RELATIONSHIP OF COAL PROPERTIES AND MACHINE PARAMETERS TO CONTINUOUS MINING MACHINE CUTTING RATES

FINAL REPORT

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PREPARED BY

MBA **MBAssociates**

Bollinger Canyon Road, San Ramon, California 94583

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1. Forward, line 3 - "Coal Mine Health and Safety" should be "Advancing Coal Mining Technology."

2. Page 26, second paragraph, lines 4 and 13 - "1/2" should be "1/4."

3. Page 26, second paragraph, line 5 - "3/4" should be "3/8."

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16. Abstract

This report is in three parts. The first part presents a review of theoretical models which have been developed to study removal operations for various types of materials. In the second part, a brief summary is given of coal cutting research. The majority of this work has been carried out in Britain using chisel type bits rather than the conical picks which are in general use in this country. However, a comparison of specific energies measured for conical picks in the laboratory and in the mine shows very encouraging correspondence. In the third part of the report, analytical models for coal cutting by picks are suggested. An extension of the British "tensile breakage" theory, developed for chisels, is given for picks. The role of crushing and cutting in coal removal is discussed as a theory which combines the two processes is shown to give reasonable predictions for specific energy requirements. Finally, directions in which the existing theory could be improved are presented and the need for further research is emphasized.

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1.0 A REVIEW OF THEORETICAL MODELS USED TO STUDY MATERIAL REMOVAL OPERATIONS

1.1 Introduction

The removal operations that we will consider are those in which material is removed from a surface by the mechanical action of one or more cutting tools. In some cases, as in most metal cutting operations, the objective is to produce a useful shape while in situations such as rock excavation the objective is to optimize the efficiency of the removal process. In either case if progress is to be made beyond mere empirical correlation of test variables it is necessary to understand the fundamental mechanisms by which material is removed.

When one attempts to develop an analysis for removal operations, it is often convenient to consider two extremes of mechanical behavior - the ideally ductile and the ideally brittle. An ideally ductile material undergoes large plastic strains before separation occurs and such materials may be shaped by the displacing or cutting action of a tool. By contrast, in an ideally brittle material only elastic deformation occurs prior to fracture. Thus, in mechanical shaping operations on brittle solids, material will be removed from the surface by the propagation and intersection of cracks ahead of and around the loaded region. The concepts of ideally ductile and ideally brittle behavior while somewhat over-simplified do describe to a close approximation the behavior of many real materials and allow analytical solutions to be obtained.

Historically, the first problem studied, of necessity was the shaping of brittle solids. Brittle solids, in the form of rocks were one of the basic materials of antiquity and indeed the very survival of our prehistoric ancestors depended on their ability to shape stones into tools and weapons. Techniques were developed, during the Stone-Age and later, for loading a block of stone and splitting off pieces of a desired shape. The methods may appear primitive but fracture in a brittle solid is hard to control and such techniques could only have been developed after a great deal of careful experiment and experience. Much later in the pre-Columbian Americas the art of shaping brittle solids became very highly developed. The Incas in Peru matched massive stone with incredible perfection over large areas while Aztec craftsmen are said to have been able to produce 100 knife blades of obsidian in an hour. Unfortunately, one cannot find any scientific articles from the Stone Age or even from the Aztec or Incas and what we know about their shaping techniques is largely based on the finished objects and in a few cases on construction sites where work was interrupted and never resumed.

With the development of steel in large quantities in the 1850's a great deal of attention was given to mechanical shaping processes for ductile metals. Many machine tools were developed in their present form at about that time and, shortly afterwards, analytical studies of the metal cutting process were undertaken. These studies continue to the present day. However, it appears that there has been a change of emphasis

in the past twenty years. Less attention is being given to the basic removal mechanism while more effort has gone into studies of machine tool automation and related problems of tool chatter and tool life. The same period of perhaps twenty years has seen a dramatic revival of interest in brittle solids. One reason for this is that brittle solids in the form of oxides, carbides, nitrides and graphite have the greatest strength at elevated temperature. With the growing emphasis on fuel economy there is a need to increase operating temperatures and extensive research on novel equipment such as ceramic gas turbines is under way. The shaping of an item such as a silicon nitride turbine blade is an expensive process, so efforts are made to form parts from powder as close as possible to their final shape. Thus the final shaping operations are carried out on a localized scale with surface integrity and surface finish being more important than large removal rates. Another area which has led to a great deal of work on the behavior of brittle solids is that of tunneling and excavation of rock. With interest in improving the environment by providing underground rapid transit systems, underground power plants etc., a lot of attention has been given to novel tunneling techniques which could replace the traditional "drill and blast" method. One method is the "cutting" of rock by tools in a manner analogous to metal cutting. This approach has been influenced by the extensive studies on the cutting of coal carried out at the British National Coal Board in the years prior to 1966. We will comment on this work in more detail later in this review.

Before turning to consider the specific solutions that have been developed, some general observations about the analysis of ductile and brittle shaping operations may be in order. There is fairly general agreement on the mechanism by which ductile metals are cut although the problem is far from completely understood. We will review the analysis of continuous cutting of ductile metals in some detail because it has been taken as the starting point by many workers interested in brittle solids. By contrast to ductile behavior, there is relatively little in the way of a concensus when it comes to analyzing shaping operations for brittle solids. This is not surprising since many aspects of the process of fracture itself in brittle solids have only become clarified in recent years. Important problems such as the trajectory a crack will follow in an arbitrary stress field are only partially understood so it would be premature to expect a detailed fundamental understanding of shaping operations in brittle solids at the present time.

1.2   General Features of Metal Cutting, Forces and Chip Geometry

In most metal cutting operations the processes involved are three dimensional. However, to simplify an analysis of the mechanics of the cutting process most investigators have studied the two dimensional case in which a tool whose edge is perpendicular to the cutting direction is moved parallel to the surface of the metal being cut. If the further restriction is made that the depth of cut is small relative to the width then there will be little side flow of metal and the process

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will be truly two dimensional or one of "plane strain". Once this process is understood the problems in extending it to the three dimensional case will be mainly geometrical.

The manner in which the chip is removed will depend on the properties of the work material as well as upon the tool geometry, velocity and depth of cut.

The classification of the various types of chip formation generally adopted is,

Discontinuous chip

Continuous chip

Continuous chip with built-up edge

These are illustrated by Figures 1-1a, b, c which are photomicrographs of the same material cut under different conditions. No sharp division exists between these types. The material may flow off in a continuous ribbon, or may suffer fracture to a varying degree. With any of these conditions cutting may take place with part of the material being cut adhering to the nose of the tool in what is known as a "built-up edge".

Examination of Figure 1-1 reveals two major processes.

1.) A highly localized shear strain extending from the tool point to the work surface. This shearing may cause only plastic deformation or may result in partial or complete shear fracture.

2.) Friction along the tool-chip interface. The process being so extreme in some cases as to cause adhesion of the chip to the tool.

The two processes meet at the tool point and are not independent of one another as can be seen by the change in the shear strain when different cutting fluids are used.

