EVALUATING THE ECONOMIC AVAILABILITY OF MESABI RANGE TACONITE IRON ORES WITH COMPUTERIZED MODELS

By R. W. Michelson, H. J. Polta, and Orin Peterson

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EVALUATING THE ECONOMIC AVAILABILITY OF MESABI RANGE TACONITE IRON ORES WITH COMPUTERIZED MODELS

by

R. W. Michelson,[1] H. J. Polta,[2] and Orin Peterson[3]

ABSTRACT

The Bureau of Mines developed computerized mathematical models to estimate the economic availability of Mesabi Range taconite concentrates; these are a quality model, a cost model, and an economic availability model. The quality model evaluates metallurgical test data; the cost model determines iron unit cost from mining and processing cost data; and the economic availability model determines quantity from geologic and ore quality data. By correlating all of the physical resource quality factors and dimensional characteristics with the economic cost factors associated with producing iron ore pellets, these three models combine to establish a final relation between iron ore reserve quantities and iron unit cost.

INTRODUCTION

Purpose

This study has a twofold purpose: (1) to develop a procedure for evaluating the economic availability of ore from a deposit, and (2) to demonstrate the procedure by making an economic availability evaluation of Mesabi Range taconite concentrates. Methodology is given special emphasis since it can be used for re-evaluating Mesabi Iron Range resources as additional data become available, and, with appropriate modifications, for evaluating other mineral deposits. The economic availability evaluation is conducted under the Bureau of Mines mineral availabilities studies guidelines.

This study uses the Mesabi Range because the Mesabi continues to be the largest source of iron ore in the United States. The Mesabi has accounted for nearly 60 pct of the U.S. past production of iron ore. While most of the past production has been from the natural iron ores, the conversion in recent years

[1]Geologist.
[2]Mining engineer.
[3]Mathematician.
All authors are with the Twin Cities Office of Mineral Supply, Bureau of Mines, Minneapolis, Minn.

to the taconite ores adds even greater importance to the Mesabi Range as a source of future production.

The Bureau of Mines and others have conducted a considerable amount of research effort towards evaluating the physical characteristics of the taconite resources (3, 7-10, 16, 18, 22, 28-29).[4] Previous studies have also been conducted concerning the economics of producing iron ore pellets from taconite (13-14, 24). This study, however, is a unique attempt to analytically establish the relation between resource quantities and iron unit cost.

MESABI RANGE, MINNESOTA (27, 29)

The location and geology of the Mesabi Range are presented as background for subsequent discussions of the scope and assumptions of the evaluation.

General Geology and Location

The Mesabi Range of Minnesota is a preglacial outcrop of the Biwabik iron formation. Now mostly covered by glacial drift, the range occupies a linear belt, one-fourth to 3 miles wide, extending for 120 miles along the strike of the formation from west of Grand Rapids to east of Babbitt, Minn. (fig. 1).

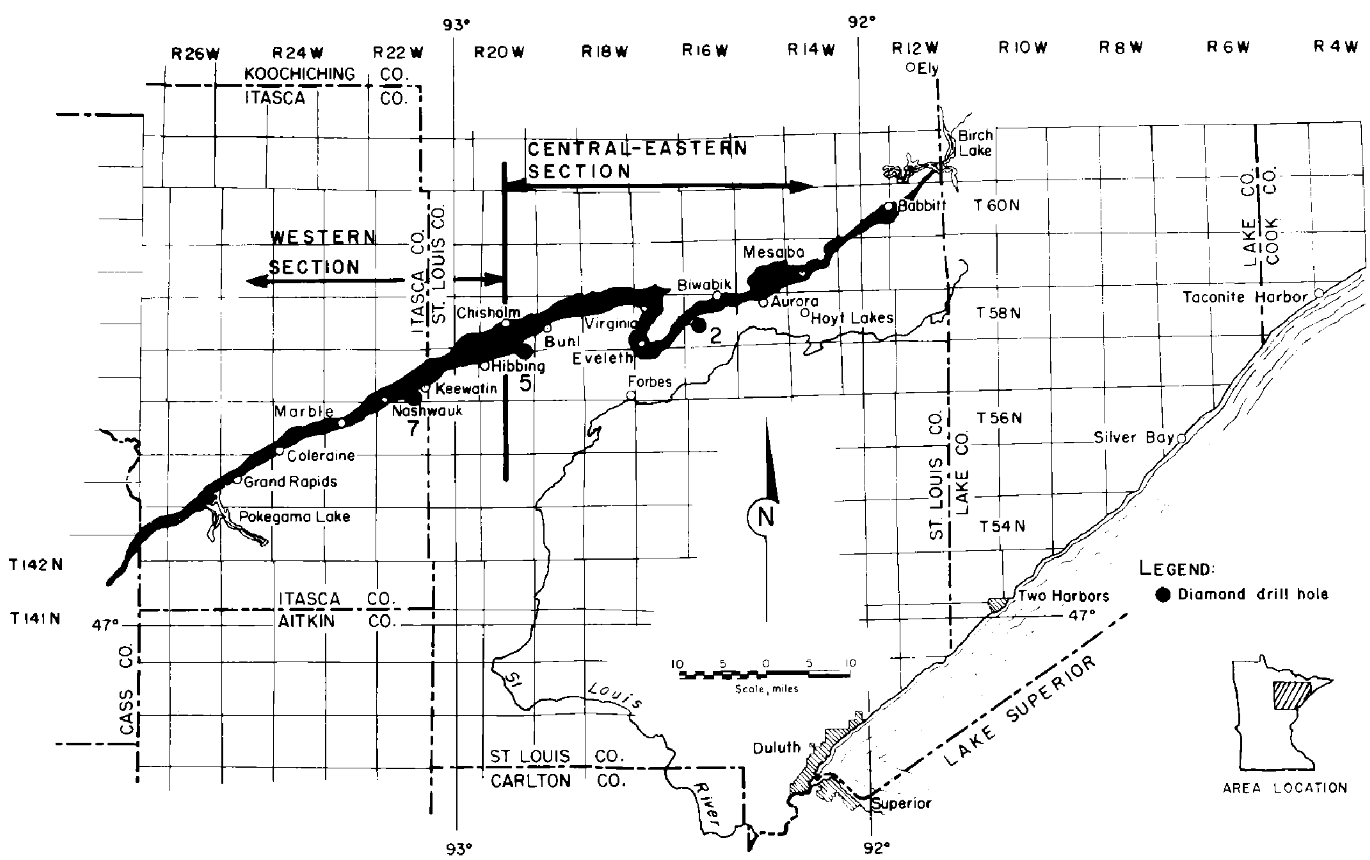

FIGURE 1. - Index Map of the Mesabi Range, Minnesota.

[4]Underlined numbers in parentheses refer to items in the list of references preceding the appendixes.

The Biwabik iron formation is the middle of three conformable sedimentary units of the Animikie Group of middle Precambrian Age. The other two units are the overlying Virginia Slate and the underlying Pokegama Quartzite. A generalized section of the Biwabik iron formation is shown in figure 2.

The Animikie beds occupy the north limb of the Lake Superior syncline. They strike generally N 75° E and dip on the average about 10 pct (6°) to the southeast. They are overlain unconformably by glacial drift that is 0 to 20 ft thick at the north outcrop, but often is over 100 ft thick a mile to the south. The thickness of this drift increases southward at an average rate of about 20 ft per 1,000 ft.

The Biwabik iron formation averages about 600 ft in thickness and is subdivided into four members: The Lower Cherty, Lower Slaty, Upper Cherty, and Upper Slaty (table 1). The formation collectively consists of various varieties of iron-bearing rock (taconite), either cherty or slaty, consisting principally of cherty quartz, iron silicates, iron oxides, and siderite.

Mesabi Range Iron Ores

Classification

Mesabi Range iron ores have been classified into two main types: natural ores and taconite ores.

Natural Ores.--Natural ores are those that are merchantable either without any processing or that can be made merchantable with processes that do not require grinding the ore to less than 20-mesh (0.0328-in. diam). These processes include washing, jigging, and heavy media separation. Natural ores, the weathered and enriched portion of the Biwabik iron formation, are largely composed of hematite, limonite, and silica. These ores range from completely loose and uncemented to well-cemented material; they have been subclassed into direct shipping or merchantable ore, wash ore, and heavy media ore.

Taconite Ores.--Taconite is classified mineralogically and/or structurally as cherty taconite, slaty taconite, mottled taconite, and "irregular or wavy banded" taconite (10). For this study, however, taconite is classified as magnetic or nonmagnetic, depending on its response to one or more beneficiation processes. The two varieties may be differentiated by several chemical and mineralogical characteristics, the most important being the chemical composition of its essential iron oxide constituent.

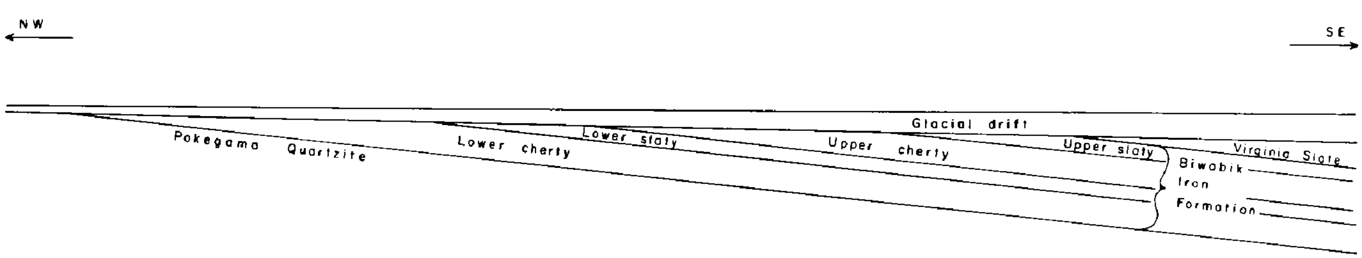

FIGURE 2. - Generalized Cross Section of the Biwabik Iron Formation.

TABLE 1. - Generalized section of subdivisions of the Biwabik formation[1]

Division	Thickness, feet
Upper Slaty division:	
Limy carbonate with silicates and slaty taconite	0 to 25
Slaty and cherty taconite, silicate taconite, and slate. On East Mesabi white quartz septaria and some beds rich in magnetite in lower 40 feet	0 to 145
Upper Cherty division:	
Cherty, banded, slaty, and silicate taconite with layers of conglomerate and algal structures. Some beds rich in magnetite. On East Mesabi many rich magnetite beds	80 to 250
Lower Slaty division:	
Slaty and silicate taconite. Slate, banded, and some cherty taconite. Considerable carbonate and scattered conglomerate in upper part. On East Mesabi largely recrystallized to amphibolitic taconite with some fayalite	0 to 250
"Intermediate Slate." Black slate, silicate slate, and paint rock	1 to 40
Lower Cherty division:	
Lean member. White cherty and silicate taconite	10 to 52
Member rich in magnetite. Irregularly banded, mottled, cherty, and silicate taconite. On East Mesabi 2 to 70 ft thick	2 to 250
Member with iron in ferric state:	
Beds of cherty and banded taconite with slaty taconite on top. Largely absent on East Mesabi	8 to 70
Red basal taconite. Absent on East Mesabi	0 to 40
Basal conglomerate and algal structures	0 to 15
Total	350 to 755

[1]Gruner, John W. The Mineralogy and Geology of the Taconite and Iron Ores of the Mesabi Range, Minnesota. Office of the Commissioner of the Iron Range Resources and Rehabilitation, St. Paul, Minn., 1946, 122 pp.

Magnetic taconite has as its principal iron oxide the naturally magnetic mineral magnetite, and therefore can be concentrated by magnetic separators. Nonmagnetic taconite, on the other hand, requires other processing such as reductive roasting to convert its iron oxide constituents (usually martite, hematite, and goethite) to magnetite before it can be concentrated by magnetic separators. The iron oxide may also be concentrated by nonmagnetic processes such as flotation.

For this evaluation, taconite is not necessarily classified as either magnetic or nonmagnetic on the basis of its present amenability to commercial concentration. The classifications are not mutually exclusive, even for a given deposit or quality of taconite. Some deposits might be economically amenable to both a magnetic and a nonmagnetic concentration process while other deposits might be amenable only to one. Applying some minimum quality criteria, the potential economic availability of both varieties are related to different levels of "cost of supply." Consequently, the entire resource base was subjected to two independent analyses: first, for the magnetic taconite process; and second, for the nonmagnetic process.

Reserves

Natural Ore Reserves.--Mining during the past 75 years has nearly depleted Mesabi natural ore reserves (fig. 3). With the decrease in natural ore reserves, taconites have been contributing increasingly larger quantities of iron ore concentrates to the Nation's economy, and taconites now occupy the dominant position in iron ore inventory.

In 1910, Mesabi Range natural ores were estimated to be 1.6 billion tons (27), an extremely conservative figure. Figure 3 shows the annual reported reserve estimates since 1910 and the cumulative natural ore shipments up to 1967. The difference in the slopes reflects the new reserves resulting from new discoveries and advances in technology. The lines converge in 1952, and since then the estimated reserves have been depleted by mining by at least a 1:1 ratio. This condition indicates that no new natural ore reserves are being reported. The cumulative shipment curve dipping below the estimated reserve curve between 1952 and 1963 indicates additional depletion of the reserves by increased quality standards for the product.

The 1967 natural ore reserve estimate of approximately 300 million tons may seem large until one considers that the annual Mesabi production of iron ore exceeds 50 million tons and has exceeded 75 million tons. The future significance of natural ores as a source of iron ore is obviously limited. It is likely that all Mesabi Range iron ore will soon be produced from the taconites. Although the production of iron ore pellets from taconite did not begin on a commercial scale until 1956, by 1968 the iron ore from taconite accounted for 59 pct of the total produced on the Mesabi Range (fig. 4). By 1975, the projected percentage will approach 100 pct. Although taconite pellets are more costly to produce than natural ore concentrates, their characteristic superiority as a blast furnace feed make them a more desirable product.

Taconite Reserves.--Mesabi Range taconites have been estimated as sufficient to supply the crude ore required to produce in excess of 57 billion tons of concentrates (5). However, without knowing the costs, such an estimate is of little use in evaluating the deposit as a future producer of iron ore in competition with other domestic and foreign sources.

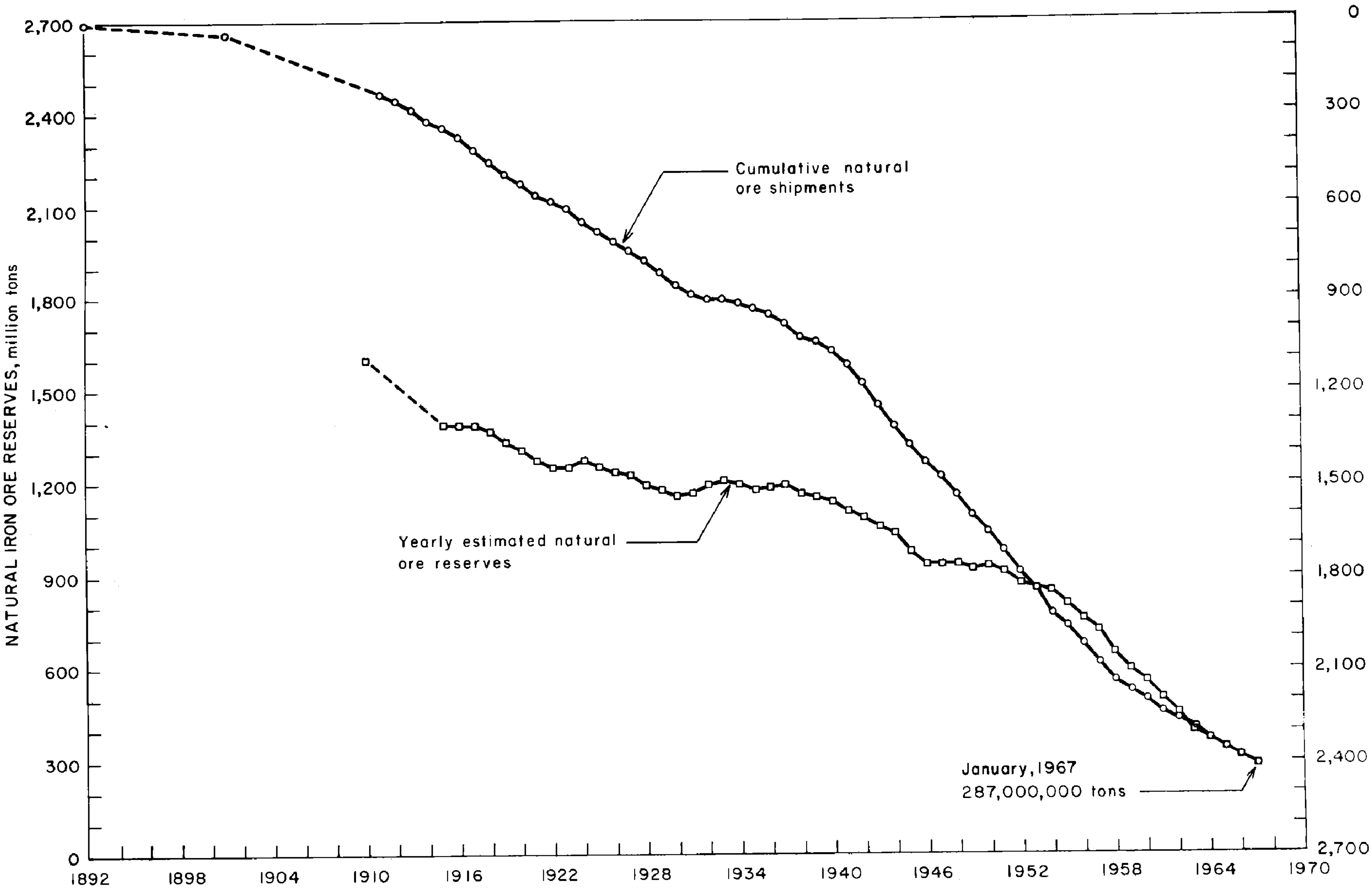

FIGURE 3. Annual Reserve Estimates and Cumulative Shipments, Mesabi Range Natural Iron Ore.

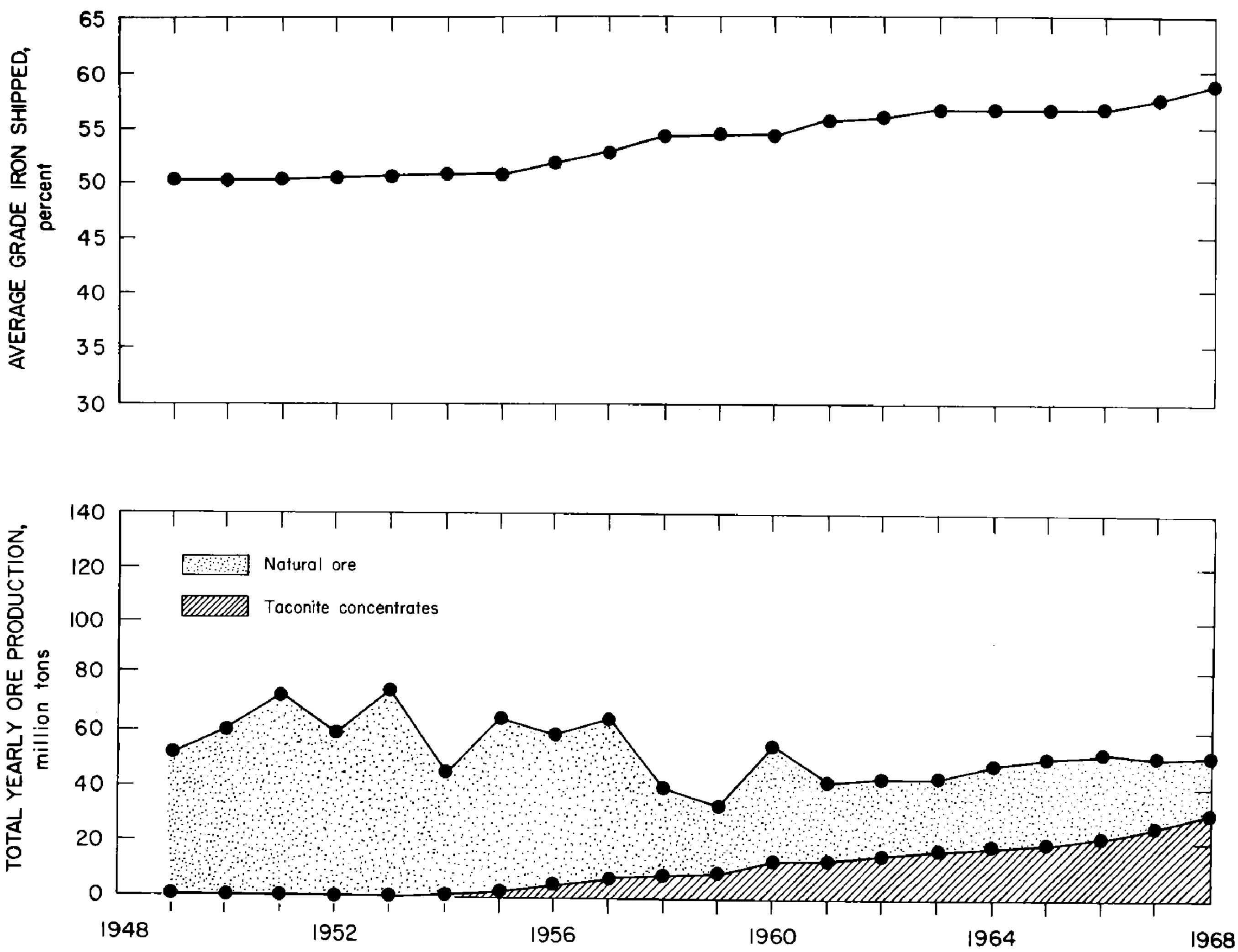

FIGURE 4. - Iron Ore Production From the Mesabi Range.

LIMITS OF THE EVALUATION

This evaluation is generalized; a detailed evaluation of Mesabi Range taconite resources would require much data not presently available. Paradoxically, the vastness of the iron resources allows valid and significant generalization. By statistical inference, a relatively small amount of data, based on actual operations and tested against past research and production performance, is used to represent several billion tons of ore. Since production from such a large resource cannot be typified by a single operation, "typical operations" are simulated with "typical production costs" to derive processing costs. Cost factors that may vary from one operation to the next, such as railroad transportation, State and local taxes, and royalties, are based on present production averages.

Several limiting assumptions are incorporated to develop a "worst possible case" approach. The generalized evaluation is validated by confining it to the areas and horizons within the scope of the data and by considering past production, independent research, and mining trends of new operations. As more data become available, these assumptions may be modified and the scope of the evaluation extended without distorting its basic methodology. Without specifying this "worst possible case" approach, several of the study's assumptions might be matters for considerable controversy.

Economic availability estimates were based on only one beneficiation process, reduction roasting using iron turnings as a reductant, probably one of the least economical processes available (14). It was used for the following reasons:

1. Test results for reduction roasting and magnetic separation are the only nonmagnetic taconite resource data available in adequate amount and areal distribution.

2. The Bureau of Mines Process Evaluation Group has made a detailed capital and cost evaluation[5] for reduction roasting and magnetic separation using iron turnings. The only similarly documented cost evaluation, also by the Process Evaluation Group, uses essentially the same method except that auto scrap is used instead of iron turnings. Since the nonmagnetic processing method that will ultimately be used will probably be less costly than the reduction roasting method, the cost-supply curve from this evaluation should set an upper limit estimate of the future cost of supply, while the magnetic taconite process costs set a lower limit.

Physical Limits of the Resource Evaluation

Areal Extent

Although the Mesabi Range is approximately 120 miles long, the evaluation was confined to a 77-mile strip extending from west of Coleraine, Minn., northeastward to Mesaba, Minn., for the following reasons:

1. All the available data originate within this area. The east and west boundaries are considered the limits to which the data can be reasonably extrapolated.

2. The greatest reserve potential lies within the area, and all future taconite operations will probably be within it. Only one magnetic taconite mining operation is outside the area, nearly 10 miles to the east, and a second taconite operation straddles the eastern boundary. All other operations have been located to the west. The two easternmost operations mine a greater proportion from the Upper Cherty member, while the other operations mine primarily from the Lower Cherty. The two eastern operations are producing a lower quality product with 1968 natural iron content averages of 60.22 and 61.83 pct, and silica content averages of 8.43 and 6.98 pct for Reserve and Erie Mining Companies, respectively (2). The four other operations produce pellets averaging in iron content from 61.68 to 64.64 pct; but, more

[5]Process Evaluation Group. Rept. No. 69-8: Pellets From Non-magnetic Taconite and Iron Tailings, a Cost Analysis of a 1.5-Million-Ton-per-Year Plant. Open File Rept., Dec. 6, 1968, 24 pp. Available for consultation at Morgantown Research Center, Bureau of Mines, Morgantown, W. Va.

important, the silica content averages only 4.61 to 5.38 pct (2). Lower silica content is desired by the iron industry, and iron ore quality standards have been rising in recent years.

3. While considerable magnetic taconite reserves are found in the Upper Cherty formations east of the area evaluated, the significant nonmagnetic taconite reserve potential is in the Lower Cherty formation (7, 10, 18). The Lower Cherty thins out to the east and the greatest potential for nonmagnetic taconite development is the western section (fig. 1) of the 77-mile strip. The western boundary of the area is set by lack of data.

Stratigraphic Unit

The evaluation considers only the Lower Cherty member of the Biwabik iron formation, which, it is generally agreed, has the greatest potential as a future reserve. The overlying Upper Cherty member presents more problems in concentrating because complex silicates and carbonates are more frequently encountered, and it lacks the consistency needed for a systematic evaluation. Concentratable taconites found in the Upper Cherty member when stripping for the Lower Cherty member below will serve as conservative elements in interpreting both costs and quantities in the final evaluation.

METHODOLOGY

The economic availability evaluation, requiring computerized statistical and simulation techniques, uses three mathematical models:

1. Quality model.
2. Discounted cash flow (DCF) cost model.
3. Economic availability model.

By correlating all of the resource quality and dimensional factors with the cost factors associated with producing iron ore pellets, these three models establish a final relation between iron ore reserve quantities and iron unit cost.

Quality Evaluation

Quality evaluation involves the identification of the physical resource factors that influence the economic availability of iron ore, their quantification, and their distribution in the Mesabi Range (table 2).

Quality Evaluation Model

The quality model describes the quantification of the quality resource factors (fig. 5). A series of computer programs calculated these factors from published data derived by testing the response of many samples to various extraction processes. Output from this model--recoverable iron content cumulative probability distribution, concentrate iron content, and ore probability factor--is used in either or both the cost model and economic availability model as described later.

TABLE 2. - Quality evaluation factors

Factor	Unit	Symbol
Input:		
Exploration data:		
Number of samples	Number	N
Vertical feet of sample	Feet	F(I) (I = 1,N)
Concentrate silica content	Percent	$C_{S(I)}$ (I = 1,N)
Concentrate iron content	...do	$C_{I(I)}$ (I = 1,N)
Grade of crude	...do	G(I) (I = 1,N)
Distribution factor	...do	R(I) (I = 1,N)
Magnetic iron content	...do	M_G(I) (I = 1,N)
Potential ore quality criteria:		
Concentrate silica maximum	...do	S_{MX}
Recoverable iron content minimum	...do	I_{MW}
Output:		
Ore probability factor	Decimal	A_5
Recoverable iron content cumulative probability function.	(Numerical coefficients)	$F(A_1)$
Concentrate iron content	Percent	A_2

Exploration and Quality Test Data Factors

Number of Samples (N).--The variable N is the total number of samples from a specific member of the Biwabik iron formation in the particular area being evaluated.

Vertical Feet of Sample (F).--The factor F weights the influence of individual samples which are not of equal thickness. Each foot of sample is considered a single occurrence of a particular associated quality test value, such as magnetic iron content. A 5-ft sample, therefore, represents five occurrences in constructing a relative frequency relationship from which the probability distributions are estimated.

Concentrate Silica Content (C_S).--The concentrate silica content factor expresses the results of a taconite sample subjected to a standardized test simulating an actual production processing operation. Silica content is related to grain size and textural characteristics that determine how finely the taconite must be ground to liberate the iron oxide particles from the quartz or iron silicate particles.

Grade of Crude (G).--The grade of crude (percent iron content) is used only in the calculations for evaluating nonmagnetic taconite. Although there are significant vertical variations and local variations where the taconite merges into the natural ore deposits, the lateral distribution of iron content in the cherty members is relatively constant--about 30 pct iron across the range (table 3).

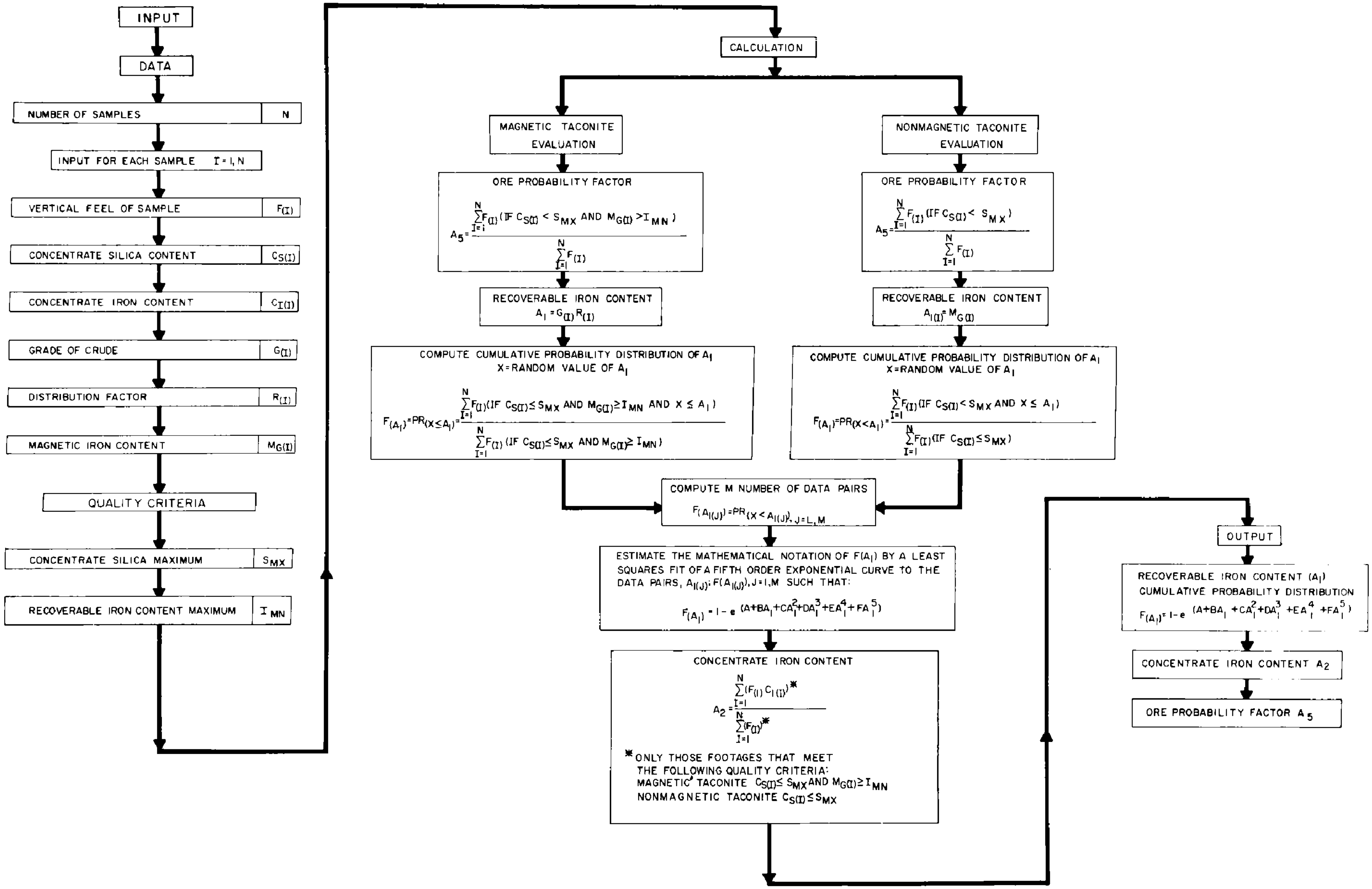

FIGURE 5. - A Summary Flow Diagram of the Quality Evaluation Model.

TABLE 3. - Grade of crude ore, Biwabik iron formation[1]

Section	Ore grade, pct iron		
	Upper Cherty	Lower Slaty	Lower Cherty
Western....................	29.2	23.3	30.6
Central-eastern............	30.5	27.7	30.2

[1]Averaged from all nonmagnetic composite samples (7, 16).

Distribution Factor (R).--The distribution factor (percent iron recovery) expresses the fraction of the total iron in the crude sample that can be recovered in the concentrate by a particular concentration process. The distribution factor was used only in the evaluation of nonmagnetic taconite. The product of the distribution factor and grade of crude is the recoverable iron content (percent) of the sample, one of the outputs of the quality evaluation.

Magnetic Iron Content (MG).--The magnetic iron content is the iron in the crude ore that can be separated by magnetic processes, after grinding to a specified fineness, expressed as a percentage of total crude. Because the iron recovery in a magnetic taconite plant approximates the recovery attained in laboratory tests, magnetic iron content is for all practical purposes equal to the recoverable iron content of the ore the sample represents.

Potential Ore Quality Criteria

The potential ore quality criteria are the acceptable limits of concentrate silica and recoverable iron beyond which the sample is no longer considered representative of a potential iron ore. This applies only to the response to a particular process. That is, samples from a deposit may show no potential for magnetic concentration but may test very well in the nonmagnetic taconite procedure. All samples that met the quality evaluation criteria were considered "potential ore" and, unless otherwise noted, subsequent probability distributions and averages calculated in the model reflect the results of only those samples. The rest were rejected and considered as waste.

Concentrate Silica Maximum (SMX).--The 10 pct cutoff maximum used here for concentrate silica content is consistent with other Mesabi Range taconite resource evaluations (7, 16, 18). Currently, concentrates containing as much as 10 pct silica are not commercially acceptable. With a maximum cutoff limit of 10 pct, however, the average or expected value of the concentrate silica would be considerably less than 10 pct. The concentrate silica maximum was the only criterion for establishing the ore probability factor for nonmagnetic iron.

