a crater of the same depth than does a 60-degree cutter. In Tennessee marble, for example, with the same vertical force on each cutter, the 60-degree cutter will produce a crater approximately 1.23 times wider than the 90-degree cutter. To calculate this constant for all the rocks, the vertical force-depth relationships in table 4 were used to substitute for depth, D, in the crater depth-crater width equations in table 2.

Then

$$\frac{W_{90}}{W_{60}} = \frac{K_{90}\ (K'_{90}\ F_v)}{K_{60}\ (K'_{60}\ F_v)}, \tag{4}$$

where K' = a constant, in/lb

and F_v = average vertical force on the disk, lb.

For the same vertical force on both cutters equation 4 reduces to:

$$\frac{W_{90}}{W_{60}} = \frac{K_{90}K'_{90}}{K_{60}K'_{60}} = C, \tag{5}$$

where C = a constant representing the effect of cutter angle on crater width for the same F_v on both cutters.

Hence for the same vertical force on both cutters, the widths of the craters produced by a 60-degree cutter were found to be 1.22, 1.33, 1.33, 1.23, and 1.51 times greater than those produced by the 90-degree cutter for type 1 and 2 Indiana limestone, Kasota stone, Tennessee marble, and Valders white rock, respectively. Furthermore, since crater width was a linear function of crater depth and crater depth was a linear function of vertical force, crater width must then be a linear function of vertical force for both cutting edges in all of the rocks tested (equation 4).

These experimental results can now be summarized:

1. Crater width is generally a linear function of crater depth for both cutter angles in all the rocks tested. Crater width is also a linear function of the vertical force on the cutter for both cutters in all the rocks tested.

2. At the same crater depth the 90-degree cutter will produce a slightly wider crater than the 60-degree cutter. For the same vertical force on both cutters however, the 60-degree cutter will produce a wider crater than the 90-degree cutter.

Crater Volume Per Unit Length as a Function of Vertical Force

Crater volume per unit length was chosen as a measure of crater size instead of volume since it was then possible to compare craters of different lengths. The volumes of the craters were calculated by dividing the weight of the chips formed during crater formation by the density of the rock. This volume was then divided by the crater length to yield volume per unit length, with units of cubic inches per foot (in^3/ft) or cubic centimeters per meter (cm^3/m).

Figure 6 shows crater volume per unit length as a function of vertical force on the disk cutter for both cutter edges operating in Tennessee marble. The equations of the curves of all the rocks tested are given in table 3. In all cases, the relation between crater volume per unit length and vertical force was found to follow the form shown in equation 6:

$$V/L = KF_v^{\,x}, \tag{6}$$

where V/L = crater volume per unit length, in^3/ft,

K = a constant, $in^3/ft\ lb^x$,

and x = an exponent which averages 1.83 for all the rocks and cutter edges tested.

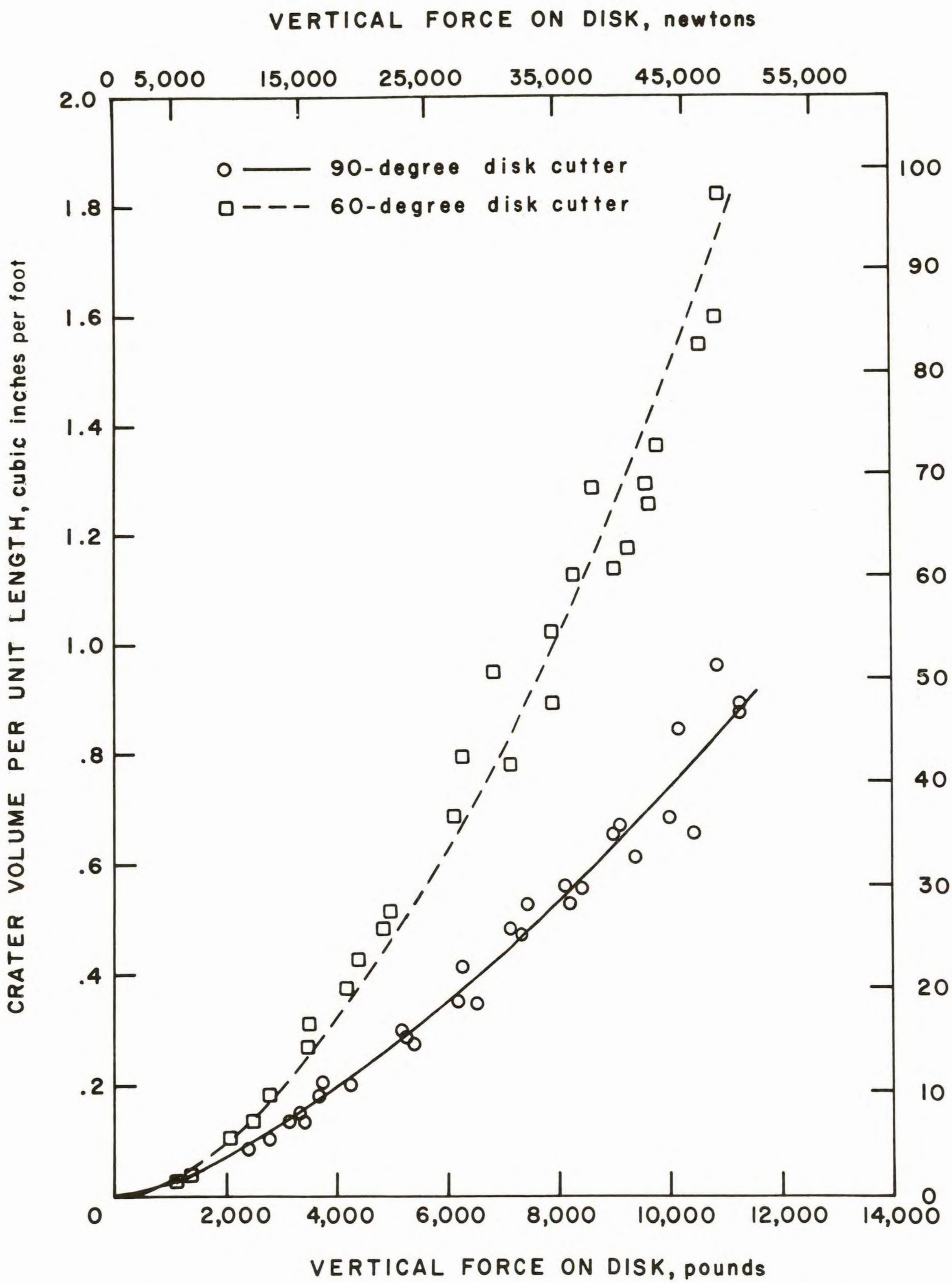

FIGURE 6. - Crater Volume-Vertical Force Relation for Tennessee Marble.

TABLE 3. - Crater volume per unit length as a function of vertical force

Rock type	Crater volume per unit length, in³/ft, as a function of vertical force, lb	S_e[1]
Indiana limestone, type 1......	$V/L_{60} = 5.162 \times 10^{-7}\ F_v^{1.689}$	0.096
	$V/L_{90} = 6.249 \times 10^{-8}\ F_v^{1.872}$	.119
Indiana limestone, type 2......	$V/L_{60} = 5.819 \times 10^{-7}\ F_v^{1.721}$	.422
	$V/L_{90} = 1.422 \times 10^{-7}\ F_v^{1.800}$	.121
Kasota stone..................	$V/L_{60} = 1.357 \times 10^{-8}\ F_v^{2.130}$	.032
	$V/L_{90} = 1.141 \times 10^{-7}\ F_v^{1.778}$	.082
Tennessee marble..............	$V/L_{60} = 2.141 \times 10^{-7}\ F_v^{1.711}$	.092
	$V/L_{90} = 1.039 \times 10^{-6}\ F_v^{1.462}$	.048
Valders white rock............	$V/L_{60} = 3.984 \times 10^{-10}\ F_v^{2.422}$	.110
	$V/L_{90} = 1.193 \times 10^{-7}\ (F_v - 2{,}000)^{1.735}$	.057

[1]Standard error of estimate.