For the case of continuous cutting as idealized in Figure 1-2 the shear process may be characterized by a shear angle $\emptyset$., the angle the zone of intense shear strain makes with the cutting velocity direction, while the friction process may be defined by a coefficient of friction. The fundamental problem then, in a study of continuous cutting, is to determine the relationship between the shear angle, the coefficient of friction, and the other variables of the process. In addition it is important to be able to predict the occurrence of fracture in the shear process and adhesion or "built-up edge" in the friction process.

Referring to the idealized picture of continuous cutting shown in Figure 1-2, the cutting force R exerted by the tool on the work is considered as the resultant of a normal component N and a frictional

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component F. The equal and opposite reaction force R' may be resolved into components F_S and N_S causing shear and normal stresses on the shear plane oa. The cutting force is usually measured by its horizontal and vertical components F_H and F_V. These will be taken as positive in the direction shown.

It has never been possible to determine the point of action of R experimentally due to the small size of the zone being deformed, thus the best that can be done when evaluating force data is to calculate average stresses.

The average coefficient of friction (μ) along the tool face is

$$\mu = \tan\beta = \frac{F}{N} = \frac{F_H \sin\alpha + F_V \cos\alpha}{F_H \cos\alpha - F_V \sin\alpha}$$

i.e.,
$$\mu = \frac{F_V + F_H \tan\alpha}{F_H - F_V \tan\alpha} \tag{1}$$

or
$$\mu = \tan(\alpha + \lambda) \text{ where } \lambda = \tan^{-1} \frac{F_V}{F_H}$$

If the depth of cut is t and the width of cut is b, then the average shear stress on the shear plane is,

$$\tau = \frac{F_S \sin\emptyset}{bt} = (F_H \cos\emptyset - F_V \sin\emptyset) \frac{\sin\emptyset}{bt}$$

or
$$\tau = \frac{R'}{bt} (\sin\emptyset) \cos(\beta - \alpha + \emptyset) \tag{2}$$

and the average normal stress on the shear plane is,

$$\sigma = N_S \frac{\sin\emptyset}{bt} = (F_H \sin\emptyset + F_V \cos\emptyset) \frac{\sin\emptyset}{bt} \tag{3}$$

The shear angle may be expressed in terms of the ratio (r) of depth of cut to chip thickness.

$$r = \frac{t}{t_c} = \frac{\sin\emptyset}{\cos(\emptyset - \alpha)}$$

Hence,
$$\tan\emptyset = \frac{r \cos\alpha}{1 - r \sin\alpha} \tag{4}$$

which is a convenient way of finding ϕ by measuring the chip length lc and the length of cut l. Since no volume changes occur during plastic deformation

$$\ell c / \ell = \frac{t}{t_c} = r$$

The shear strain (γ) may be expressed by two equivalent expressions,

$$\gamma = \cot \phi + \tan (\phi - \alpha)$$

or,

$$\gamma = \frac{\cos \alpha}{\sin \phi \cos (\phi - \alpha)}$$

The energy consumed in the cutting process per unit volume of material removed is,

$$u = V F_H / btV = F_H / bt$$

where V is the velocity of cutting.

1.3   Analyses of Chip Formation
           Discontinuous Chip Formation

The first attempt to explain the formation of a discontinuous rather than continuous chip in terms of the mechanics of the cutting process and the properties of the material being cut was made by Piispanen[1] in 1948. He concluded that the shear angle decreased during chip formation with the shear strain consequently increasing and finally reaching a limiting or fracture shear strain. However, as can be seen in Figure 1-1, shear is not confined to a single zone but is distributed over the entire chip. Field and Merchant[2] in 1949 extended this analysis by assuming that this decreased shear angle was due to an increasing coefficient of friction on the tool face. While they actually observed the coefficient of friction to decrease they attributed this to the lack of frequency response of their dynamometer. In 1952, Cook, Finnie, and Shaw[3] showed that completely discontinuous chip formation is quite different from continuous cutting and is one of periodic extrusion rather than simple shear. They pointed out that the coefficient of friction was static rather than dynamic in nature and showed that the important variable in determining when a continuous chip would become discontinuous was the normal stress on the shear plane. Later, Lee[4] studied, theoretically, the early stage of discontinuous chip formation as a case of the flow of a perfectly plastic solid (i.e. a material which does not strain harden). Only with this assumption can the stress strain relationships for a plastic flow be made

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mathematically tractable. His solutions cover only the early stages of chip formation and do not appear to agree with the observed facts that, first, the chip does not flow up the tool as in continuous cutting, rather the friction is static, and secondly that plastic flow is not confined initially to a small zone bounded by a line from the tool point to the fracture surface as he assumes but takes place from the very beginning all along the original fracture surface.

Thus, the most desirable form of chip formation from a point of view of tool wear and surface finish is by its continuous nature more amenable to analysis than the other forms discussed and has received more attention from investigators. The various analyses will be outlined briefly along with the reasoning followed.

In 1941 Ernst and Merchant[5] presented the following analysis for the shear angle. In a review article by Finnie[6] it was shown that similar analyses date back to the previous century. Here we reference the more generally available work. In Figure 1-2, the forces on a tool are shown. Assuming the shear stress on the shear plane τ to be uniformly distributed, it was shown in Section 1.2 that

$$\tau = \frac{F_s}{A_s} = \frac{R' \cos(\phi + \beta - \alpha)\sin \phi}{A} \qquad (2)$$

where A_s and A are the areas of the shear plane and that corresponding to the width (b) times the depth of cut (t) respectively. Ernst and Merchant reasoned that ϕ should be an angle such that τ would be a maximum. A relationship for ϕ was obtained by differentiating equation (2) with respect to ϕ and equating the resulting expression to zero. This led to the result

$$\phi = 45 - \beta/2 + \alpha/2 \qquad (6)$$

However, it is to be noted that in differentiating, both R' and β were considered independent of ϕ.

In 1945 Merchant[7] presented a different derivation that led to equation (6). This time, an expression for the total power consumed in the cutting process was first written:

$$P = F_H V = \tau AV \frac{\cos (\beta - \alpha)}{\sin\phi \cos(\phi + \beta - \alpha)} \qquad (7)$$

It was then reasoned that ϕ would be such that the total power would be a minimum. An expression identical to equation (6) was obtained when P was differentiated with respect to ϕ, this time considering τ and β to be independent of ϕ.

This is what Piispanen[8] had done previously in a graphical way. However, Piispanen immediately carried this line of reasoning one step further and assumed that the shear stress τ would be directly influenced by normal stress on the shear plane σ as follows:

$$\tau = \tau_o + K\sigma \tag{8}$$

Piispanen then incorporated this observation into his graphical solution for the shear angle.

Upon finding equation (6) to be in poor agreement with experimental data, Merchant also assumed a relationship as given by equation (8) and proceeded to work this into his second analysis as follows.