Recoverable Iron Content Minimum (IMW).--The reduction roasting process reduces nearly all of the iron oxide to magnetite, yielding a much higher iron recovery than for magnetic taconite where only the natural magnetite is concentrated. The concentrates from the roasted samples of nonmagnetic taconite with silica contents under 10 pct almost always had recoverable iron contents exceeding 25 pct. Although the process for concentrating magnetic taconite may produce a high-quality concentrate (silica content, <4 pct; iron content,

>68 pct) from taconite having less than 10 pct magnetic iron content, this low recovery ore cannot be processed at a profit. The minimum recoverable iron content is an economic rather than a quality cutoff factor. An arbitrary cutoff of 17 pct recoverable iron content was used in deriving the magnetic taconite ore probability factors. Although a 17-pct or even an 18-pct ore would probably not compete economically with the higher grades, their inclusion does not detract from the analysis.

Quality Evaluation Output Factors

The quality evaluation model output consists of the following factors: ore probability factor, recoverable iron content cumulative probability function, and concentrate iron content.

Ore Probability Factor (A_5).--The ore probability factor is the probability that a random sample from a specific deposit meets the quality criteria that specifies its consideration as either magnetic or nonmagnetic taconite ore.

This factor is calculated as the proportion of the number of occurrences (sample footage) that meet the quality criteria (minimum recoverable iron and/or maximum concentrate silica content) to the total number of occurrences.

For nonmagnetic taconite the ore probability factor is the probability that ore from a specific source (iron formation member in a particular area) will meet the 10 pct concentrate silica cutoff. For magnetic taconite the criterion of recoverable iron minimum must be met also.

When the ore probability factor is finally applied to the economic availability evaluation of the resource, it establishes the parts of the deposit to be computed as crude ore and as waste rock, which must be stripped. It is also used to determine whether specific members of the formation have any real ore reserve potential.

Recoverable Iron Content Cumulative Probability Function ($F(A_1)$).--Recoverable iron content is the percentage of the total crude sample that can be recovered as iron. It can be considered as the effective grade. In evaluating magnetic taconites, recoverable iron content is considered equal to magnetic iron content as determined by the laboratory test; in evaluating nonmagnetic taconite, it is calculated by taking the product of the grade of crude and the distribution factor.

For a measured ore deposit, the estimated average recoverable iron content would serve well as input to the DCF cost model to find the average iron unit cost for mining the deposit. The evaluation of the Mesabi Range taconites, however, requires a more open-ended approach since the limit of the resource, at least for the next century, is determined only by costs. Recoverable iron content determined from the quality data is an input factor to both the cost and economic availability evaluation models which establish a unit cost-quantity relationship. This relationship is greatly affected by the natural variations of the recoverable iron contents (appendix A). The relation between iron unit cost and quantity can be represented as a curve connecting a series of single point evaluations for each input value including recoverable iron content. To effectively evaluate the economic availability of iron from Mesabi Range taconites, not only the average but the full range of probabilities of recoverable iron content must be considered. Recoverable iron content is therefore output as a cumulative probability function.

The distribution function for recoverable iron content (A_1) is approximated from sample tests by using the following techniques:

a. The sample values are sorted into equal grade intervals by recoverable iron content. In a sufficiently large sample, the proportion of values falling within each interval to the total number of values will be an estimate of the probability density function (fig. 6).

b. The cumulative relative frequency for each grade interval is calculated:

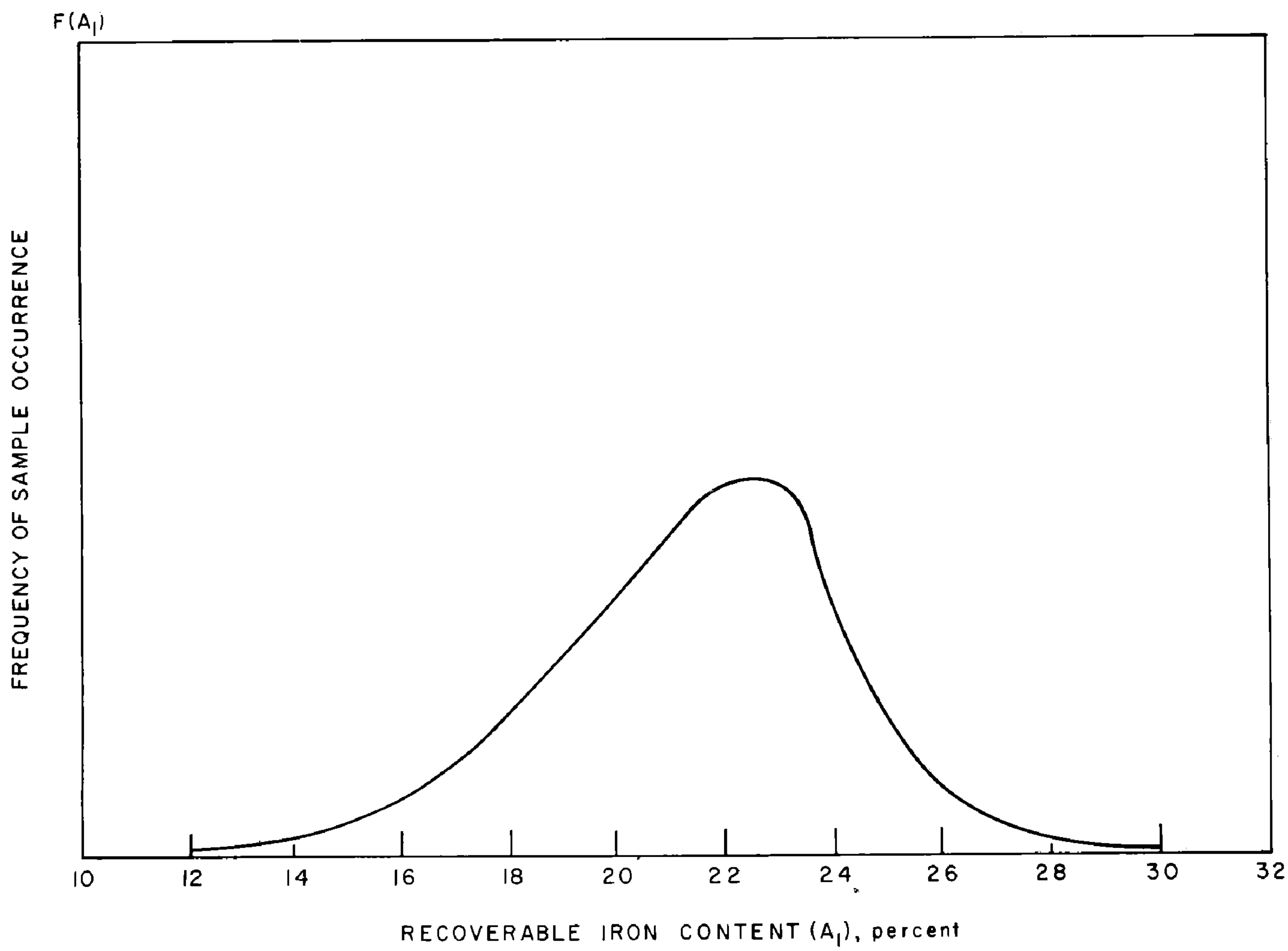

FIGURE 6. - Typical Probability Density Function of Recoverable Iron Content.

c. The mathematical notation of the probability distribution function (fig. 7) is approximated from the cumulative relative frequency.

The numerical form of the cumulative relative frequency is represented by a series of A_1, Y coordinates where

$A_{1(I)}$ = grade,

$Y_{(I)}$ = the proportion of values greater or equal to $A_{1(I)}$.

By a computer curve-fitting routine, the numerical representation can be readily converted to a mathematical notation that will closely approximate the actual cumulative probability function. The curve used to approximate the cumulative probability function provides an adequate approximation in all instances and is easily derived with the computer:

$$F(A_1) = 1-e^{(A+BA_1+CA_1^2+DA_1^3+EA_1^4+FA_1^5)}$$

= cumulative probability function of recoverable iron content ($F(A_1)$) is output as the six coefficients of the exponential function: A, B, C, D, E, and F. The maximum error between actual and calculated cumulative relative frequency relates to allowable difference in grade of less than 0.5 pct.

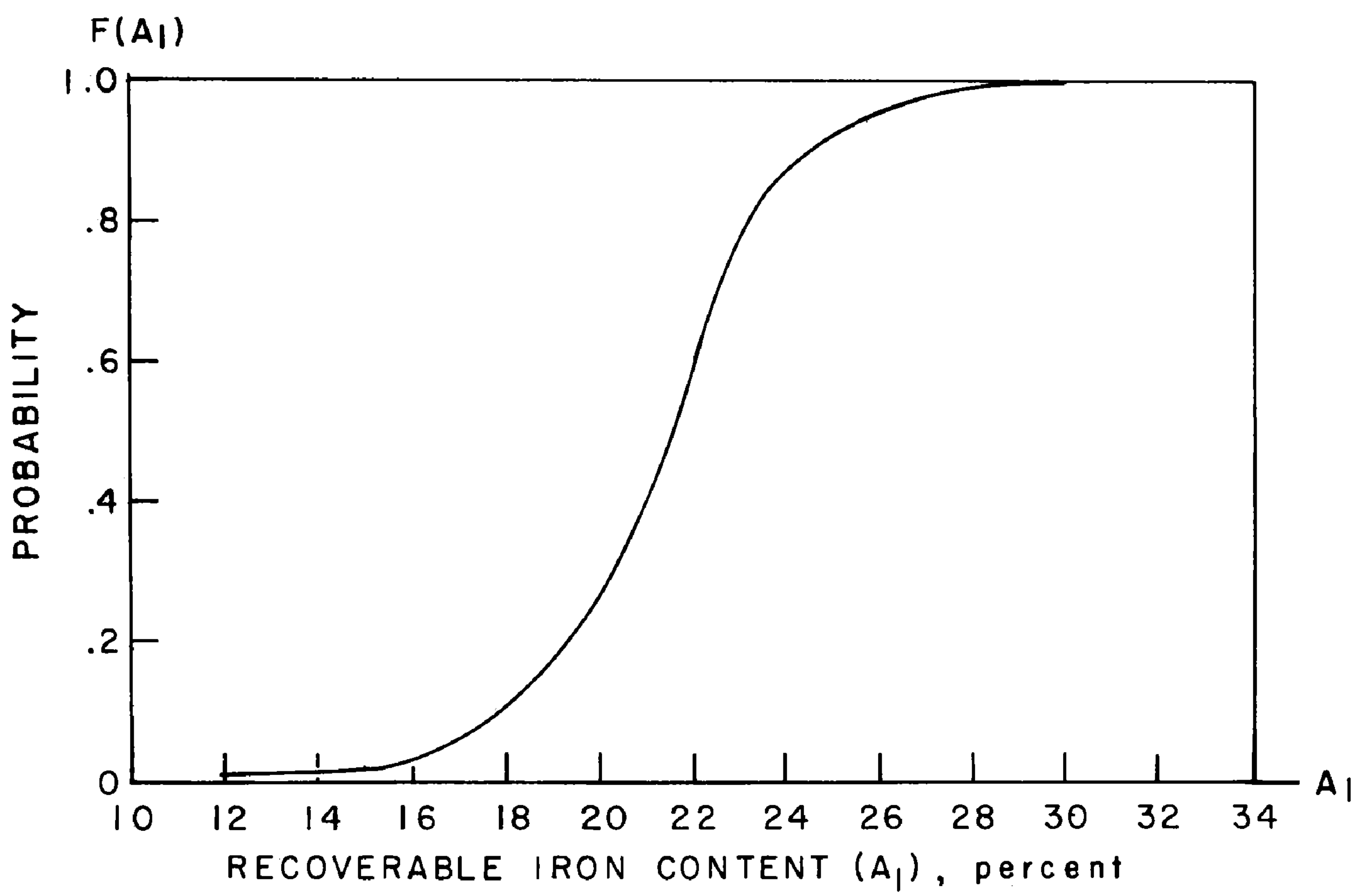

FIGURE 7. - Typical Cumulative Probability Function of Recoverable Iron Content.

The cumulative probability function is derived only from those samples meeting the quality criteria--maximum concentrate silica and minimum recoverable iron. It will be shown in a following section how this function is particularly suited to the application of the Monte Carlo simulation technique to analyze the effect of the probability distribution of recoverable iron content upon the iron unit cost-quantity relationship.

Concentrate Iron Content (A_2).--The concentrate iron content is the weighted average concentrate iron from all of the samples meeting the quality criteria.

The concentrate iron content thus determined must be adjusted slightly to represent the final pellet iron content since the pellet contains moisture. Because of moisture, the actual concentrate iron is approximately 2 pct less than the weighted average calculated from the dried samples.

Quality Evaluation of Mesabi Range Taconites

Table 4 shows values of weighted averages of economically significant quality data of Mesabi Range Lower Cherty ore samples. Each sample was from a 5- or 6-ft vertical drill core or a cut in a pit bank for nonmagnetic taconites; however, the analyses are of composites of two or more original samples combined to represent a vertical dimension up to 40 ft. While each sample represents a particular horizon within a member, they are considered to be random samples of specific blocks of the area being evaluated. The results of tests on the samples are used collectively to define the probability distributions of the quality factors in a given area.

TABLE 4. - Average Lower Cherty quality data for potential ore[1]

Sample	Grade of crude, pct (G)	Recoverable iron content, pct[2] (A_1)	Concentrate silica content, pct (C_s)	Concentrate iron content, pct[3] (A_2)
MAGNETIC RESOURCES				
Hole:				
No. 2	30.5	24.6	6.91	63.5
No. 5	28.8	18.8	3.10	63.5
No. 7	30.3	21.4	1.90	63.5
NONMAGNETIC RESOURCES				
Section:				
Western	32.3	30.4	7.8	65.0
Central-eastern	32.8	30.9	7.7	66.3

[1]Averages based only on samples that met quality evaluation criteria ($A_1 \geq 17$ and/or $C_s \leq 10$).

[2]Magnetic taconite: A_1 is the weighted average magnetic iron content. Nonmagnetic taconite: A_1 is the weighted average of the calculated product of the grade of crude and the distribution factor.

[3]For magnetic taconites this value was derived as the weighted average from all 3 holes adjusted from dry iron content to natural iron content.

The assumption of randomness was made with considerable caution, the statistical pitfalls being recognized, but the limited data available made this the only practical technique for making a comprehensive quality evaluation.

Although the 350 sample composites from 7,700 vertical ft of pit bank were from a belt stretching 75 miles along the strike of the Biwabik iron formation, there may still be the question as to whether they are true random samples of the entire Biwabik iron formation. The samples were necessarily taken from the banks of natural iron ore mines, and are therefore more truly samples of at least partially oxidized taconite, that is taconite not altered and enriched to natural ore, but at least partly oxidized so as not to be completely fresh taconite either. However, the results obtained by considering the samples as random samples of the entire Biwabik iron formation seem reasonable (see appendix B).

Magnetic Taconite Data

Magnetic iron content, the factor of primary importance in analyzing the magnetic taconites, was not adequately analyzed in past production and development testing. Results from exploration and development drilling in magnetic taconite deposits are generally unavailable to the Bureau of Mines. Nearly all of the Bureau's research effort on the Mesabi Range taconites concerned evaluation of the nonmagnetic taconites.

A progress report published in January 1968 by the Minnesota Geological Survey and the University of Minnesota School of Mineral and Metallurgical Engineering (21) discusses four holes drilled south of the Mesabi Range outcrop (fig. 1). The Biwabik iron formation dips well under the surface at the location of the holes and is encountered at depths ranging from less than 500 ft to more than 1,600 ft. Results of metallurgical tests on over 1,900 ft of Biwabik iron formation core from three of the holes (2, 5, and 7) included the factors needed for a quality evaluation (no metallurgical testing was reported on the fourth hole).

The procedure for testing the quality of magnetic taconite consisted of stage crushing to minus 150-mesh and magnetic separation. The difference in the degree of fineness might seem very significant; however, the fraction passing 150-mesh sieve contains a considerable quantity that also passes the 325-mesh sieve, on the average over 80 percent. The amount of data at first appears insufficient on which to base a generalized evaluation of the Mesabi Range magnetic taconite, but their use exclusively for the quality evaluation is justified for several reasons:

1. These were the only metallurgical data on magnetic taconite available in significant quantity.

2. Even though the sample locations are at depth, several thousand feet down-dip from the present open-pit operation, the Biwabik iron formation is fairly consistent in thickness and mineralogical composition along the dip of the formation (22).

3. In retrospect, these magnetic taconite data gave a weighted average magnetic iron content of 20.8 pct in the economic availability evaluation, nearly identical to the average recoverable iron content from several past years' actual magnetic taconite production (table 5). This equivalence is the best evidence that can be offered that the data are reliably representative of the magnetic taconite deposits considered in a generalized evaluation. It is further assumed that if the average value is a good estimate, the complete probability distribution is likewise reasonably accurate.

TABLE 5. - Recoverable iron content and concentrate iron content calculated from past years magnetic taconite production data

Year	Production: Crude ore (tons)	Production: Concentrate (tons)	Concentrate[1] iron content (pct)	Recoverable[2] iron content (pct)
1962...........	44,820,352	14,734,766	61.25	20.14
1963...........	50,166,649	16,486,601	61.60	20.24
1964...........	55,146,845	18,298,333	61.41	20.04
1965...........	54,956,359	18,898,183	61.17	21.04
1966...........	62,819,017	21,599,420	60.99	20.97
1967...........	75,874,402	24,326,683	[3]61.61	19.8
1968...........	90,309,482	30,255,395	[3]62.02	20.8
Weighted average 1962-68................			61.48	20.5

[1] University of Minnesota Mining Directories.
[2] Calculated by:

$$\text{Recoverable iron content} = \frac{\text{tons conc.}}{\text{tons crude}} \times \text{conc. iron content.}$$

[3] Bureau of Mines.

Nonmagnetic Taconite Data

As with the magnetic taconite evaluation, the data most relevant to the quality evaluation of nonmagnetic taconites are those developed by test procedures simulating the beneficiation process being considered. For the economic availability evaluation of nonmagnetic taconites, a reductive roasting and magnetic separation process is the only beneficiation process considered.

Considerable research has been done by the U.S. Bureau of Mines and others to analyze the nonmagnetic taconite resources of the Mesabi Range (3, 6-7, 16, 18, 20, 23, 27-28). Two Bureau of Mines papers (7, 16) report the response of several hundred taconite samples to reductive roasting, fine grinding, and magnetic separation. The samples represent locations well distributed over the Mesabi Range.

In this evaluation, the Mesabi Range is divided at Chisholm into a western section and a central-eastern section (fig. 1).

Western Section.--Evaluation data for the western section of the Mesabi Range were developed in a U.S. Bureau of Mines sampling and testing program (7, 18). Of 898 samples collected between Grand Rapids and Chisholm, Minn.

(fig. 1), 865 were from open-pit bank faces, and 33 were taken from rock dumps. Over 3,900 vertical ft of bank was sampled at different locations. The 898 field samples were combined into 159 composites for metallurgical evaluation. Each of the composites included samples from only one of the four members of the iron formation.

The composite crude samples averaged 30 pct iron and 52 pct silica. While the iron content ranged from as little as 6 pct to as much as 45 pct, the 30 pct average is consistent with most studies of Mesabi Range taconites.

The beneficiation potential of the composite samples was analyzed by the following procedure:

1. A portion of a sample was crushed to minus 8-mesh.

2. The minus 8-mesh material was roasted at 500° C in hydrogen to reduce the iron oxide to a magnetic form.

3. The roasted ore was then ground to one or more sizes in the minus 100- to minus 400-mesh range.

4. The roasted ground ore was magnetically separated.

The quality evaluation used principally the results from tests made on material ground to minus 325-mesh, but if grinding did not proceed to minus 325-mesh and if acceptable silica content was found at the coarser grinds, these data were included. Seven of the composites were rejected from the evaluation because they represented rock dumps and their relative influence on the other composite from one in situ could not be accurately weighted.

The input data for the quality evaluation model cover the following five factors from each composite sample:

1. The number of vertical feet (N) represented by the composite.

2. Concentrate silica content (SMX), the SiO_2 analysis of the Davis Tube concentrate.

3. Concentrate iron content (A_2), the Fe analysis of the Davis Tube concentrate.

4. The iron content of the crude (G) taconite sample.

5. The distribution factor (R), the percentage of the total amount of iron recovered in the Davis Tube concentrate after roasting.

Central-Eastern Section.--The quality evaluation data for the central-eastern half of the Mesabi Range were developed in a similar U.S. Bureau of Mines research program (16). Between Chisholm and Aurora, Minn. (fig. 1) 771 5-ft vertical bank samples were taken and combined into 191 composites for metallurgical evaluation with each composite originating from a specific member of the Biwabik iron formation. The metallurgical procedure used to evaluate the beneficiation potential was the same as that described for the western composites.

Mesabi Range Quality Evaluation Output

Ore Probability Factors (A_5).--The distribution of ore probability factors, associated with both magnetic and nonmagnetic taconite processing, were derived for the Upper Cherty, Lower Slaty, and Lower Cherty members of the Biwabik iron formation (table 6). The evaluation demonstrated that taconite with an ore probability factor of less than 0.15 has little effect on total resource quantity because the concentrate becomes available at a relatively high cost. This cost factor alone eliminated all but the Lower Cherty members as potential ore reserves for the nonmagnetic process. On the other hand, the ore probability factors relating to the magnetic taconite evaluation indicate a significant potential reserve in the Upper Cherty member. Economic Upper Cherty horizons found in stripping will result in a more favorable cost-quantity relationship for the Lower Cherty member.

TABLE 6. - Ore probability factors (A_5)

Taconite samples	Member		
	Upper Cherty	Lower Slaty	Lower Cherty
Magnetic:			
Hole No.:			
7	0.417	0.000	0.430
5	.080	.000	.220
2	.330	.000	.670
Nonmagnetic:			
Section:			
Western	.082	.000	.358
Central-eastern	.130	.015	.244

Recoverable Iron Content Cumulative Probability Function ($F(A_1)$).--The output for recoverable iron content is expressed as a fifth order exponential curve that approximates the cumulative probability function for both the magnetic and nonmagnetic evaluations (figs. 8 and 9).

Concentrate Iron Content (A_2).--The average Lower Cherty values for concentrate iron content have been given in table 4. For this evaluation, however, past production performance was taken into account (table 5), and values of 62 pct and 65 pct were used for the magnetic and nonmagnetic taconite evaluations, respectively.

Cost Evaluation

Cost evaluation involves identifying and quantifying all the factors, including profit, that collectively determine the total cost of producing a product.

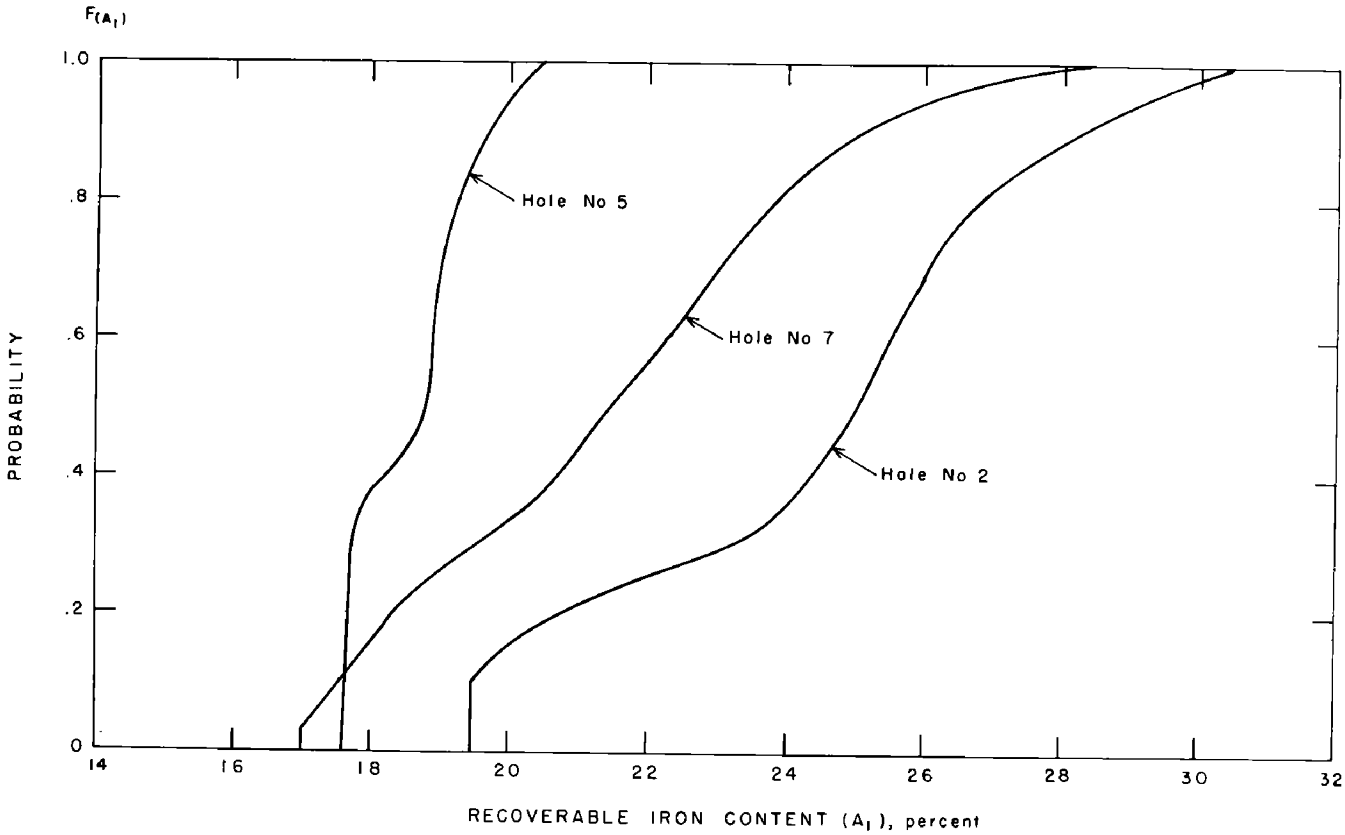

FIGURE 8. - Cumulative Probability Distribution of Recoverable Iron Content, Magnetic Taconite.

Discounted cash flow analysis, commonly used to evaluate a business' profit-making potential (17, 30), can also analyze the effects of varying physical and economic conditions upon the real cost of supplying a mineral commodity. If the flow of cash, discounted at the specified rate of return on capital investment, is just adequate to repay the investment over the life of the venture, the net present value of the venture is zero. Under this condition, the unit cost of producing the mineral will be exactly equal to the selling price required to calculate a zero net present value.

Discounted Cash Flow (DCF) Cost Evaluation Model

Procedure

The DCF cost evaluation model (fig. 10) mathematically relates all the cost factors for producing iron ore pellets to determine iron unit cost. Whereas the ore quality evaluation model is used only once to determine ore quality factors either as probability functions or probability values, the cost evaluation model is used repeatedly.

Singular Evaluation.--In the singular evaluation, or one time evaluation of each drill hole section, costs are calculated for each section as called for by the economic availability model described later. For each drill hole section, the economic availability model generates a random value for recoverable iron content. This necessitates the calculation of iron unit costs for a

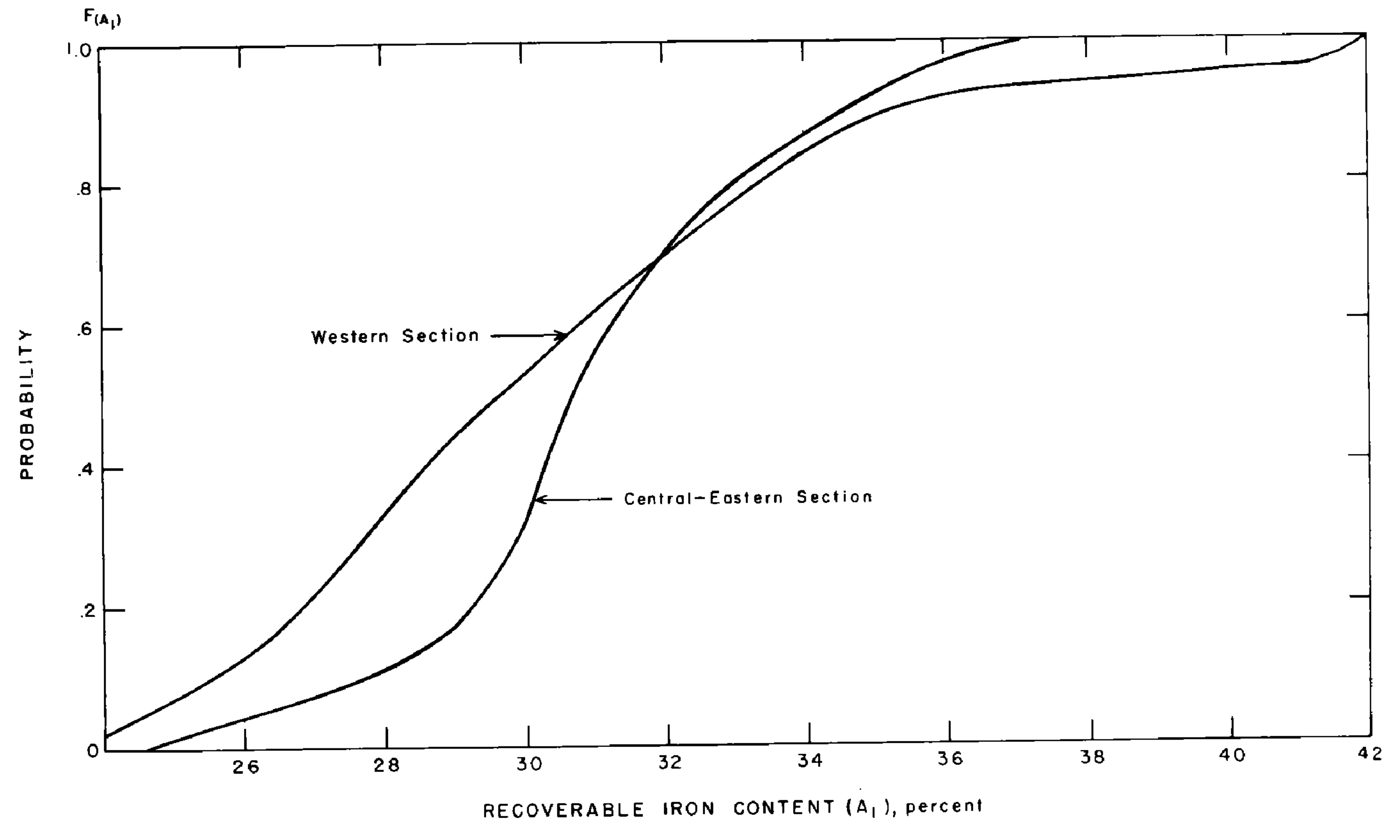

FIGURE 9. - Cumulative Probability Distribution of Recoverable Iron Content, Nonmagnetic Taconite.

range of equivalent stripping ratios with a different randomly selected recoverable iron content for each section.

Monte Carlo Repetition.--In the Monte Carlo simulation routine, the singular evaluation is repeated a number of times and the results appraised until statistical confidence of the results is satisfactory.

Cost Evaluation Model Input

The DCF cost evaluation model uses 19 input factors. Eighteen of the factors are used to calculate the annual cash flow over the expected life of a magnetic taconite processing operation. The nineteenth variable specifies the method for calculating the depreciation allowance in the cash flows.

The component cost variables (table 7) include the collective arrays of component costs that are used in part to evaluate the magnetic and nonmagnetic taconite resources. They are used in total to provide a basis of comparison for evaluating the relative influence of each factor on total unit cost (see appendix A).

TABLE 7. - Component cost input to DCF cost evaluation model

Component	Symbol	Units	Input values[1]	
			Magnetic taconite	Nonmagnetic taconite
Capital investment	K	Dollars	60 million	43 million
Working capital	WK	...do	2.76 million	1.9585 million
Plant capacity	P	Tons per year (crude)	6 million	2.8 million
Production payments:				
Mining	C_1	Dollars per ton (crude)	0.40	0.40
Beneficiation	C_2	...do	.85	2.90
Supervision and management	C_3	...do	.17	.42
Pelletizing	C_4	Dollars per ton (product)	1.15	1.15
Stripping	C_5	Dollars per cu yd (equivalent surface).	.40	.40
Transportation	C_6	Dollars per ton (product)	3.80	3.80
Franchise payments:				
Average State and local taxes.	S_1	...do	.31	.31
Royalty costs	S_2	...do	.65	.65
Computation factors for Federal income tax:				
Federal income tax rate	F_1	Percent (taxable income)	50	50
Gross rate for depletion	F_2	Percent (gross income for depletion)	15	15
Net rate for depletion	F_3	Percent (net income for depletion)	50	50
Cash flow parameters:				
Return on investment	I	Percent (capital investment)	12	12
Repayment period	N	Years	25	25
Resource variables:				
Recoverable iron content	A_1	Percent (crude)	21	[2]34.6
Concentrate iron content	A_2	Percent (concentrate)	62	65
Equivalent stripping ratio	A_3	Cu yd (equivalent surface) per ton (product).	1.	1.
Depreciation method indicator.	L	L = 1, 2, or 3 (straight line, double declining, or sum of years digits methods, respectively).	1	1

[1]Cash cost values were derived principally from the following publications:

Hammes, J. K. The Economics of Producing and Delivering Iron Ore Pellets From North American Taconite. Proc. of the 27th Annual Mining Symposium, University of Minnesota, 1966.

Alm, Mildred. Mining Directory Issue, University of Minnesota Bulletin No. 14, v. 71, Mines Experiment Station, Minn., 1960-68, 255 pp.

Process Evaluation Group. Report No. 69-8, Pellets From Nonmagnetic Taconite and Iron Tailings, a Cost Analysis of a 1.5 Million Ton per Year Plant. Morgantown Research Center, Bureau of Mines, Morgantown, W. Va.