An analysis of the effect of vertical force on crater volume showed that the crater volume per unit length varied with the vertical force on the cutter raised to an average power of 1.83 for both cutter edges in all of the rocks tested. It can be shown theoretically that for a triangular-shaped crater, the crater volume per unit length varies as the square of the vertical load.[9] Hence, the differences in the values of the experimentally determined exponents from 2.0 reflects the deviation of the craters from the ideal triangular cross-sectional shape.

The force-volume equation for Valders white rock, the hardest rock tested, also showed that a vertical load of 2,000 lb was required before the 90-degree disk cutter produced a measurable crater. This is the first instance where the conditions of rock hardness and cutter angle required the application of a threshold force before any measurable penetration occurred.

[9]Since $V = 1/2\ (WDLC)$,

$V/L = 1/2\ (WDC)$,

where V/L = crater volume per unit length, in³/ft,

V = crater volume, in³,

W = crater width, in,

D = crater depth, in,

L = crater length, ft,

and C = conversion factor equal to 12 in/ft.

It is shown elsewhere in this report that

$W = K_1 F_v$

and $D = K_2 F_v$.

Substituting the relationships for W and D in the equation for V/L gives:

$$V/L = 1/2(CK_1 K_2 F_v^2).$$

Next, a study of the effect of cutter angle on volume per unit length showed that the volume of material removed by the sharper 60-degree cutter is always larger than that removed by the 90-degree cutter for the same load in the same rock. To determine this effect more precisely, the average volume per unit length was calculated for both cutter angles and the ratio of these volumes became the measure of the average difference between the cutters. From a practical standpoint we decided to compare the curves from 7,500 to 10,000 lb (approximately three-quarters load to full load) vertical force since in actual use cutters are loaded quite heavily. To do this the curves were integrated between the load limits mentioned to obtain the area under each curve which then represented the average volume broken per unit length over that range. The ratio of these areas then yielded a proportionality constant between the 60-degree and 90-degree cutter over that range.

Since
$$\text{Area} = K\int_{7,500}^{10,000} F_v^x \, dF_v , \tag{7}$$

then
$$\frac{V/L_{60}}{V/L_{90}} = \frac{\text{Area}_{60}}{\text{Area}_{90}} = C, \tag{8}$$

where C = a constant representing the average difference in volume per unit length produced by a 60-degree and 90-degree cutter between 7,500 lb and 10,000 lb.

These ratios were calculated at 1.61, 1.98, 2.84, 2.10, and 1.76 for type 1 Indiana limestone, type 2 Indiana limestone, Kasota stone, Tennessee marble, and Valders white rock, respectively. These ratios can be used to estimate the crater volume per unit length for either a 60-degree or 90-degree disk cutter if the crater volume per unit length for either one is known.

Finally, the effect of rock type on crater volume per unit length was analyzed. Again the same problems that prevented a precise comparison of the effect of cutter angle also prevented us from determining the precise effect of rock type on crater volume per unit length. Consequently it was decided to evaluate the force-volume equations in table 3 at 7,500 lb (three-quarters load) and compare the volumes produced. It was found that the volume per unit length for the 90-degree cutter decreased from 1.30 in^3/ft to 0.35 in^3/ft as the compressive strength of the rock increased from 9,000 to 27,000 psi. The variation in volume with compressive strength was not linear. For the 60-degree cutter, the volume per unit length decreased from 2.70 in^3/ft to 0.90 in^3/ft as the compressive strength of the rocks tested increased.

Other attempts to correlate the volume per unit length for different rocks using rock physical properties were not made for two reasons. First, the variability in the exponents would probably make such a correlation inaccurate. Second, since volume per unit length is more sensitive to indexing distance between cutters, condition of rock, etc., it is probably not as useful as crater depth in defining penetration rate.

Crater Depth as a Function of Vertical Force on the Cutter

Because of the variation in the vertical force acting on the disk cutters during a run, the vertical force was defined as the average force acting on the cutter over the length of a run. The crater depth was similarly defined as the average depth of the crater measured at intervals along its entire length.

A force-depth relationship, typical of that found for all of the rocks, is shown in figure 7 for cutters with 60-degree and 90-degree cutting edges in Tennessee marble. The vertical force is plotted as the independent variable and the crater depth as the dependent variable. Each data point in figure 7 represents the average vertical force acting on a cutter and the average depth of crater for each crater produced. The curves for the other four rocks tested are represented by the equations of their curves in table 4. The force-depth relationship for the majority of the rocks tested was found to follow a linear form:

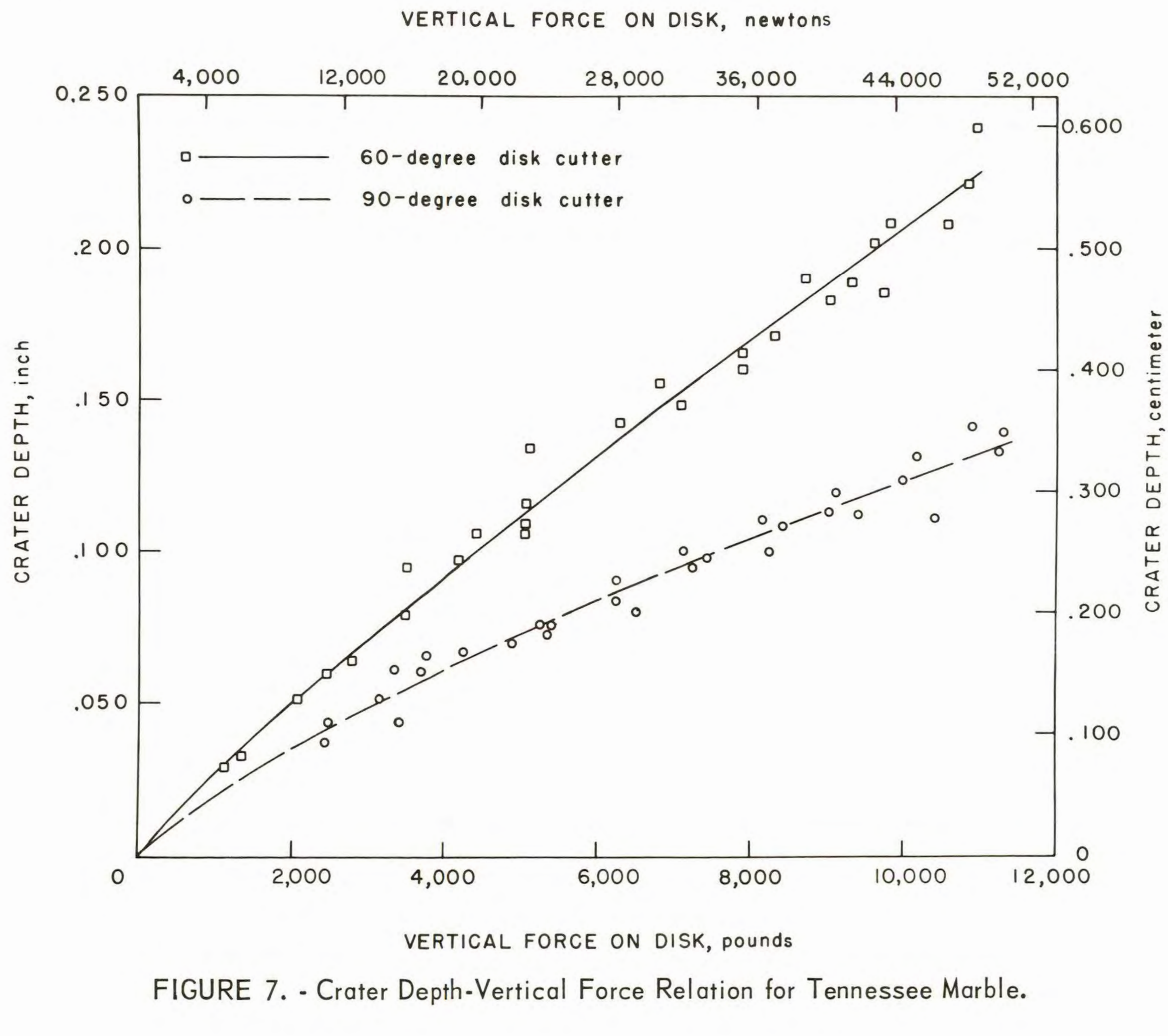

FIGURE 7. - Crater Depth-Vertical Force Relation for Tennessee Marble.

$$D = D_o + KF_v , \quad (9)$$

where D = crater depth, in,

D_o = intercept, in,

K = constant, in/lb,

and F_v = average vertical force on disk, lb.