From Figure 1-2 it may be seen that

$$\sigma = \tau \tan(\phi + \beta - \alpha) \tag{9}$$

or from equation (8)

$$\tau = \tau_o + K\tau \tan(\phi + \beta - \alpha) \tag{10}$$

hence,

$$\tau = \frac{\tau_o}{1 - K \tan(\phi + \beta - \alpha)} \tag{11}$$

When this is substituted into equation (7) we have,

$$P = \frac{\tau_o AV \cos(\beta - \alpha)}{[1 - K \tan(\phi + \beta - \alpha)] \sin\phi \cos(\phi + \beta - \alpha)} \tag{12}$$

Now when P is differentiated with respect to ϕ with τ_o and β considered independent of ϕ we obtain

$$\phi = \frac{\cot^{-1}(K)}{2} - \frac{\beta}{2} + \frac{\alpha}{2} \tag{13}$$

Merchant called the quantity $\cot^{-1}(K)$ the "machining constant" (C).

Stabler[9] has presented an analysis in which it was assumed that the maximum shear on the shear plane and the resultant shear strain were collinear, then it followed from purely geometrical considerations that

$$\phi = 45 - \beta + \frac{\alpha}{2} \tag{14}$$

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Lee and Shaffer[10] have derived still other relationship for ϕ. These authors assumed; 1) that the material cut behaves as an ideal plastic which does not strain harden, and 2) that the shear plane represents a direction of maximum shear stress in the material cut. On this basis, they obtained

$$\phi = 45 - \beta + \alpha \qquad (15)$$

Upon finding equation (15) to be in disagreement with experimental data, Lee and Shaffer assumed that a small built-up edge was normally present and that this built-up edge was responsible for the lack of agreement. They then extended their analysis to include an idealized built-up edge bounded by straight lines and a circular arc, and obtained

$$\phi = 45 + \theta - \beta + \alpha \qquad (16)$$

This differs from equation (15) only by the angle θ, which is the angle subtended by the arc of the built-up edge at point a in Figure 1-2.

More elaborate slip-line field analyses, which include strain hardening, have also been proposed. However, these require such an extensive amount of prior information on chip geometry that they hardly seem practical for predictive purposes. An extensive evaluation of these and other slip-line field solutions has been given by Johnson, Sowerby and Haddow.[11]

<u>Evaluation of Continuous Chip Theories</u>

When the various equations for shear angle prediction are tested against a wide range of experimental data, some are found to be in better agreement than others, but none are found to be really satisfactory. When the observed angle ϕ is plotted against $(-\beta + \alpha)$ for a wide range of rake angles, the points are found to lie along a straight line whose slope indicates that

$$\phi = \text{constant} - \beta + \alpha \qquad (17)$$

However, the value of the "constant" is not always the same, nor does it correspond to the values given by the various theories. For this reason it may be worth while to examine the assumptions on which the derivations have been based.

In Ernst and Merchant's analysis leading to equation (6), the following assumptions were made:

1. That the shear stress is a maximum in the direction of the shear plane.

2. That the coefficient of friction is independent of the shear angle.

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3. That the resultant force R is independent of the shear angle. The first appears reasonable, but the second and third are incorrect since an examination of cutting data reveals a strong dependence of R and β on ϕ.

In the second derivation of equation (6), a different set of assumptions were made:

4. That the angle ϕ is such that the total power is a minimum.

5. That the coefficient of friction is independent of the shear angle.

6. That the shear stress on the shear plane is independent of the shear angle.

There is no clear physical reason why item 4 should be true. Speaking of the derivation of equation (6) Hill[12] said "The comparative failure of this theory is almost certainly due to the inadequacy of the minimum work Hypothesis." The 5th item has already been discussed. The 6th assumption is also questionable. The shear stress on the shear plane is usually observed to increase or decrease, significantly as the shear angle is decreased.

In deriving equation (13) assumption 7 below was introduced in addition to 4, 5, and 6.

7. That the flow stress of the metal is increased by the presence of a normal compressive stress on the shear plane.

While it is well established that the rupture stress of both brittle and ductile materials is significantly increased by the presence of a compressive stress, as suggested by Coulomb and Mohr, there is no clear evidence that a similar relationship holds for flow stress as assumed in 7. Although several isolated test results are to be found in the literature which seem to indicate a dependence of flow stress on normal stress, these tests are generally so complex that one cannot be sure that they are being interpreted correctly. On the other hand, several fairly clear cut experimental observations, such as the fact that the yield points of metals in tension and compression are equal, may be cited in support of the view that flow stress is virtually uneffected by normal stress.

As already mentioned Stabler's equation (14) is based upon the assumption:

8. That the maximum shear stress and shear strain are collinear on the shear plane in three dimensional cutting operations.

Since it was experimentally demonstrated that assumption 8 is incorrect, equation (14) need not be considered further.

In deriving equation (15) is was assumed that:

9. The material cut is non-strain hardening.

10. The shear plane is along the direction of maximum shear stress. The strain hardening that occurs when metals are cut is very important. However, despite this fact assumption 9 does not lead to a poor approximation for the reason that it is only applied to a region in which the material is essentially rigid. All of the strain hardening that takes place occurs as the metal crosses the idealized shear plane and this is before the ideal plastic assumtion is applied to the material. Equation (15) could be derived directly without introducing the ideal plastic concept by making another assumption that is equivalent to 9.

11. That a uniform stress field exists in the vicinity of the tool point. This assumption, together with 10 will yield equation (15).

Equation (16) differs from (15) only by the introduction of a built-up edge. In the treatment of the built-up edge the non-strain hardening assumption must be considered as an approximation, since the material within the built-up edge is actually highly deformed. However, if the built-up edge is small the degree of approximation associated with assumption 9 is probably slight.

From the foregoing discussion of the assumptions on which the several equations for ϕ are based, it would appear that all but equation (15) should be discarded on the basis of serious objections to the underlying assumptions. However, the constant in equation (15) is not found to be $45°$ nor in fact is it always the same.

From this rather concise summary it is clear that the almost century old objective of relating the shear angle to other variables has yet to be realized. A major problem is that the conditions in metal cutting are not easily simulated in other materials tests. The small "specimen" geometry and large strains may be responsbile for behavior not observed in conventional materials tests. Shear and strain hardening are very localized. Purely plastic deformation can hardly be separated from intermittent shear fracture and rewelding on the shear plane under the high normal stresses. If the later process is occuring, a normal stress effect on the shear stress such as equation (8) may well be involved. Friction along the tool-chip interface occurs under conditions quite different from conventional sliding friction and appears to be influenced by the prior shear deformation.

Built-Up Edge Formation

For completeness we make a few comments on this third type of chip formation. Most investigators have pointed out that the built up edge leads a precarious existence, parts being carried off occasionally by the tool and by the work surface with consequent poor surface

finish. The major role in the existence of a built up edge was at first attributed to the velocity. It was thought that the decreased friction at high velocities had the same effect as adding a lubricant. Rapatz[14] however in a detailed study of built up edge and resulting surface finish concluded that temperature of cutting was the significant variable and velocity only important in that it modified the temperature. This conclusion was supported by experiments in which the built up edge disappeared in cutting preheated specimens at low speeds. Ernst and Martellotti[15] re-emphasized that the built up edge is not a collection of particles gathered by the tool as it passes over the work but is part of the chip which develops in a shape similar to a cutting tool due to the distribution of stress in the cutting process. They mentioned that the built up edge is continually being built up and breaking off and also that in many cases, as when fracture occurs, the built up edge is not carried to completion but passes off with the chip segment. This further emphasizes the lack of a sharp subdivision between the three types of chip formation.