[2]The nonmagnetic taconite process uses iron turnings as a reductant. The recoverable iron content of 34.6 pct is derived from a natural recoverable iron content of approximately 31 pct plus the estimated amount of iron incorporated into the ore from the turnings.

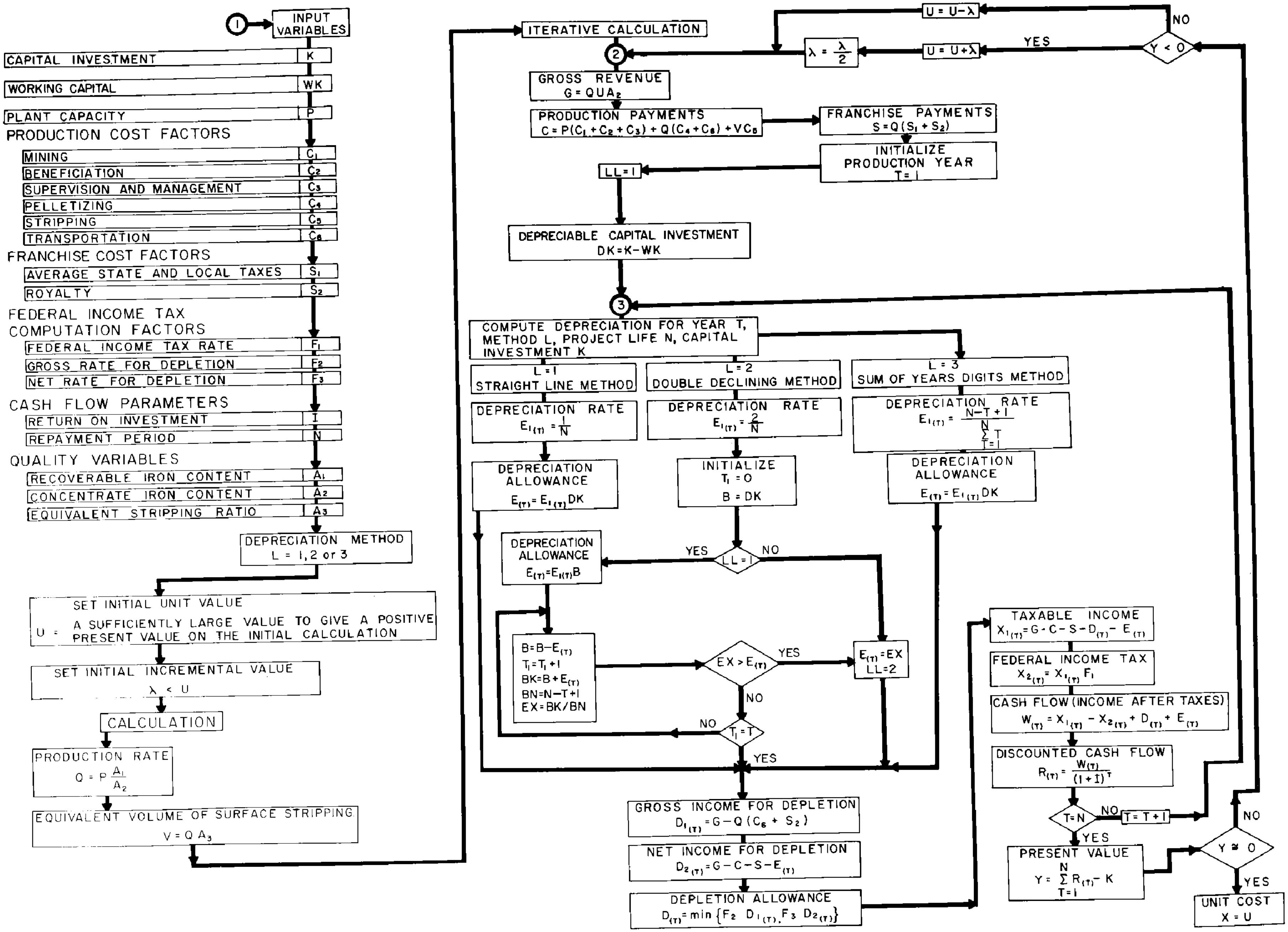

FIGURE 10. - Flow Diagram of DCF Cost Evaluation Model.

Capital Investment (K).--For a generalized evaluation, capital investment includes (1) the initial investment for the mining operation and processing plant, (2) initial working capital, and (3) total accrued interest before production. Any additional investment normally required during the project life is either projected back on the initial investment or allocated to the operation and maintenance costs. Another assumption, also due to the generalized nature of the evaluation, is that none of the capital investment is borrowed from commercial sources but that all is equity capital earning the full rate of return. Hence, no financial or interest payments are considered.

Working Capital (WK).--Working capital includes the operating cash for building inventories of raw materials and products, and maintaining an accounts receivable account. Because it is an undepreciable item in income tax calculations, it must be subtracted from the capital investment when figuring depreciation allowances.

Plant Capacity (P).--Taconite plant capacity, commonly defined as annual production in long tons of pellets, is defined here as annual crude ore processing capacity in long tons. The quantity of concentrate production may then vary slightly depending on iron recoverability characteristics of the ore. For a given processing method, the ratio of new plant capacity to capital investment is held constant so that the size of the operation is not significant, assuming, of course, that it is large enough to be efficient.

Mining Costs (C_1).--Mining costs include only the following variable direct costs for delivering the ore to the first processing station: Operating and maintenance labor, and supplies for drilling, blasting, loading, and hauling. Mining costs do not include the stripping of waste materials or franchise payments.

The total cost for mining is related to the amount of crude ore actually processed; and the variable, C_1, is introduced in the cash flow evaluation as dollars per ton of crude ore.

Beneficiation Costs (C_2).--Beneficiation costs are those from the production of the concentrate from taconite ore. They include the operating labor and supplies and maintenance labor and supplies expended in crushing, grinding, sizing, and magnetic separation, and in addition for nonmagnetic taconite, reductive roasting.

Supervision and Management (C_3).--Costs associated with supervision and management are assumed to relate to the size of the operation and are input as average dollars per ton of crude processed.

Pelletizing Cost (C_4).--The labor and supplies costs associated with producing pellets from a taconite concentrate include the costs of blending, balling, and indurating.

The variable, C_4, is input as dollars per ton of pellet production.

Stripping Cost (C_5).--The variable labor and supply cost of stripping (removing of waste material associated with mining) is expressed as dollars per cubic yard of equivalent surface material. The removal of rock waste is considered to be twice as costly as the removal of unconsolidated waste. The cost of stripping rock is calculated at the rate for stripping unconsolidated overburden by doubling the rock volume in the calculation of the equivalent stripping ratio.

Transportation Cost (C_6).--Transportation cost, input as dollars per ton of concentrate, is the average charge for loading, hauling, handling, and delivering taconite pellets to lower lake ports.

State and Local Taxes (S_1).--This component cost variable is a composite of the following taxes:

1. Taconite tax or production tax,
2. Occupation tax,
3. Royalty tax,
4. Excise taxes.

These taxes are calculated either as rates per ton produced or as a percentage of income. The composite tax structure varies for different operations since ore may be produced from State-owned land, privately owned land, and company-owned deposits.

For a generalized evaluation, the best approximation of State and local taxes is the average current tax payments (1). State and local taxes are input as average dollars per ton of product.

Royalty Cost (S_2).--Royalty is the money paid for mining iron ore from deposits owned by other individuals, corporations, or by the State. It is generally paid on a per ton of concentrate basis with certain annual minimums.

Federal Income Tax Rate (F_1).--Federal income tax rate is the rate that is applied to the taxable income of the operating company. The maximum corporate rate of 50 pct is commonly used when evaluating taconite operations.

Gross Rate for Depletion (F_2).--The gross rate for depletion is set by law and is applied to the gross income when calculating the depletion allowance. The rate is 15 pct for iron ore production.

Net Rate for Depletion (F_3).--The net rate for depletion is similar to the gross rate, except that it applies to the net income for depletion. The rate is 50 pct for iron ore production.

Return on Investment (I).--Return on investment is the interest rate used in discounting the annual cash flows.

Repayment Period (N).--Repayment period is the assumed life of the project and is the number of years in which the project pays back the original capital investment.

Recoverable Iron Content (A_1).--Recoverable iron content is the product of the crude ore iron content and the percent iron recovery. For example, if crude ore iron analysis is 30 pct, and 70 pct of the total iron is recovered by a magnetic concentration process, the recoverable iron content is 21 pct (70 pct x 30 pct = 21 pct). Recoverable iron content of magnetic taconites is approximately the same as the magnetic iron content determined by laboratory test.

Concentrate Iron Content (A_2).--Concentrate iron content is the iron analysis of the pellets. Since the maximum concentrate silica content, the other major component in iron ore pellets, is subject to industry standards, the concentrate iron content must fall within a relatively short range of values.

In the iron ore industry, pellets are bought and sold on an iron unit basis. Defined as 1 pct of a long ton, a unit of iron is 22.4 pounds of available metal. The concentrate iron content is the number of iron units in one ton of pellets.

Equivalent Stripping Ratio (A_3).--As used in iron mining on the Mesabi Range, the equivalent stripping ratio reflects the volume, of waste material in cu yds that must be stripped to produce one ton of concentrate. Since rock is about twice as expensive to remove as unconsolidated surface material, equivalent volume of surface stripping is the number of cu yds of unconsolidated material plus two times the number of cu yds of rock. The equivalent stipping ratio, then, is the cu yds of equivalent surface stripping per ton of concentrate produced and is a function of the amounts of glacial drift, overlying rock, rock associated with the ore,and recoverable iron content.

Depreciation Method Selector (L).--The depreciation method selector, L, designates whether the value of the plant (depreciable capital investment) will be depreciated by the straight line, double declining, or sum of years digits methods (1, 2, or 3, respectively). The straight line method (L=1) is used for evaluating long range investments; the relative effect is discussed later in the sections on "Depreciation Rate" and "Depreciation Allowance."

Intermediate Accounting Calculations

Between the component cost input and the total cost per iron unit output, several identifiable steps are defined as intermediate accounting variables. The objective variable in intermediate accounting is the net present value, Y, of the venture. Table 8 summarizes the intermediate accounting variables and their interrelations. To fully comprehend the model, however, each variable must be defined or interpreted as it is used in the evaluation.

Iron Unit Value (U).--Iron unit value is the value of one iron unit at lower lake ports, generally implying the actual selling price. In the DCF model, however, it is an initially assumed value, incremented positively or negatively, to calculate a zero net present value.

TABLE 8. - Intermediate accounting variables in DCF cost evaluation model

Intermediate accounting variables	Symbol	Units	Formula
Unit value..................	U.....	Dollars per iron unit	
Production rate.............	Q.....	Tons per year........	$P\frac{A_1}{A_2}$
Stripping, equivalent surface volume.	V.....	Cu yd per year.......	QA_3
Gross revenue...............	G.....	Dollars per year.....	QUA_2
Production payments.........	C.....	...do................	$P(C_1+C_2+C_3)+Q(C_4+C_6)+VC_5$
Franchise payments..........	S.....	...do................	$Q(S_1+S_2)$
Production period...........	T.....	Year.................	$(T=1,N)$
Depreciable capital investment.	DK....	Dollar...............	$DK=K-WK$
Depreciation rate...........	$E_1(\tau)$.	Decimal for year T...	Straight line method $\frac{1}{N}$ Double declining method $\frac{2}{N}$ Sum of years digits method $\frac{N-T+1}{\sum_{T=1}^{N} T}$
Depreciation allowance......	$E(\tau)$..	Dollars in year T....	Straight line method $E_1(\tau)DK$ Double declining method $E_1(\tau)\left(DK - \sum_{J=2}^{N} E(J-1)\right)$ Sum of years digits method $E_1(\tau)DK$
Gross income for depletion..	$D_1(\tau)$.	...do................	$G-Q(C_6+S_2)$
Net income for depletion....	$D_2(\tau)$.	...do................	$G-C-S-E(\tau)$
Depletion allowance.........	$D(\tau)$..	...do................	$\min\{F_2D_1(\tau);F_3D_2(\tau)\}$
Taxable income..............	$X_1(\tau)$.	...do................	$G-C-S-E(\tau)-D(\tau)$
Federal income tax..........	$X_2(\tau)$.	...do................	$F_1X_1(\tau)$
Cash flow...................	$W(\tau)$..	...do................	$X_1(\tau)-X_2(\tau)+E(\tau)+D(\tau)$
Discounted cash flow........	$R(\tau)$..	...do................	$\frac{W(\tau)}{(1+I)^T}$
Net present value...........	Y.....	Dollars..............	$\sum_{T=1}^{N} R(\tau)-K$

Production Rate ($Q = PA_1/A_2$).--The quantity of pellets (tons) produced in one year is the product of the total crude processed and the recoverable iron content divided by the concentrate iron content.

Equivalent Stripping Volume ($V = QA_3$).--The quantity of pellets produced times the equivalent stripping ratio is the equivalent stripping volume. It is the number of cu yds of equivalent surface stripped for 1 year's production.

Gross Revenue ($G = QUA_2$).--The gross value of 1 year's production is the product of quantity of pellets, lower lake port unit value, and the number of iron units per ton of the concentrate pellets.

Production Payments ($C = P(C_1+C_2+C_3)+Q(C_4+C_6)+VC_5$).--Production payments are the annual total cost of operating labor, and supply and maintenance labor, and supply associated with stripping, mining, beneficiation, pelletizing, transportation, and supervision and management.

Franchise Payments ($S = Q(S_1+S_2)$).--Franchise costs, the charges for the privilege of operating, include the total annual State and local taxes and royalty costs.

Production Period ($T=1,N$).--Several accounting variables which are a function of depreciation allowance may vary from year to year in the DCF model except with the straight line depreciation method. T identifies the year in the repayment period. All capital investment is assumed to have occurred by the end of year 0. All other financial transactions in the cash flow determination are assumed to have occurred at the end of the year for which calculations are being made. In actual practice, some cost factors such as franchise costs do not actually become expenditures until the following year, in year T+1. Since production and franchise rates are constant in each evaluation, the advancement need not be made.

Depreciable Capital Investment ($DK=K-WK$).--Depreciable capital investment is the capital investment minus working capital.

Depreciation Rate ($E_{1(T)}$).--The depreciation rate is the average rate that applies to the depreciable capital investment or its balance at the end of year T to compute the depreciation allowance. The rate varies with the depreciation method.

Straight Line Method.--The depreciation is the same each year over the repayment period;

$$E_{1(T)} = \frac{1}{N}.$$

Double Declining Method.--The rate is double that of the straight line method;

$$E_{1(T)} = \frac{2}{N}.$$

Sum of Years Digits Method.--The depreciation rate varies from year to year. It is the highest in the first year and is calculated from 1 to N. For example, if the project repayment period is 10 years, the sum of years is

1+2+3...+10 = 55. The first year's rate is calculated to be (10-1+1)/55 = 10/55, the second year's rate 9/55, and so on;

$$E_1(\tau) = \frac{N-T+1}{\sum_{T=1}^{N} T} .$$

Depreciation Allowance ($E(\tau)$).--The depreciation allowance is the amount deductible from the annual income to compensate for the depreciating value or the wearing out of the plant when calculating Federal income tax. The initial value of the plant, that is capital investment minus working capital, is effectively "written off" over the repayment period according to one of the legally sanctioned formulas. The depreciation rates for three methods used in the DCF model have been discussed. The annual depreciation allowance is calculated by the three methods as follows:

Straight Line Method.--The depreciation allowance does not vary from year to year. It is simply the depreciation rate times the depreciable capital investment;

$$E(\tau) = E_1(\tau)DK.$$

Double Declining Method.--The allowance is greater in the early years of the repayment period. The rate does not vary but applies only to the balance of the depreciable capital investment after all of the previous years' "writeoffs" have been deducted;

$$E(\tau) = E_1(\tau)\left(DK - \sum_{J=2}^{T} E(J-1)\right) .$$

Because the Internal Revenue Service allows a change to straight line depreciation at any time, the model changes from the use of double declining depreciation to straight line depreciation when the use of straight line depreciation results in a greater depreciation amount.

Sum of Years Digits Method.--The allowance is greater in the early years of the repayment period because the rate is higher;

$$E(\tau) = E_1(\tau)DK.$$

Gross Income for Depletion ($D_1(\tau) = G-Q(C_8+S_2)$).--Referred to in the tax manuals as "gross income from mining," the gross income for depletion is computed as gross revenue minus annual transportation and royalty charges.

Net Income for Depletion ($D_2(\tau) = G-(C+S+E(\tau))$).--The net income for depletion is the net revenue before depletion and Federal tax. It is the gross revenue minus all production, franchise, and depreciation costs.

Depletion Allowance ($D(\tau) = \min \{F_2 D_1(\tau); F_3 D_2(\tau)\}$).--In a sense, the depletion allowance is a depreciation allowance that applies to the resource rather than to equipment and facilities. The percentage depletion allowance for iron ore production is computed by taking the gross rate for depletion times gross income for depletion, or the net rate for depletion times net income for depletion, whichever is lower. Cost depletion (relating to either the cost of the deposit or the value on a particular day divided by the total remaining units in the deposit to give depletion cost per ton) was not used in this evaluation because of the serious disadvantage: ore deposits, generally, cannot be estimated as precisely as this depletion method requires.

Taxable Income ($X_1(\tau) = G-C-S-E(\tau)-D(\tau)$).--The taxable income is the net income subject to Federal income tax. It utilizes all of the deductions allowed for computing net income for depletion purposes, and in addition incorporates the depletion allowance.

Federal Income Tax ($X_2(\tau) = F_1 X_1(\tau)$).--The Federal income tax is the taxable income times the Federal income tax rate.

Cash Flow ($W(\tau) = X_1(\tau)-X_2(\tau)+E(\tau)+D(\tau)$).--The annual cash flow is the taxable income minus Federal income tax plus the depreciation and depletion allowances.

Discounted Cash Flow $\left(R(\tau) = \frac{W(\tau)}{(1+I)^{\tau}} \right)$.--The discounted cash flow is the present value of the cash flow in year T after discounting by the return on investment I

Net Present Value $\left(Y = \sum_{T=1}^{N} R(\tau) - K \right)$.--The net present value is the difference between the sum of all discounted cash flows and the capital investment.

Solution for Unit Cost (X) by Iterative Computation

Since the return on investment is considered a real cost to the economy, it is one of the component cost variables determining total iron unit cost. Therefore, when the net present value of a venture is zero, the unit value U in the DCF model is exactly equal to the unit cost. In the model, the unit value is given an arbitrary initial value and is thereafter systematically incremented until it results in a calculated net present value of approximaterly zero. Unit cost X is then set equal to unit value U.

The basic output from the model is unit cost. Final annual values for several intermediate accounting variables are also available as output providing better comprehension of the evaluation. These include the annual depreciation and depletion allowances, taxable income, Federal income tax, cash flow, and discounted cash flow. Tables 9 through 14 show schedules of the cash flows as determined by each of the three depreciation methods. For program efficiency, the model is designed to converge net present value on zero only until the error in unit cost becomes insignificant.

TABLE 9. - Discounted cash flow evaluation of magnetic taconite with straight-line depreciation[1]

(Capital investment--60 million dollars; working capital--2.76 million dollars; rate of return--12 percent)

Year, T	Cash flow (million dollars)								
	Gross revenue, G	Production cost, C	Franchise cost, S	Depreciation allowance, E(T)	Depletion allowance, D(T)	Taxable income, X1(T)	Federal income tax, X2(T)	Cash flow, W(T)	Discounted cash flow, R(T)
1........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	6.829
2........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	6.098
3........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	5.444
4........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	4.861
5........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	4.340
6........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	3.875
7........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	3.460
8........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	3.089
9........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	2.758
10........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	2.463
11........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	2.199
12........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	1.963
13........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	1.753
14........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	1.565
15........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	1.397
16........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	1.248
17........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	1.114
18........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	.995
19........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	.888
20........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	.793
21........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	.708
22........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	.632
23........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	.564
24........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	.504
25........	31.051	19.393	1.951	2.290	3.301	4.117	2.058	7.649	.450

[1] Present value of cash flow............. 59.992
Capital investment...................... -60.000
Net present value....................... -0.008
Unit cost (X)....dollars per iron unit.. 0.24636

TABLE 10. - Discounted cash flow evaluation of magnetic taconite with double declining depreciation[1]

(Capital investment--60 million dollars; working capital--2.76 million dollars; rate of return--12 percent)

Year, T	Cash flow (million dollars)								
	Gross revenue, G	Production cost, C	Franchise cost, S	Depreciation allowance, E(T)	Depletion allowance, D(T)	Taxable income, X1(T)	Federal income tax, X2(T)	Cash flow, W(T)	Discounted cash flow, R(T)
1........	30.679	19.393	1.951	4.579	2.378	2.378	1.189	8.146	7.273
2........	30.679	19.393	1.951	4.213	2.561	2.561	1.281	8.055	6.421
3........	30.679	19.393	1.951	3.876	2.730	2.730	1.365	7.970	5.673
4........	30.679	19.393	1.951	3.566	2.885	2.885	1.442	7.893	5.016
5........	30.679	19.393	1.951	3.281	3.027	3.027	1.514	7.821	4.438
6........	30.679	19.393	1.951	3.018	3.159	3.159	1.579	7.756	3.929
7........	30.679	19.393	1.951	2.777	3.245	3.313	1.657	7.678	3.473
8........	30.679	19.393	1.951	2.554	3.245	3.535	1.768	7.567	3.056
9........	30.679	19.393	1.951	2.350	3.245	3.740	1.870	7.465	2.692
10........	30.679	19.393	1.951	2.162	3.245	3.928	1.964	7.371	2.373
11........	30.679	19.393	1.951	1.989	3.245	4.101	2.050	7.285	2.094
12........	30.679	19.393	1.951	1.830	3.245	4.260	2.130	7.205	1.849
13........	30.679	19.393	1.951	1.684	3.245	4.406	2.203	7.132	1.634
14........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	1.452
15........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	1.297
16........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	1.158
17........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	1.034
18........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	.923
19........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	.824
20........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	.736
21........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	.657
22........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	.587
23........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	.524
24........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	.468
25........	30.679	19.393	1.951	1.613	3.245	4.476	2.238	7.097	.417

[1] Present value of cash flow.............. 59.999
Capital investment...................... -60.000
Net present value....................... -0.001
Unit cost (X)....dollars per iron unit.. 0.243481

TABLE 11. - Discounted cash flow evaluation of magnetic taconite with sum of years digits depreciation[1]

(Capital investment--60 million dollars; working capital--2.76 million dollars; rate of return--12 percent)

Year, T	Cash flow (million dollars)								
	Gross revenue, G	Production cost, C	Franchise cost, S	Depreciation allowance, E(T)	Depletion allowance, D(T)	Taxable income, X1(T)	Federal income tax, X2(T)	Cash flow, W(T)	Discounted cash flow, R(T)
1........	30.568	19.393	1.951	4.403	2.411	2.411	1.205	8.019	7.160
2........	30.568	19.393	1.951	4.227	2.499	2.499	1.249	7.975	6.358
3........	30.568	19.393	1.951	4.051	2.587	2.587	1.293	7.931	5.645
4........	30.568	19.393	1.951	3.875	2.675	2.675	1.337	7.887	5.012
5........	30.568	19.393	1.951	3.699	2.763	2.763	1.381	7.843	4.450
6........	30.568	19.393	1.951	3.522	2.851	2.851	1.425	7.799	3.951
7........	30.568	19.393	1.951	3.346	2.939	2.939	1.470	7.755	3.508
8........	30.568	19.393	1.951	3.170	3.027	3.027	1.514	7.711	3.114
9........	30.568	19.393	1.951	2.994	3.115	3.115	1.558	7.667	2.765
10........	30.568	19.393	1.951	2.818	3.203	3.203	1.602	7.623	2.454
11........	30.568	19.393	1.951	2.642	3.229	3.354	1.677	7.547	2.170
12........	30.568	19.393	1.951	2.466	3.229	3.530	1.765	7.459	1.915
13........	30.568	19.393	1.951	2.290	3.229	3.706	1.853	7.371	1.689
14........	30.568	19.393	1.951	2.113	3.229	3.882	1.941	7.283	1.490
15........	30.568	19.393	1.951	1.937	3.229	4.058	2.029	7.195	1.315
16........	30.568	19.393	1.951	1.761	3.229	4.234	2.117	7.107	1.159
17........	30.568	19.393	1.951	1.585	3.229	4.411	2.205	7.019	1.022
18........	30.568	19.393	1.951	1.409	3.229	4.587	2.293	6.931	.901
19........	30.568	19.393	1.951	1.233	3.229	4.763	2.381	6.843	.795
20........	30.568	19.393	1.951	1.057	3.229	4.939	2.469	6.755	.700
21........	30.568	19.393	1.951	.881	3.229	5.115	2.558	6.667	.617
22........	30.568	19.393	1.951	.704	3.229	5.291	2.646	6.579	.544
23........	30.568	19.393	1.951	.528	3.229	5.467	2.734	6.491	.479
24........	30.568	19.393	1.951	.352	3.229	5.643	2.822	6.403	.422
25........	30.568	19.393	1.951	.176	3.229	5.820	2.910	6.315	.371

[1] Present value of cash flow............. 60.007
Capital investment..................... -60.000
Net present value...................... 0.007
Unit cost (X)....dollars per iron unit.. 0.242603

TABLE 12. - Discounted cash flow evaluation of nonmagnetic taconite with straight-line depreciation[1]

(Capital investment--43 million dollars; working capital--1.959 million dollars; rate of return--12 percent)

Year, T	Cash flow (million dollars)								
	Gross revenue, G	Production cost, C	Franchise cost, S	Depreciation allowance, E(T)	Depletion allowance, D(T)	Taxable income, X1(T)	Federal income tax, X2(T)	Cash flow, W(T)	Discounted cash flow, R(T)
1........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	4.896
2........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	4.372
3........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	3.903
4........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	3.485
5........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	3.112
6........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	2.778
7........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	2.481
8........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	2.215
9........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	1.977
10........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	1.766
11........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	1.576
12........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	1.408
13........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	1.257
14........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	1.122
15........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	1.002
16........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	.895
17........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	.799
18........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	.713
19........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	.637
20........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	.568
21........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	.508
22........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	.453
23........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	.405
24........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	.361
25........	26.585	18.390	1.431	1.642	2.561	2.561	1.281	5.484	.323

[1] Present value of cash flow............. 43.010
Capital investment..................... -43.000
Net present value...................... 0.010
Unit cost (X)....dollars per iron unit.. 0.274414

TABLE 13. - Discounted cash flow evaluation of nonmagnetic taconite with double declining depreciation[1]

(Capital investment--43 million dollars; working capital--1.959 million dollars; rate of return--12 percent)

Year, T	Cash flow (million dollars)								
	Gross revenue, G	Production cost, C	Franchise cost, S	Depreciation allowance, E(T)	Depletion allowance, D(T)	Taxable income, X1(T)	Federal income tax, X2(T)	Cash flow, W(T)	Discounted cash flow, R(T)
1........	26.417	18.390	1.431	3.283	1.657	1.657	0.828	5.768	5.150
2........	26.417	18.390	1.431	3.021	1.788	1.788	.894	5.703	4.546
3........	26.417	18.390	1.431	2.779	1.909	1.909	.954	5.642	4.016
4........	26.417	18.390	1.431	2.557	2.020	2.020	1.010	5.587	3.550
5........	26.417	18.390	1.431	2.352	2.122	2.122	1.061	5.535	3.141
6........	26.417	18.390	1.431	2.164	2.216	2.216	1.108	5.488	2.781
7........	26.417	18.390	1.431	1.991	2.303	2.303	1.151	5.445	2.463
8........	26.417	18.390	1.431	1.832	2.382	2.382	1.191	5.405	2.183
9........	26.417	18.390	1.431	1.685	2.456	2.456	1.228	5.369	1.936
10........	26.417	18.390	1.431	1.550	2.523	2.523	1.262	5.335	1.718
11........	26.417	18.390	1.431	1.426	2.585	2.585	1.293	5.304	1.525
12........	26.417	18.390	1.431	1.312	2.642	2.642	1.321	5.275	1.354
13........	26.417	18.390	1.431	1.207	2.695	2.695	1.347	5.249	1.203
14........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	1.072
15........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.957
16........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.854
17........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.763
18........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.681
19........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.608
20........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.543
21........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.485
22........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.433
23........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.386
24........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.345
25........	26.417	18.390	1.431	1.157	2.720	2.720	1.360	5.237	.308

[1] Present value of cash flow.............. 42.999
Capital investment...................... -43.000
Net present value....................... 0.001
Unit cost (X)....dollars per iron unit.. 0.272681

TABLE 14. - Discounted cash flow evaluation of nonmagnetic taconite with sum of years digits depreciation[1]

(Capital investment--43 million dollars; working capital--1.959 million dollars; rate of return--12 percent)

Year, T	Cash flow (million dollars)								
	Gross revenue, G	Production cost, C	Franchise cost, S	Depreciation allowance, E(T)	Depletion allowance, D(T)	Taxable income, X1(T)	Federal income tax, X2(T)	Cash flow, W(T)	Discounted cash flow, R(T)
1........	26.368	18.390	1.431	3.157	1.695	1.695	0.847	5.699	5.089
2........	26.368	18.390	1.431	3.031	1.758	1.758	.879	5.668	4.518
3........	26.368	18.390	1.431	2.904	1.821	1.821	.911	5.636	4.012
4........	26.368	18.390	1.431	2.778	1.884	1.884	.942	5.605	3.562
5........	26.368	18.390	1.431	2.652	1.947	1.947	.974	5.573	3.162
6........	26.368	18.390	1.431	2.526	2.011	2.011	1.005	5.542	2.808
7........	26.368	18.390	1.431	2.399	2.074	2.074	1.037	5.510	2.492
8........	26.368	18.390	1.431	2.273	2.137	2.137	1.068	5.478	2.213
9........	26.368	18.390	1.431	2.147	2.200	2.200	1.100	5.447	1.964
10........	26.368	18.390	1.431	2.021	2.263	2.263	1.132	5.415	1.744
11........	26.368	18.390	1.431	1.894	2.326	2.326	1.163	5.384	1.548
12........	26.368	18.390	1.431	1.768	2.389	2.389	1.195	5.352	1.374
13........	26.368	18.390	1.431	1.642	2.453	2.453	1.226	5.321	1.219
14........	26.368	18.390	1.431	1.515	2.516	2.516	1.258	5.289	1.082
15........	26.368	18.390	1.431	1.389	2.579	2.579	1.289	5.257	.961
16........	26.368	18.390	1.431	1.263	2.642	2.642	1.321	5.226	.852
17........	26.368	18.390	1.431	1.137	2.705	2.705	1.353	5.194	.757
18........	26.368	18.390	1.431	1.010	2.768	2.768	1.384	5.163	.671
19........	26.368	18.390	1.431	.884	2.831	2.831	1.416	5.131	.596
20........	26.368	18.390	1.431	.758	2.895	2.895	1.447	5.100	.529
21........	26.368	18.390	1.431	.631	2.958	2.958	1.479	5.068	.469
22........	26.368	18.390	1.431	.505	2.960	3.081	1.541	5.006	.414
23........	26.368	18.390	1.431	.379	2.960	3.208	1.604	4.943	.365
24........	26.368	18.390	1.431	.253	2.960	3.334	1.667	4.880	.321
25........	26.368	18.390	1.431	.126	2.960	3.460	1.730	4.817	.283

[1] Present value of cash flow............. 43.004
Capital investment.................... -43.000
Net present value...................... -0.004
Unit cost (X)....dollars per iron unit.. 0.272168

Cost Evaluation of Mining and Processing Mesabi Range Taconites

Input for Cost Evaluation

Mesabi Range cost data used in the DCF cost evaluation (table 7) were derived from both published and unpublished sources. Some of the derived figures were not in suitable form for direct input into the cost model and had to be adjusted, possibly losing their exact identity with the data in the referenced publication. In other cases, estimates were somewhat modified as a result of impressions gained while analyzing other related information. These modifications and adjustments are clarified as much as possible.

The primary production cost data for evaluating magnetic taconite operations were derived from production cost estimates for seven magnetic taconite operations reported by J. K. Hammes (13), two of which are located on the Mesabi Range. The nonmagnetic taconite production cost data were developed, in part, by the Bureau of Mines Process Evaluation Group, Morgantown, W. Va., for a process for making iron ore pellets from nonmagnetic taconite and iron turnings in a 1.5 million tons pellets per year plant. Other component cost factor input values are averages from the annual University of Minnesota Mining Directories. All of the values used should be interpreted as the authors' best estimate based on the interpretation of all available data. Where cost items vary greatly between specific operations, any value chosen for a generalized evaluation may seem somewhat arbitrary.

Capital Investment (K), Working Capital (W), and Plant Capacity (P). Magnetic Taconite.--The input estimate for capital investment is based on several sources of data which require considerable interpretation. A generalized value for a "typical operation" is particularly difficult to establish, not only because of the physical uniqueness of separate operations but also because the investments for actual mining and processing facilities are incorporated in a total "capital investment" estimate that includes complete townsites, railroad facilities, and other facilities for a full plant complex. An analysis of several Mesabi Range operations (table 15) shows how greatly two capital investment estimates for the same operation can vary. The Erie Mining Company Hoyt Lakes plant is an example of a full plant complex including townsite and rail facilities. The extent to which these factors are included in the estimates accounts for the variation in estimates. Both the lower estimate for the Hoyt Lakes plant and the estimate for the Reserve Mining Company, E. W. Davis Works, undoubtedly include some investment for the support facilities. This fact accounts for the relatively larger capital investment per annual ton capacity over the smaller Butler Taconite and National Steel Pellet plants.

For the generalized evaluation, \$60 million capital investment, which includes working capital of \$2.76 million, is used as an estimate for a plant designed to process 6 million tons of crude ore per year and produce approximately 2 million tons of iron ore pellets. The \$30 per ton of annual concentrate capacity is not only consistent with the data but is a widely quoted figure in the taconite industry (24). The capital investment per annual ton of crude ore capacity, approximately $10, is of greater significance in comparing the capital investment factor than that estimated for nonmagnetic taconite processing. Plant capacity and capital investment are closely interrelated in the generalized evaluation. In this case, plant capacity is no more than a scale factor, and no other relationships are affected in the DCF evaluation, including total unit cost.