TABLE 4. - Crater depth as a function of vertical force

Rock type	Force-depth relationship	Se[1]
Indiana limestone, type 1....	$D_{60} = 7.901 \times 10^{-5} F_v^{0.892}$ $D_{90} = 0.001 + 1.914 \times 10^{-5} F_v$	0.011 .003
Indiana limestone, type 2....	$D_{60} = 0.026 + 3.074 \times 10^{-5} F_v$ $D_{90} = 0.009 + 2.056 \times 10^{-5} F_v$	.015 .011
Kasota stone................	$D_{60} = 0.012 + 3.519 \times 10^{-5} F_v$ $D_{90} = 0.002 + 1.797 \times 10^{-5} F_v$	.016 .009
Tennessee marble............	$D_{60} = 5.536 \times 10^{-5} F_v^{0.893}$ $D_{90} = 9.368 \times 10^{-5} F_v^{0.779}$	.024 .0056
Valders white rock..........	$D_{60} = -0.033 + 2.785 \times 10^{-5} F_v$ $D_{90} = -0.024 + 1.705 \times 10^{-5} F_v$	.0098 .029

[1]Standard error of estimate.

Inspection of the force-depth equations in table 4 and the force-depth curves in figure 7 showed three important effects. First, in a given rock, the depth of the crater is essentially a linear function of the vertical force on the disk for both cutters tested. This effect holds even for those rocks represented by the power relationship (table 4), for vertical loads in excess of 3,000 lb. Second, the depth of crater produced in a given rock by the 60-degree cutting edge is always greater than that produced by the 90-degree cutting edge at the same level of vertical force. Third, the depth of the crater produced by some fixed combination of cutter edge and vertical load will generally be greater in rocks with the lowest density and Shore scleroscope hardness values.

The linear force-depth relationship was found for all the rocks tested, with three exceptions. For the 60-degree cutter in type 1 Indiana limestone, a nonlinear force-depth relationship occurred at vertical loads below 2,000 lb; above this load the force-depth curve became linear. A similar deviation from linearity occurred in Tennessee marble which showed the power form of the force-depth equation, with exponents of less than 1.0 for both cutters tested. This nonlinear behavior in Tennessee marble occurred only at vertical loads below 3,000 lb; above this load the force-depth curves became linear.

In summary, the relationship between vertical force on the disk and crater depth is linear for both cutter angles in all the rocks tested. A minimum vertical force of 3,000 lb, however, is required in Tennessee marble for both cutting edges before linearity is achieved while a minimum vertical force of 2,000 lb is required in type 1 Indiana limestone for the 60-degree cutter only.

Next the effect of cutter edge on the depth of cratering was analyzed. The typical effect of cutter edge (fig. 7) shows the craters produced by the 60-degree edge to be deeper in a given rock than those produced by the 90-degree edge for the same level of vertical force on the cutter. To determine more precisely the effect of disk edge angle, the slopes of the best-fit straight line for the force-depth data were used as the standard of comparison, since within the optimum range of variables tested this straight-line fit gave the best approximation of the force-depth curves for all the rocks tested. When calculated for each rock type, the ratio of the slopes for each cutting edge yielded a constant representing the difference in effect between the two cutter angles tested.

Since

$$D = KF_v , \tag{10}$$

$$\frac{D_{60}}{D_{90}} = \frac{K_{60}\ F_v}{K_{90}\ F_v} . \tag{11}$$

At the same value of F_v for both cutting edges, equation 11 reduces to:

$$\frac{D_{60}}{D_{90}} = \frac{K_{60}}{K_{90}} = C, \tag{12}$$

where D = crater depth, in,

K = slope of straight line, in/lb,

F_v = average vertical force on the cutter, lb,

and C = a constant representing the difference between cutting edges.

Using equation 12, this ratio was calculated to be 1.59, 1.50, 1.87, 1.78, and 1.69 for type 1 and type 2 Indiana limestone, Kasota stone, Tennessee marble, and Valders white rock, respectively. These ratios can be used as a simple method of approximating the depth of a crater for either cutting edge if the crater depth of one is known.

For rocks other than those tested but with approximately the same physical properties, the crater depth or depth of penetration for the 90-degree cutter can be calculated from equation 23. The depth of penetration for the 60-degree cutter can then be estimated using equation 12 and the ratio that most nearly matches the properties of the rocks tested. In equation 23 the most important properties are Shore scleroscope hardness and rock density.

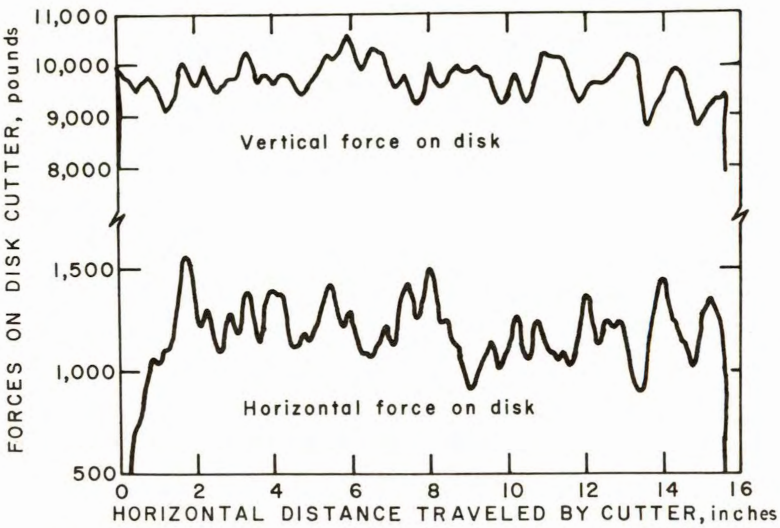

FIGURE 8. - Typical Force-Distance Recordings for Simple Disk Cutter.

Finally, the effect of rock type on the crater depth was investigated. To analyze this effect more accurately, each rock was represented by its unique set of physical properties, and a statistical regression analysis was then used to determine which rock properties were significantly related to the crater depth. The result was an equation for predicting crater depth as a function of vertical force and the rock properties of Shore scleroscope hardness and density (equation 23).

Horizontal Force as a Function of Vertical Force

Figure 8 shows some typical records of the horizontal and vertical forces acting on a disk cutter during these experiments. These traces were analyzed to obtain the average vertical and horizontal forces acting on a cutter during a run. Although only the average forces were used in these analyses, knowledge of the peak forces acting on a cutter is also important from a design standpoint. Analysis showed the peak vertical forces on the disk to be 10 to 40 percent greater than the average vertical force while the peak horizontal forces were found to be 30 to 80 percent greater than the average horizontal force. These peak forces were generally independent of cutter angle and rock type. Although the peak forces just given are probably characteristic of our testing machine, no doubt a similar force fluctuation is also active on the cutters of an actual boring machine.