1.4   The Extension of Metal Cutting Analyses to Other Materials

An interesting application of the analysis of continuous metal cutting to another material is the work on two organic polymers by Rao, Cumming and Thomsen[16]. For these materials the friction angle β is extremely low. It was argued that the minimum energy criterion should be valid in this case and equation 6 was used in the form

$$\phi = 45 + \frac{\alpha}{2}$$

On this basis using a constant (dynamic) value for the shear stress, excellent predictions were made for the cutting forces.

Several workers have applied equation 13, the extension of equation 6 in which the shear stress is assumed to depend on normal stress, to the cutting of brittle solids. There is certainly more justification for the use of a Coulomb "failure" criterion for brittle solids than for metals. However, the cutting process in brittle solids is intermittent with crack propagation occuring rather than plastic flow. By contrast to metals, in which the flow condition has to be satisfied continuously all along a shear plane, in brittle materials the fracture criterion need only be satisfied at the tip of a propagating crack.

Perhaps the most complete example of the direct application of metal cutting analysis to brittle solids (rock and cement) is that of Nishimatsu[17]. As he points out, the tool first crushes the rock and then detaches a chip. Unlike discontinuous metal cutting, the detached chip, Figure 1-3, is formed by a fracture surface which propagates down below the nominal depth of cut. Such behavior has also been observed in cutting brittle plastics and cast iron, as well as by other workers in rock cutting[18]. Nishimatsu relates the peak values

of the fluctuating cutting force F_H and thrust force F_V to the depth of cut τ by

$$F_H = a_1 + b_1 t \, , \; F_V = a_2 + b_2 t \, .$$

The terms a_1 and a_2 are related to crushing while the terms b_1 and b_2 are expressed using the modified metal cutting analysis.

A similar approach was taken by Paul and Sikarskie[19] in studying the chipping that occurs under a rigid two dimensional wedge. They were apparently unaware of the metal cutting literature and the equation they obtained for the angle of chipping is identical to equation 13. Later, Miller and Sikarske[20] extended the analysis to three dimensional indenters.

Merchants metal cutting analysis was also used by Evans[21] to estimate the shear strength of coal from cutting tests. He concluded that it seemed applicable for friable coking coal but not for hard bituminous coals.

1.5   <u>Material Removal in Brittle Solids Based on Deterministic Fracture Criteria</u>

The fracture pattern produced when a wedge indents a brittle solid has been studied extensively by Evans[21] and his colleagues. This work is summarized in the book by Evans and Pomeroy[22]. The situation considered is shown in Figure 1-4. A large number of assumptions have to be made. Perhaps the least satisfactory are the "a-priori" assumption of the fracture path and the requirement that a fracture criterion be satisfied all along the path at the moment of fracture. These limitations were recognized by Sikarskie and Altiero in a series of papers (e.g., Refs. 23, 24). In a detailed study of wedge indentation, which involves numerical solution for the stresses in the brittle solid, they predicted the initiation of fracture using the Coulomb-Mohr criterion (the simplest version of this is Eq. 8). Subsequent stable and then unstable crack growth was predicted using a fracture function obtained from the Coulomb-Mohr criterion. The theory is an interesting one and deserves further experimental study. A somewhat similar approach was followed by Glenn[25] who studied failure under impulsive load by allowing cracking to occur in any elemental region in which the Coulomb-Mohr criterion was satisfied. One could argue that once a crack is formed, its subsequent propagation is a problem of linear elastic fracture mechanics (LEFM). However, the difficulty of making crack trajectory predictions using LEFM is formidable. For this reason the approaches taken by Sikarskie and Altiero and by Glenn may represent a practical alternative to the LEFM method for predicting crack trajectories or damaged regions in the brittle solid. However, it would be of interest to compare the predictions of the Sikarskie and Altiero method for the situation shown in Figure 1-4 not only with experiment but also with a LEFM calculation of the fracture path. The latter calculation would involve extensive amounts of computer time.

1.6   Material Removal in Brittle Solids Based on
        Probablistic Fracture Calculations

When brittle solids are tested under tensile states of stress, the strength values obtained from apparently identical specimens generally show considerable scatter. This is explained by noting that brittle solids contain a distribution of flaws of varying severity. Thus, nominally identical specimens will contain flaws of different size and break at different stress levels. Another consequence of the flaw distribution is the "size-effect." Because of the decreased probability of finding a large flaw in a small specimen, the average stress at fracture increases as the size of the test specimen is reduced. Similarly, if identical specimens are tested in tension and bending, the probability of failure at the same maximum stress is less in the bending specimen. This is due to the fact that the entire volume of the tensile specimen is subjected to the maximum stress while only the outer fiber of the bending specimen (a vanishingly small volume) is exposed to this stress. All of these qualitative observations can be treated quantitatively using the Weibull[26] probabalistic approach to brittle strength. An extension of Weibull's approach by Oh and Finnie[27] allows the distribution of fracture location to be predicted in certain problems (Hertzian fracture, three point bending, spalling of a rod).

In finishing operations for ceramic material (e.g., grinding, abrasion, ultrasonic cutting) small hard particles are used to chip away the surface and produce a fine surface finish. Under these conditions the size effect may be very important in determining the influence of particle size and load on material removal. In the limit, some brittle solids (e.g., glass, tungsten carbide) have a high enough strength when loaded by small particles that "plastic" flow will occur rather than fracture.

We will not go into the details of the Weibull approach which would require a lengthy discussion. However, the existence of the size effect should be kept in mind if small scale experiments are planned to simulate material removal operations.

1.7   The Energy Approach in Studying Removal Operations

An important although not the only consideration in material removal operations is "the specific energy". That is, the energy required per unit volume of material removal. While convenient in that different material removal operations can be compared using a single number, energy approaches have not been very helpful in shedding light on the mechanisms of material removal. We have seen that energy approaches to calculating the shear angle in metal cutting have not been fruitful except perhaps for polymeric materials with extremely low friction coefficients. In brittle solids the essential energy requirement, that to create fresh surface, is inevitably a small and unpredictable fraction of the total energy consumed.

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In metal cutting the specific energy is found to be approximately equal to the Brinell hardness of the material being cut[28]. In brittle solids the correlation often found is that the specific energy $E_{sp} \approx .3$ to 3 times the unconfined compressive strenght (σ_c). For example, Bailey and Dean[29] quote the following values in studying the cutting of ice:

<p align="center">Table I: Specific Energies for Various Processes
for the Disengagement of Ice
($\sigma_c = 3.45 \frac{mN}{m^2}$, Assumed) Ref. 29</p>

Method	E_{sp} (MJ/m^3)	E_{sp}/σ_c	Remarks
Coal Cutter	12.0	3.5	Joy 10 RV machine
Coal cutter, auger drilling & blasting	2.8	0.82	Cut four sides of tunnel + cutter power and explosive power
Drill & blast	12.0	3.5	Drill power + explosive energy
Manual picking	0.16	0.046	Manpower, assumed 1/10 hp steady
Melting	289	84	Fusion power only

Similar comparisons have been made for other brittle materials and two important points emerge. The first is that "exotic" processes involving melting require extremely large specific energies. The second, illustrated by the low value for manual picking is that the energy required for "breaking into" the material is greater than that required for "breaking it off". The first requires overcoming the compressive strength while the second involves the relatively much lower tensile strength. A man with a pick can presumably optimize the "breaking into" phase by looking for cracks or holes into which to direct the pick. These conclusions are confirmed by the work of Hughes[30]. As other workers, he finds the specific energy decreasing as the size of the material removed increases.