TABLE 15. - Capital investments, based on published data, for some Mesabi Range magnetic taconite plants

Plant	Company	Capital investment, million dollars	Concentrate capacity, annual million tons	Capital investment per annual ton of concentrate capacity, dollars	Capital investment per annual ton of crude ore capacity, dollars[1]
E. W. Davis Works	Reserve Mining Co.	[2]337	10.7	31.5	10.5
Hoyt Lakes.......	Erie Mining Co....	[2]438	10.3	42.5	14.2
		[3]350		34.0	11.3
Butler Taconite..	Itasca Pellet & Inland Steel.	[4]60	2.0	30.0	10.0
National Steel Pellet.	National Steel and Hanna Mining.	[4 5 6]65	2.4	27.1	9.0

[1]Estimate based on 3:1 concentration ratio (tons of crude/tons concentrate).
[2]Estimate by J. K. Hammes. The Economics of Producing and Delivering Iron Ore Pellet From North American Taconite Type Resources.
[3]Undated Erie Mining Co. brochure.
[4]The Hanna Miner, May 1967, p. 5.
[5]Skillings Mining Review, July 8, 1967, pp. 1, 4-6.
[6]The Hanna Miner, May 1969, p. 4.

If actual capital investment data have been established for a specific operation, the total amount and actual crude ore capacities can be input to estimate the relative cost efficiencies of large and small plants.

Nonmagnetic Taconite.--Both capital investment and plant capacity input estimates used in the generalized evaluation were derived by the Bureau of Mines Process Evaluation Group. These estimates, based on the nonmagnetic process described in the section, "Limits of the Evaluation," are as follows:

1. Capital investment is $43 million, or $27.8 per annual ton of concentrate capacity. The concentrate ratio for this process, about 1.9, makes the capital investment about $15.4 per annual ton of crude ore processed. This value is logically comparable to the corresponding $10 per annual ton for the less complex magnetic plant.

2. Working capital included in the capital investment above was estimated to be $1.9585 million, approximately 4.6 percent of the capital investment.

3. Plant capacity is 2.8 million tons of crude ore per year. This is calculated as the product 1.5 million tons of pellets and a concentration ratio of approximately 1.9 which has been derived from quality data discussed in a following section of the report.

Mining Cost (C_1).--The mining cost estimate is identical for magnetic and nonmagnetic taconite economic availability evaluations. It is based on Hammes' cost estimates for magnetic taconite operations on the Mesabi Range (table 16).

TABLE 16. - Cost data for mining taconite estimated for two Mesabi Range operations,[1] in dollars per ton of crude

Mining	Erie Mining Co.	Reserve Mining Co.
Stripping..............................	$0.20	$0.10
Drilling..............................	.09	.09
Blasting..............................	.08	.10
Loading..............................	.07	.06
Haulage..............................	.14	.14
General..............................	.06	.05
Total..............................	.64	.54

[1]Hammes, J. K. The Economics of Producing and Delivering Iron Ore Pellets From North American Taconite. Proc. of the Twenty-Seventh Annual Mining Symposium, University of Minnesota, 1966, pp. 9-16.

Since stripping cost is calculated separately in the DCF model, it is deducted from the estimated mining cost totals given in table 16. The resulting cost is $0.44 per ton of crude. Further, the use of larger and more efficient mining equipment was assumed, reducing this estimate to $0.40 per ton of crude ore mined.

Beneficiation Cost (C_2).--The input estimate for beneficiation cost varies greatly between the magnetic and nomagnetic taconite evaluations because the processing of nonmagnetic taconite requires more sophisticated beneficiation methods. Although several processes are technically feasible for processing nonmagnetic taconite, only a reduction roasting process was considered for reasons discussed in the introduction of this report.

Magnetic Taconite.--The beneficiation cost input for magnetic taconite is also based on Hammes' estimates (table 17). The estimated cost for the beneficiation input variable is $0.85 per ton of crude ore.

TABLE 17. - Cost data for beneficiating magnetic taconite estimated for two Mesabi Range operations[1]

Beneficiation	Erie Mining Co., dollars per ton (crude)	Reserve Mining Co., dollars per ton (crude)
Coarse grinding........................	0.10	0.11
Fine grinding........................	.16	.17
Concentration........................	.58	.55
Total........................	.84	.83

[1]Hammes, J. K. The Economics of Producing and Delivering Iron Ore Pellets From North American Taconite. Proc. of the Twenty-Seventh Annual Mining Symposium, University of Minnesota, 1966, pp. 9-16.

Nonmagnetic Taconite.--The beneficiation cost estimate, \$2.90 per ton, was derived from the Bureau of Mines Process Evaluation Group production cost estimates for the nonmagnetic process discussed previously.

Supervision and Management Cost (C_3).--The greater complexity of a nonmagnetic taconite beneficiation plant is reflected in the supervision and management costs per ton of crude ore processed.

Magnetic Taconite.--The supervision and management cost is derived from Hammes' overhead costs of \$0.45 and \$0.40 per ton of pellet for the Erie Mining Co. plant and Reserve Mining Co. plant, respectively (13). The estimate for DCF cost evaluation input, \$0.50 per ton of pellet, is \$0.167 per ton of crude ore processed.

Nonmagnetic Taconite.--The supervision and management cost factor value is derived from the indirect cost item estimated by the Bureau of Mines Process Evaluation Group. The indirect cost of \$1,193,600 is divided by the 2.8 million tons of crude ore to give approximately \$0.42 per ton of crude ore processed.

Pelletizing Cost (C_4).--The input for pelletizing cost is based on Hammes' estimate of \$1.15 per ton of pellets for both Reserve Mining Co. and Erie Mining Co., and is used for magnetic and nonmagnetic taconite.

Stripping Cost (C_5).--Hammes' stripping cost estimates are \$0.20 and \$0.10 per ton of crude ore for Erie Mining Co. and Reserve Mining Co., respectively (table 16). At a concentration ratio of 3:1, these values become \$0.60 and \$0.30 per ton of pellets. A stripping cost of \$0.40 per cu yd of equivalent surface at an equivalent stripping ratio of 1:1, gives a stripping cost of \$0.40 per ton of pellets, well within the estimated range of \$0.30 and \$0.60 per ton of pellets.

Transportation Cost (C_6).--Hammes (13) gives a rather detailed breakdown of transportation cost including rail costs at both the shipping and receiving ports, handling costs, and lake freight costs. These costs vary by companies and destination (Chicago or Pittsburgh).

For a generalized evaluation, it is difficult to project the traffic pattern other than to assume that it will remain essentially the same. Therefore, rather than attempt to derive a single transportation cost value from the estimates of two operations, the estimate selected for the DCF cost evaluation is the published current transportation cost rate including rail charges to the upper lake ports, handling, and lake freight charges. Since the value of the pellets is calculated only to the lower lake ports, handling and rail transportation costs beyond these points are not included in the cost factor estimate. The 1968 average transportation cost rate quoted in the University of Minnesota Mining Directory is \$3.805 per ton of pellets.

State and Local Taxes (S_1).--The State and local tax cost in this evaluation is the sum of Minnesota taxes on taconite operations--the sum of the production tax, occupation tax, excise tax, and royalty tax. The railroad tax was not included because it relates only to specific taconite operations. The production, occupation, and excise taxes used are the average 1966 taxes as published in the 1968 University of Minnesota Mining Directory (the average tax per ton remained constant for the period 1963-68 (1)). Royalty tax was calculated as the average effective royalty tax rate imposed on the royalty cost used in the model. Finally, the total tax cost was increased by $0.06 to include a tax increase of approximately that amount voted by the 1969 State Legislature:

Average 1966 taconite production tax.............	$0.059 per ton (pellets)
Average 1966 occupation tax......................	.088 per ton
Average 1966 excise tax..........................	.079 per ton
Average effective 1966 royalty tax, 3.75 pct 0.0375 X $0.65 (royalty) = ..	.024 per ton
	.250 per ton
Approximate 1969 increase in production tax......	.06 per ton
Total....................................	.31 per ton

Royalty Costs (S_2).--Because royalty charges range from zero to over a dollar per ton, no fixed value can be given for an evaluation of future production. According to the Minnesota State Division of Waters, Soil, and Minerals, recent new State leases call for royalties averaging about $0.65 per ton of pellets, for both magnetic and nonmagnetic resource evaluations.

Federal Income Tax Computation Factors (F_1, F_2, and F_3).--The Federal income tax rate, F_1, gross rate for depletion, F_2, and net rate for depletion, F_3, are values fixed by law at 50, 15, and 50 pct, respectively.

Return on Investment (I).--An arbitrary 12 pct was used for the return on investment cost or interest rate.

Repayment Period (N).--A repayment period of 25 years was used because long project life minimizes costs and the large Mesabi Range taconite resources warrant consideration of project amortization over that period.

Recoverable Iron Content (A_1).--Recoverable iron contents were derived as described under "Quality Evaluation."

Concentrate Iron Content (A_2).--Concentrate iron contents were 62 and 65 pct, respectively, for magnetic and nonmagnetic as described under "Quality Evaluation."

Equivalent Stripping Ratio (A_3).--The direct physical factor that determines the economic availability of ore from an open pit operation, all other considerations being equal, is equivalent stripping ratio. In the final economic availability evaluation, a quality-quantity cost relationship was established for a range of equivalent stripping ratios.

Depreciation Method Selector (L).--Straight line depreciation was used in the evaluation of both magnetic and nonmagnetic taconite reserves and the depreciation method selector L = 1 was therefore used.

Output--Iron Unit Cost (U)

Output from the cost evaluation model, iron unit cost, consists of a schedule of costs (for varying recoverable iron content and a range of equivalent stripping ratios) as requested by the economic availability model described later.

Economic Availability Evaluation

Economic Availability Evaluation Model

Procedure

The economic availability evaluation model describes the procedures used to incorporate output from the quality and cost evaluations with the dimensional and geometric data of the mineral deposit to calculate a relation between equivalent stripping ratio, concentrate quantity, and iron unit cost (fig. 11).

Singular Evaluation.--As in the cost evaluation model, singular evaluation refers to a one-time evaluation of all cross sections. A different value for recoverable iron content is randomly selected for each cross section from the related probability distribution. The evaluation is summarized by accumulating quantities and calculating the weighted average iron unit cost for each stripping ratio. The summary of the singular evaluation is a schedule of stripping ratio, quantity, and iron unit cost.

Monte Carlo Repetition.--The singular evaluation is repeated several times by a simulation method called the Monte Carlo technique (fig. 12). A distribution of the relation between equivalent stripping ratio, iron unit cost, and resource quantity can be generated to describe the different distributions and distribution patterns of recoverable iron content.

Economic Availability Evaluation Model Input

Input for the economic evaluation model consists of the output from the quality evaluation model, the output from the cost evaluation model, and the geology of the mineral deposit as determined by geologic exploration and drill logs (fig. 13).

Quality Factors.--The following quality factors have been defined in "Quality Evaluation Output":

1. Recoverable iron content, cumulative probability function $F(A_1)$;

2. Concentrate iron content, A_2;

3. Ore probability factor, A_5.

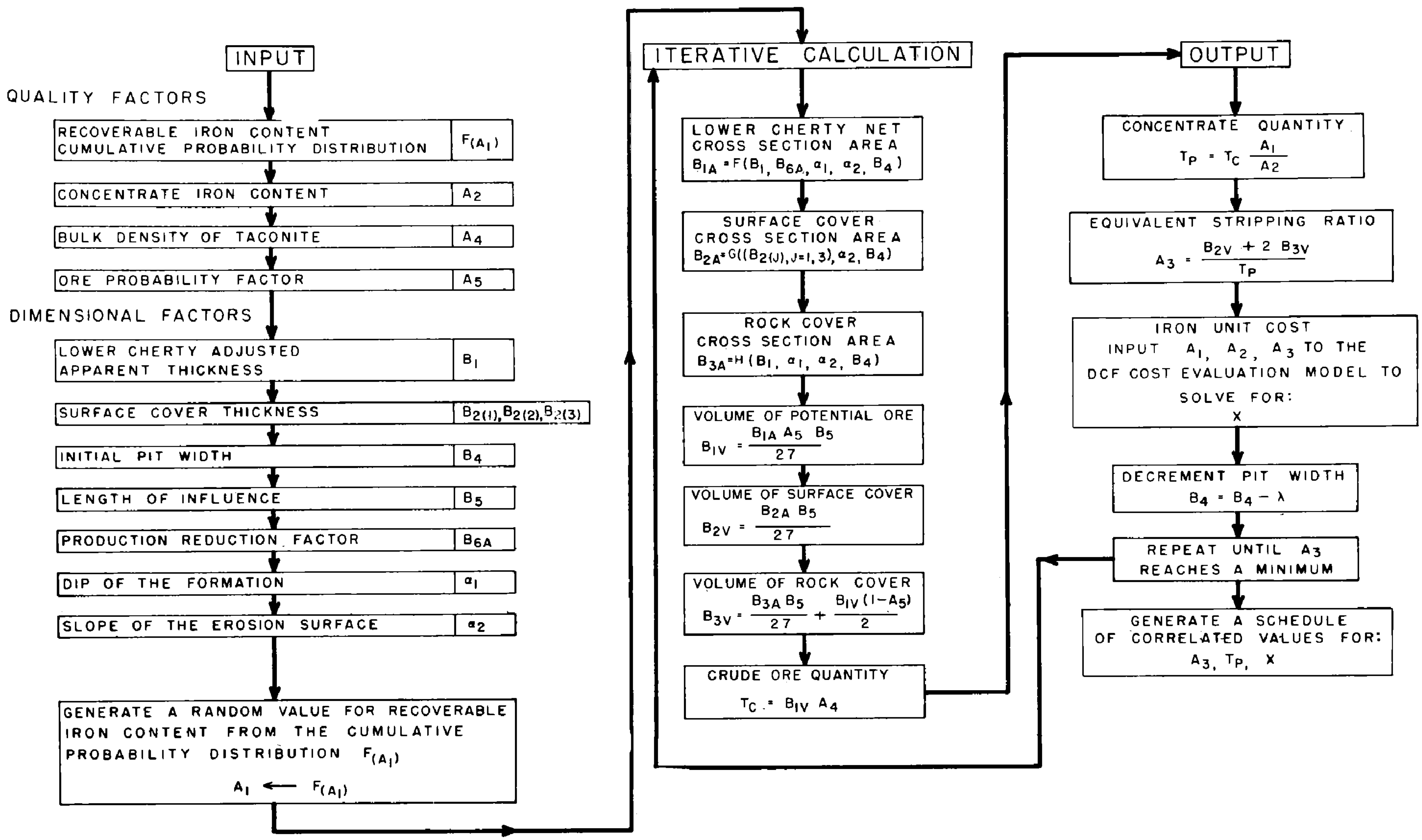

FIGURE 11. - Summary Flow Diagram of Economic Availability Model.

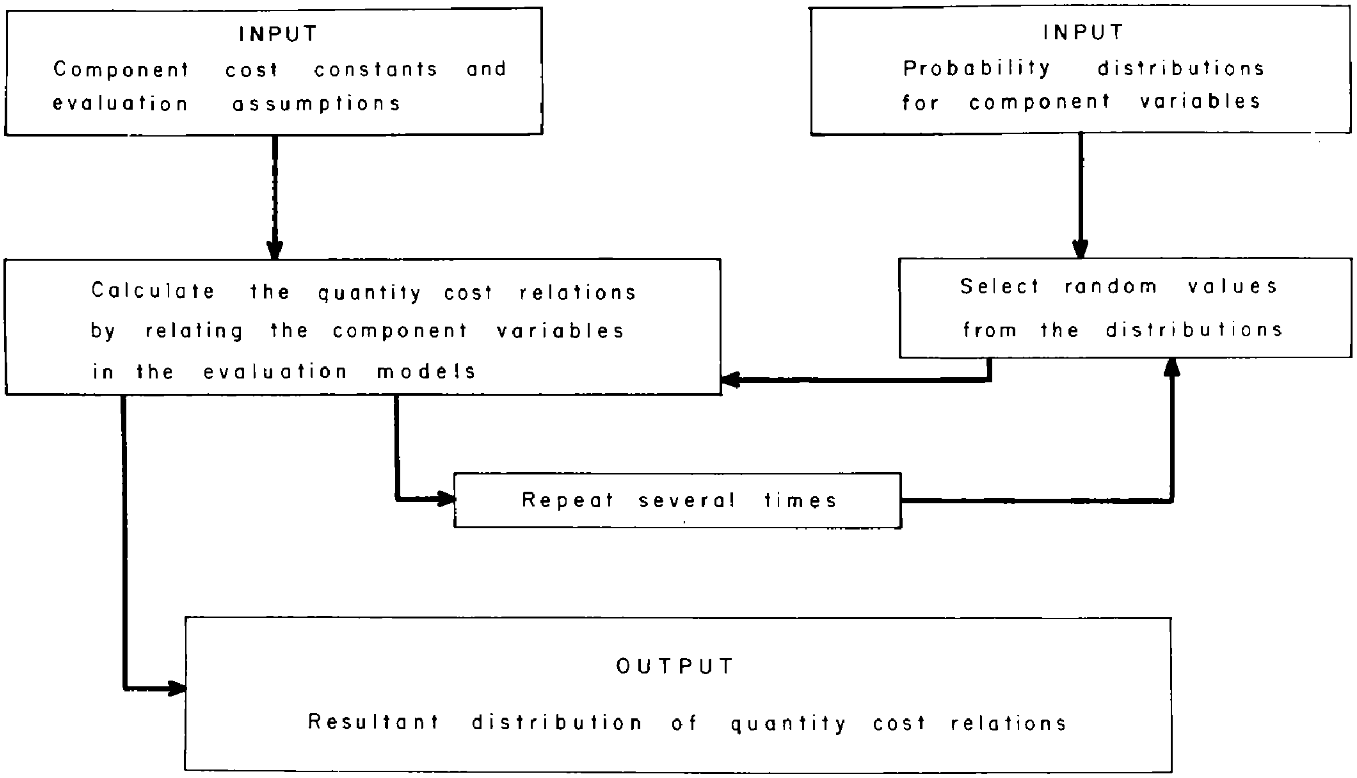

FIGURE 12. - Flow Diagram of Monte Carlo Simulation Method.

Bulk Density (A_4).--The bulk density factor converts taconite volumes to tonnages; this is 11 cu ft per ton.

Production Reduction Factor (B_{6A}).--The production reduction factor represents a cross section area constant derived to correct the economic availability evaluation for previous iron ore production from the Mesabi Range.

It is nearly impossible to estimate the original stratigraphic distribution of previously mined ore. To maintain the conservative character of the evaluation, it is assumed that all of the previous production originated in the Lower Cherty member. Although ore removals have been from individual mines and groups of mines scattered throughout the 77 miles under evaluation, the evaluation assumes uniform removals along the entire length. Therefore, 2.44 billion tons of ore over 77 miles gives approximately 6,000 tons per linear ft of the range. Converting to volume gives approximately 66,000 cu ft per linear foot or a production reduction factor of 66,000 sq ft per cross section.

Dip of the Formation (α_1).--The geologic dip is the southward slope of the iron formation and adjacent rock formations measured perpendicular to the strike of the formations. While dip is usually given in degress, in this case a percent slope is used. The dip of iron formation generally varies between 8 and 12 pct and averages about 10 pct.

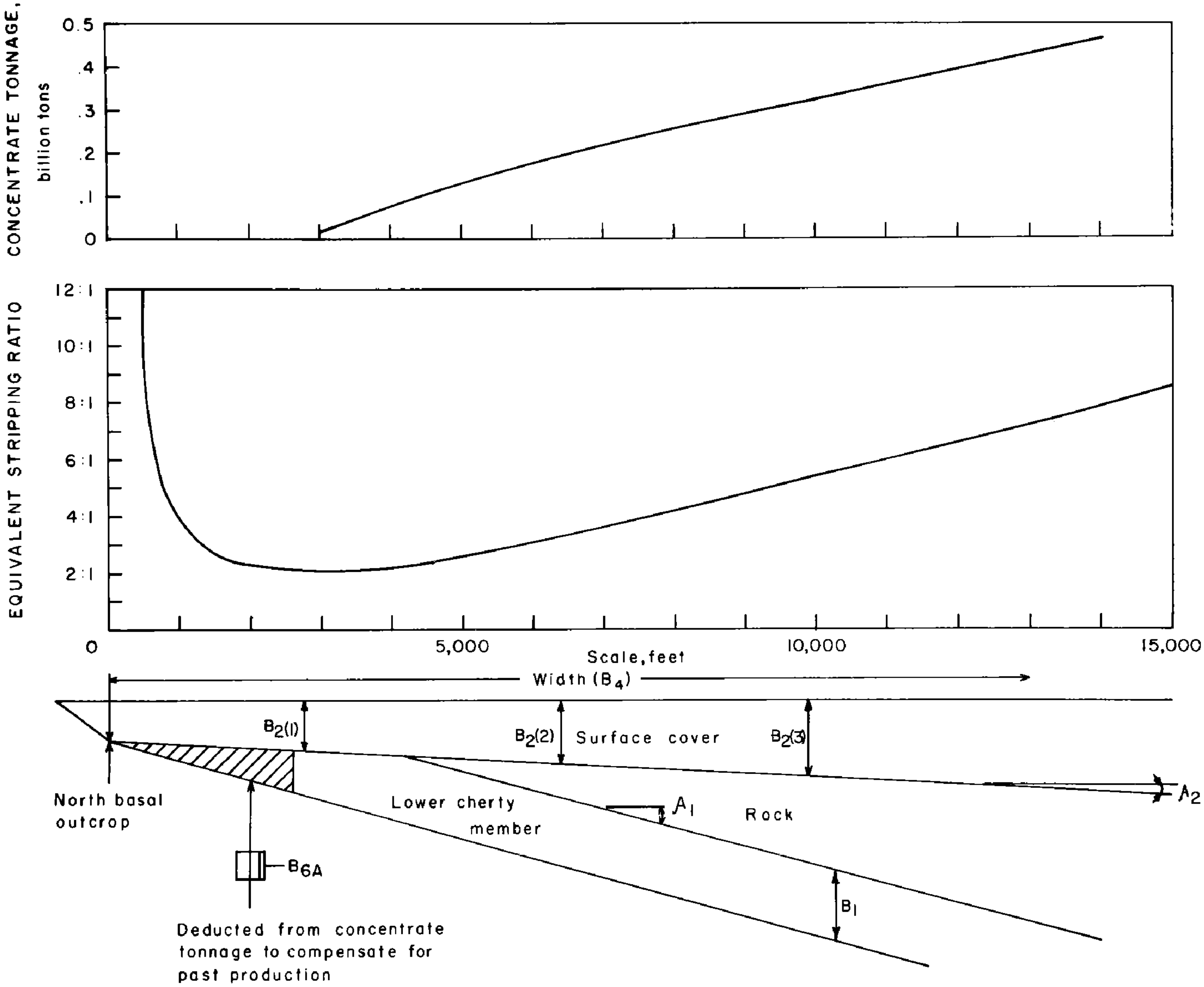

FIGURE 13. - The Relation Between Pit Width, B_4, Equivalent Stripping Ratio, A_3, and Concentrate Tonnage, T_p, Derived From the Evaluation of a Single Cross Section (Length Factor = 8,500 ft).

Slope of the Erosion Surface (α_2).--The erosion surface is the unconformable contact between the glacial drift and the underlying rock formations, including the iron formation. The slope is approximately 2 pct to the south, perpendicular to the strike of the formation.

Lower Cherty Apparent Thickness (Adjusted) (B_1).--Apparent thickness is the geological expression of the vertical thickness of a sloping bed. For the evaluation, the Lower Cherty thickness is decreased by 0 to over 50 ft to exclude the slates and conglomerate material occurring at the base of the member.

Surface Cover Thicknesses ($B_{2(1)}$, $B_{2(2)}$, $B_{2(3)}$).--Surface cover is the glacial drift overlying the iron formation. Surface cover thickness data are an estimated array of averages for one-half mile increments over each cross section. The increase in thickness southward, perpendicular to the strike, averages about 2 pct of the horizontal distance. The first three thicknesses are estimated from actual data. Beyond these, southward additional thicknesses to the south are calculated at 2 pct of the horizontal distance ($B_{2(n)} = B_{2(n-1)} + (.02)(2640)$).

Pit Width (B_4).--Pit width is the independent variable that is incremented to calculate the relation between equivalent stripping ratio, A_3, and concentrate quantity, T_p. It is measured horizontally perpendicular to the strike of the formation, southward from the north basal contact (fig. 13). The contact is the northernmost extension of the Lower Cherty member, usually buried beneath glacial drift, but calculated geometrically from other dimensional factors in the computer model.

Length of Influence (B_5).--The length influence is the horizontal dimension parallel to the strike of the formation and is used to determine the quantitative influence of each cross section. The length influence was determined for each cross section by summing the half distances to adjacent cross sections.

Intermediate Calculated Values

Between the input and output there are several identifiable steps which calculate significant values such as volume of stripping and crude ore which are used to determine the final output: concentrate quantity, and iron unit cost at a specific equivalent stripping ratio.

Recoverable Iron Content (A_1).--The first step in evaluating each section is to generate a random value for recoverable iron content from the appropriate cumulative probability function, $F(A_1)$, according to the following procedure:

A distribution of random values can be generated one at a time from a cumulative probability function such that the probability distribution of the generate values is nearly identical to the distribution of the sample test values. Known as the uniform probability transformation (4), this technique is based on the fact that the cumulative probability function for any continuous variate is uniformly distributed over the interval (0,1). That is, for any random variable, Y, with probability density, F(Y), the variate

$$F(Y) = \int_{-\infty}^{Y} F(X)DX$$

is uniformly distributed over (0,1), or F(Y) has the probability density function $G[F(Y)] = 1,\ 0 \leq Y \leq 1$.

If $Y = A_1$ (recoverable iron content), a random value can be obtained as follows:

1. Generate a random value, Ru, from a uniform distribution over (0,1);

2. Set $Ru = F(A_1) = 1 - e^{(A+BA_1+CA_1^2+DA_1^3+EA_1^4+FA_1^5)}$;

3. Solve for (A_1).

The computer routines for each step followed the procedure illustrated in figure 14.

Cross Section Areas (B_{1A}, B_{2A}, B_{3A}).--Cross section areas are calculated for the Lower Cherty member, surface cover, and rock as functions of the dimensional factors and the variable pit width (fig. 13).

Lower Cherty net cross section area (B_{1A})

$B_{1A} = F(B_1, \alpha_1, \alpha_2, B_4) - B_{6A}$

Surface cover cross section area (B_{2A})

$B_{2A} = G(B_{2(J)}, J=1,3, \alpha_2, B_4)$

Rock cover cross section area (B_{3A})

$B_{3A} = H(B_1, \alpha_1, \alpha_2, B_4)$

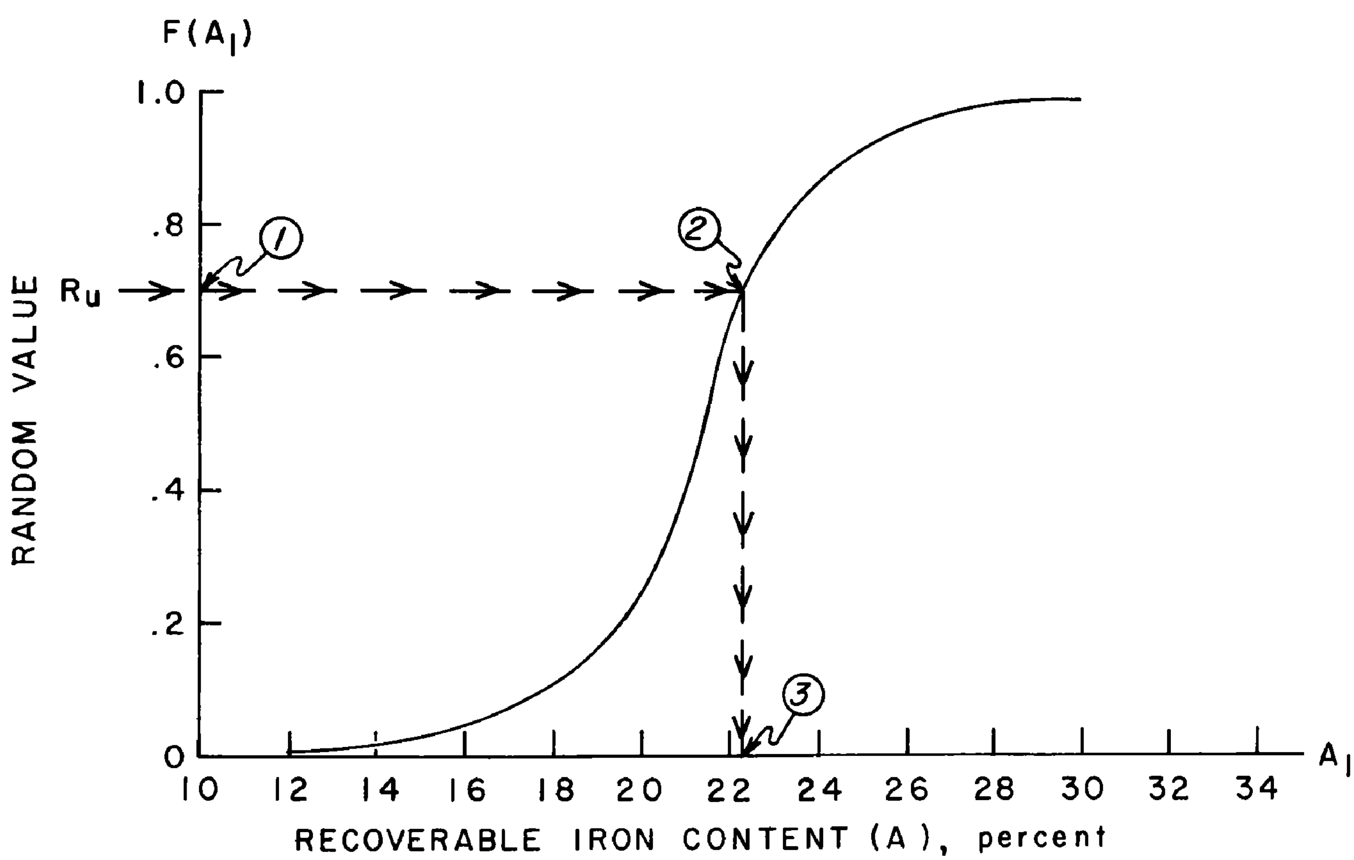

FIGURE 14. - Generation of Random Variate by Uniform Probability Transformation.

Volume of Potential Ore (B_{1V}).--The volume of potential ore is the product of net cross section area, ore probability factor, and length of influence converted from cubic feet to cubic yards. The ore probability factor in this calculation establishes the expected fraction of the total Lower Cherty that is potential ore. The rest is discarded as waste. Assuming that one-half of the waste lies on top of the potential ore, this amount is added to the rock cover and included in calculating stripping.

Volume of Surface Cover (B_{2V}).--Volume of surface cover is the product of the cross section area of surface cover and length of influence converted from cu ft to cu yd.

Volume of Rock Cover (B_{3V}).--Volume of rock cover is the product of the cross section area of rock and length influence. Additional rock cover is calculated from one-half the waste from the Lower Cherty member.

Crude Ore Quantity (T_C).--Crude ore quantity is calculated as the product of volume of potential ore and bulk density of taconite.

Economic Availability Evaluation Output

Concentrate Quantity (T_P).--Concentrate quantity is the actual tonnage of iron ore pellets that can be produced. It is calculated as the product of crude ore quantity and a weight recovery factor, the ratio of recoverable iron content to concentrate iron content.

Equivalent Stripping Ratio (A_3).--The equivalent stripping ratio has been defined as the ratio of the volume of unconsolidated surface cover plus two times the volume of rock cover to tons of concentrate (pellets) produced.

Equivalent Stripping Ratio--Quantity-Cost Relationship.--The equivalent stripping ratio (corresponding to the concentrate quantity as determined above) together with the recoverable iron content and concentrate iron content used in the calculations are input to the cost evaluation model to obtain a corresponding iron unit cost. The result is an equivalent stripping ratio--quantity-cost relation.

Economic Availability Evaluation of Mesabi Range Taconites

Singular Evaluation

Input.--Data required to calculate Mesabi Range cross section areas perpendicular to the strike of the Biwabik iron formation were obtained from the logs of 40 drill holes included in a report by Gruner (10). These 40 holes are distributed along the strike of the iron formation between Coleraine and Aurora, Minn. (fig. 15). Table 18 lists the dimensional data as interpreted from these logs.

TABLE 18. - Mesabi Range dimensional data

Hole No.	Location			Lower Cherty adjusted apparent thickness,[1] feet, B_1	Surface cover thicknesses,[2] feet			Length of influence, feet, B_5
	Sec.	Twp.	R.		B_2 (1)	B_2 (2)	B_2 (3)	
1.............	31	56	24	265	70	90	110	15,200
2.............	29	56	24	190	60	120	160	6,000
3.............	28	56	24	209	70	130	170	4,700
4.............	27	56	24	260	50	130	190	10,500
5.............	19	56	24	290	60	120	180	13,000
6.............	21	56	23	345	40	120	160	8,500
7.............	15	26	23	288	90	100	200	6,200
8.............	14	26	23	273	30	70	120	6,300
9.............	12	56	23	278	40	40	80	7,500
10.............	6	56	22	311	20	80	120	7,000
11.............	32	57	22	315	20	80	130	5,900
12.............	33	57	22	303	40	120	160	5,900
13.............	27	57	22	315	80	100	160	6,700
14.............	23	57	22	233	40	40	120	5,500
15.............	24	57	22	270	30	40	150	7,700
16.............	17	57	21	245	30	40	80	7,200
17.............	17	57	21	245	20	80	120	5,000
18.............	15	57	21	220	20	40	130	8,600
19.............	12	57	21	215	20	30	40	9,300
20.............	6	57	20	235	20	40	80	13,500
21.............	34	58	20	188	120	80	120	12,400
22.............	26	58	20	248	20	50	120	5,900
23.............	25	58	20	230	80	80	80	8,800
24.............	17	58	19	205	60	120	160	7,600
25.............	21	58	19	228	40	60	120	4,700
26.............	15	58	19	240	20	60	120	8,300
27.............	13	58	19	205	30	80	100	7,500
28.............	7	58	18	223	20	40	40	5,400
29.............	8	58	18	223	20	40	80	9,000
30.............	3	58	18	214	20	20	60	10,700
31.............	1	58	18	203	20	40	80	10,700
32.............	4	57	17	182	20	40	70	19,000
33.............	24	58	17	225	20	80	140	14,000
34.............	18	58	16	208	40	100	150	12,000
35.............	10	58	16	135	20	40	120	9,500
36.............	12	58	16	158	20	20	40	13,300
37.............	5	58	15	190	20	60	100	16,000
38.............	23	59	15	122	20	20	20	15,100
39.............	29	59	14	130	20	20	25	9,300
40.............	21	59	14	118	20	20	20	8,400

[1] Gruner, John W. The Mineralogy and Geology of the Taconites and Iron Ores of the Mesabi Range, Minn. Office of the Commissioner of the Iron Range Resources and Rehabilitation, St. Paul, Minn., 1946, 122 pp.