Figure 9 shows the horizontal versus vertical force relationship for disk cutters with both 60-degree and 90-degree cutting edges operating in Tennessee marble. These curves are typical of those produced for the other four rocks tested in these experiments. The vertical force is plotted as the independent variable and the horizontal force as the dependent variable. The equations representing the curves for all the rocks tested are listed in table 5. The force relationships for all the rocks tested were found to follow the form:

$$F_h = KF_v^{\,x}, \qquad (13)$$

where x = an exponent which differs for each rock and cutter combination,

F_h = the average horizontal force on the disk, lb,

and K = a constant, lb^{1-x}.

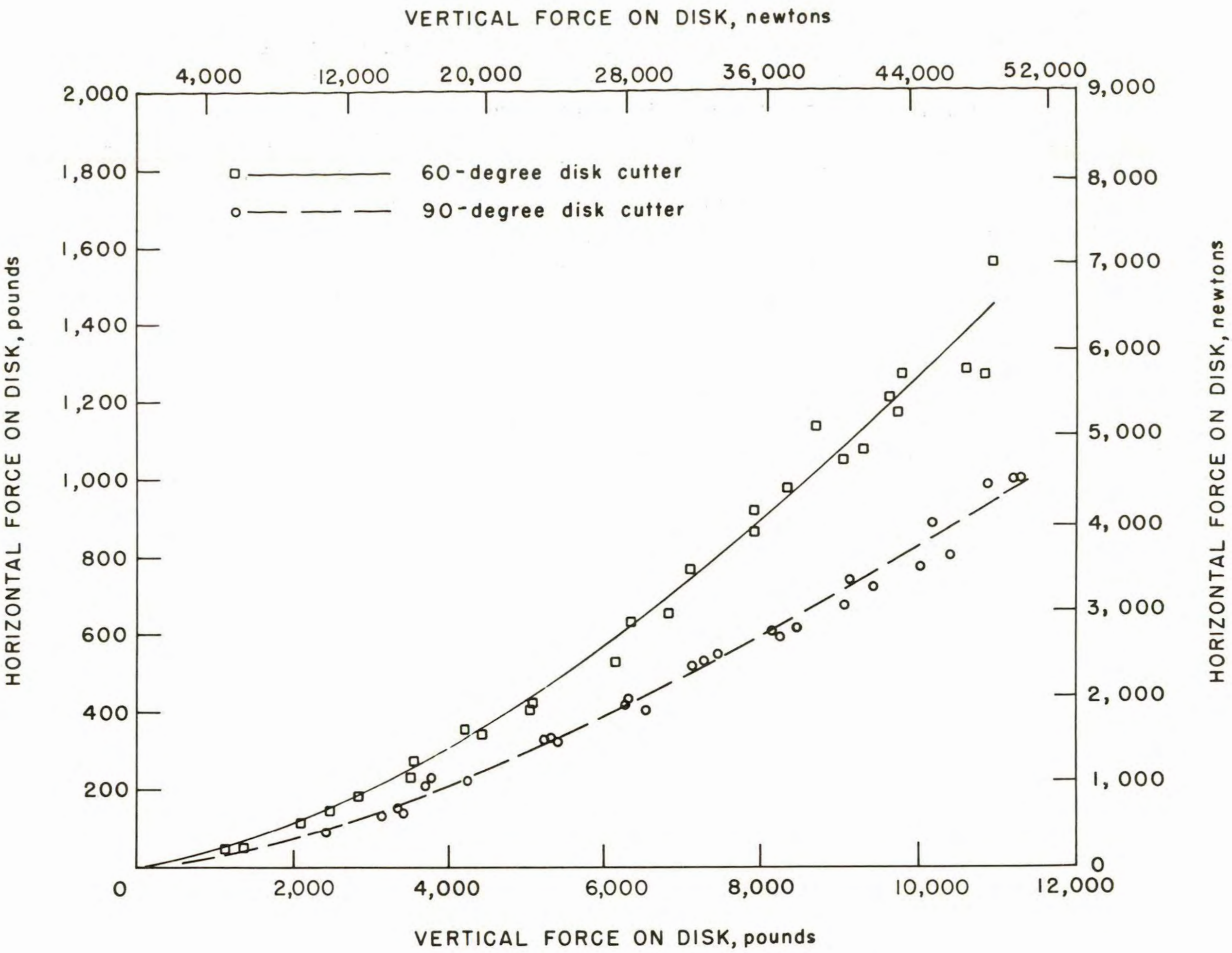

FIGURE 9. - Horizontal Force-Vertical Force Relation for Tennessee Marble.

TABLE 5. - Horizontal force as a function of vertical force

Rock type	Horizontal force-vertical force relationship	S_e[1]
Indiana limestone, type 1......	$F_{h\,60} = 9.379 \times 10^{-4}\ F_v^{1.551}$ $F_{h\,90} = 5.796 \times 10^{-4}\ F_v^{1.577}$	57.4 41.5
Indiana limestone, type 2......	$F_{h\,60} = 2.321 \times 10^{-3}\ F_v^{1.449}$ $F_{h\,90} = 1.443 \times 10^{-4}\ F_v^{1.724}$	65.8 59.0
Kasota stone..................	$F_{h\,60} = 3.126 \times 10^{-4}\ F_v^{1.694}$ $F_{h\,90} = 3.536 \times 10^{-4}\ F_v^{1.644}$	12.9 29.0
Tennessee marble..............	$F_{h\,60} = 7.985 \times 10^{-4}\ F_v^{1.549}$ $F_{h\,90} = 8.427 \times 10^{-4}\ F_v^{1.497}$	50.6 24.7
Valders white rock............	$F_{h\,60} = 1.937 \times 10^{-5}\ F_v^{1.971}$ $F_{h\,90} = 2.352 \times 10^{-2}\ (F_v - 2{,}000)^{1.163}$	60.7 24.3

[1]Standard error of estimate.

An analysis of the horizontal force-vertical force relationships in table 5 gave three important results. First, the horizontal force on a disk cutter increases at an increasing rate with increasing vertical force. Second, the horizontal force required to roll a disk cutter with a 90-degree cutting edge across a given rock surface will be less than that required for a disk with a 60-degree edge for the same level of vertical load on the disk. To give some idea of the horizontal force necessary to roll a disk across a rock surface, the force equations in table 5 were all evaluated at three-quarters load (approximately 7,500 lb). It was then calculated that the 60-degree cutting edge required a horizontal force of from 0.10 to 0.14 times the vertical force on the disk, depending on the rock type, whereas the horizontal force necessary with the 90-degree edge ranged from 0.07 to 0.10 times the vertical force. The third effect noted from an analysis of the force equations was that the horizontal force necessary to roll a disk across the harder rocks was generally less than that required for the softer rocks at a given level of vertical load. Rock hardness here is synonymous with increasing rock density and tensile strength as defined in prediction equation 27.

Since the force equations for Valders white rock were substantially different from those for the other rocks, they require some additional discussion. The force equation for the 60-degree cutter differed in that the horizontal force increased at a much faster rate with increasing vertical force (an exponent of 1.97) than for the other rocks tested. The magnitude of the horizontal force, however, because of the smaller constant, remained in the range of from 0.10 to 0.15 times the vertical force on the disk established for the 60-degree cutter in all of the rocks tested. The force equation for the 90-degree cutter in the same rock showed the horizontal force rising at the lowest rate of all the rocks (an exponent of 1.16) and also showed that a vertical force of approximately 2,000 lb was required before any measurable horizontal force was registered. The threshold force of 2,000 lb was believed to be due to the combination of hard rock (compressive strength = 27,000 psi) and the 90-degree disk which at low levels of vertical force produced no measurable craters and hence required little horizontal force for moving the cutter. Again the magnitude of the horizontal force fell within the range of from 0.07 to 0.10 times the vertical force typical of that found for the other rocks tested for the 90-degree disk.