There is little in the way of analytical studies which relate specific energy to fracture properties. However, a recent study[31] of the grinding of brittle solids presents a model which relates specific energy to the fracture toughness and the extent of the loaded region. The extension of this approach to more general cutting operations is not a trivial problem, but should be worth pursuing.

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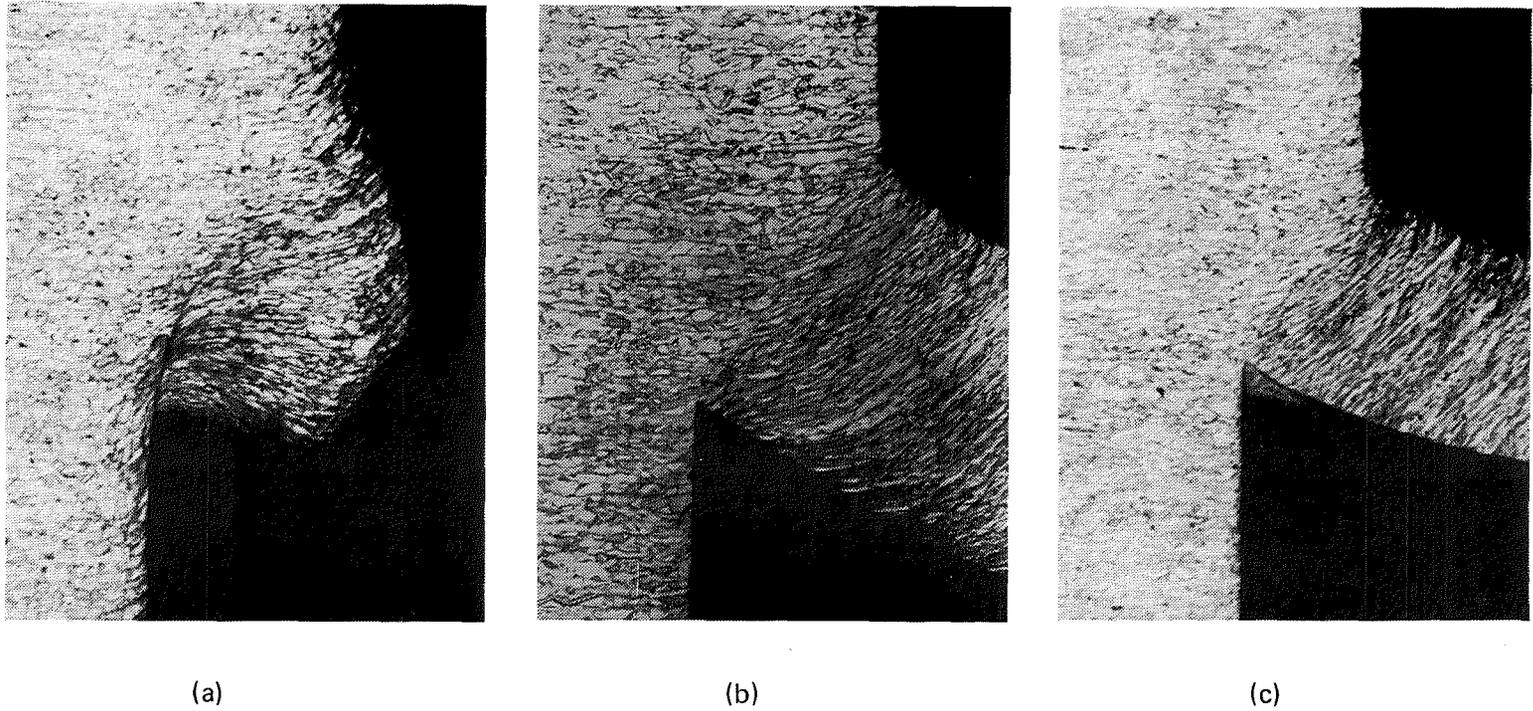
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(a)

(b)

(c)

FIGURE 1-1

Photomicrographs of β brass being cut at 0.5 inches/min with no cutting fluid.

(a) Discontinuous chip. Rake Angle 15°, Depth of cut 0.008 inches.

(b) Continuous chip. Rake Angle 30°, Depth of cut 0.010 inches.

(c) Continuous chip with built up edge. Rake Angle 15°, Depth of cut 0.007 inches.

FIGURE 1-2
Forces and Chip Geometry for Two-Dimensional Cutting.

FIGURE 1-3
An Illustration of the Failure Process
of Rock Cutting, Nishimatsu[17].

FIGURE 1-4
The Chipping Configuration
Studied by Evans[21].

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2.0 SUMMARY OF COAL CUTTING RESEARCH

2.1   Basic Coal Properties

Coal is a brittle anistropic material which varies in
composition and properties from one geographical region to the next.
Although research in the field of coal cutting and removal operations
has been active for many years, the bulk of the investigations are
based on empirical observations related to a particular area. The
properties of coal, relating to its fracture and the energy required to
break it from the seam are greatly dependent on its innate weaknesses.
Most specimens of coal have a layered structure. The bands, having
little cohesive strength, lie parallel to the planes of the bedded material.
In addition, there are primary and secondary cleat planes, also weaker
than the bulk, running roughly perpendicular to each other and to the
bedding planes. Due to the inherent structure of the coal, therefore, the
seam contains a maze of internal cracks which give rise to the distinctly
different compressive and tensile characteristics of coal. While coal
properties do vary quite dramatically, there are various generalizations
which lead to a better understanding of the coal winning requirements.

Young's Modulus has been measured for coal in a number
of different ways[1] (i.e., compression, tension, static bending, and
dynamic techniques) as well as for various geometrical shapes and orien-
tations with respect to the bedding and cleat planes. The range of E is
from 5.4 GN/m^2 for anthracite coals to 3.3 GN/m^2 for bituminous coals.
While Young's Modulus has been determined the measurement of
Poisson's ratio is more difficult. Tests indicate, however, that this
value varies between .35 to .50. Since both of these material constants
are affected by the moisture content of the coal, they are a function of
the environment in which the testing is carried out. Thus, transporting
coal from the seam to the laboratory may in itself change the properties
of the coal due to the different environmental conditions. Generally, the
specific gravity of most coals range from 1.3 to 1.5 regardless of the
source.