[2] White, David A. The Stratigraphy and Structure of the Mesabi Range, Minnesota. Univ. of Minnesota Press, 1954, 92 pp.

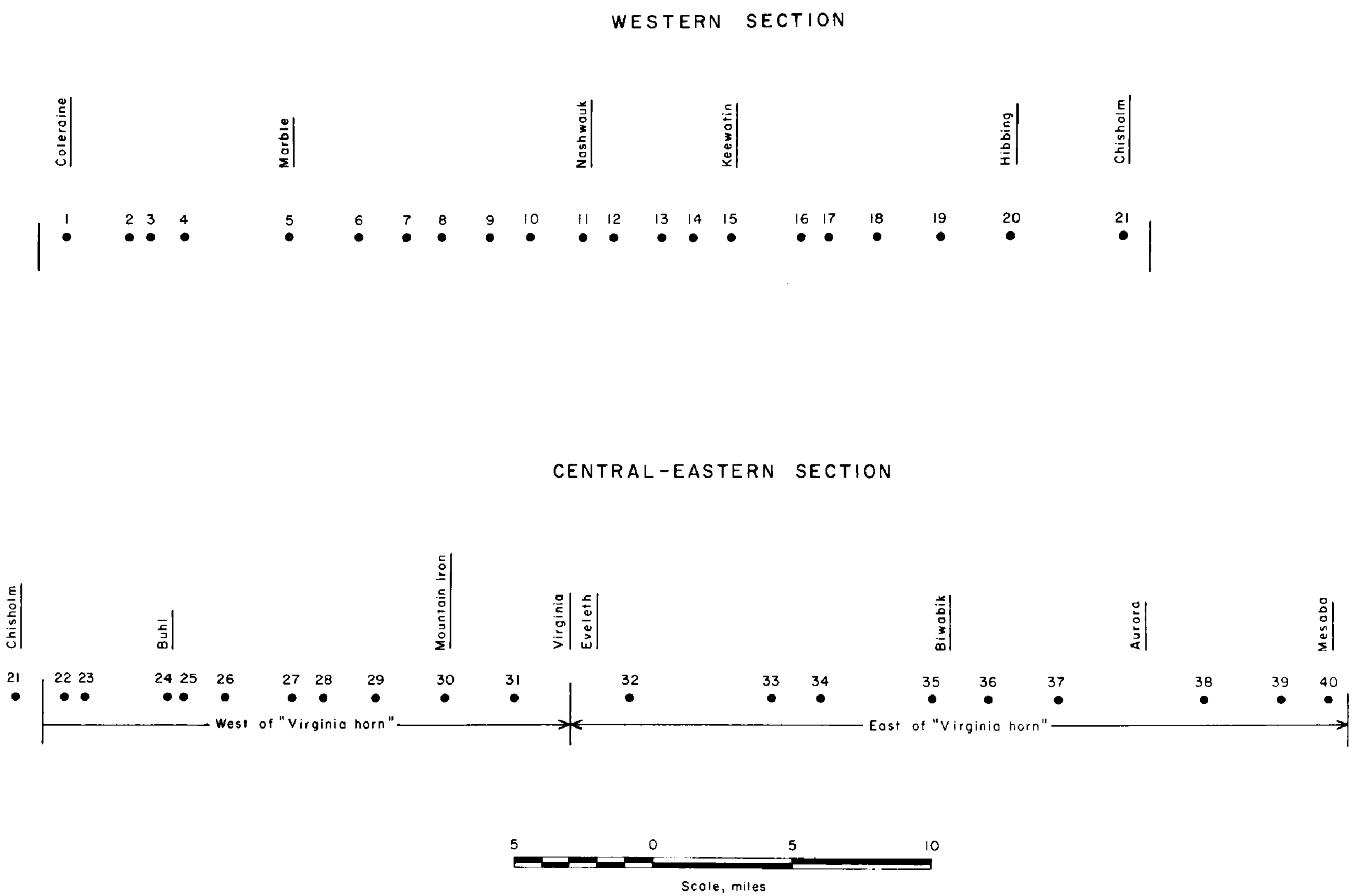

FIGURE 15. - Distributions of Cross Sections Projected on a Line Parallel With the Strike of the Biwabik Iron Formation.

Equivalent Stripping Ratio--Quantity-Cost Relationship.--The economic availability calculation results for a single section evaluation are shown in table 19. A singular or one time evaluation of the entire Mesabi Range consists of a summation of the evaluations of 40 sections, table 20.

TABLE 19. - Schedule of equivalent stripping ratio, concentrate quantity, and iron unit cost for a typical cross section

Equivalent stripping ratio, A_3	Concentrate quantity (million tons), T_P	Iron unit cost, X	Equivalent stripping ratio, A_3	Concentrate quantity (million tons), T_P	Iron unit cost, X
2.50	165.03	0.235266	7.50	502.85	0.267529
3.00	196.84	.238488	8.00	538.05	.270751
3.50	223.92	.241723	8.50	566.48	.273974
4.00	264.54	.244946	9.00	600.33	.277197
4.50	302.46	.248168	9.50	634.18	.280419
5.00	332.24	.251391	10.00	663.97	.283642
5.50	367.45	.254614	10.50	697.82	.286877
6.00	404.01	.257836	11.00	731.67	.290100
6.50	432.44	.261071	11.50	765.52	.293334
7.00	467.64	.264294	12.00	759.31	.296557

TABLE 20. - Singular evaluation schedule of economic availability

Equivalent stripping ratio	Magnetic taconite ore		Nonmagnetic taconite	
	Quantity of pellets (million tons)	Cost (dollars per iron unit)	Quantity of pellets (million tons)	Cost (dollars per iron unit)
1.50.............	933.226	0.24792	-	-
2.00.............	1453.214	.25111	819.453	0.26916
2.50.............	2348.385	.25453	1842.658	.27313
3.00.............	3254.693	.25771	2811.181	.27606
3.50.............	4204.758	.26089	4364.494	.27862
4.00.............	5041.392	.26410	5639.184	.28146
4.50.............	5747.475	.26734	6602.522	.28448
5.00.............	6413.267	.27056	7665.170	.28747
5.50.............	7109.465	.27378	8469.133	.29054
6.00.............	7782.704	.27701	9273.429	.29361
6.50.............	8492.997	.28023	10135.620	.29668
7.00.............	9218.038	.28362	10991.388	.29974
7.50.............	10050.506	.28729	11834.987	.30281
8.00.............	10788.362	.29050	12671.934	.30588
8.50.............	11492.704	.29388	13517.042	.30896
9.00.............	12196.116	.29714	14304.573	.31203
9.50.............	12867.641	.30039	15112.971	.31511
10.00.............	13542.009	.30366	15909.608	.31819
10.50.............	14217.117	.30691	16743.325	.32126
11.00.............	14888.610	.31015	17564.073	.32433
11.50.............	15566.416	.31338	18399.546	.32741
12.00.............	16226.637	.31659	19213.031	.33050

Monte Carlo Simulation

Input.--Input used in the Monte Carlo simulation procedure was exactly the same as for the singular evaluation, but the evaluation was repeated 10 times.

Equivalent Stripping Ratio--Quantity-Cost Relationship.--The repetition of the singular evaluation resulted in 10 different results because recoverable iron content was represented by a probability function. While only 10 repetitions did not provide an adequate distribution to be used for a good confidence evaluation, a schedule of distribution parameters was developed showing the high, low, and average iron unit costs and their standard deviations related to various quantity levels (table 21). The data thus developed provide the means for analyzing the expected variance of the quantity-cost relationship that results from the variance in recoverable iron content. The weighted averages for recoverable iron content from the 10 evaluations are distributed over a relatively small range. Figure 16 shows that the relation between recoverable iron content and iron unit cost is nearly linear at each quantity level although it has been shown previously that over larger ranges of values the relation between recoverable iron content and iron unit cost is definitely curvilinear. The linear relationship can be determined for each quantity level with a schedule of linear coefficients, table 22.

$$X = A + BA_1$$

X = Iron unit cost (dollars per iron unit)

A_1 = Average recoverable iron content (pct)

These coefficients are valid only in the ranges of about 19 to 23 pct average recoverable iron content for the magnetic taconite process, and about 29 to 32 pct for the nonmagnetic process. These ranges, however, would probably encompass most estimates for either case.

TABLE 21. - Schedule of quantity and distribution factors of iron unit cost

Pellets, billion tons	Iron unit cost, dollars							
	Magnetic taconite				Nonmagnetic taconite			
	Low	High	Average	Std. Dev.	Low	High	Average	Std. Dev.
0.5........	0.2313	0.2533	0.2409	0.0064	0.2370	0.2700	0.2553	0.0094
1.0........	.2337	.2546	.2433	.0061	.2450	.2746	.2604	.0091
2.0........	.2382	.2571	.2477	.0056	.2595	.2800	.2686	.0084
3.0........	.2427	.2597	.2517	.0050	.2660	.2835	.2736	.0072
4.0........	.2470	.2642	.2560	.0051	.2695	.2875	.2766	.0068
5.0........	.2513	.2690	.2602	.0054	.2715	2915	.2793	.0068
6.0........	.2556	.2739	.2649	.0057	.2745	.2940	.2818	.0065
7.0........	.2559	.2791	.2696	.0061	.2795	.2950	.2849	.0056
8.0........	.2614	.2837	.2742	.0061	.2825	.2965	.2881	.0051
9.0........	.2684	.2883	.2786	.0061	.2865	.2990	.2913	.0048
10.0........	.2728	.2931	.2832	.0062	.2900	.3020	.2948	.0048
11.0........	.2770	.2978	.2877	.0064	.2930	.3060	.2979	.0048
12.0........	.2813	.3027	.2923	.0067	.2970	.3100	.3018	.0053
13.0........	.2856	.3081	.2970	.0070	.3005	.3135	.3054	.0054
14.0........	.2899	.3132	.3016	.0072	.3040	.3175	.3091	.0056
15.0........	.2941	.3182	.3068	.0072	.3075	.3215	.3129	.0056

TABLE 22. - Linear coefficients relating iron unit cost and average recoverable iron content at various quantity levels (billion tons)

Pellets, billion tons	Magnetic taconite		Nonmagnetic taconite	
	A	B	A	B
0.5	0.47432	-0.01089	0.55332	-0.00966
1.0	.47329	-.01073	.57615	-.01023
2.0	.46487	-.01013	.59944	-.01072
3.0	.44947	-.00922	.59575	-.01045
4.0	.45800	-.00942	.58337	-.00994
5.0	.47506	-.01002	.57819	-.00969
6.0	.49496	-.01073	.57402	-.00947
7.0	.51499	-.01145	.54590	-.00846
8.0	.52023	-.01148	.53361	-.00796
9.0	.52652	-.01156	.52724	-.00765
10.0	.53426	-.01171	.53266	-.00771
11.0	.54679	-.01209	.52886	-.00749
12.0	.56141	-.01255	.56418	-.00850
13.0	.57794	-.01311	.57241	-.00866
14.0	.59240	-.01356	.58668	-.00900
15.0	.58970	-.01320	.58739	-.00890

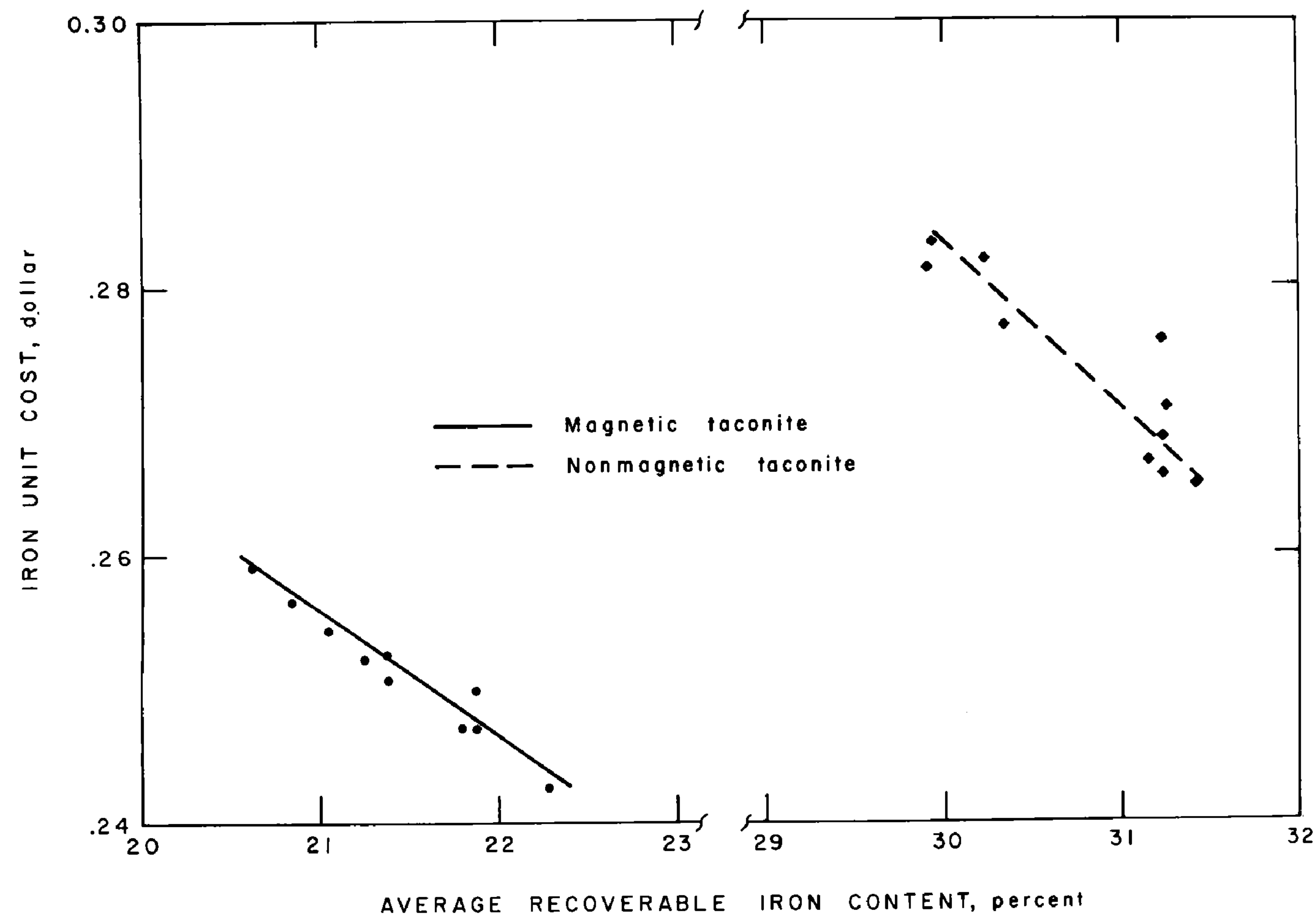

FIGURE 16. - Plotted Weighted Average Recoverable Iron Contents Showing Their Linear Relationship With Iron Unit Cost.

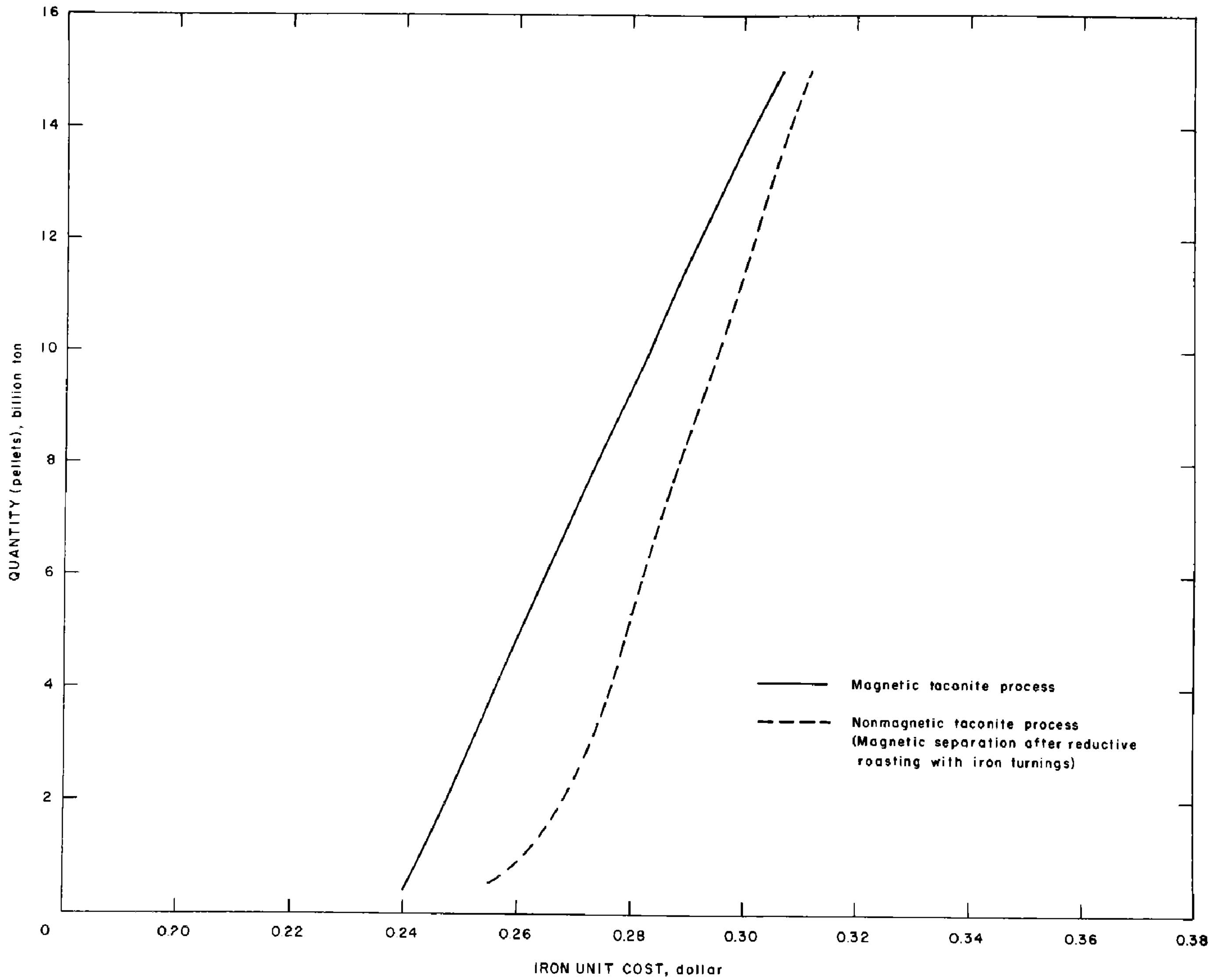

FIGURE 17. - Summary Economic Supply Curves for Iron Ore From Mesabi Range Taconite by the Magnetic and Nonmagnetic Taconite Processes.

The economic availability reserves of iron ore from the taconite resources of the Mesabi Range is summarized by economic supply curves for both the magnetic and nonmagnetic taconite processes (fig. 17). These curves are developed by plotting the concentrate quantities and average iron unit cost from table 21. Since quantities of the overall taconite resource base might be amenable to working by either the magnetic or nonmagnetic taconite processes, the respective quantities corresponding to any unit cost, are not cumulative. That is, while an average unit cost of, for example, \$0.26 corresponds to a five billion ton reserve by the magnetic process and a one billion ton reserve by the nonmagnetic process, the total quantity available, all other factors being correct, is not six billion tons but rather between five and six billion.

SUMMARY

The methodology developed for evaluating the economic availability of Mesabi Range iron ore consists of three mathematical models: (1) quality evaluation, (2) discounted cash flow cost evaluation, and (3) economic availability. The quality evaluation model uses exploration data to determine recoverable iron content (as a cumulative probability function), concentrate iron content, and ore probability factor. The discounted cash flow model uses the output from the quality evaluation model together with the cost related factors to calculate iron unit cost at specific levels of availability (equivalent stripping ratios). The economic availability model also uses the output from the quality evaluation model, integrates it with the dimensional characteristics of the Mesabi Range taconite ore body to determine quantities of concentrate available at a range of equivalent stripping ratios. The economic availability model then combines these quantities with the appropriate iron unit costs calculated in the discounted cash flow model to develop a quantity-cost schedule.

The quantity-cost schedule of Mesabi Range iron ore presented in table 21 and figure 19 was developed by using available Mesabi Range exploration data, geologic setting of the Biwabik iron formation, and available cost data in the models using Monte Carlo simulation techniques. The relative sensitivity of iron unit cost to individual cost related items is presented in a series of graphs in appendix A; the effect of changes in the physical parameters on Mesabi Range reserves in appendix B. The computer programs are in appendix C.

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APPENDIX A.--RELATIVE INFLUENCE OF COST RELATED FACTORS ON IRON UNIT COST

Measuring the effects of individual cost changes on total iron unit cost is useful in (1) assessing the confidence of the DCF cost evaluation, (2) adjusting the result of the evaluation to give a better estimate of a component variable, and (3) identifying the most sensitive cost factors for which more accurate data should be obtained. The effect of changes in specific component cost factors on total costs is difficult to estimate intuitively because of the factors' interrelationship.

Apart from its application in the economic availability evaluation, the DCF model is useful in establishing the relation between individual component cost variables and total iron unit cost. The value of any component cost can be incremented over a reasonable range and corresponding values of total unit cost calculated while all of the other variables are held constant. Except for the variable being analyzed, or unless otherwise noted, the values of the remaining component cost variables are given in table 7. The relation analyzed by this procedure will be discussed and illustrated in the following paragraphs. While these relations are relatively straightforward, several have characteristics that are relevant to the overall evaluation and deserve clarification. Included among these is an apparent break in slope on some of the curves representing the component cost-unit cost relationship. The break in slope identifies a point in the DCF evaluation where the basis for depletion allowance shifts from 15 pct of the gross income for depletion to 50 pct of the net income for depletion as the latter value becomes the smaller of the two.

In the following figures, a horizontal arrow identifies the point at which the inputs to the DCF model are the values listed in table 7. Unless otherwise noted, the relationships discussed refer only to the magnetic taconite evaluation. The nonmagnetic taconite component cost-iron unit cost curves are also illustrated, and their interpretation is identical to those dealing with the magnetic taconite.

Capital Investment

Capital investment, used in the evaluation as a total dollar value, was converted to dollars per ton of annual crude ore capacity in figure A-1. The reason for the different slopes of the magnetic and nonmagnetic curves is that higher iron recoveries are associated with the nonmagnetic taconite process. This difference in slope is noted wherever the relation involves crude ore tonnages or costs.

The depletion shift point is apparent on the magnetic taconite curve but is absent within the range analyzed on the nonmagnetic curve. Several of the other component cost-iron unit cost relationships show this characteristic also. A $1.00 increase of capital investment per annual ton of crude ore capacity results in a $0.0089 increase in iron unit cost above the shift point and $0.0074 below, or $0.55 and $0.46 per ton of pellets, respectively, for the magnetic taconite process.

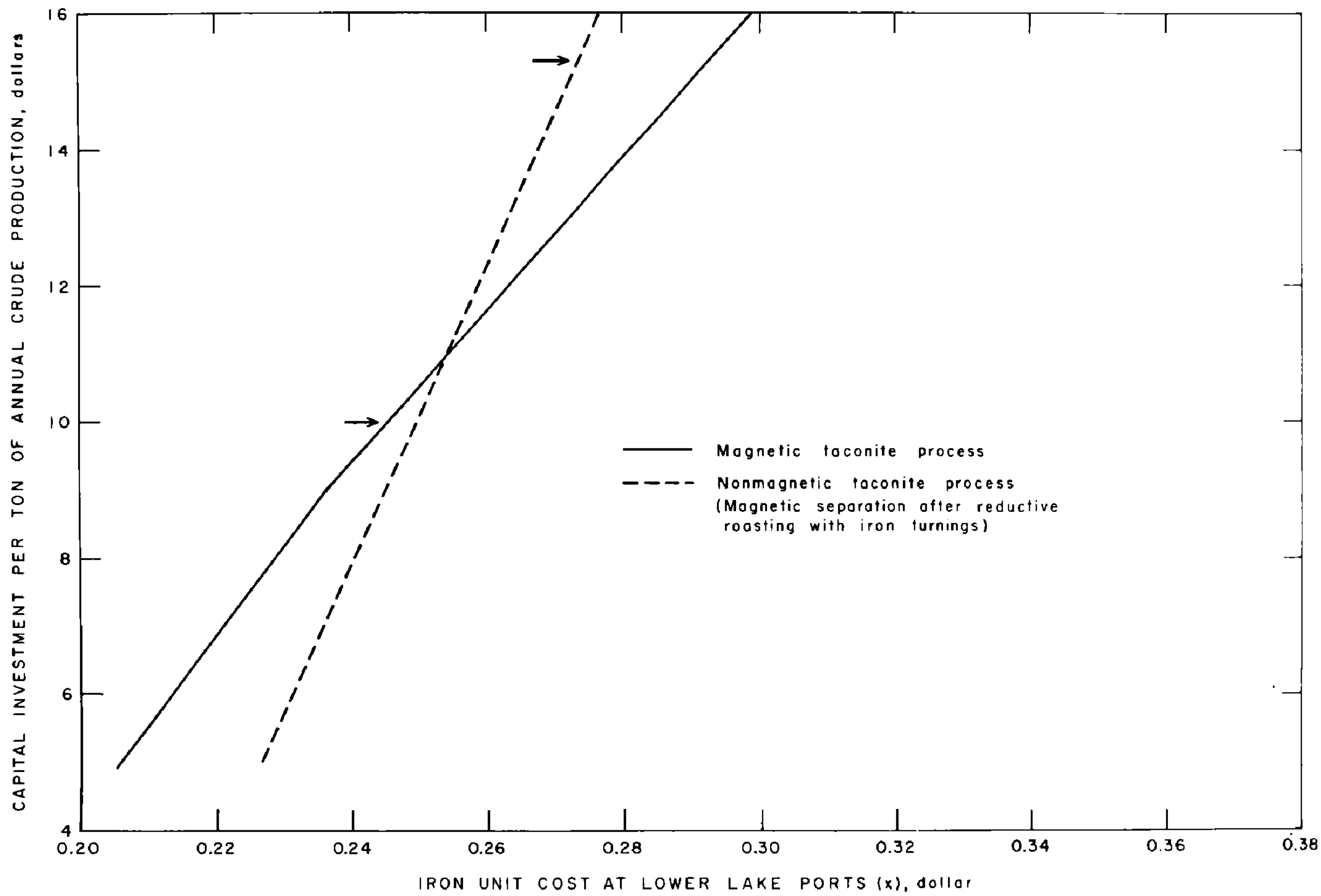

FIGURE A-1. - Relation Between Capital Investment and Iron Unit Cost.

Beneficiation Cost

When considering the cost per ton of crude ore production, the relation between beneficiation cost and iron unit cost is dependent on the recoverable iron content factor (fig. A-2). For the magnetic taconite process and the data in table 1, the effect is about \$0.0476 per iron unit for every dollar variation above the depletion shift point and \$0.0414 per iron unit below the point, or approximately \$2.95 and \$2.56 per ton of pellets, respectively.

Mining Cost

Mining cost, like beneficiation cost, is input as dollars per ton crude. Therefore, changes in mining cost have the same effect as changes in beneficiation cost.

Supervision and Management Cost

Because this cost is also input as dollars per ton crude, its relation to total iron unit cost is the same as that of mining and beneficiation cost.

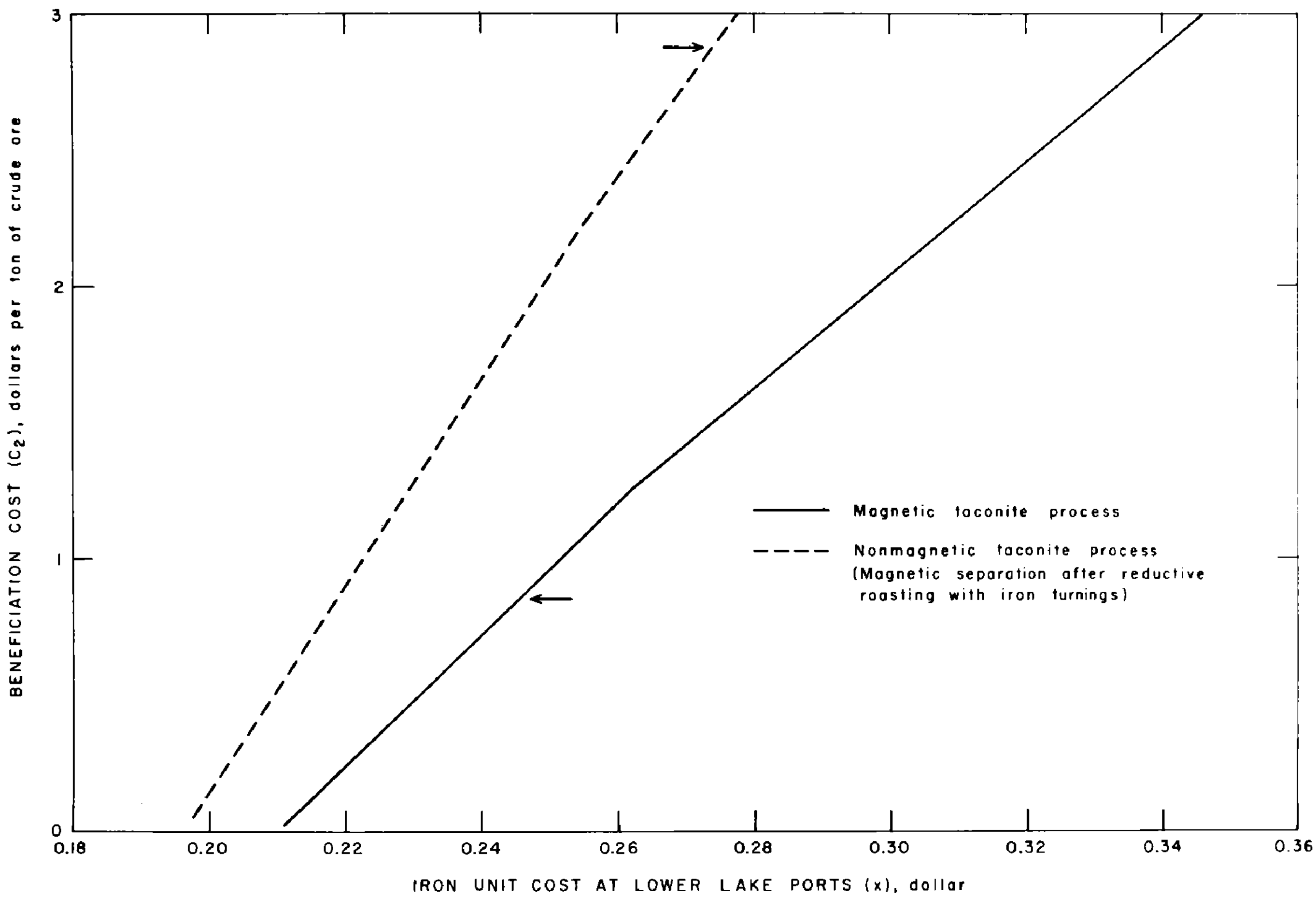

FIGURE A-2. - Relation Between Beneficiation Cost and Iron Unit Cost.

Pelletizing Cost

Changes in pelletizing cost affect total concentrate cost on nearly a one to one basis above the depletion shift point on the magnetic taconite curve (fig. A-3) and approximately 0.87 per dollar below. The cost per iron unit varies by $0.0161 and $0.014, respectively. For the nonmagnetic taconite process data input used the variation is at a 1:1 ratio over the entire range.

Transportation Cost

The relation between transportation cost and iron unit cost is completely linear with no depletion shift point (fig. A-4) because transportation cost is excluded in the calculation of gross income for depletion. Therefore the relationship is also a direct one. A transportation cost increase of $0.50 per ton results in a pellet cost increase of $0.50 per ton ($0.00806 per iron unit with the magnetic taconite data used).

State and Local Taxes

The effect of changes in State and local taxes on total iron unit cost is the same as that of changes in pelletizing cost, both factors being input as dollars per ton of concentrate.