Next the effect of cutting edge was analyzed. As already stated, the 60-degree cutter required more horizontal force to traverse a rock surface than did the 90-degree cutter. Determining the effect of cutting edge on horizontal force with any degree of accuracy was not possible because of the differences in the value of the exponent for each combination of rock and cutter angle. To provide a basis of comparison, the force equations in table 4 were integrated between 0 and 10,000 lb vertical force. This operation yielded the area under each curve and the ratio of the areas for the two cutting edges in the same rock then served as a measure of the average effect of cutter edge on the horizontal force for that particular rock.

Since $$F_h = KF_v^{\,x}, \qquad (14)$$

and $$\text{Area} = K\int_0^{10,000} F_v{}^x \, dF_v \,, \tag{15}$$

and $$\frac{F_{h60}}{F_{h90}} = \frac{\text{Area}_{60}}{\text{Area}_{90}} = C, \tag{16}$$

then $$\frac{F_{h60}}{F_{h90}} = C, \tag{17}$$

where C = a constant comparing the effect of cutter angle on horizontal force.

These ratios were calculated at 1.49, 1.42, 1.29, 1.37, and 1.73 for types 1 and 2 Indiana limestone, Kasota stone, Tennessee marble, and Valders white rock[10] respectively. Given F_{h60} or F_{h90}, either by experimental observation of by equation 27, one can approximate the other, at the same level of F_v and in the same rock type, by using equation 17 and the constant corresponding to the proper rock type.

Energy-Volume Relationships

The energy or work used to form a crater with a disk cutter was defined as the sum of the vertical and horizontal work done by the disk. The vertical work was calculated by multiplying the average vertical force on the cutter times the average depth of crater and horizontal work was calculated by multiplying the average horizontal force on the cutter times the length of the run. The vertical work in these experiments was found in all cases to be a small fraction of the horizontal work, usually from 6 to 10 percent. Note that because energy is a function of the length of run, the energy per unit volume we calculate in this report will be correct only for runs of approximately 20 in. Since the vertical work is only a small fraction of the total work, however, the largest error that could be involved in any case would be about ± 10 percent of the total work.

A crater volume versus energy relationship for Tennessee marble, typical of that found for all of the rocks tested, is shown in figure 10. Energy is plotted as the independent variable and crater volume as the dependent variable. The energy-volume relationships for the other four rocks tested are represented by the equations of their curves in table 6. The energy-volume relationships for a majority of the rocks and cutter edges tested were found to have this form:

$$V = V_o + KE, \tag{18}$$

where V = the crater volume, in^3,

V_o = intercept, in^3,

K = a constant, in^2/lb,

and E = total input energy to the disk, in lb.

[10] Valders white rock was compared between 7,500 and 10,000 lb F_v.

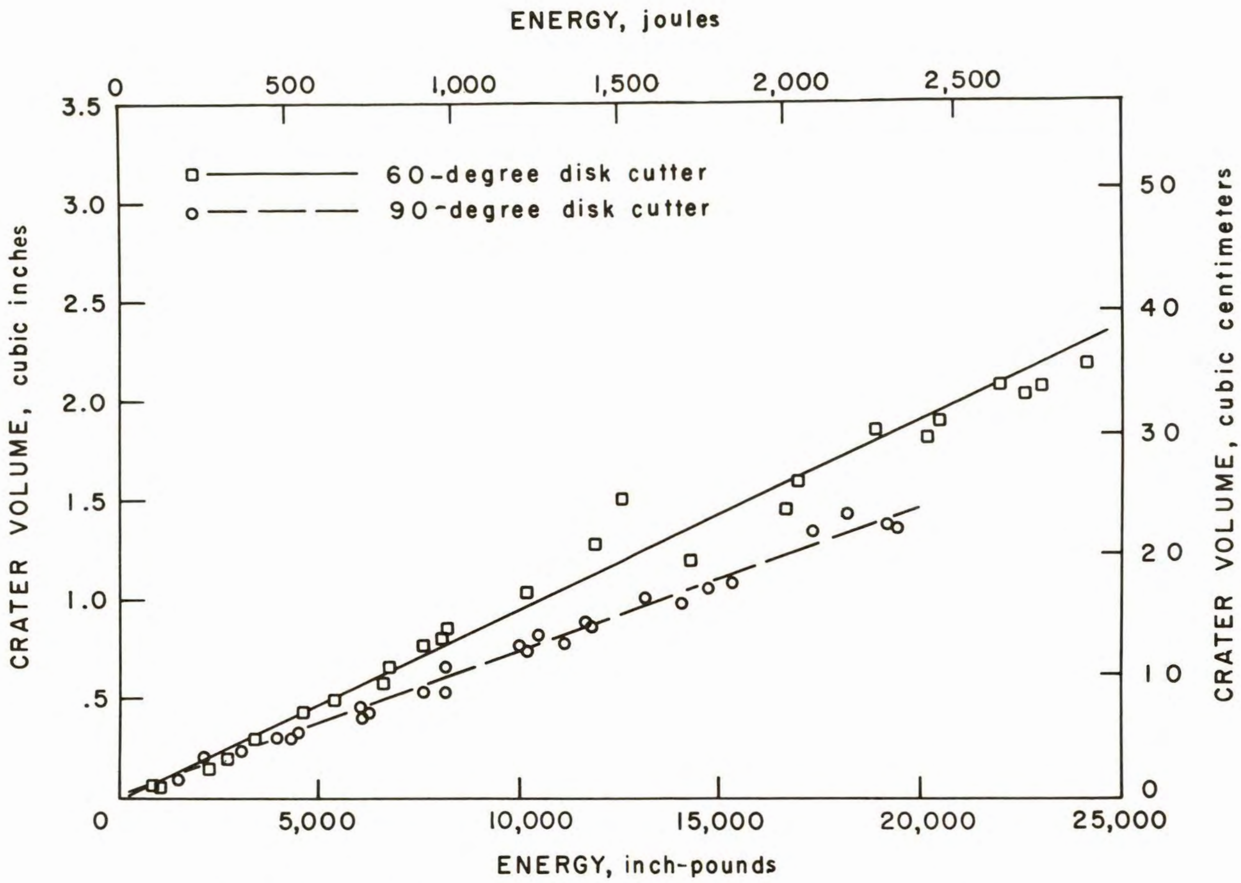

FIGURE 10. - Volume-Energy Relation for Tennessee Marble.

TABLE 6. - Crater volume as a function of input energy

Rock type	Crater volume, in³, as a function of energy, in lb	S_e[1]	Energy per unit volume, in lb/in³
Indiana limestone, type 1.....	$V_{60} = 4.904 \times 10^{-5}E^{1.122}$	0.357	[2]6,550
	$V_{90} = 0.073 + 1.305 \times 10^{-4}E$	.158	8,200
Indiana limestone, type 2.....	$V_{60} = 3.368 \times 10^{-5}E^{1.195}$	.479	[2]4,100
	$V_{90} = 0.085 + 1.702 \times 10^{-4}E$	.234	6,350
Kasota stone..................	$V_{60} = 0.176 + 1.708 \times 10^{-4}E$	.252	6,850
	$V_{90} = 0.059 + 1.041 \times 10^{-4}E$	.174	10,100
Tennessee marble..............	$V_{60} = 0.015 + 9.607 \times 10^{-5}E$	.114	10,400
	$V_{90} = 0.003 + 7.279 \times 10^{-5}E$	.043	13,800
Valders white rock............	$V_{60} = -0.165 + 1.040 \times 10^{-4}E$	.164	10,430
	$V_{90} = -0.096 + 6.617 \times 10^{-5}E$	.082	16,290

[1] Standard error of estimate.
[2] Energy per unit volume for the power forms are approximations.

An inspection of the energy-volume equations in table 6 gives three important results. First, the volume of a crater is a linear function of the input energy to the disk for both cutter edges and for all rock types tested. This linear relationship serves as a good approximation even for the two equations which follow the power form (table 6), since the exponents are close to 1.0. Second, the volume of a crater produced by a 60-degree cutting edge in a given rock is greater than that produced by the 90-degree edge at any given energy level. Third, the energy required to produce a given crater volume with a fixed cutter angle generally increased as the Shore scleroscope hardness of the rocks increased. The results of a statistical regression analysis on the effect of rock properties on crater volume is shown in equation 24.