Compression tests on coal are probably the oldest and
most common means of determining its strength and workability.
Through the various types of uniaxial compression tests, information
related to the tensile properties and internal crack size distribution may
also be deduced. On the average, the compressive strength of a coal
specimen will vary between 14 MN/m^2 and 70 MN/m^2 depending on the
specimen size, orientation and type of coal. Using different size cubes
and rectangular blocks, all having a prescribed orientation, Evans and
Pomeroy[1] found that the mean crushing strength of a coal sample,
although showing large scatter, decreases as the side of the cube in-
creases. If P_a is the probability of survival of a sample of side dimension
a, and P_b the survival probability of a sample of size dimension b, then
the test results for a given load may be summarized by:

$$P_b = P_a^{\alpha(b/a)}$$

where in general $\alpha \approx 1$. This result leads to an important observation relating the breakage mechanism to a characteristic length of the coal sample. Using the size of the specimen to describe the probable possible internal crack length, the basis for a weakest link theory of coal breakage is established. Work by others[2] has also confirmed the size dependence of the compressive strength - however, actual relationships vary somewhat due to differences in size, shape and kind of sample used. In the Brazilian Test[3] for example it is known that even with uniform compressive loading, tensile stresses are developed within the specimen, Figure 2-1. In this test the statistical treatment for strength predictions leads to a very satisfactory correlation between diametrical compression of a disc and bending tests for certain rocks[3]. The statistical treatment based on weakest link theory thus predicts failure of the sample when the strength of a single crack having the highest tensile stress-concentration is reached. To this end, if samples which have already been cracked in this manner are reused for successive tests, the maximum required crushing force increases since the previous weakest link has been removed by the fracture process.

Direct tensile tests on coal specimens present difficulties to the inherent weaknesses and the difficulty of machining suitable samples. Due to the lack of strength between the bedding and cleat planes the most consistent set of tensile data is found using the 3-point-bending and Brazilian disc tests. The highest tensile strength is parallel to the bedding and main cleat planes, but is only about 1/20 of the compressive strength of similar samples. Strength perpendicular to the bedding planes is 1/2 to 1/3 of the tensile strength parallel to these planes. These very low values of the tensile strength correspond well with the weakest link concept of strength.

While it may be instructive to investigate the tensile and compressive properties in an idealized laboratory situation, coal in a seam is under a rather complex system of stresses[1] which in general does not relate to simple laboratory tests. Coal on the face of a seam experiences a biaxial compression which may vary considerably depending on the overlying strata. A typical value would be 15 MN/m^2. Just slightly in advance of the coal face there exists a condition of biaxial tension, due to the relief of the coal face, with the third principal stress being compressive due to the overburden. Remote from the face, stress often approaches triaxial compression. In this case it has been observed[1,2] that for higher confining pressures the coal will support higher axial loads, the strength of the coal thus being affected by the intermediate stress (of the three principal stresses). The large increases in the fracture stresses for rather small increases in this intermediate stress show the importance of the strata and face loading. In addition, coal under high confining pressures tends to become less brittle and more flow is experienced. Evans and Pomeroy had little success in applying the strain energy of distortion theory as a failure criterion[1] in an attempt to explain the role of the intermediate principal stress. Other failure theories (i.e., Coulomb, maximum tensile stress

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or maximum shear stress theory) predict that the intermediate principal stress has no effect.

As a final property which must be understood to better investigate the coal winning process, we consider the friction coefficient at the cutter-coal interface. For pressures exceeding .3 MN/m^2, the type and hardness of steel used seem to have little effect. The coefficient of friction varies widely depending on the type of coal and normal pressure with values between .4 and .8 being typical. As the normal pressure is increased the coefficient of friction drops, presumably due to the removal of asperities on the coal surface. Additionally, the effect of wetting with water may reduce friction by 20-40%[1].

2.2   Coal Cutting with Bits

Due, apparently, to the difference in evolution of the mining industries in the United States and Great Britain, two schools of thought developed regarding the configuration of the individual bits for the continuous mining operation. The American mining industry has opted for the point attack bits with a conical body (picks), whereas British research has emphasized cutting with chisel type bits. The differences between these two approaches is not simply a matter of which cuts coal the easiest, but must be based on an overall evaluation considering both efficiency and durability.

The British Coal Board has undertaken numerous investigations into the cutting of coal by wedge action. The wedge, simulating the head of a chisel, is forced into blocks of coal while the penetration force and distance are recorded. The initial contact of the wedge is along the edge. After some penetration though, the shoulders of the wedge carry an increasingly higher percentage of the load, while the edge may come out of contact with the specimen. For two different British coals tested the penetration force was found to be related to the depth of penetration, the wedge angle, friction, and a penetration resistance of the coal:

$$P = 2q \, wh(\tan(\theta) + \mu)$$

where:

P = penetration force

q = penetration resistance

w = wedge width

h = depth

θ = 1/2 wedge angle

μ = friction coefficient.

The interesting result of this is that for roughly constant mean identuation pressures on wedges of 10° and 140° the penetration resistance is approximately the compressive strength of the coal[1]. This result is

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quite different from that found in ductile metals where hardness is related to tensile stress at low strains. As the loading rate was decreased in these tests the effective compressive strength increased. This is contrary to what is observed when the loading rate is varied for other material. In addition, the investigations demonstrate that for small wedge angles, θ, the mean cutting (breaking) force is reduced[2].

In tests more cosely representing the in-mine cutting of coal, chisel bits are dragged over blocks of coal in a linear fashion. For these tests the depth of cut, bit angle, and coal orientation may all be varied. In general, deeper cuts are more efficient and produce less dust. Changing the orientation of the bedding and cleat planes of the sample resulted in marked variation of the mean peak cutting force* and specific energy. The cutting force is maximum when the cleat planes are perpendicular to the cutting direction and least when the planes are oriented at 45°. The mean peak cutting force for a 1 cm deep cut (1.27 cm width chisel) ranged from 250N to 700N - increasing roughly linearly with increasing coal tensile strength. Although the strength characteristic which appears to control breakage is tensile, cleat and bedding orientation and overburden pressure may have pronounced effects. Care must be taken in predicting breakage forces from laboratory strength measurements.

The chisel shape has a major role in the overall efficiency of coal cutting. Three important factors in the design of a chisel bit are: 1) back clearance angle, 2) rake angle, and 3) face width, Figure 2-2a. For back clearance angles of greater than about 6° there is no decrease in the energy requirement or cutting force. Insufficient back clearance will also result in enhanced bit wear. Increasing this angle much beyond 8° simply reduces strength of the chisel without any beneficial effect, Figure 2-2b.

A change in the rake angle influences the mean cutting force, specific energy, and the normal force on the bit. Larger rake angles result in a continuous decrease of cutting force and energy. Beyond $\approx 40^{\circ}$ though, the beneficial effects taper off and the weakening of the bit is of primary importance, Figure 2-2c. One to two J/g represents a rough limit of specific energy which can be achieved in the laboratory with single chisel type bits with cut depths 1-2 cms. The normal force on the bit simply represents the force required to maintain a given depth of cut as the bit progresses across the coal face. At very low rake angles this force may be substantial (1000-2000N) but drops rapidly with increasing rake angle, Figure 2-2d. For rake angles above 20° this force was between +40N and -50N using Barnsley Hard coal samples with bit clearance of 5°. In the tests carried out on chisel type bits, the broader chisels produced greater side splay and resulted in better overall efficiency.