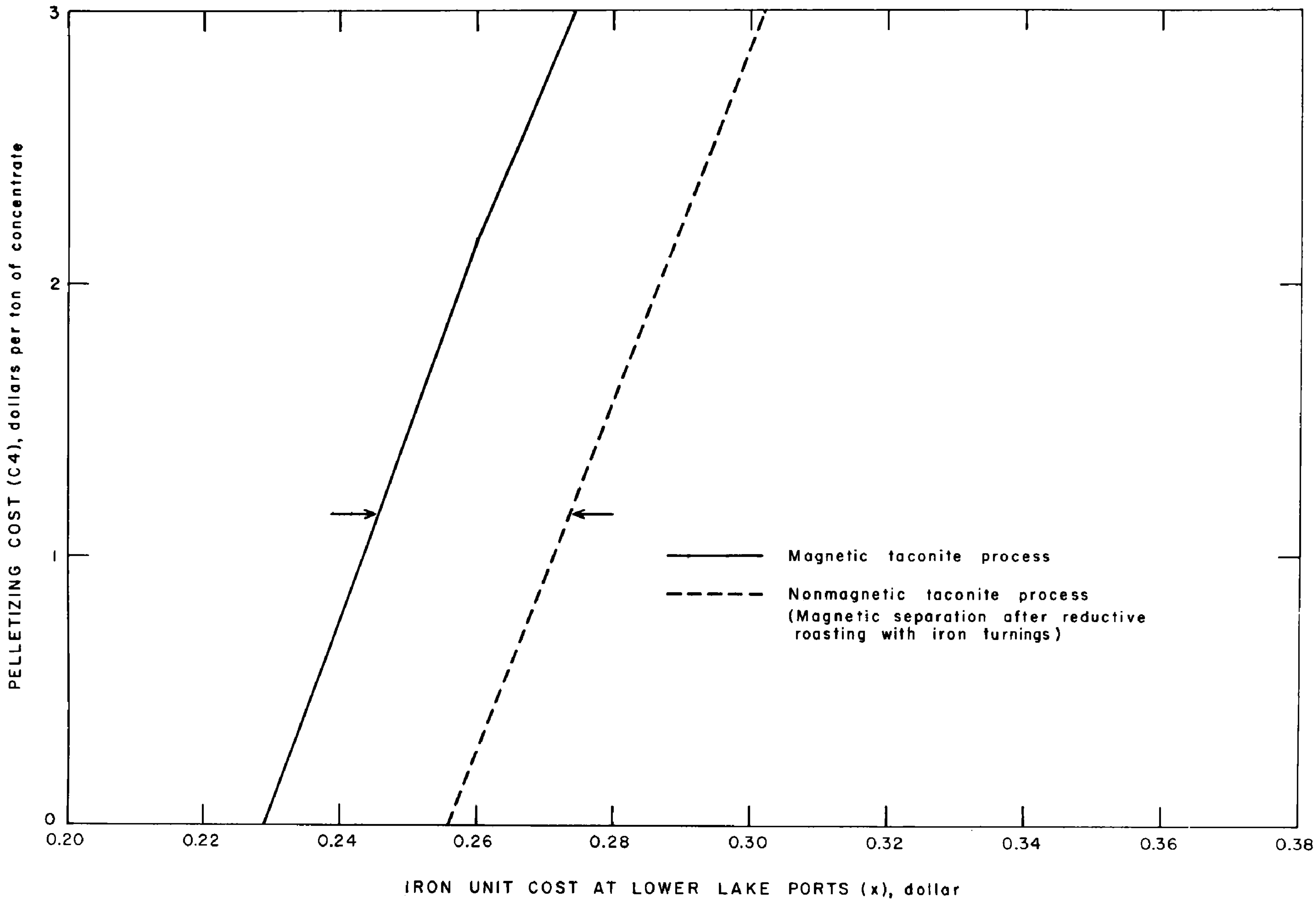

FIGURE A-3. - Relation Between Pelletizing Cost and Iron Unit Cost.

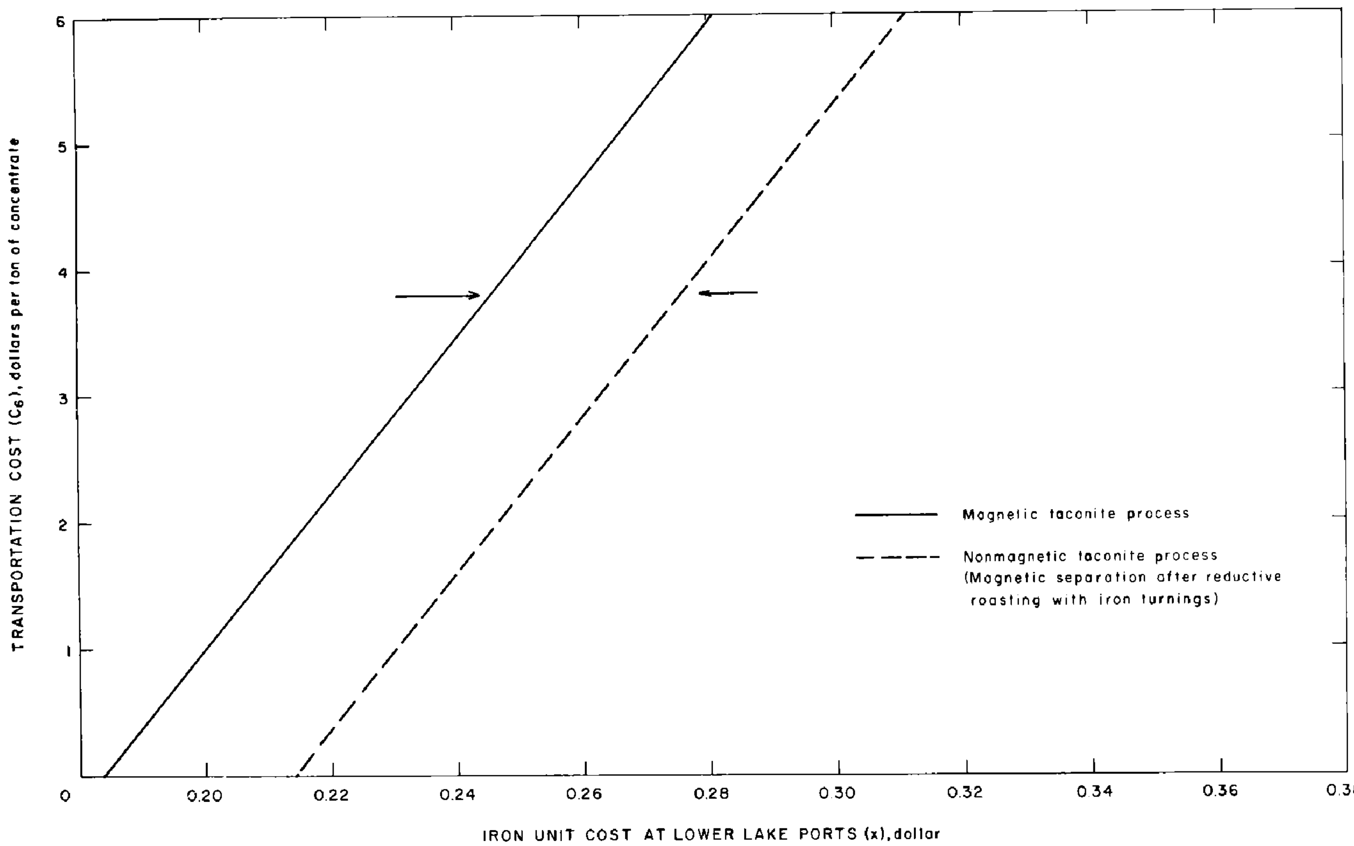

FIGURE A-4. - Relation Between Transportation Cost and Iron Unit Cost.

Royalty

Royalty, also being a function of tons of concentrate, is related to iron unit cost the same as pelletizing and taxes.

Federal Income Tax Rate

The Federal income tax rate is a relatively insignificant component cost variable in calculating iron unit cost (fig. A-5). If no other variables are changed, decreasing the tax rate from 50 to 35 pct decreases iron unit cost by only $0.0067 or about $0.41 per ton of pellets (magnetic taconite process). The relative unimportance of the Federal income tax rate in determining iron unit cost is due primarily to the low ratio of the taxable income to the combined production cost in taconite operations. Even at a 50-pct rate, total Federal income tax, as a cost factor, averages less than $1.00 per ton of pellets ($0.016 per iron unit).

Depletion Allowance Rates

The influence of the interrelationship of gross rate for depletion and net rate for depletion on total iron unit cost is represented as a series of eight curves each having a vertical and a negatively sloping component (except F_3 = 0, which has only the vertical component). The complete F_3 = 50 pct curve is drawn, however, on the graphs (figs. A-6 and A-7).

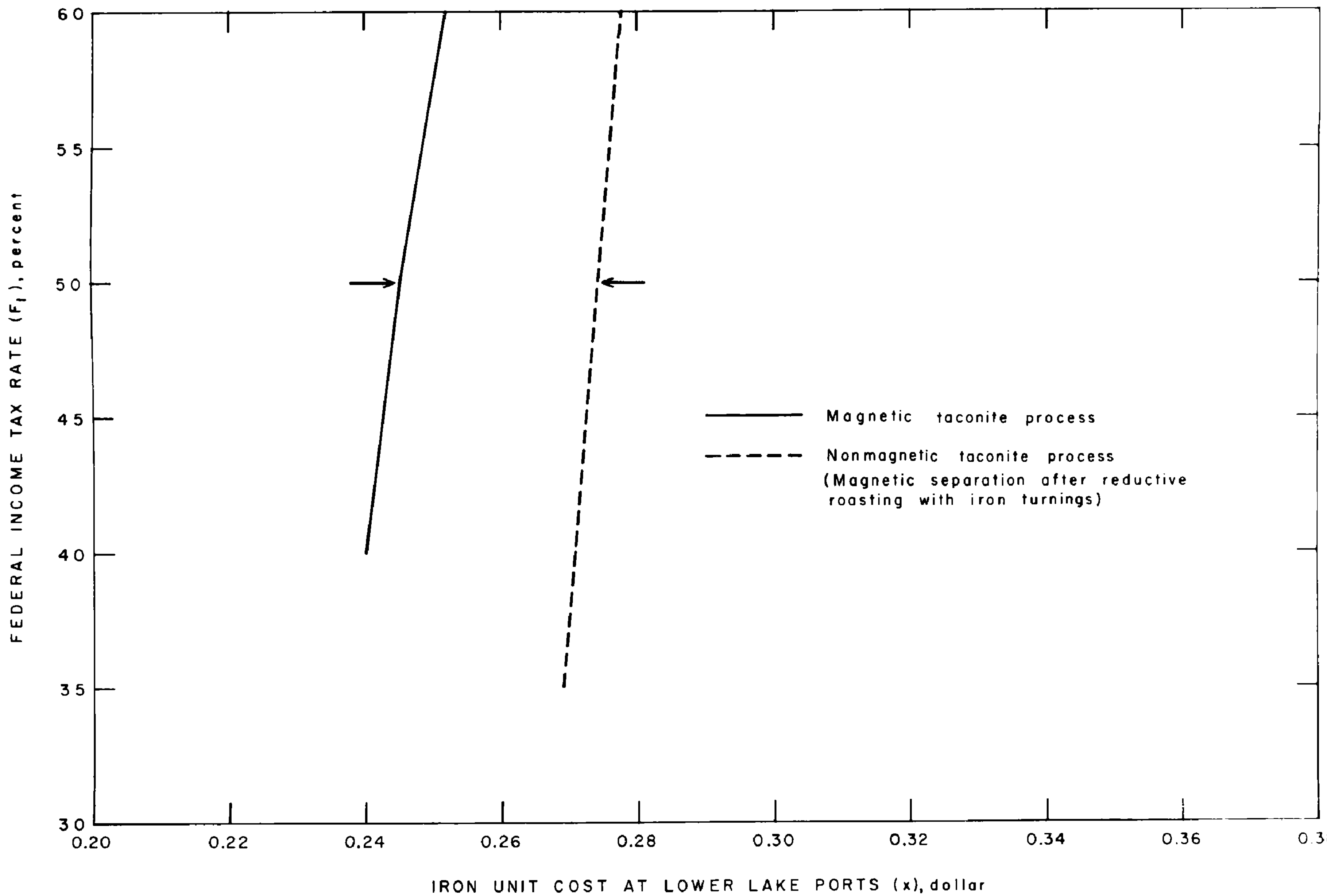

FIGURE A-5. - Relation Between Federal Income Tax Rate and Iron Unit Cost.

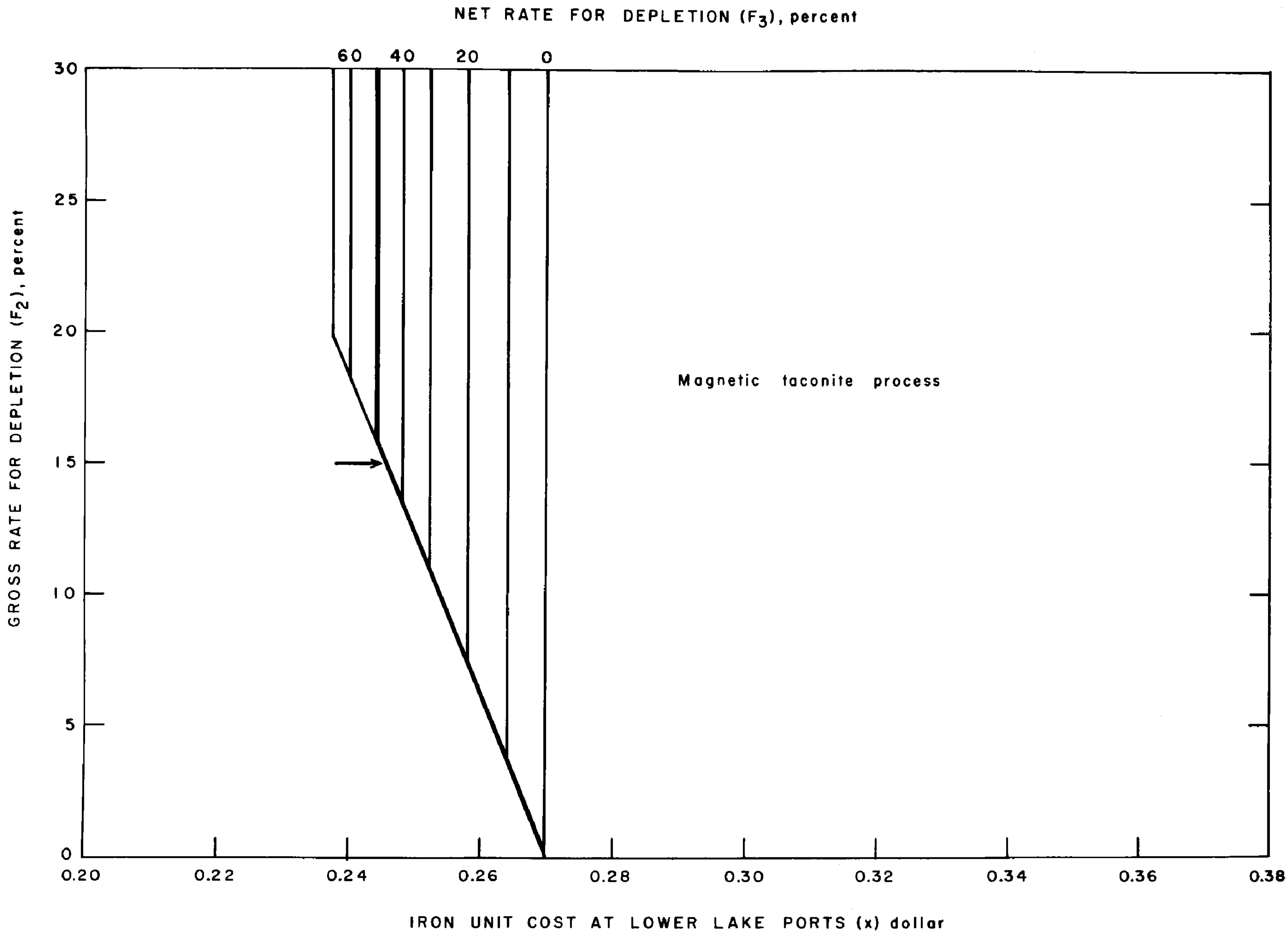

FIGURE A-6. - Relation Between Gross Rate for Depletion, Net Rate for Depletion, and Iron Unit Cost (Magnetic Taconite).

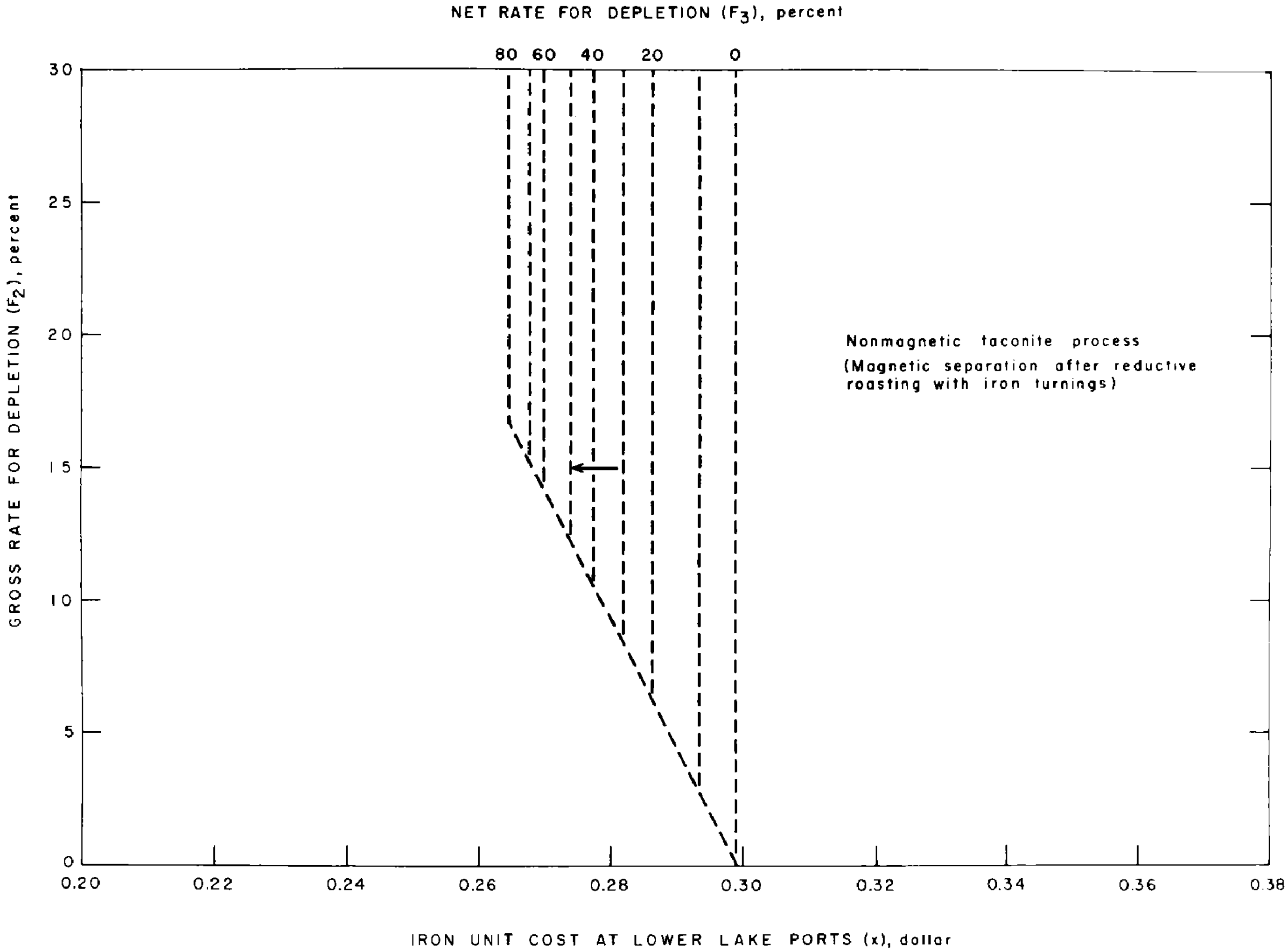

FIGURE A-7. - Relation Between Gross Rate for Depletion, Net Rate for Depletion, and Iron Unit Cost (Nonmagnetic Taconite).

Magnetic Taconite Process

From the base value (arrow) it is apparent that a decrease from 50 to 45 pct in net rate for depletion has no effect on total iron unit cost, while a decrease in the gross rate from 15 to 10 pct increases iron unit cost by approximately $0.008, or about $0.49 per ton of pellets.

Nonmagnetic Taconite Process

The situation is reversed from the previous example. At the base value (arrow) the net rate for depletion is the controlling factor for F_3 = 50 pct; the gross rate F_2 could vary anywhere upward from about 12.5 pct with no effect on iron unit cost. At the same net rate, gross rates below 12.5 pct are used in determining depletion allowance.

Return on Investment

A 12-pct return on investment is an arbitrary value used in the DCF economic evaluation for comparative purposes (fig. A-8).

It is apparent that this variable has a considerable influence on unit cost. For instance, in evaluating the magnetic taconite process, if 10 pct were a satisfactory rate of return, the total iron unit cost would fall about $0.0127 per iron unit or $0.79 per ton of pellets, enough to allow the equivalent stripping ratio to rise from 1:1 to over 3.3:1 at the original iron unit cost. Conversely, the rate of return-unit cost curve can be used to determine the actual rate of return at a given selling price. For example, the current selling price for taconite pellets is $0.252 per iron unit. This price gives a rate of return of about 12.86 percent according to the magnetic taconite data in table 1. In an actual profitability evaluation, a similar curve can be derived to solve for rate of return at the current selling price after all of the other variables have been set at their real values.

Repayment Period

The relation between repayment period and the iron unit cost is an extremely significant one up to about 20 years (fig. A-9). Between 20 and 50 years, however, variations in this component cost factor have little effect on total iron unit cost. The optimum repayment period for this particular set

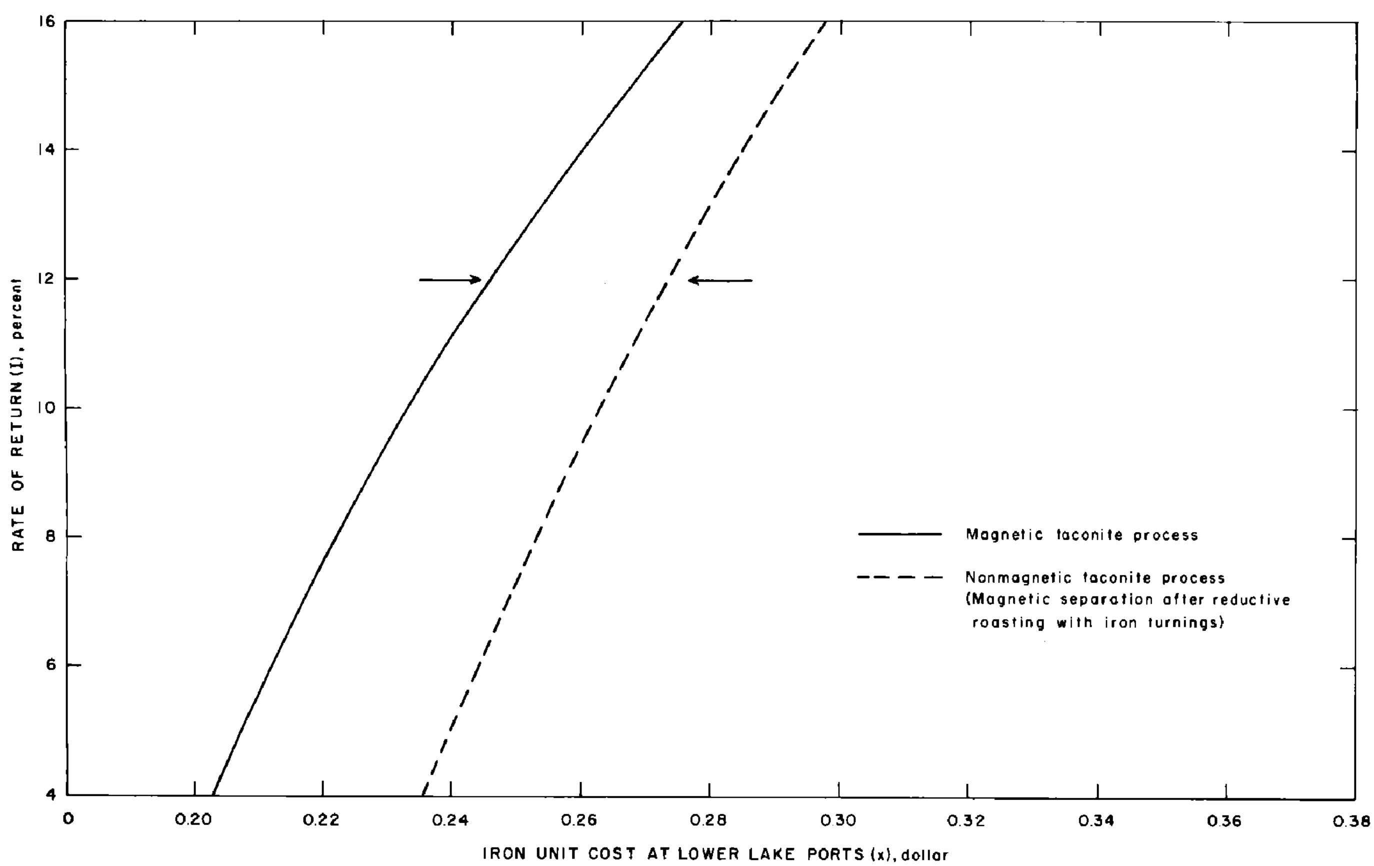

FIGURE A-8. - Relation Between Rate of Return and Iron Unit Cost.

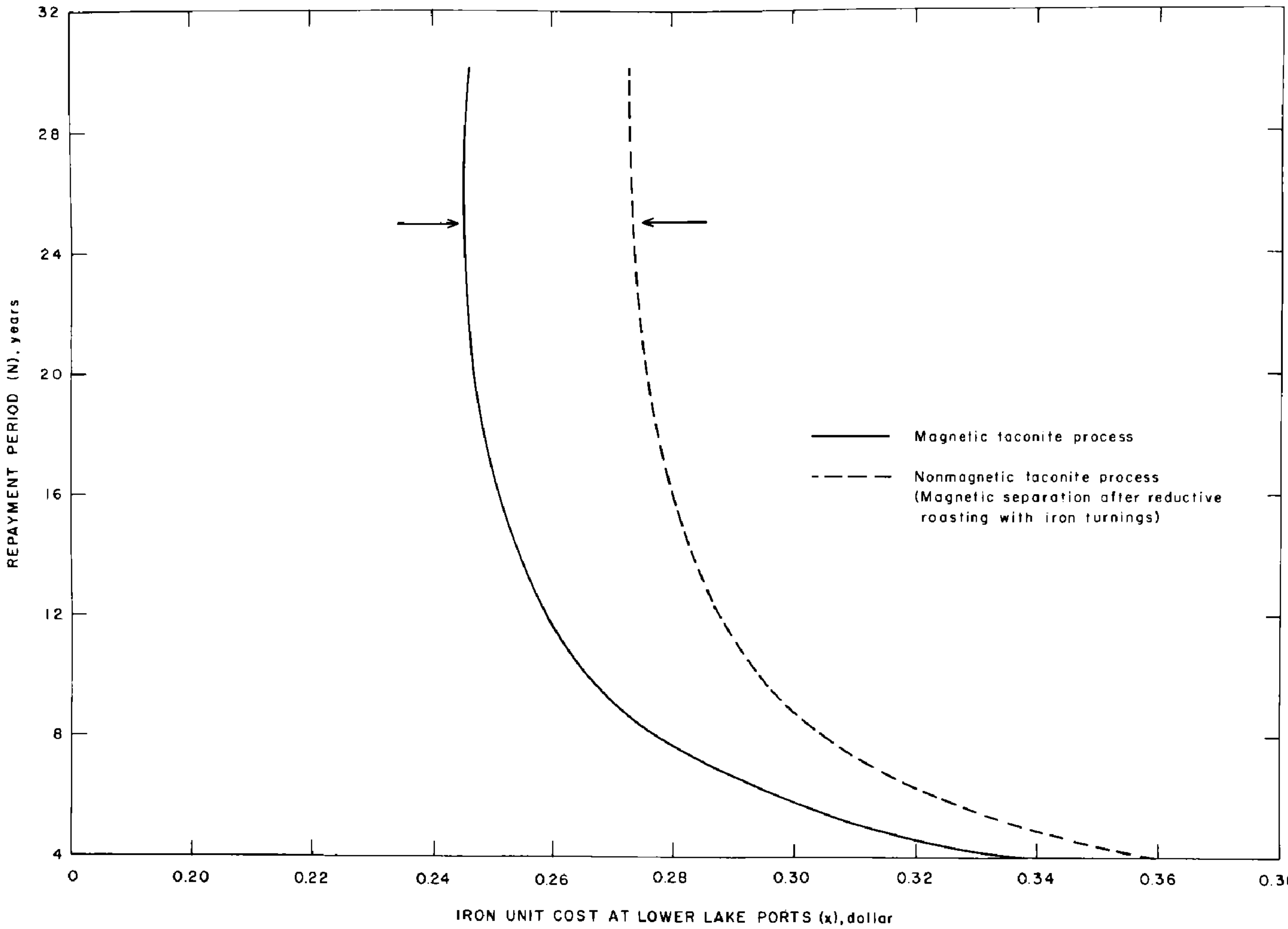

FIGURE A-9. - Relation Between Repayment Period and Iron Unit Cost.

of data (table 1) is between 25 and 30 years. Beyond that point, the cost rises slightly because the basis for calculating the depletion allowance shifts from net income to gross income. Below the depletion shift, the depletion allowance increases with an increase in repayment period and reflects the decreasing depreciation allowance in the calculation of net income for depletion. Above the shift point, the depletion allowance is constant.

Recoverable Iron Content

Recoverable iron content has an inverse curvilinear relation with total iron unit cost (fig. A-10) because of its interrelationships with several other input variables. The two curves associated with the magnetic taconite and nonmagnetic processes demonstrate how important the increased recoveries are in making the nonmagnetic process even remotely competitive. At any given recoverable iron content, the unit cost differential between the two processes is great. The recoverable iron content--iron unit cost curve can be used to adjust an economic availability evaluation for different estimates of recoverable iron content, assuming of course, that other component cost factors remain relatively constant.

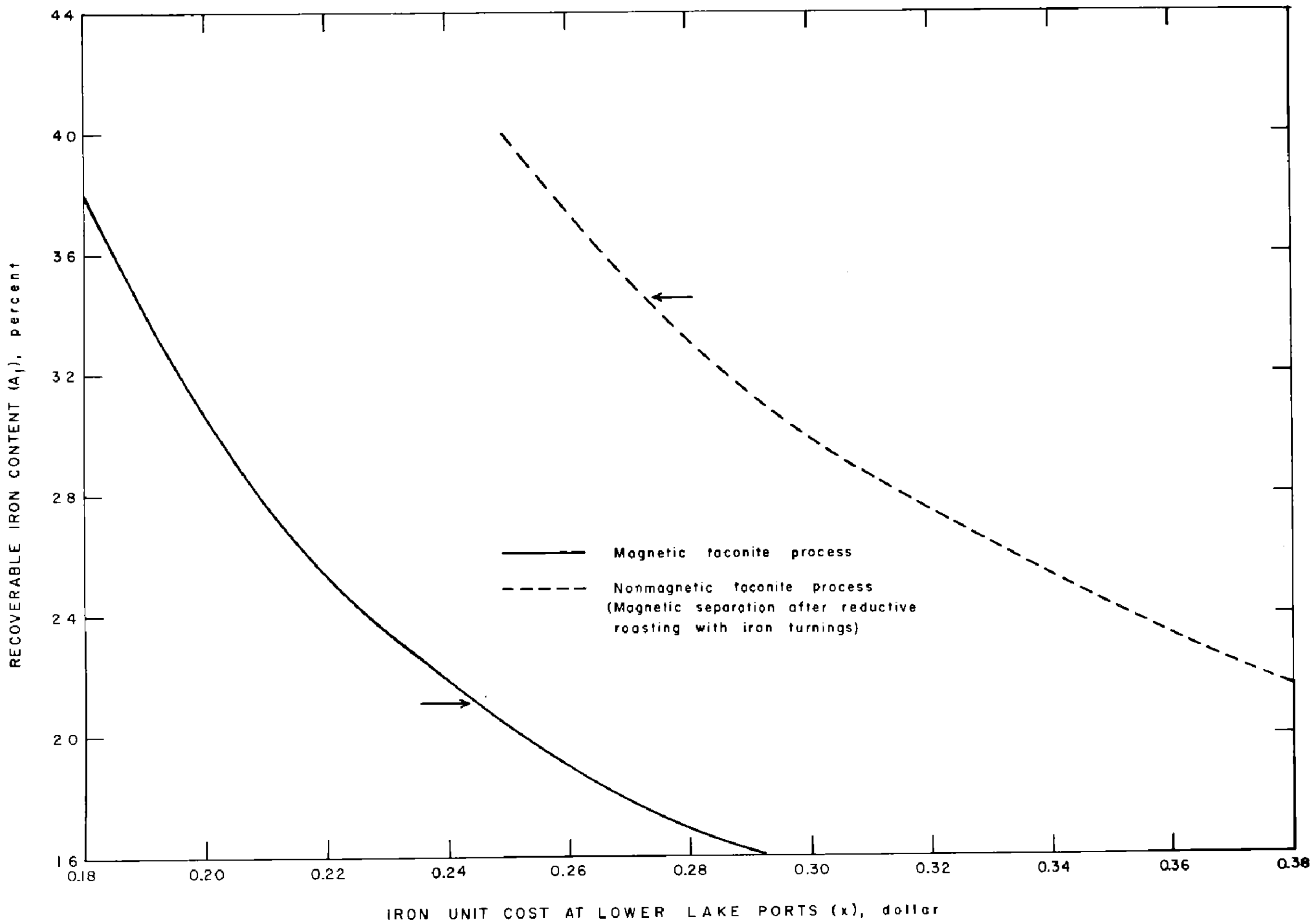

FIGURE A-10. - Relation Between Recoverable Iron Content and Iron Unit Cost.

Concentrate Iron Content

The effect of variations in concentrate iron content upon iron unit cost is related to the various component cost factors estimated on a per ton of concentrate basis. The higher the concentrate iron content, the more iron units per ton of pellets and conversely the less tonnage per iron unit. Therefore, such costs as are directly related to the concentrate tonnage, for example, pelletizing and transportation, are reduced. The total iron unit cost is relatively insensitive to variations in the concentrate iron content (fig. A-11). The relation between the two is inverse and nearly linear.

Equivalent Stripping Ratio

Above the depletion shift point on the magnetic taconite curve (fig. A-12) and for all of the nonmagnetic taconite curve, in the range illustrated, the relationship between equivalent stripping ratio and iron unit cost is linear. A one-unit increase in the equivalent stripping ratio increases the total pellet cost per ton by an amount exactly equal to the stripping cost per cu yd C_5.