Further analysis of the energy-volume relationships showed a deviation from linearity occurred with the 60-degree cutter in both type 1 and type 2 Indiana limestones. This deviation from linearity evidently reflects the fact that at low vertical loads (less than 4,000 lb in most cases) energy per unit volume decreases as vertical load increases. This behavior was found in all the rocks tested. As the vertical load increased beyond a few thousand pounds, crater volume quickly became a linear function of energy. It is not known at this time, whether this behavior is caused by a fundamental change in the rock-breakage mechanism or if it is simply the result of errors caused by the difficulty in measuring small crater volumes at low levels of vertical load. For all practical purposes, however, crater volume will be a linear function of energy over the range of loads usually encountered in actual tunneling.

Since volume was found to be a linear function of energy, with a given cutter angle and rock type, it followed that the energy per unit volume was a constant with the exception of very small vertical forces as noted above. Subsequent analysis showed that the energy per unit volume required for the 90-degree cutter was 20 to 35 percent greater than that required for the 60-degree cutter. Further, it was found that the energy per unit volume required for the 90-degree cutter generally approached 60 to 80 percent of the compressive strength of the rocks tested. The energy per unit volume for each rock and cutter combination is given in table 6.

To determine the effect of the cutter edge angle on crater volume, the slopes of the linear energy-volume relations were compared directly wherever possible. For the power form of the energy-volume relation the slope of the best-fit straight line was used. Then the ratio of the slopes for the two different cutting edges in the same rock served as a measure of the effect of cutter edge on crater volume for that rock type.

Since
$$V = KE, \tag{19}$$

$$\frac{V_{60}}{V_{90}} = \frac{K_{60}\ E_{60}}{K_{90}\ E_{90}} = C; \tag{20}$$

at the same value of E for both cutting edges, equation 20 reduces to:

$$\frac{V_{60}}{V_{90}} = \frac{K_{60}}{K_{90}} = C, \tag{21}$$

where C = a constant comparing the effect of cutter angle on crater volume.

With equation 21, ratios were calculated at 1.28, 1.40, 1.55, 1.30, and 1.56 for types 1 and 2 Indiana limestone, Kasota stone, Tennessee marble, and Valders white rock, respectively. These ratios can be used with equation 21 to estimate the crater volume for either cutter if the volume for the other is known. For rocks other than those tested but with similar physical properties, the crater volume could be calculated for the 90-degree cutter with equation 24. The volume for the 60-degree cutter could then be estimated using equation 21 and the ratio that most nearly matches the properties of the rocks tested.

Prediction of Disk Cutter Performance Using Rock Physical Properties

A stepwise multiple linear regression analysis was used to devise a prediction formula for disk cutter performance. This procedure entered one independent variable at a time to give a series of regression equations with each equation containing one more independent variable than the equation before it. F-testing at the 95-percent level was used to determine the significance of the regression coefficients found during the regression analysis.

Because the prediction equations for the 60-degree and 90-degree cutter edges would be essentially the same, only the equations for the 90-degree edge were developed. The constants necessary to convert the results obtained for the 90-degree to the 60-degree edge were given previously. From the fundamental relation to be fitted, a statistical regression model was derived with the following form:

$$\hat{Y}_{90} = \hat{\beta}_0 + X\left[\frac{\hat{\beta}_1}{RP_1} + \frac{\hat{\beta}_2}{RP_2} + \ldots\ldots + \frac{\hat{\beta}_n}{RP_n}\right], \tag{22}$$

where $\hat{Y}$ = dependent variable to be estimated, either crater depth or crater volume,

X = independent variable, vertical force or energy,

$\hat{\beta}_i$ = the desired regression coefficients from regression analysis,

and RP_i = physical properties of the rock.

The terms in brackets in equation 22, representing the effect of rock type, comprise a more detailed breakdown of the constants given in the original equations. The reciprocals of the rock properties were used in the prediction equations since this transformation would cause a logical change in

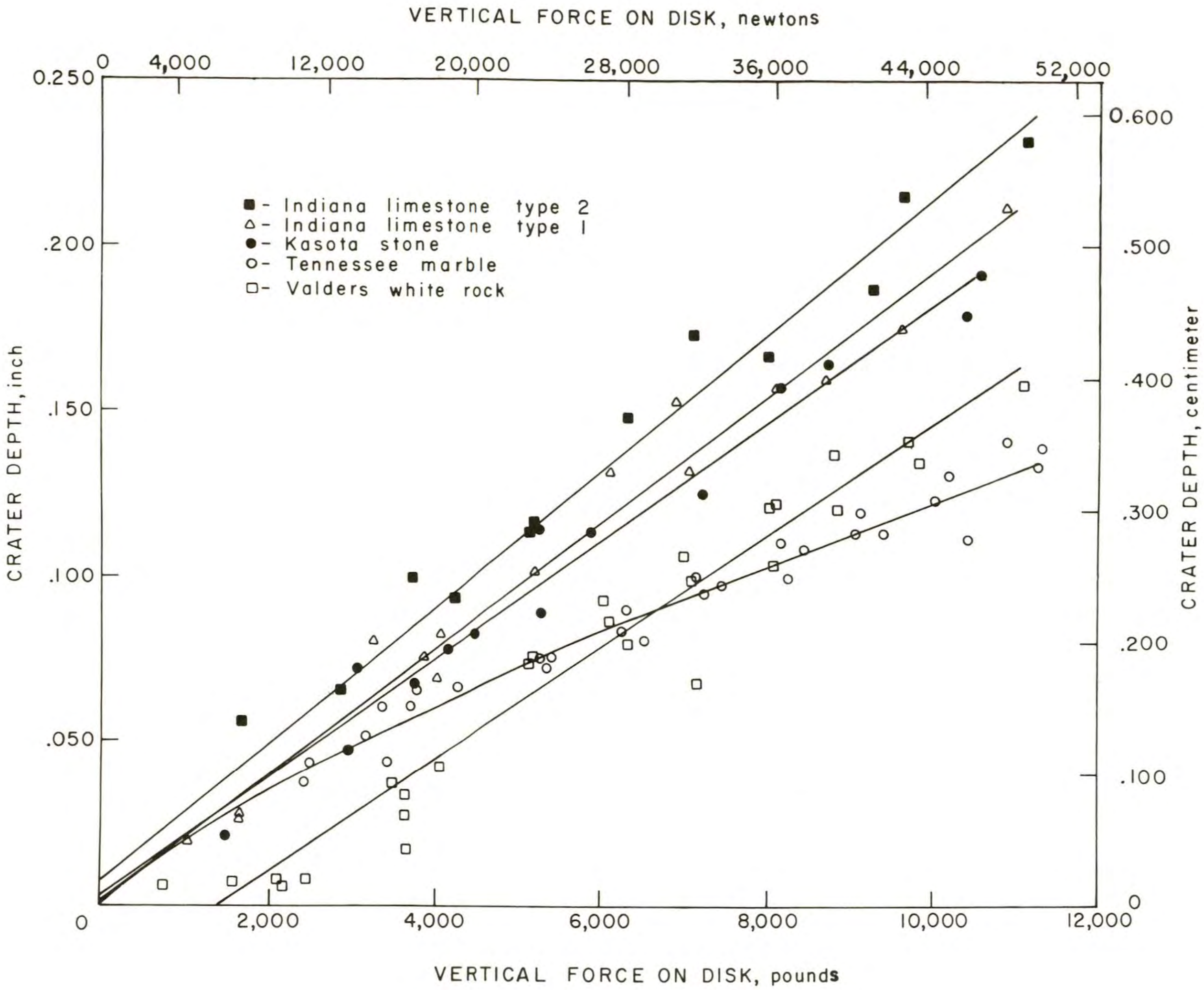

FIGURE 11. - Crater Depth-Vertical Force Relation for All Rocks Tested.

the value of the dependent variable as the value of the rock properties changed. For example, as the rock became harder to penetrate, due to a change in rock type, the reciprocals of the rock properties would decrease and cause a logical decrease in the crater depth and crater volume.