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*Mean of several adjacent force peaks on force-distance record.

Little work has been carried out using single bit action of conical point attack bits (most work includes multiple parallel grooving). While the same trends appear to be prevalent for the point attack bits and the chisel type, a few differences should be noted. The angle of attack (as opposed to rake angle) of conical picks are generally from 35° to 55° with 45° giving the best overall performance, Figure 2-3. Tests showed that the normal force on the point attack bit to be the same order as the cutting force. This is in contrast to the low normal forces associated with chisel bits. While this force is not associated with supplying energy to break the coal, the coal winning machine must be capable of keeping the bit in the coal face at a pre-scribed depth. Conical bits with the smallest tip angle seemed to be energetically most efficient but the wear and durability aspect will once again influence the choice of a configuration which gives the most economic results.

A direct comparison of point attack bits and chisel bit is not currently possible due to differences in methods of testing and more importantly the types of coals used. Each type of bit has its particular advantage though. The forces associated with the chisel type bits seem to be generally lower (even when testing hard coals) while the difference in specific energy is much less pronounced[1,2]. The tip of point attack bits are subjected to considerably less bending stresses than the face type bits due to the way the carbide tip is mount-ed and the angle of attack. "Many mines report much longer bit life that mainly can be attributed to the better retention of the carbide in the bit. The rotating feature of the point attack bit is also very important in equalizing wear of the carbide or the backup metal behind the tip."[4]

2.3   Multiple Bits and Depth of Cut

The specific energy realized in continuous mining machines is largely due to the interaction of multiple bits. While one bit may remove a volume of coal somewhat larger than the volume it sweeps out, the interaction by side splay of two adjacent bits signifi-cantly increases the amount of coal broken from a seam per pass of bit. The efficiency of the multiple bit mining depends on the correct groove spacing for the particular depth of cut, coal, bit size and other operating conditions. The chipping between the bit paths results in a coarse bulk product mined in addition to reducing the fine dust particles.

Evans[5] has developed an elementary theory for the relation of bit spacing, depth of cut and bit size for chisel shaped bits. Using the tensile breakage theory[1] Evans showed that an optimum separation of bits for promoting lateral break-out (side splay) between the cut channels can be achieved, Figure 2-4. Assuming that the break-out due to side splay starts from the corners of the bit and cracks at an angle α to the surface, a moment balance may be made for all forces

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acting on the coal perpendicular to the cutting direction, Figure 2-5.
Making the additional assumption that the angle α of the side splay
will be such that a minimum force for breakage is required, $dP/d\alpha = 0$,
then:

$$\tan \alpha = \frac{1}{5} (k + \sqrt{k^2 + 20}$$

where: $k = w/d =$ bit width/depth of cut.

If the requirement of interaction of the side breakage from two adjacent
bits is now imposed, an optimum spacing can be obtained:

$$S = \frac{w}{2} (1 + \sqrt{1 + 20/k^2})$$

Comparing the spacing and breakage pattern achieved in tests to those
predicted by this elementary theory, in view of the assumptions and un-
certainties, the agreement seems resonably good.

While Evan's work specifically refers to chisel-type bits,
studies of multiple bit action in both point attack bits and bits of rectan-
gular shaft construction agree that bit penetration must be related to the
bit spacing in order to achieve proper fragmentation. As a general rule
to be followed in optimizing cutting efficiency, the bit penetration should
be a maximum commensurate with the bit spacing, strength and force
capabilities of the mining machine[1].

2.4   Point Attack Bits - Laboratory Tests

In USBM Report on Investigations 8185, Roepke, Lindroth,
and Myren[6] carried out laboratory tests on various point attack bits,
Figure 2-6. Although large scatter was reported for most of the tests
using four different bit types*, the specific energy showed a decrease as
the depth of cut increased for a given S/D (spacing/depth of cut). For
S/D's of 2 and 3 and cut depths greater than 1 cm, specific energies of
1-2 J/g were realized. Although the coal used in these tests are
Pittsburgh seam and Illinois No. 6 the energy range is comparable to
those reported earlier for chisel type bits in British coals. This simi-
larity lends credence to Warner's[4] assumption that while geometry
may vary considerably, the same progressive crushing and chipping
action applies to both kinds of bits.

Our earlier discussion indicated that the greater the
depth the more energetically efficient was the process. While the
Bureau of Mines data does bear this out, there appears to be a point
at which greater cut depths no longer reduce the specific energy for a
given S/D. Tests on sandstone do in fact demonstrate an optimum spac-
ing and depth relationship for a minimum of specific energy[7]. Both
Pittsburgh and Illinois No. 6 coal show a marked decrease in energy
dependence on depth as the cut depth is increased from .6 to 2.5 cm.

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*The four bit types used were: 1) Small bullet, 2) Pencil, 3) Minibob
and 4) Plumb bob. A 45° attack angle is used in all tests.

Each of the four bits used had a different included tip angle ranging from 24° for the pencil to 88° on the plumb-bob. The results indicated a general trend that the smaller the tip angle the lower the mean specific energy. Although this trend is in part due to the pick body geometry*, it is interesting to note that a similar trend is found as the rake angle of chisel bits is reduced.

In unpublished (13) data from the Bureau of Mines, plumb bob bit attack angles of 35°, 45° and 55° were investigated with respect to the normal and cutting forces and the specific energy. Additionally, bit wear was modeled by running similar tests with a new and 1/2" and 3/4" tip radii bits. The scatter of data for any particular tests was in many cases several times the mean measured values. Due to this, extraction of specific trends become uncertain. In all the tests, however, the ratio of normal force to the cutting force is approximately 1. For both the new and 1/4" radius bits these forces ranged from 500N to 2000N, increasing as the depth of cut increased. The forces associated with the 3/8" radius bit, while showing an increase with depth, ranged from 1300N to 6000N. Specific energy for the 3/8" radius bits was also generally higher than for the new and 1/2" radius bits - though this difference was not as pronounced. Energy measurements ranged from 4 to 4.5 J/g compared with 1 to 4 J/g for the latter.

2.5   Point Attack Bits - In-Mine Operation

Whereas in-lab test provide valuable information into the methods and design of the mining equipment, the full benefit can only be realized if it adequately predicts the in-mine operation. To this end, Ingersoll Rand Corp. has instrumented its large continuous miner to provide energy, dust and coal cutting data (14). The continuous miner contains 49 conical picks mounted on a cylindrical rotating drum. The picks are about 11 cm long, have an included angle of 35°, attack angle of 45° and scribe parallel paths with a 10 cm spacing. The depths of cuts range from 2 to 10 cms. Since the spacing of picks is fixed for this type of continuous miner, varying the depth of cut effects the S/D equally. All instrumented test runs were in the Pittsburgh coal seam.