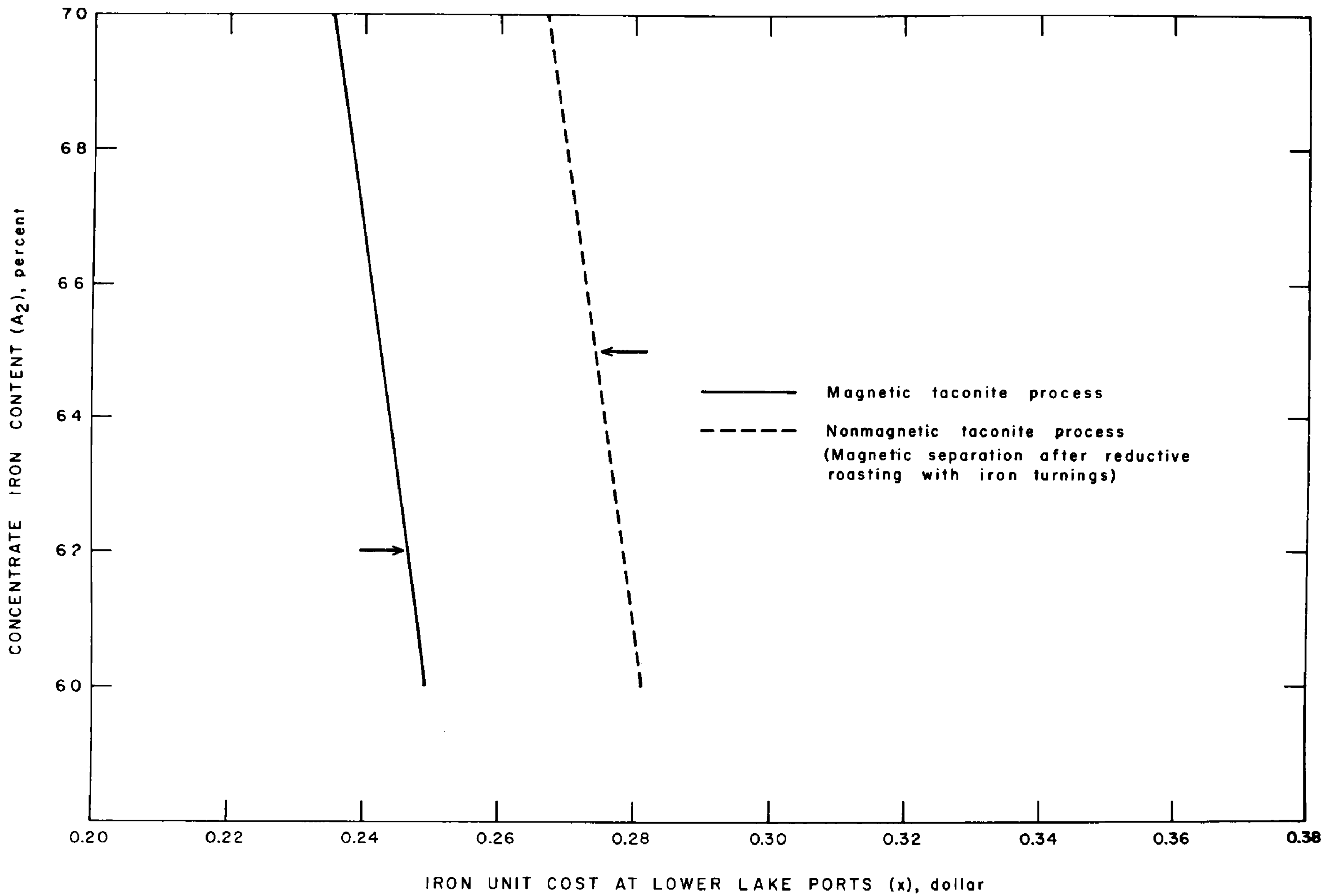

FIGURE A-11. - Relation Between Concentrate Iron Content and Iron Unit Cost.

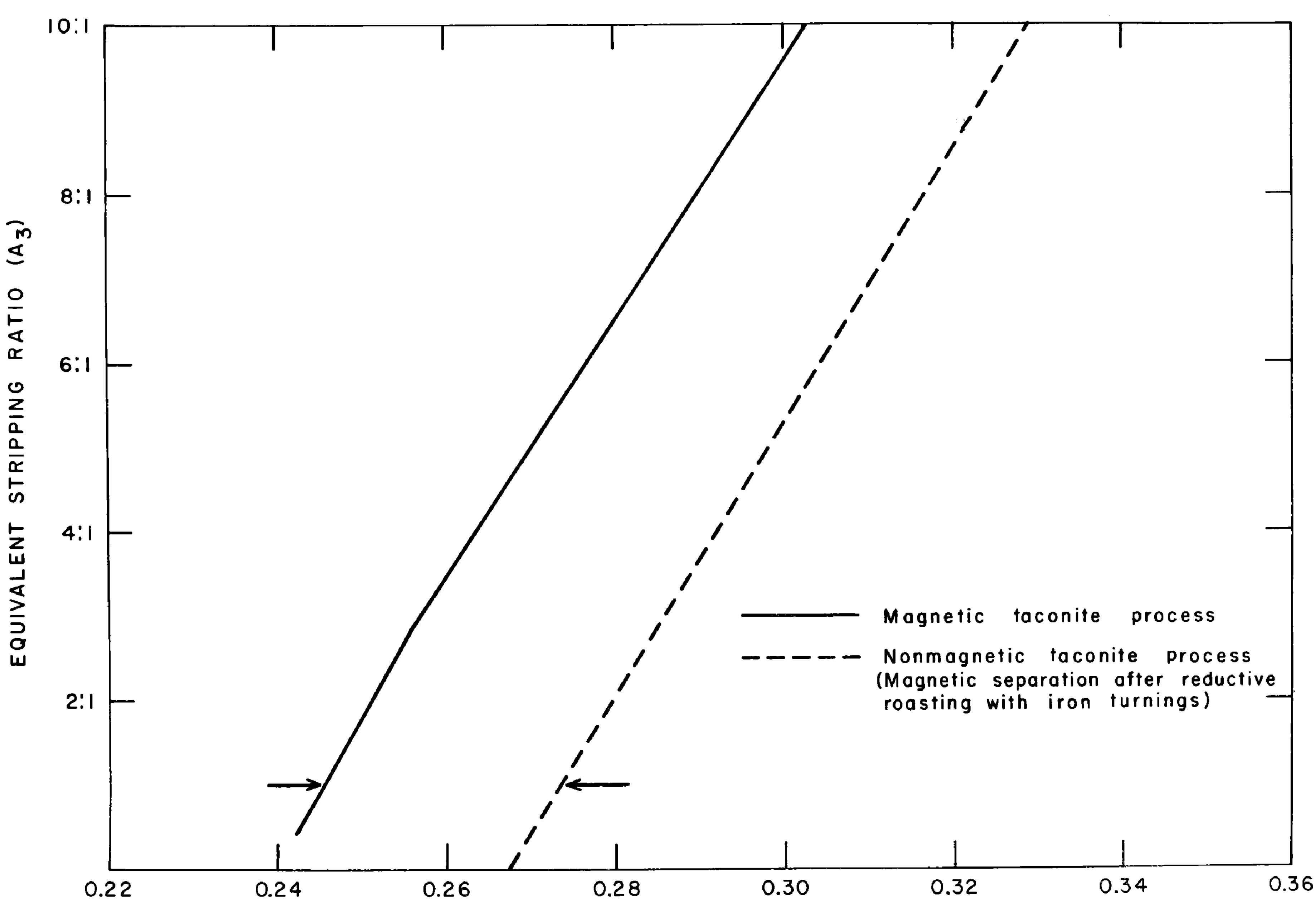

FIGURE A-12. - Relation Between Equivalent Stripping Ratio and Iron Unit Cost.

APPENDIX B.--EFFECT OF CHANGES IN THE PHYSICAL PARAMETERS ON MESABI RANGE RESERVES

The relative effect of changes in the physical parameters that limit Mesabi Range iron ore reserves is shown in figures B-1, B-2, and B-3. The charts show the importance of stripping ratio, percent iron recovery, and Lower Cherty ore probability factor on quantity of Mesabi Range iron ore reserves. Technological advances that increase any of these factors increase reserve quantity significantly.

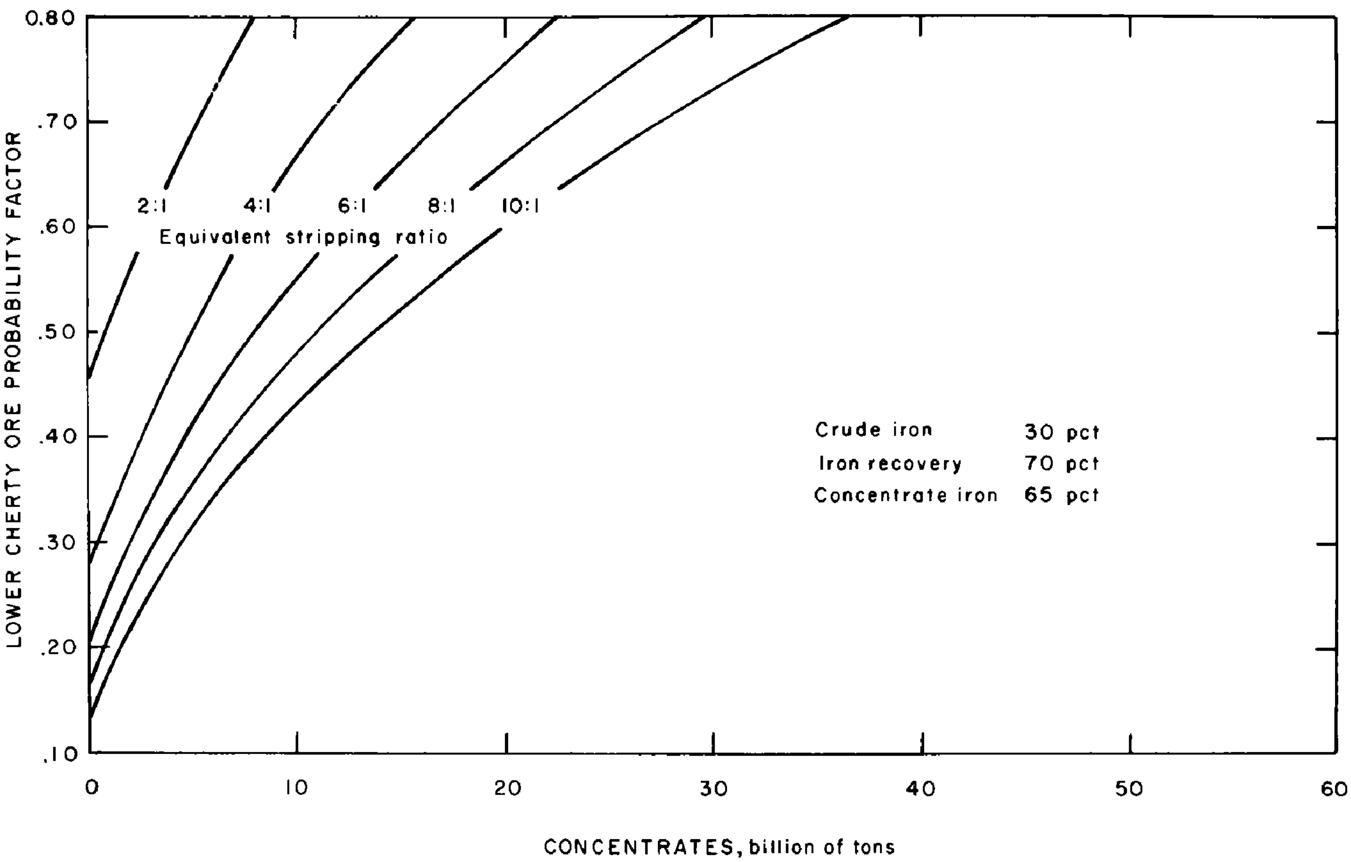

FIGURE B-1. - Mesabi Range Lower Cherty Taconite Reserves, 1968, Based on Selected Parameter Values With 70 Percent Iron Recovery.

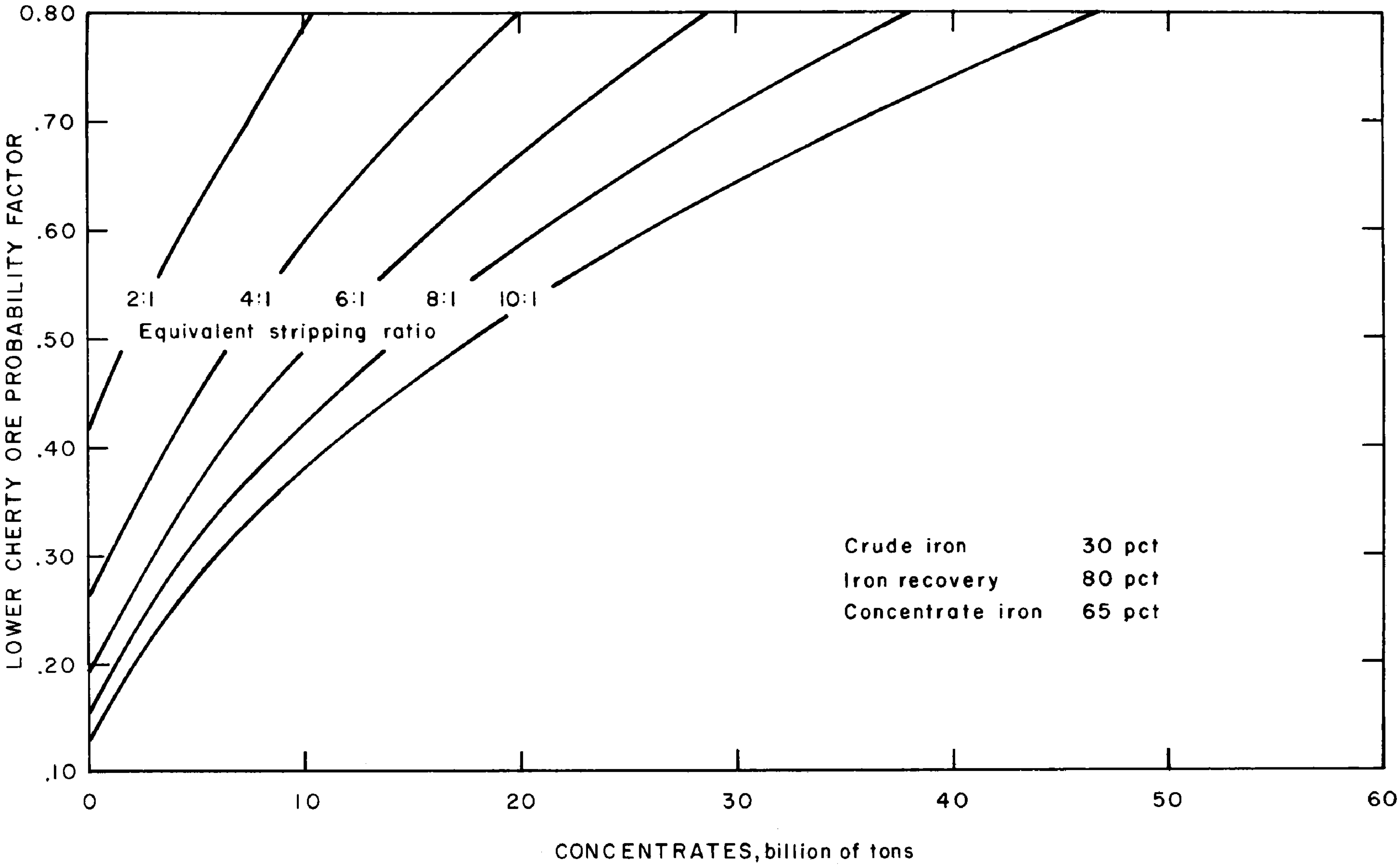

FIGURE B-2. - Mesabi Range Lower Cherty Taconite Reserves, 1968, Based on Selected Parameter Values With 80 Percent Iron Recovery.

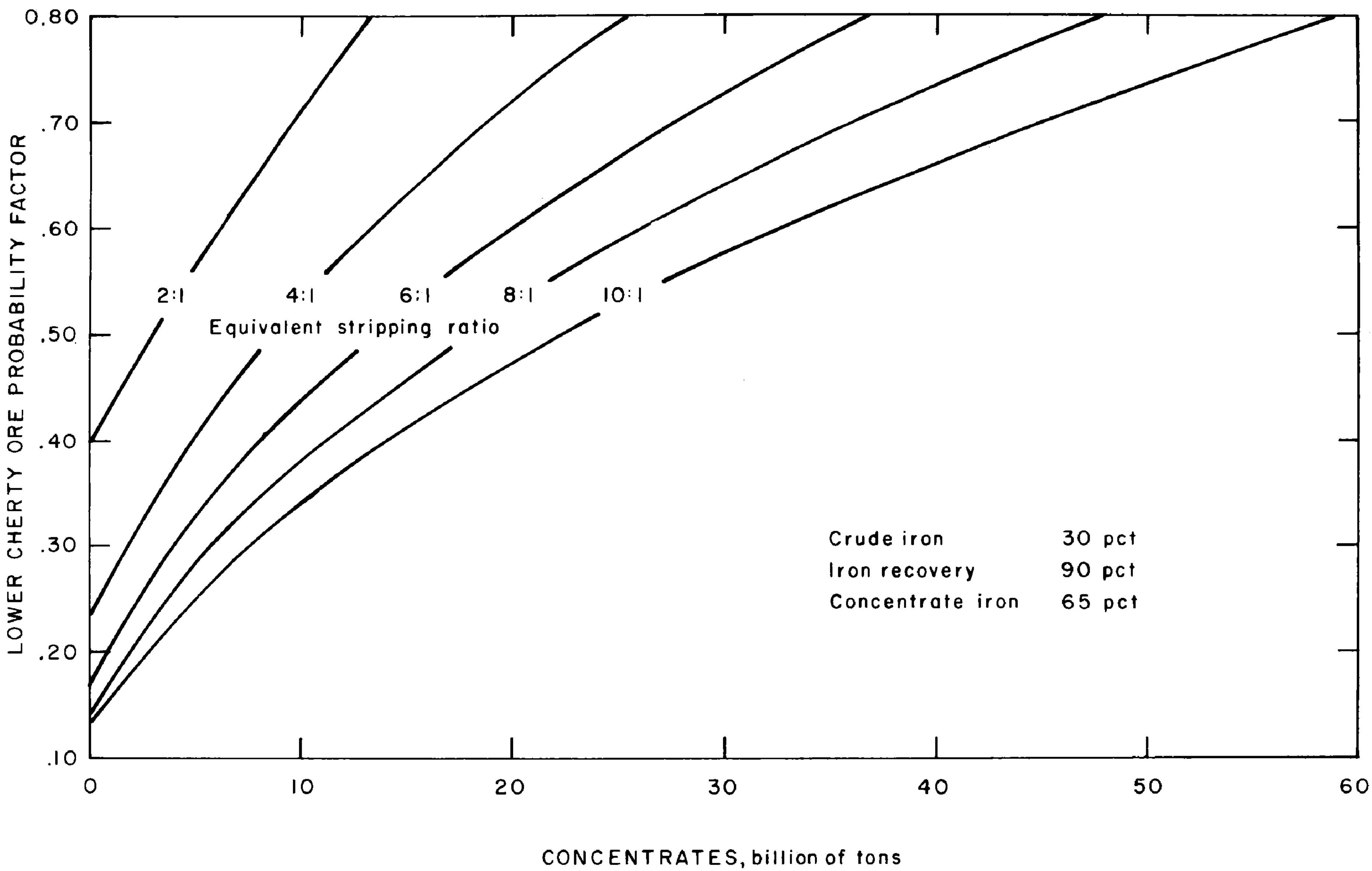

FIGURE B-3. - Mesabi Range Lower Cherty Taconite Reserves, 1968, Based on Selected Parameter Values With 90 Percent Iron Recovery.