Prediction of Depth of Penetration or Crater Depth

Figure 11 shows the force-depth relationship for the 90-degree cutter edge in all five of the rocks tested. The difference in the slopes of the lines represents the effect of rock type since all of the other independent variables were held constant. Using the stepwise regression technique described earlier, the following prediction equation for crater depth was found:

$$D_{90} = 0.0016 + F_v \left[\frac{4.686 \times 10^{-4}}{SH} + \frac{2.441 \times 10^{-5}}{\rho} \right], \qquad (23)$$

where D_{90} = crater depth for a 90-degree cutting edge, in,

F_v = average vertical force on the cutter, lb,

SH = Shore scleroscope hardness, scleroscope units,

and ρ = density of the rock, slugs/ft^3.

This equation has a multiple correlation coefficient of 0.936 and a standard error of estimate of 0.018 in. The rock properties in equation 23 are given in their order of importance with Shore hardness the most significant property followed by the density of the rock. With the crater depth for the 90-degree cutter known from equation 23, the crater depth for the 60-degree cutter can be estimated from equation 12. Note that because the constant in equation 12 may be a function of the rock tested,[11] it should be used only for rocks falling within the physical property range of the rock types considered in this investigation.

Although tensile and compressive strength were also found to be significant properties at the 95-percent confidence level, they were not incorporated in prediction equation 23 for two reasons. First, their inclusion in the predictor equation did not make any practical improvement in prediction accuracy and, second, determining tensile and compressive strengths required elaborate equipment whereas the two properties used (Shore hardness and density) could be readily determined with simple equipment under field conditions.

Prediction of Crater Volume

Figure 12 shows the energy versus volume relationship for the 90-degree cutter edge in all five rocks. The difference in the slopes of the lines represents the effect of rock type since all the other independent variables were held constant. Using the technique described earlier, the following prediction equation for crater volume was developed:

$$V_{90} = -0.069 + E \left[\frac{4.304 \times 10^{-3}}{SH} \right], \qquad (24)$$

where V_{90} = crater volume for 90-degree cutting edge, in^3,

and E = total input energy to the cutter, in lb.

This equation has a multiple correlation coefficient of 0.987 and a standard error of 0.151 in^3. Again, Shore hardness was found to be the most significant property while all other properties were rejected at the 95-percent level of significance.

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[11] Hartman, H. L. Crater Geometry Relations in Percussive Drilling--Single Blow Studies. Mine & Quarry Engineering, v. 28, No. 12, December 1962, pp. 530-536.

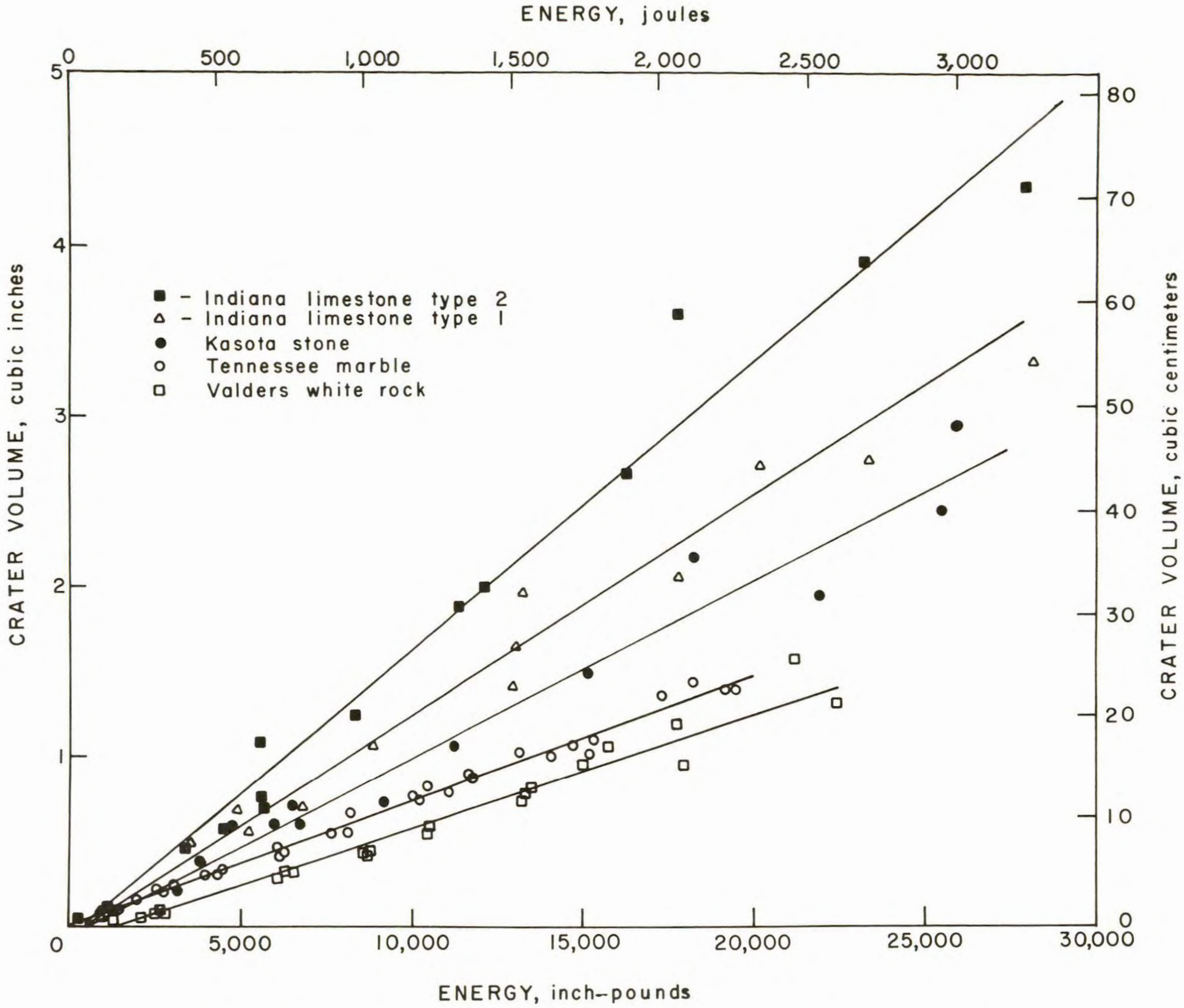

FIGURE 12. - Crater Volume-Energy Relation for All Rocks Tested.

With the crater volume for the 90-degree cutter known from equation 24, the crater volume for the 60-degree cutter can be estimated from equation 21. Since the constant in equation 21 may be a function of the rock tested,[12] it should only be used for rocks falling within the physical property range of the rocks considered in this investigation.

Prediction of Horizontal Force

The development of a predictor equation for horizontal force involved a more complex model than was previously encountered since the vertical-horizontal force relationship took the following power form:

$$F_h = KF_v^x . \quad (25)$$

[12] Page 47 of work cited in footnote 11.

Further, since the exponent varied for each rock tested, the possibility existed that it and the constant were both functions of rock properties. Since it was not possible to simultaneously define both the exponent and constant as functions of rock properties, it was necessary to simplify the model. It was found after a thorough investigation that the model which would best fit the data would be of this linear form:

$$F_h = KF_v . \tag{26}$$

Although this model had the obvious disadvantage of providing a linear fit to a curvilinear relationship, it nevertheless provided (with the exception of Kasota stone) a predicted value of F_h within ± 10 percent of the actual value of F_h for vertical loads of from 7,500 lb to 10,000 lb.