APPENDIX C.--COMPUTER PROGRAMS

```
*NAME QEM
*ONE WORD INTEGERS
*IOCS(CARD,1132 PRINTER,DISK)
*LIST ALL
      REAL IMN,K,I
      DIMENSION X(200),Y(200),BMX(6),XMX(6)
      COMMON K,WK,P,C1,C2,C3,C4,C5,C6,S1,S2,F1,F2,F3,I,N,A1,A2,A3,LL,L,BMX,A5
      READ(2,10) L,NN,SMX,IMN
   10 FORMAT(I5,I5,2F6.2)
      WRITE(3,100)
  100 FORMAT(1H1,11X,35HECONOMIC AVAILABILITY EVALUATION OF,/)
      GO TO(20,30),L
   20 WRITE(3,101)
  101 FORMAT(1H ,21X,17HMAGNETIC TACONITE,/)
      GO TO 40
   30 WRITE(3,102)
  102 FORMAT(1H ,20X,20HNONMAGNETIC TACONITE,/)
   40 WRITE(3,110) NN,SMX
  110 FORMAT(1H ,18X,24HQUALITY EVALUATION INPUT,//,1H ,10X,9HNUMBER OF,
     /8H SAMPLES,18X, I5 ,/,1H ,10X,26HCONCENTRATE SILICA MAXIMUM,9X,
     /F5.2,9H  PERCENT,/)
      GO TO(50,60),L
   50 WRITE(3,120) IMN
  120 FORMAT(1H ,10X,32HRECOVERABLE IRON CONTENT MINIMUM,3X,F5.2,9H  PERCENT,/)
      WRITE(3,123)
  123 FORMAT(1H ,14X,1HF,9X,2HMG,8X,2HCI,8X,2HCS)
      GO TO 70
   60 WRITE(3,124)
  124 FORMAT(1H ,13X,1HF,7X,1HG,7X,1HR,7X,2HCI,6X,2HCS)
   70 A2=0
      SUM=0
      SUM1=0
      M=0
      DO 90 II=1,NN
      READ(2,135) FT,G,R,MG,CI,CS
  135 FORMAT(F5.0,F6.2,F5.3, I9, F7.2,F6.2)
      GO TO(61,62),L
   61 WRITE(3,125)FT,MG,CI,CS
      GO TO 63
   62 WRITE(3,126)FT,G,R,CI,CS
   63 CONTINUE
      GO TO(80,85),L
   80 IF(CI-IMN)89,81,81
   81 IF(CS-SMX)82,82,89
   82 SUM1=SUM1+FT
      M=M+1
      X(M)=MG
      Y(M)=FT
      A2=A2+CI*FT
```

```
      GO TO 89
 85   IF(CS-SMX)86,86,89
 86   SUM1=SUM1+FT
      M=M+1
      X(M)=G*R
      Y(M)=FT
      A2=A2+CI*FT
 89   SUM=SUM+FT
 90   CONTINUE
      A2=A2/SUM1
      A5=SUM1/SUM
      WRITE(3,140)
140   FORMAT(1H1,20X,19HINTERMEDIATE OUTPUT,/)
      CALL SORTA(X,Y,M,20.,.5,1)
      CALL LST(X,Y,M,XMX,0)
      DO 145 III=1,6
145   BMX(III)=XMX(III)
      WRITE(3,150)
150   FORMAT(1H ,18X,25HQUALITY EVALUATION OUTPUT)
      WRITE(3,142) A5
142   FORMAT(1H ,10X,24HORE PROBABILITY FACTOR =,F9.2,//,1H ,10X,4HCUMU,
     /31HLATIVE PROBABILITY DISTRIBUTION,/,1H ,10X,16HFOR RECOVERABLE ,
     /12HIRON CONTENT/1H ,10X,38HF(A1)=1-EXP(A+B*A1+C*A1**2+D*A1**3+E*A,
     /13H1**4+F*A1**5)/)
      WRITE(3,143) BMX,A2
143   FORMAT(1H ,10X,2HA=,E14.6,/,1H ,10X,2HB=,E14.6/1H ,10X,2HC=,E14.6/
     /1H ,10X,2HD=,E14.6/1H ,10X,2HE=,E14.6/1H ,10X,2HF=,E14.6//,1H ,
     /10X,26HCONCENTRATE IRON CONTENT =,F7.2)
125   FORMAT(1H ,7X,4F10.2)
126   FORMAT(1H ,8X,5F8.2)
      READ(2,160)K,WK,P,C1,C2,C3,C4,C5,C6,S1,S2,F1,F2,F3,I,N,A1,A2,A3,LL
160   FORMAT(F6.0,F9.0,F6.0,8F4.2,4F3.0,I2,F4.1,2F3.0,I1)
      WRITE(3,163) K,WK,P,C1,C2
163   FORMAT(1H1,31X,25HDCF COST EVALUATION INPUT//17H CAPITAL INVESTME,
     /2HNT,21X,F11.0/16H WORKING CAPITAL,24X,F11.0/15H PLANT CAPACITY,
     /26X,F10.0/12H MINING COST34X,F7.2/19H BENEFICIATION COST, 27X,F7.2)
      WRITE(3,165) C3,C4,C5,C6
165   FORMAT(27H SUPERVISION AND MANAGEMENT, 19X,F8.3/15H PELLETIZING CO,
     /2HST,29X,F7.2/15H STRIPPING COST,31X,F7.2/15H TRANSPORTATION,31X,F8.3)
      WRITE(3,167) S1,S2,F1,F2,F3,I,N,LL
167   FORMAT(30H AVERAGE STATE AND LOCAL TAXES16X,F7.2/13H ROYALTY COST,
     /33X,F7.2/24H FEDERAL INCOME TAX RATE,22X,F5.0/16H GROSS RATE FOR ,
     /9HDEPLETION,21X,F5.0/23H NET RATE FOR DEPLETION,23X,F5.0/6H RATE ,
     /9HOF RETURN,31X,F5.0/17H REPAYMENT PERIOD,29X,I4/13H DEPRECIATION,
     /16H METHOD SELECTOR,20X,I1)
      CALL LINK(VOL)
      END
```

```
*NAME VOL
      REAL K
      DIMENSION TLC(43),BNG(43),B1(43),B2(43,3),B5(43),FA(6),SUM(6),C(6)
      COMMON K,WK,P,C1,C2,C3,C4,C5,C6,S1,S2,F1,F2,F3,R,N,A1,A2,A3,L,LL
      COMMON FA,A5
      DEFINE FILE 1(1500,14,U,NREC)
      KX=7
      M=0
      DO 2 I=1,6
    2 C(I)=FA(I)
C
C TRIGOMETRIC CONSTANTS
C    A = SLOPE OF PIT BANK
C    Z = DIP OF EROSION SURFACE
C    W = DIP OF FORMATION
C
      A=ATAN(.50)
      Z=ATAN(.02)
      W=ATAN(.10)
      PI=3.1415926
      B=ATAN(10.0)
      AC=PI-(A+B)
      D=ATAN(W)-ATAN(Z)
      F=PI-AC
      E=PI-(F+D)
      G=A
      RR=ATAN(50.)
      O=D
      PP=F
      Q=E
C
C  IX  = SEED FOR RANDOM NUMBER GENERATOR
C     U = ACCURACY DESIRED IN CALCULATING EQUIVALENT STRIPPING RATIO
C    NX = NUMBER OF SECTIONS
C
      READ(2,1) IX,U,NX
    1 FORMAT(I5,F5.2,I5)
C
C QUALITY FACTORS
C   A1 = RECOVERABLE IRON CONTENT
C   A2 = CONCENTRATE IRON CONTENT
C   A3 = EQUIVALENT STRIPPING RATIO
C   A4 = BULK DENSITY OF TACONITE
C   A5 = ORE PROBABILITY FACTOR
C
```

```
C DIMENSIONAL FACTORS
C  TLC = TOTAL LOWER CHERTY THICKNESS
C  BNG = BOTTOM SLATES AND CONGLOMERATES
C   B1 = LOWER CHERTY ADJUSTED APPARENT THICKNESS
C   B2 = SURFACE COVER THICKNESSES
C   B4 = INITIAL PIT WIDTH
C   B5 = LENGTH OF INFLUENCE
C  B6A = PRODUCTION REDUCTION FACTOR
C
      READ(2,7) (TLC(I),BNG(I),B5(I),(B2(I,J),J=1,3),I=1,NX)
    7 FORMAT(2F5.0,F10.0,3F5.0)
      A4=11.
      B6A=6000. * A4
      DO 101 I=1,NX
      IN=0
      A3M=0.4
      GO TO(50,55),LL
   50 A2=62.00
      IF(I-22) 51,52,53
   51 A5= 0.43
      GO TO 60
   52 A5= 0.22
      GO TO 60
   53 IF(I-32) 52,54,54
   54 A5= 0.67
      GO TO 60
   55 A2= 65.00
      IF(I-22) 56,57,57
   56 A5= 0.358
      GO TO 60
   57 A5= 0.244
C
C   CALL F(A1) FOR RANDOM VALUE OF RECOVERABLE IRON CONTENT
C
   60 CALL RVAL(IX,KX,C,A1)
      RE=A1
      GO TO(6,8),LL
    8 A1=A1*1.14
    6 PWRC = A1/A2 * 100.
      IJK=1
      DO 101 J=1,24
      ZIT=J
      A3 = ZIT/2.
      IF(A3-A3M) 31,42,42
   42 CONTINUE
      CC=0.0
      GO TO(10,11),IJK
   10 DISS=10000.
      GO TO 12
   11 DISS=IDISS
   12 Y=10000.
      XX= 10000.
      B1(I)= TLC(I) - BNG(I)
```

```
   25  CONTINUE
C
C    ROCK COVER CROSS SECTION AREA
C
       XP=DISS/SIN(RR)
       XH=SIN(O)*XP/SIN(PP)
       B3A = 0.5 * XH * XP * SIN(Q)
C
C    LOWER CHERTY NET CROSS SECTION AREA
C
       XB = SIN(B) * B1(I) / SIN(AC)
       XF = SIN(F) * XB / SIN(D)
       ALCN = 0.5 * XB * XF * SIN(E)
       XG = XH * SIN(G)
       ADIS = DISS - XG
       IF(ADIS-0.0001) 62,62,63
   63  CONTINUE
       ALCS = ADIS * B1(I)
       GO TO 64
   62  ALCS = 0.
   64  CONTINUE
       B1A = ALCN + ALCS - B6A
C
C    SURFACE COVER
C
       ASN = (XF + 20. + B2(I,1)) * B2(I,1)
       IF(DISS - 2640.) 21,21,22
   21  ASS = DISS * B2(I,2)
       GO TO 27
   22  IF(DISS -5280.) 23,23,24
   23  ASS = 2640. *B2(I,2) + (DISS - 2640.) * B2(I,3)
       GO TO 27
   24  AA = DISS - 5280.
       ASS = 2640. * (B2(I,2) + B2(I,3)) + AA * (B2(I,3) + 0.02 * AA)
   27  CONTINUE
       B2A = ASN + ASS
C
C    B1V = VOLUMN OF POTENTIAL ORE
C
       B1V = B5(I) * A5 * B1A / 27.
C
C    B2V = VOLUMN OF SURFACE COVER
C
       B2V = B5(I) * B2A / 27.
C
C    B3V = VOLUMN OF ROCK COVER
C
       B3V = B5(I) * (B3A + B1A * (1.-A5) / 2.) / 27.
C
C    TC = CRUDE ORE QUANTITY
C
       TC = B1V * 27. / A4
C
C    TP = CONCENTRATE QUANTITY
C
```

```
      TP = TC * A1 / A2
C
C   A3X = EQUIVALENT STRIPPING RATIO
C
      A3X = (B2V + (2. * B3V)) / TP
C
C   WHEN IN=0 CALCULATE MINIMUM EQUIVALENT STRIPPING RATIO  (A3M)
C
      IF(IN-1) 17,43,43
  17  CONTINUE
      IF(ABS(CC-A3X)-U) 15,15,13
  13  CONTINUE
      IF(CC-A3X) 14,15,30
  14  DISS = DISS - 2. * XX
      IF(DISS- 0.00001) 80,80,16
  80  DISS = 0.00001
  16  CONTINUE
      XX = XX/ 10.
      GO TO 71
  30  DISS = DISS + XX
  71  CONTINUE
      CC = A3X
      GO TO 25
  15  CONTINUE
      A3M = A3X
      IN = IN + 1
      IDISS = DISS
      IJK = 2
      WRITE(3,125) I,RE,A5,A3M,IDISS
 125  FORMAT(1H1,8H SECTION,I3,10X,24HRECOVERABLE IRON CONTENT,F7.2,10X,
     /22HORE PROBABILITY FACTOR,F6.3,/,1H ,21X,11HMINIMUM ESR,F5.1,3H IS
     / I6,25H FEET SOUTH OF PR OUTCROP,//,1H 2X,3HESR,8X,3HTCR,7X,4HPWRC,
     /6X,4HTCON,7X,2HSY,8X,2HRY,5X,3HRFE,5X,3HFEP,4X,3HAUV,4X,3HNTP,7X,
     /4HNDIS,/)
  43  CONTINUE
C
C  MINIMUM EQUIVALENT STRIPPING RATIO HAS BEEN CALCULATED
C
      IF(A3 - A3M) 31,35,35
C
C CALCULATE QUANITIES FOR GIVEN EQUIVALENT STRIPPING RATIOS
C
  35  CONTINUE
      IF(ABS(A3X-A3) - U) 98,98,5
   5  CONTINUE
      IF(A3X -A3) 9,98,3
   9  DISS =DISS + Y
      GO TO 25
   3  DISS = DISS - Y
      Y = Y / 10.
      IF(Y-0.1) 98,25,25
C
C WHEN GIVEN ESR IS LESS THAN MINIMUM ESR ALL QUANTITIES = ZERO
C
```

```
  31   CONTINUE
       ATC = 0.
       AWRC = 0.
       ATP = 0.
       AB2V = 0.
       AB3V = 0.
       X = 0.
       XXTP = 0.
       TCXA1 = 0.
       NTP = 0.
       NDIS = 0.
       GO TO 39
  98   CONTINUE
       ATC = TC / 1.E6
       AWRC = PWRC
       ATP = TP / 1.E6
       AB2V = B2V / 1.E6
       AB3V = B3V / 1.E6
       CALL DCF(X)
       XXTP = X * ATP
       TCXA1 = ATC * A1
       NTP = XF + DISS
       NDIS= DISS
  39   CONTINUE
       WRITE(3,28) A3,ATC,AWRC,ATP,AB2V,AB3V,RE,A2,X,NTP,NDIS
  28   FORMAT(1H ,F5.1,5X,F9.3,F9.2,1X,3F10.3,2F7.2,F6.3,I8,I10)
       M=M+1
       WRITE(1'M) A3,ATC,ATP,AB2V,AB3V,XXTP,TCXA1
 101   CONTINUE
C
C   SUMMARY PRINTOUT
C
       WRITE(3,130)
 130   FORMAT(1H1,36X,7HSUMMARY,//,1H ,2X,3HESR,9X,3HTCR,7X,4HPWRC,6X,
      /4HTCON,6X,2HSY,8X,2HRY,6X,3HRFE,4X,3HFEP,4X,3HAUV,/)
       DO 150 J=1,24
       DO 135 I=1,6
 135   SUM(I)=0.
       DO 140 I=1,NX
       M=J+24*(I-1)
       READ(1'M) A3,ATC,ATP,AB2V,AB3V,XXTP,TCXA1
       SUM(1) = SUM(1) + ATC
       SUM(2) = SUM(2) + ATP
       SUM(3) = SUM(3) + AB2V
       SUM(4) = SUM(4) + AB3V
       SUM(5) = SUM(5) + XXTP
       SUM(6) = SUM(6) + TCXA1
 140   CONTINUE
       AUV=SUM(5)/SUM(2)
       ARFE=SUM(6)/SUM(1)
       PWRC=SUM(2)/SUM(1)
       WRITE(3,145)  A3,SUM(1),PWRC,(SUM(JJ),JJ=2,4),ARFE,A2,AUV
 145   FORMAT(1H ,F5.1,5X,F9.3,F9.2,1X,3F10.3,2F7.2,F6.3)
 150   CONTINUE
       CALL EXIT
       END
```

```
      SUBROUTINE SORTA(X,Y,N,XM,XIN,LP)
C
C     PROGRAMMER - RONALD W. MICHELSON
C                  U.S. BUREAU OF MINES
C
C     SUBROUTINE CALCULATES CUMULATIVE FREQUENCY DISTRIBUTION
C
C        INPUT ARGUMENTS
C           X(I) = SAMBLE VARIATE
C           Y(I) = NUMBER OF OBSERVATIONS
C           M    = NUMBER OF X(I),Y(I) PAIRS
C                XM = MINIMUM OUTPUT VALUE
C                XIN = SORTING INTERVAL
C                LP = OUTPUT INDICATOR
C                   LP = 0;  Y(I) = ACTUAL CUMULATIVE FREQUENCY
C                   LP = 1,  Y(I) = RELATIVE CUMULATIVE FREQUENCY
C        OUTPUT ARGUMENTS
C           X(I) = VARIABLE
C           Y(I) = 1. - CUMULATIVE FREQUENCY DISTRIBUTION
C           M    = NUMBER OF X(I),Y(I) PAIRS
C
      DIMENSION X(200),Y(200),XINT(201),NNT(201)
      DO 5 I=1,201
      XINT(I)=0
    5 NNT(I)=0
      DO 100 I=1,N
      X1=XM
      X2=0
      DO 80 J=1,200
      IF(X(I)-X1)15,25,25
   15 IF(X(I)-X2)25,20,20
   20 XINT(J)=XINT(J)+Y(I)
      NNT(J)=NNT(J)+1
      GO TO 100
   25 X2=X1
      X1=X1+XIN
   80 CONTINUE
      XINT(201)=XINT(201)+Y(I)
  100 CONTINUE
      SX=0
      DO 105 I=1,201
  105 SX=SX+XINT(I)
      IF(LP) 120,120,115
  115 DO 110 I=1,201
  110 XINT(I)=XINT(I)/SX*100.
      SX=100.
```

```
120   CONTINUE
      X1=XM
      N=0
      DO 125 I=1,200
      IF(X1-40.) 126,126,127
126   N=N+1
      X(N)=X1
      SX=SX-XINT(I)
      Y(N)=SX
130   X1=X1+XIN
125   CONTINUE
127   CONTINUE
      WRITE(3,143)
      DO 140 I=1,N
      Y(I)=Y(I)/100.
140   WRITE(3,141) X(I),Y(I)
141   FORMAT(1H ,2F20.2)
143   FORMAT(10X,38H1. - CUMULATIVE FREQUENCY DISTRIBUTION,/,
     /17X,4HX(I),17X,4HY(I),/,
     /14X,29H----------------------------,/)
      RETURN
      END
```

```
      SUBROUTINE LST(XVL,Y,N,BMX,KKK)
C
C     SUBROUTINE CALCULATES COEFFICIENTS FOR A FIFTH ORDER POLYNOMIAL OR
C     A FIFTH ORDER EXPONENTIAL FUNCTION
C
C     PROGRAMMER - JON F. VOEDISCH
C                  U.S. BUREAU OF MINES
C
C            INPUT ARGUMENTS
C               XVL = ARRAY OF INDEPENDENT VARIABLES
C               Y   = ARRAY OF DEPENDENT VARIABLES
C               N   = NUMBER OF VARIABLE PAIRS
C               KKK = CURVE INDICATOR
C                  KKK = 0, EXPONENTIAL FUNCTION
C                  KKK = 1, FIFTH ORDER POLYNOMIAL
C            OUTPUT ARGUMENTS
C               BMX = ARRAY OF FUNCTION COEFFICIENTS

      DIMENSION XVL(200),YVL(200),Y(200),AMX(6,6),BMX(6),CMX(200)
      IF(KKK)3,3,4
    3 M=1
      GO TO 10
    4 M=2
   10 CONTINUE
      DO 5 I=1,N
      GO TO (20,30),M
   20 YVL(I)=ALOG(Y(I))
      GO TO 5
   30 YVL(I)=Y(I)
    5 CONTINUE
      SUM=0.0
      TOT=0.0
      DO 100 I=1,N
      SUM=SUM+XVL(I)
      TOT=TOT+YVL(I)
  100 CONTINUE
      BMX(1)=TOT
      CMX(1)=N
      CMX(2)=SUM
      DO 120 I=1,5
      TOT=0.0
      DO 110 J=1,N
      TOT =TOT+YVL(J)*XVL(J)**I
  110 CONTINUE
      BMX(I+1)=TOT
  120 CONTINUE
      DO 140 I=2,10
      SUM=0.0
      DO 130 J=1,N
      SUM=SUM+XVL(J)**I
```

```
130   CONTINUE
      CMX(I+1)=SUM
140   CONTINUE
      DO 160 I=1,6
      DO 150 J=1,6
      K=I+J-1
      AMX(I,J)=CMX(K)
150   CONTINUE
160   CONTINUE
      DO 21 I=1,6
      ARG=1.0/AMX(I,I)
      BMX(I)=BMX(I)*ARG
      DO 170 J=1,6
      AMX(I,J)=AMX(I,J)*ARG
170   CONTINUE
      DO 15 J=1,6
      IF(I-J)180,15,180
180   ARG=AMX(J,I)
      BMX(J)=BMX(J)-BMX(I)*ARG
      DO 190 K=1,6
      AMX(J,K)=AMX(J,K)-AMX(I,K)*ARG
190   CONTINUE
 15   CONTINUE
 21   CONTINUE
      WRITE(3,331)
331   FORMAT(1H1)
      GO TO(97,98),M
 98   WRITE(3,99)
      GO TO 32
 97   WRITE(3,31)
 32   CONTINUE
      WRITE(3,33)(BMX(I),I=1,6)
      WRITE(3,35)
      DO 136 J=1,N
      X=XVL(J)
      TOT=BMX(1)+X*(BMX(2)+X*(BMX(3)+X*(BMX(4)+X*(BMX(5)+X*BMX(6)))))
      GO TO(102,122),M
102   RR=EXP(TOT)
      GO TO 36
122   RR=TOT
 36   CONTINUE
      PCT=(RR-Y(J))/Y(J)*100.
136   WRITE(3,38)XVL(J),Y(J),RR,PCT
      RETURN
 99   FORMAT(9X,46HY= A + B*X + C*X**2 + D*X**3 + E*X**4 + F*X**5)
 31   FORMAT(9X,52HY=EXP**(A + B*X + C*X**2 + D*X**3 + E*X**4 + F*X**5))
 33   FORMAT(//,9X,1HA,14X,1HB,14X,1HC,14X,1HD,14X,1HE,14X,1HF,//,6E15.6,//)
 35   FORMAT(/,15X,1HX,8X,8HY-ACTUAL,5X,12HY-CALCULATED,5X,14HPCT-DIFFERENCE,//)
 38   FORMAT(5X,3F15.5,F12.2)
      END
```

```
      SUBROUTINE RVAL (IX,K,C,A1)
      DIMENSION C(6)
  100 CALL RANDU(IX,IY,Y)
      IF(Y-.05) 100,101,101
  101 IF(Y-.95) 102,102,100
  102 CONTINUE
      XNT=10.
      X=20.
    5 YY=1.-CURV(K,C,X)
      IF(ABS(YY-Y)-.01)20,20,10
   10 IF(YY-Y)11,20,12
   11 X=X+XNT
      IF(X-40) 16,16,15
   15 X=25.
      XNT=2.
   16 GO TO 5
   12 X=X-1.5*XNT
      XNT=XNT/2.
      IF(X-20.) 17,18,18
   17 X=30.
      XNT=2.
   18 GO TO 5
   20 A1=X
      RETURN
      END
```

```
C     RANDU
C     PURPOSE
C          COMPUTES UNIFORMLY DISTRIBUTED RANDOM FLOATING POINT NUMBERS
C          BETWEEN 0 AND 1.0 AND INTEGERS IN THE RANGE 0 TO 2**15.
C     USAGE
C          CALL RANDU(IX,IY,YFL)
C     DESCRIPTION OF PARAMETERS
C            IX - FOR THE FIRST ENTRY THIS MUST CONTAIN ANY ODD POSITIVE
C                 INTEGER LESS THAN 32,768.  AFTER THE FIRST ENTRY, IX
C                 SHOULD BE THE PREVIOUS VALUE OF IY COMPUTED BY THIS
C                 SUBROUTINE.
C            IY - A RESULTANT INTEGER RANDOM NUMBER REQUIRED FOR THE NEXT
C                 ENTRY TO THIS SUBROUTINE.  THE RANGE OF THIS NUMBER IS
C                 FROM ZERO TO 2**15.
C           YFL - THE RESULTANT UNIFORMLY DISTRIBUTED, FLOATING POINT, RANDOM
C                 NUMBER IN THE RANGE 0 TO 1.0.
C     REMARKS
C          THIS SUBROUTINE IS SPECIFIC TO THE IBM 1130.
C          THIS SUBROUTINE SHOULD NOT REPEAT ITS CYCLE IN LESS THAN 2 TO
C          THE 13TH ENTRIES.
C               NOTE  IF RANDOM BITS ARE NEEDED, THE HIGH ORDER BITS OF
C          IY SHOULD BE CHOSEN.
C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C          NONE.
      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*899
      IF(IY)5,6,6
    5 IY=IY+32767+1
    6 YFL=IY
      IX=IY
      YFL=YFL/32767.
      RETURN
      END
      FUNCTION CURV(K,C,X)
      DIMENSION C(6)
      IF(K-6)8,8,80
    8 A=C(1)
      B=C(2)
      GO TO (1,2,3,4,5,6),K
    1 CURV=A+B*X
      GO TO 7
    2 CURV=A*EXP(B*X)
      GO TO 7
    3 CURV=A*(X**B)
      GO TO 7
    4 CURV=A+(B/X)
      GO TO 7
    5 CURV=1/(A*B*X)
      GO TO 7
    6 CURV=X/(A*X+B)
      GO TO 7
   80 TOT=C(1)+X*(C(2)+X*(C(3)+X*(C(4)+X*(C(5)+X*C(6)))))
      IF(K-7)7,20,30
   20 CURV=EXP(TOT)
      GO TO 7
   30 CURV=TOT
    7 RETURN
      END
```

```
      SUBROUTINE DCF(X)
C
C  THE SUBROUTINE DCF COMPUTES UNIT COST (X) AS A FUNCTION OF SEVERAL
C  COMPONENT COST VARIABLES LISTED IN THE COMMON STATEMENT.  THE METHOD
C  DISCOUNTS THE FLOW OF CASH OVER THE PROJECT REPAYMENT PERIOD.  IT IS
C  NECESSARY TO DETERMINE, BY ITERATION, THE UNIT VALUE(SELLING PRICE)
C  REQUIRED, THAT THE SUM OF THE DISCOUNTED CASH FLOWS IS JUST ADEQUATE
C  TO REPAY THE CAPITAL INVESTMENT.  AT THIS CONDITION UNIT VALUE=UNIT COST
C
      REAL K,I
      INTEGER T,T1
      DIMENSION E1(50),E(50),D1(50),D2(50),D(50),X1(50),X2(50),W(50),
     /R(50)
      COMMON K,WK,P,C1,C2,C3,C4,C5,C6,S1,S2,F1,F2,F3,I,N,A1,A2,A3,L
C
C  K  = CAPITAL INVESTMENT                   $
C  WK = WORKING CAPITAL                      $
C  P  = PLANT CAPACITY                       TONS/YEAR
C  C1 = MINING COST                          $/TON (CRUDE)
C  C2 = BENEFICIATION COST                   $/TON (CRUDE)
C  C3 = SUPERVISION AND MANAGEMENT COST      $/TON (CRUDE)
C  C4 = PELLETIZING COST                     $/TON (PRODUCT)
C  C5 = STRIPPING COST                       $/CU.YD. (EQUIVALENT SURFACE)
C  C6 = TRANSPORTATION COST                  $/TON (PRODUCT)
C  S1 = AVERAGE STATE AND LOCAL TAXES        $/TON (PRODUCT)
C  S2 = ROYALTY COST                         $/TON (PRODUCT)
C  F1 = FEDERAL INCOME TAX RATE              PERCENT
C  F2 = GROSS RATE FOR DEPLETION             PERCENT
C  F3 = NET RATE FOR DEPLETION               PERCENT
C  I  = RETURN ON INVESTMENT                 PERCENT
C  N  = REPAYMENT PERIOD                     YEARS
C  A1 = RECOVERABLE IRON CONTENT             PERCENT (CRUDE)
C  A2 = CONCENTRATE IRON CONTENT             PERCENT (CONCENTRATE)
C  A3 = EQUIVALENT STRIPPING RATIO           YDS./TON (SURFACE/PRODUCT)
C
C  U  = UNIT VALUE                           $/IRON UNIT
      U=.5
C  Z  = INCREMENT OF U                       $/IRON UNIT
      Z=.1
      Q=P*(A1/A2)
      V=Q*A3
C  G  = GROSS REVENUE                        $/YEAR
   10 G=Q*U*A2
C  C  = PRODUCTION COST                      $/YEAR
      C=P*(C1+C2+C3)+Q*(C4+C6)+V*C5
C  S  = FRANCHISE COST                       $/YEAR
```

```
      S=Q*(S1+S2)
C  T  = 1,N                                      YEAR
C  T  = YEAR ONE, INITIALIZES TIME
      T=1
      LL=1
C  DK = DEPRECIABLE CATITAL                      $
      DK=K-WK
C  CALCULATE DEPRECIATION ALLOWANCE
C  E1(T) = DEPRECIATION RATE                     DECIMAL
C
 20   IF(L-2)30,40,50
 30   XN=N
      E1(T)=1./XN
      E(T)=E1(T)*DK
      GO TO 60
 40   XN=N
      GO TO(42,45),LL
 42   E1(T)=2./XN
      B=DK
      DO 41 T1=1,T
      E(T)=E1(T)*B
 41   B=B-E(T)
      BK=B+E(T)
      BN=N-T+1
      IF(BK/BN-E(T)) 60,45,45
 45   LL=2
      E(T)=BK/BN
      GO TO 60
 50   XN=N
      XT=T
      SUMT=0
      DO 51 J=1,N
      XJ=J
 51   SUMT=SUMT+XJ
      E1(T)=(XN-XT+1.)/SUMT
      E(T)=E1(T)*DK
 60   CONTINUE
C
C  D1(T) = GROSS INCOME FOR DEPLETION            $/YEAR(T)
C
      D1(T)=G-Q*(C6+S2)
C
C  D2(T) = NET INCOME FOR DEPLETION              $/YEAR(T)
      D2(T)=G-C-S-E(T)
      DA=D1(T)*(F2/100.)
      DB=D2(T)*(F3/100.)
      IF(DA-DB)70,80,80
C
C  D(T)   = DEPLETION ALLOWANCE                  $/YEAR(T)
C
```

```
   70   D(T)=DA
        GO TO 90
   80   D(T)=DB
   90   CONTINUE
C
C  X1(T) = TAXABLE INCOME                      $/YEAR(T)
C
        X1(T)=G-C-S-D(T)-E(T)
C
C  X2(T) = FEDERAL INCOME TAX                  $/YEAR(T)
C
        X2(T)=X1(T)*(F1/100.)
C
C  W(T)  = CASH FLOW                           $/YEAR(T)
C
        W(T)=X1(T)-X2(T)+D(T)+E(T)
C
C  R(T)  = DISCOUNTED CASH FLOW                $/YEAR(T)
C
        R(T)=W(T)/(1.+(I/100.))**T
        IF(T-N)100,110,110
  100   T=T+1
        GO TO 20
  110   SUMRT=0
        DO 120 J=1,N
  120   SUMRT=SUMRT+R(J)
C Y     = NET PRESENT VALUE
        Y=SUMRT-K
        IF(ABS(Y)-10000.)150,150,125
  125   IF(Y)130,150,140
  140   U=U-Z
        GO TO 10
  130   U=U+Z
        Z=Z/2.
        GO TO 10
C
C X     = TOTAL UNIT COST                      $/IRON UNIT
  150   X=U
C
        RETURN
        END
```

APPENDIX D.--EXAMPLE OF PRINTOUT

Example of Printout of Economic Availability Evaluation of Nonmagnetic Taconite

Quality Evaluation Input

Number of Samples 83

Concentrate Silica Maximum 10.00 Percent

F	G	R	CI	CS
9.00	42.30	0.96	64.70	8.90
32.00	45.20	0.95	67.80	4.40
18.00	32.50	0.97	66.10	6.30
12.00	31.00	0.95	63.30	9.40
23.00	28.60	0.95	65.40	6.90
20.00	29.10	0.92	63.90	8.20
16.00	29.50	0.97	64.40	10.00
14.00	30.00	0.92	63.50	9.00
50.00	32.60	0.91	66.10	6.10
40.00	36.00	0.93	66.50	5.60
11.00	36.00	0.83	64.20	9.00
47.00	29.60	0.93	64.80	8.00
10.00	34.90	0.93	57.60	16.40
9.00	33.20	0.64	65.50	7.60
26.00	29.80	0.95	64.90	8.40
21.00	34.50	0.94	60.20	14.00
10.00	35.90	0.94	66.70	5.80
24.00	29.90	0.90	63.90	8.90
25.00	42.30	0.93	63.80	8.30
21.00	35.10	0.97	66.40	6.30
10.00	29.10	0.97	64.00	8.70
21.00	27.10	0.96	66.40	6.50
14.00	30.00	0.87	63.70	8.90
28.00	29.10	0.91	64.80	8.50
28.00	32.90	0.87	63.00	10.50
16.00	37.00	0.95	64.80	9.20
18.00	30.80	0.94	62.30	12.80
0.00	33.60	0.95	64.00	9.30
7.00	29.60	0.94	61.80	12.20
12.00	32.20	0.97	65.10	7.60
9.00	29.00	0.97	63.50	9.90
16.00	36.00	0.91	65.10	6.90
18.00	30.90	0.93	66.40	6.10
10.00	6.10	0.77	64.70	8.70
0.00	31.80	0.96	63.60	10.10
15.00	38.90	0.91	58.70	8.60
20.00	31.00	0.97	67.00	5.00
22.00	31.70	0.87	62.70	11.00
30.00	27.50	0.96	63.80	9.40
50.00	34.20	0.97	66.50	7.40
10.00	32.40	0.93	62.70	9.30
23.00	30.10	0.87	63.00	9.80

Example of Printout of Economic Availability Evaluation of Nonmagnetic Taconite (Continued)

Quality Evaluation Input

Number of Samples 83
Concentrate Silica Maximum 10.00 Percent

F	G	R	CI	CS
25.00	32.80	0.96	64.10	8.70
35.00	32.00	0.96	62.40	11.30
25.00	32.10	0.95	63.30	9.80
50.00	28.40	0.94	51.20	29.30
30.00	29.80	0.67	55.30	20.80
7.00	38.40	0.87	58.50	13.40
40.00	25.10	0.86	55.90	18.20
25.00	30.00	0.94	55.30	21.00
26.00	27.00	0.93	57.10	18.40
20.00	30.20	0.96	59.70	15.20
28.00	31.80	0.92	59.20	15.70
55.00	31.50	0.94	58.40	17.10
25.00	39.30	0.96	60.40	15.00
20.00	31.20	0.94	54.10	23.40
25.00	27.80	0.92	52.80	24.60
30.00	31.10	0.95	55.50	21.60
25.00	29.30	0.95	64.20	9.70
25.00	28.90	0.92	57.50	15.50
50.00	29.50	0.95	62.10	12.50
90.00	32.40	0.92	53.90	23.00
70.00	28.30	0.92	50.90	28.00
40.00	28.40	0.95	60.90	14.00
55.00	28.10	0.94	61.80	12.10
25.00	30.00	0.95	59.90	15.10
25.00	25.60	0.94	62.40	10.20
26.00	27.30	0.95	63.00	11.00
39.00	31.80	0.94	60.20	14.20
38.00	30.30	0.97	59.00	18.00
45.00	29.80	0.90	54.70	20.20
40.00	22.40	0.81	52.00	25.70
33.00	28.80	0.79	45.70	33.90
11.00	28.20	0.65	44.80	34.20
33.00	29.60	0.93	47.20	32.00
10.00	36.10	0.96	58.70	15.70
15.00	27.90	0.78	43.60	37.30
55.00	31.80	0.92	58.90	15.20
15.00	25.10	0.80	49.90	26.50
40.00	29.80	0.92	48.80	30.00
38.00	27.70	0.69	56.70	21.40
42.00	30.60	0.90	60.60	13.40
26.00	19.70	0.83	56.20	19.30

Intermediate Output

1. - Cumulative Frequency Distribution

X(I)	Y(I)
20.00	0.98
20.50	0.98
21.00	0.98
21.50	0.97
22.00	0.97
22.50	0.97
23.00	0.97
23.50	0.97
24.00	0.97
24.50	0.97
25.00	0.97
25.50	0.97
26.00	0.97
26.50	0.86
27.00	0.79
27.50	0.74
28.00	0.66
28.50	0.57
29.00	0.53
29.50	0.53
30.00	0.45
30.50	0.40
31.00	0.36
31.50	0.35
32.00	0.29
32.50	0.29
33.00	0.27
33.50	0.21
34.00	0.15
34.50	0.12
35.00	0.12
35.50	0.10
36.00	0.08
36.50	0.08
37.00	0.08
37.50	0.08
38.00	0.08
38.50	0.08
39.00	0.08
39.50	0.08
40.00	0.05

Y=EXP**(A + B*X + C*X**2 + D*X**3 + E*X**4 + F*X**5)

A	B	C	D	E	F
-0.132860E 02	0.141969E 01	-0.618597E-01	0.201758E-02	-0.532787E-04	0.586599E-06

X	Y-Actual	Y-Calculated	PCT-Difference
20.00000	0.98724	0.86684	-12.19
20.50000	0.98724	0.92041	-6.76
21.00000	0.98724	0.96680	-2.07
21.50000	0.97576	1.00471	2.96
22.00000	0.97576	1.03311	5.87
22.50000	0.97576	1.05124	7.73
23.00000	0.97576	1.05868	8.49
23.50000	0.97576	1.05536	8.15
24.00000	0.97576	1.04158	6.74
24.50000	0.97576	1.01793	4.32
25.00000	0.97576	0.98533	0.98
25.50000	0.97576	0.94490	-3.16
26.00000	0.97576	0.89797	-7.97
26.50000	0.86352	0.84594	-2.03
27.00000	0.79719	0.79027	-0.86
27.50000	0.74234	0.73237	-1.34
28.00000	0.66454	0.67359	1.36
28.50000	0.57525	0.61514	6.93
29.00000	0.53188	0.55806	4.92
29.50000	0.53188	0.50323	-5.38
30.00000	0.45280	0.45131	-0.32
30.50000	0.40050	0.40282	0.57
31.00000	0.36862	0.35807	-2.86
31.50000	0.35331	0.31723	-10.21
32.00000	0.29846	0.28034	-6.07
32.50000	0.29846	0.24733	-17.13
33.00000	0.27806	0.21806	-21.57
33.50000	0.21428	0.19230	-10.25
34.00000	0.15050	0.16982	12.83
34.50000	0.12372	0.15034	21.51
35.00000	0.12372	0.13358	7.97
35.50000	0.10331	0.11928	15.45
36.00000	0.08418	0.10719	27.33
36.50000	0.08418	0.09706	15.30
37.00000	0.08418	0.08871	5.38
37.50000	0.08418	0.08196	-2.63
38.00000	0.08418	0.07667	-8.91
38.50000	0.08418	0.07275	-13.57
39.00000	0.08418	0.07015	-16.66
39.50000	0.08418	0.06886	-18.19
40.00000	0.05229	0.06897	31.89

Quality Evaluation Output

Ore Probability Factor = 0.35

Cumulative Probability Distribution
For Recoverable Iron Content

```
F(A1)=1-EXP(A+B*A1+C*A1**2+D*A1**3+E*A1**4+F*A1**5)
A= -0.132860E 02
B=  0.141969E 01
C= -0.618597E-01
D=  0.201758E-02
E= -0.532787E-04
F=  0.586599E-06
```

Concentrate Iron Content = 64.95

DCF Cost Evaluation Input

Capital Investment	43000008.
Working Capital	1958500.
Plant Capacity	2800000.
Mining Cost	0.40
Beneficiation Cost	2.90
Supervision and Management	0.420
Pelletizing Cost	1.15
Stripping Cost	0.40
Transportation	3.800
Average State and Local Taxes	0.31
Royalty Cost	0.65
Federal Income Tax Rate	50.
Gross Rate for Depletion	15.
Net Rate for Depletion	50.
Rate of Return	12.
Repayment Period	25
Depreciation Method Selector	1

Section 1

Recoverable Iron Content 31.87 Ore Probability Factor 0.358
Minimum ESR 2.5 IS 400 Feet South of PR Outcrop

ESR	TCR	PWRC	TCON	SY	RY	RFE	FEP*	AUV	NTP	NDIS
0.5	0.000	0.00	0.000	0.000	0.000	31.87	65.00	0.000	0	0
1.0	0.000	0.00	0.000	0.000	0.000	31.87	65.00	0.000	0	0
1.5	0.000	0.00	0.000	0.000	0.000	31.87	65.00	0.000	0	0
2.0	0.000	0.00	0.000	0.000	0.000	31.87	65.00	0.000	0	0
2.5	0.000	0.00	0.000	0.000	0.000	31.87	65.00	0.000	0	0
3.0	429.328	55.90	240.010	235.977	243.136	31.87	65.00	0.278	5326	2000
3.5	555.447	55.90	310.516	290.697	397.086	31.87	65.00	0.281	6326	3000
4.0	670.216	55.90	374.676	347.050	574.682	31.87	65.00	0.284	7236	3910
4.5	782.462	55.90	437.426	402.164	782.938	31.87	65.00	0.287	8126	4800
5.0	889.663	55.90	497.356	456.342	1013.745	31.87	65.00	0.290	8976	5650
5.5	991.820	55.90	554.465	520.638	1262.701	31.87	65.00	0.293	9786	6460
6.0	1088.932	55.90	608.754	595.457	1525.612	31.87	65.00	0.296	10556	7230
6.5	1186.044	55.90	663.044	683.627	1814.107	31.87	65.00	0.299	11326	8000
7.0	1276.850	55.90	713.808	778.151	2107.016	31.87	65.00	0.302	12046	8720
7.5	1367.656	55.90	764.572	884.348	2422.294	31.87	65.00	0.305	12766	9440
8.0	1457.200	55.90	814.631	1000.502	2755.098	31.87	65.00	0.308	13476	10150
8.5	1546.745	55.90	864.690	1128.007	3109.657	31.87	65.00	0.312	14186	10860
9.0	1633.768	55.90	913.338	1262.797	3475.069	31.87	65.00	0.315	14876	11550
9.5	1720.791	55.90	961.988	1408.309	3861.025	31.87	65.00	0.318	15566	12240
10.0	1807.812	55.90	1010.636	1564.541	4267.526	31.87	65.00	0.321	16256	12930
10.5	1893.573	55.90	1058.580	1728.998	4688.236	31.87	65.00	0.324	16936	13610
11.0	1980.596	55.90	1107.229	1906.517	5135.528	31.87	65.00	0.327	17626	14300
11.5	2065.096	55.90	1154.468	2089.151	5589.514	31.87	65.00	0.330	18296	14970
12.0	2150.857	55.90	1202.411	2284.845	6070.083	31.87	65.00	0.333	18976	15650

* Includes iron from scrap.

Section 2 Recoverable Iron Content 32.81 Ore Probability Factor 0.358
Minimum ESR 3.0 IS 400 Feet South of PR Outcrop

ESR	TCR	PWRC	TCON	SY	RY	RFE	FEP*	AUV	NTP	NDIS
0.5	0.000	0.00	0.000	0.000	0.000	32.81	65.00	0.000	0	0
1.0	0.000	0.00	0.000	0.000	0.000	32.81	65.00	0.000	0	0
1.5	0.000	0.00	0.000	0.000	0.000	32.81	65.00	0.000	0	0
2.0	0.000	0.00	0.000	0.000	0.000	32.81	65.00	0.000	0	0
2.5	0.000	0.00	0.000	0.000	0.000	32.81	65.00	0.000	0	0
3.0	0.000	0.00	0.000	0.000	0.000	32.81	65.00	0.000	0	0
3.5	77.502	57.54	44.601	68.602	43.604	32.81	65.00	0.276	3725	1340
4.0	104.629	57.54	60.212	88.869	75.780	32.81	65.00	0.279	4485	2100
4.5	128.544	57.54	73.975	107.891	112.305	32.81	65.00	0.282	5155	2770
5.0	149.247	57.54	85.889	128.513	150.099	32.81	65.00	0.285	5735	3350
5.5	170.307	57.54	98.008	149.491	194.423	32.81	65.00	0.288	6325	3940
6.0	191.366	57.54	110.127	170.469	244.677	32.81	65.00	0.291	6915	4530
6.5	212.783	57.54	122.452	191.802	301.863	32.81	65.00	0.294	7515	5130
7.0	233.842	57.54	134.571	213.640	364.076	32.81	65.00	0.298	8105	5720
7.5	254.545	57.54	146.485	238.026	431.014	32.81	65.00	0.301	8685	6300
8.0	273.820	57.54	157.578	263.418	498.486	32.81	65.00	0.304	9225	6840
8.5	293.095	57.54	168.670	291.402	570.926	32.81	65.00	0.307	9765	7380
9.0	312.012	57.54	179.557	321.388	646.853	32.81	65.00	0.310	10295	7910
9.5	330.930	57.54	190.444	353.871	727.566	32.81	65.00	0.313	10825	8440
10.0	349.491	57.54	201.125	388.168	811.405	32.81	65.00	0.316	11345	8960
10.5	367.695	57.54	211.601	424.140	898.106	32.81	65.00	0.319	11855	9470
11.0	385.899	57.54	222.077	462.424	989.238	32.81	65.00	0.322	12365	9980
11.5	404.460	57.54	232.759	503.839	1086.718	32.81	65.00	0.325	12885	10500
12.0	422.307	57.54	243.029	545.928	1184.792	32.81	65.00	0.328	13385	11000

* Includes iron from scrap.

Summary of Two Sections

ESR	TCR	WRC	TCON	SY	RY	RFE*	FEP*	AUV
0.5	0.000	0.00	0.000	0.000	0.000	0.00	65.00	0.000
1.0	0.000	0.00	0.000	0.000	0.000	0.00	65.00	0.000
1.5	0.000	0.00	0.000	0.000	0.000	0.00	65.00	0.000
2.0	0.000	0.00	0.000	0.000	0.000	0.00	65.00	0.000
2.5	0.000	0.00	0.000	0.000	0.000	0.00	65.00	0.000
3.0	429.328	0.55	240.010	235.977	243.136	36.33	65.00	0.278
3.5	632.949	0.56	355.117	359.300	440.690	36.46	65.00	0.280
4.0	774.845	0.56	434.888	435.919	650.462	36.48	65.00	0.283
4.5	911.006	0.56	511.401	510.055	895.243	36.48	65.00	0.286
5.0	1038.911	0.56	583.245	584.856	1163.844	36.49	65.00	0.289
5.5	1162.127	0.56	652.473	670.130	1457.124	36.49	65.00	0.292
6.0	1280.298	0.56	718.882	765.926	1770.289	36.49	65.00	0.295
6.5	1398.827	0.56	785.496	875.430	2115.971	36.50	65.00	0.299
7.0	1510.692	0.56	848.380	991.791	2471.092	36.50	65.00	0.302
7.5	1622.201	0.56	911.057	1122.375	2853.309	36.50	65.00	0.305
8.0	1731.020	0.56	972.209	1263.921	3253.584	36.50	65.00	0.308
8.5	1839.840	0.56	1033.360	1419.410	3680.583	36.50	65.00	0.311
9.0	1945.780	0.56	1092.896	1584.186	4121.922	36.50	65.00	0.314
9.5	2051.721	0.56	1152.432	1762.181	4588.591	36.50	65.00	0.317
10.0	2157.304	0.56	1211.762	1952.710	5078.931	36.51	65.00	0.320
10.5	2261.269	0.56	1270.182	2153.139	5586.342	36.51	65.00	0.323
11.0	2366.496	0.56	1329.307	2368.942	6124.766	36.51	65.00	0.326
11.5	2469.557	0.56	1387.227	2592.990	6676.232	36.51	65.00	0.329
12.0	2573.165	0.56	1445.441	2830.773	7254.875	36.51	65.00	0.332

* Includes iron from scrap.

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