Using the linear model we followed the same stepwise regression as before to derive an equation to predict the horizontal force acting on a 90-degree disk cutter as functions of vertical force and rock physical properties:

$$F_{h90} = -158 + F_v \left[\frac{0.312}{\rho} + \frac{20.72}{\sigma_t} + \frac{0.701}{SH} \right], \tag{27}$$

where F_{h90} = the horizontal force necessary to roll a 90-degree disk cutter across a smooth rock surface, lb,

ρ = density of the rock, slugs/ft^3,

and σ_t = tensile strength of the rock, psi.

Equation 27 has a multiple correlation coefficient of 0.939 and a standard error of estimate of 122 lb. The density, tensile strength, and Shore scleroscope hardness of the rock were found to be the most important rock properties.[13]

With the horizontal force on the 90-degree cutter known from equation 27, the horizontal force acting on a 60-degree cutter could be estimated from equation 17.

SUMMARY

Crater Width

Crater width, W, was generally found to be a linear function (a power function in two instances) of crater depth, D, for both cutters tested in all of the rocks tested. The constant, K, varies from approximately 5 to 9 depending on the cutter and rock being tested:

$$W = KD. \tag{28}$$

[13] Although all rock properties were found significant at the 95-percent confidence level, they contributed no practical improvement in the accuracy of equation 27. Hence the model was simplified by using only the two most important properties.

Crater width was also a linear function (a power function in three instances) of the vertical force on the cutter for both cutters in all the rocks tested where K varies from about 9×10^{-5} to about 3×10^{-4} in/lb:

$$W = KF_v . \qquad (29)$$

For the same vertical force on both cutters ($F_{v60} = F_{v90}$) the 60-degree cutter will produce a crater approximately 1.32 times wider than that produced by a 90-degree cutter:

$$\frac{W_{60}}{W_{90}} = 1.32. \qquad (30)$$

For the same depth of crater ($D_{60} = D_{90}$), the 90-degree cutter will produce a crater approximately 1.27 times wider than that produced by a 60-degree cutter:

$$\frac{W_{90}}{W_{60}} = 1.27. \qquad (31)$$

Crater Volume per Unit Length

Crater volume per unit length, V/L, was found to be a power function of vertical force, F_v, for both cutters in all the rocks tested. The exponent averages 1.83 for all rocks and cutters tested:

$$V/L = KF_v^{1.83}. \qquad (32)$$

For vertical loads ($F_{v60} = F_{v90}$) between 7,500 lb and 10,000 lb, the 60-degree cutter will produce a crater volume about twice as large as that produced by the 90-degree cutter:

$$\frac{V/L_{60}}{V/L_{90}} = 2.06. \qquad (33)$$

Crater Depth

Crater depth was found to be a linear function of vertical force for both cutters in all the rocks tested. The value of K varies with rock type and cutter angle but is in the order of 10^{-5} in/lb:

$$D = KF_v . \qquad (34)$$

Under identical conditions ($F_{v60} = F_{v90}$), the crater depth produced by the 60-degree cutter will be about 1.69 times the depth produced by the 90-degree cutter:

$$\frac{D_{60}}{D_{90}} = 1.69. \qquad (35)$$

Finally, crater depth was found as a function of vertical force and various rock properties. This yielded a prediction equation for crater depth having a standard error of estimate of 0.02 in:

$$D_{90} = 0.0016 + F_v \left[\frac{4.686 \times 10^{-4}}{SH} + \frac{2.441 \times 10^{-5}}{\rho} \right]. \quad (36)$$

Horizontal Force

The horizontal force necessary to roll a disk cutter across a smooth rock surface was found to be a power function of vertical load for both cutters in all of the rocks tested. The exponent averages 1.58 for all the rocks tested:

$$F_h = KF_v^{1.58}. \quad (37)$$

Under identical conditions, the horizontal force required by the 60-degree cutter will be about 1.46 times that required for the 90-degree cutter:

$$\frac{F_{h60}}{F_{h90}} = 1.46. \quad (38)$$

Finally, the horizontal force on a 90-degree disk cutter was found as a function of the vertical force and various rock properties. This yielded a prediction equation for horizontal force having a standard error of estimate of 122 lb:

$$F_{h90} = -158 + F_v \left[\frac{0.312}{\rho} + \frac{20.718}{\sigma_t} + \frac{0.701}{SH} \right]. \quad (39)$$

It was found that as a quick estimate of F_h the following relations can be used:

$$F_{h60} = [0.10 \text{ to } 0.14] \; F_v, \quad (40)$$

and

$$F_{h90} = [0.07 \text{ to } 0.10] \; F_v. \quad (41)$$

Energy-Volume Relationship

Crater volume, V, was found to be a linear function of energy input, E, for both cutters in all the rocks tested:

$$V = KE. \quad (42)$$

Since crater volume was a linear function of energy, then energy per unit volume was a constant for each cutter and rock combination:

$$\frac{E}{V} = \frac{1}{K} = C. \quad (43)$$

This constant, C, for the 90-degree cutter was generally found to be from 60 to 80 percent of the compressive strength of the rock tested.

Using the same amount of energy, the crater volume produced by the 60-degree cutter was about 1.42 times greater than that produced by the 90-degree cutter:

$$\frac{V_{60}}{V_{90}} = 1.42. \qquad (44)$$

Finally, to determine the effect of rock on the energy-volume relation, an equation to predict crater volume as a function of energy and various rock properties was developed:

$$V_{90} = -0.069 + E\left[\frac{4.304 \times 10^{-3}}{SH}\right]. \qquad (45)$$

This equation had a standard error of estimate of 0.151 in^3.

CONCLUSIONS

Since these disk-cutter experiments were designed primarily to define the fundamental relationships governing disk-cutter performance, they were performed under a narrow range of experimental conditions including linear motion, single-crater studies, absence of indexing effects, and the constant cutter geometry. Hence, the constants of proportionality found in the equations presented will not be appropriate under conditions other than those just defined. However, some of the more important experimental results which may be of immediate value to those engaged in the design or use of rock disintegration equipment utilizing disk type cutters should be pointed out.

The 60-degree cutter edge was found to have superior penetrating characteristics as compared to the 90-degree disk cutter as it penetrated an average of 1.67 times deeper than the 90-degree cutter at the same level of vertical force. The 60-degree cutter was also superior from an energy standpoint since it required 20 to 35 percent less energy to break out a unit volume of rock than did the 90-degree cutter. Hence, using penetrating ability and energy consumption as the criteria of selection, the 60-degree edge was found to be best.

The penetration rate of a multicutter boring head can be defined as the depth of penetration per revolution. Since all cutters on the head must penetrate to the same depth, it is only necessary to define the depth of penetration of a single cutter to define penetration rate.

Under closely controlled laboratory conditions the depth of penetration of a disk cutter can be accurately determined from a prediction equation involving vertical force and Shore scleroscope hardness and rock density (equation 23).

Under actual operating conditions it is not yet possible to define a general crater depth predictor equation. We can reasonably assume from the results of our experimentation, however, that the crater depth will continue to vary as a linear function of vertical force and that within the range of rocks tested Shore scleroscope hardness and rock density will continue to be important properties for predicting crater depth.

Since the horizontal force acting on a cutter ultimately determines the torque necessary to turn a cutting head of a boring machine, it is particularly important that this horizontal force be accurately defined.

Although, under laboratory conditions, we can determine the horizontal force acting on a disk cutter (from equation 27), we cannot as yet define a general equation to predict the horizontal force on a disk cutter in actual operations. We can expect, however, that the horizontal force will continue to vary as a power function of vertical force on the cutter.

From a practical standpoint, the horizontal force acting on a 7-in-diameter disk cutter can be estimated to be from 0.07 to 0.14 times the vertical force on the cutter. It should be noted that these values pertain only to linear cutter motion and that the horizontal force acting on cutters near the center of the cutting head may be substantially greater than predicted. The fact that the peak horizontal force can be as much as 80 percent higher than the average horizontal force should also be considered in calculating and applying the horizontal force on a cutter.

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