Mining is achieved by driving the drum into the coal face in alternating sumping and shearing motion. As the drum is sumped into the coal face the picks have zero depth of cut at the entry and exit, and achieve a maximum depth at the center of the cut, Figure 2-7. During shearing, however, while the entry depth is still zero, the maximum depth of cut is reached at the exit as coal is being broken from the face, Figure 2-7. Due to the different conditions during sumping and shearing, slightly different energies are obtained. A gross energy is also recorded which includes both the sumping and shearing cycle as well as the energy required to return the drum head for the start of a new cycle and advancement of the machine.

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*Since the plumb bob and minibob have a .635 cm tip height, the .635 cm deep cuts are not necessarily indicative of the energy and force relationship for the pick body shape.

The data recorded are for three different drum rotation speeds; 8.75, 18 and 51 rpm. The 18 and 51 rpm results show little difference in the gross mean specific energy, while for an 8.75 rpm head rotation speed the energies are somewhat higher for the shallow depths of cut, Figure 2-8. As the depth of cut increased from 2 to 10 cm the mean specific energy decreased continually but leveled out, however, for the deep cuts at ≈ 1.6 J/g. It is interesting to note that while the depth increases from 2 cm to 10 cm the S/D is decreasing from 4 to 1. This counter effect of decreasing the S/D should increase the specific energy and is probably responsible for the specific energy leveling out. The fact that the specific energy decreases initially is believed to be due to the dominant effect of increasing depth.

In all cases the mean shear specific energy is less than the corresponding sump specific energy, Figure 2-9. On individual sump and shear curves characteristics similar to the gross energy curve are followed. The shear appears to level off to ≈ 0.8 J/g at depths greater than 5 cm while the sump levels at ≈ 1.3 J/g at similar depths. The differences between these two energies can possibly be accounted for in the manner of coal breaking. As shown, Figure 2-7, the bulk of the sump cutting takes place in the middle of the cut where the coal has support from the surrounding material. In addition the coal must break through at the exit of the cut where the effective cutting depth is zero. In shearing, however, the majority of the coal being mined is a result of breaking coal from near the face in a tensile manner. There is no bulk of coal to give additional support as in the case of sumping.

Since both Ingersoll Rand and the Bureau of Mines have conducted tests using Pittsburgh seam coal, these data will be used as the basis for comparison of in-lab vs. in-mine observations. The geometry of the plumb bob and the minibob somewhat approximate the I-R pick and will be used in the comparison. When plotted with respect to the depth of cut the minibob gives values of the specific energy consistently less than the in-mine points. The plumb bob, though, shows energy data well within the range of the Ingersoll Rand equipment. It must be remembered, however, that while these tests can compare depth of cut or S/D, the two factors cannot be compared simultaneously. Such a comparison would give the best overall picture of the mining operation.

Looking at the Bureau of Mines unpublished data for an S/D of 2.5 using a plumb bob and I R's data at a cut depth of 4 cm (S/D = 2.5) a reasonable comparision can be made. In this case a plot of specific energy versus depth of cut takes on meaning since all points have a control S/D, Figure 2-10. The in-lab and in-mine test data show surprisingly close correlation with respect to the shearing cycle. The gross and sumping both have somewhat higher specific energies, but even this is well within the uncertainties of the different tests. If we are to consider that the I R depths reported are the maximum depths so that a corresponding true test depth would be somewhat less, then the shearing comparison to the lab data is that much better.

In reality, the fact that the laboratory data and the actual test data compare at all is rather exciting. While it may be due to a variety of offsetting circumstances as discussed, it is encouraging to know that laboratory test can apparently be related to the true mining operation.

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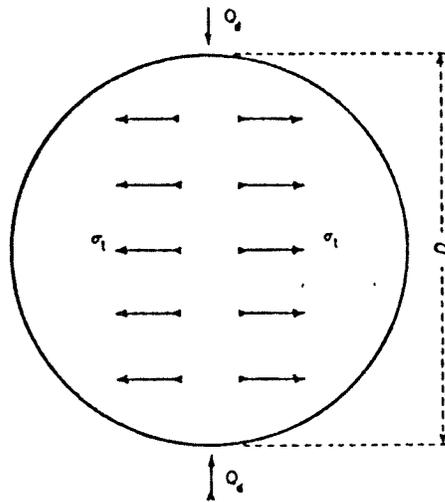
Tensile stress in a disc subjected to compressive loading.
$\sigma_t = 2Q_d/\pi D$, where Q_d = load per unit length at right
angles to plane of diagram.

(a)

(b)
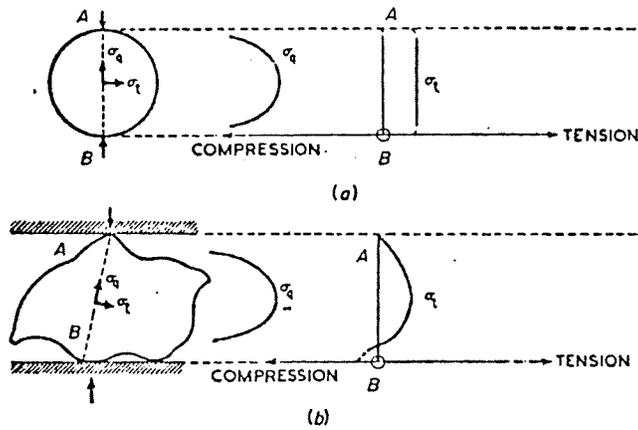
Principal stresses on line joining points of application of load.
(a) Disc in diametral compression, (b) irregularly shaped piece
compressed between parallel plates.

FIGURE 2-1
TENSILE STRESSES ARISING FROM
COMPRESSIVE LOADING (1)
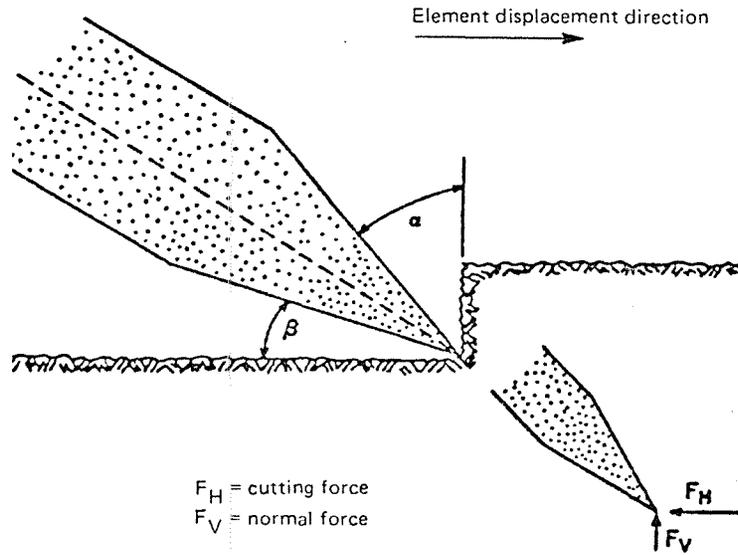
30

α = rake angle
β = clearance angle

FIGURE A. Coal cutting element.

F_H = cutting force
F_V = normal force

FIGURE B. Effect of element clearance on cutting force.

FIGURE C. Effect of rake angle on cutting force.
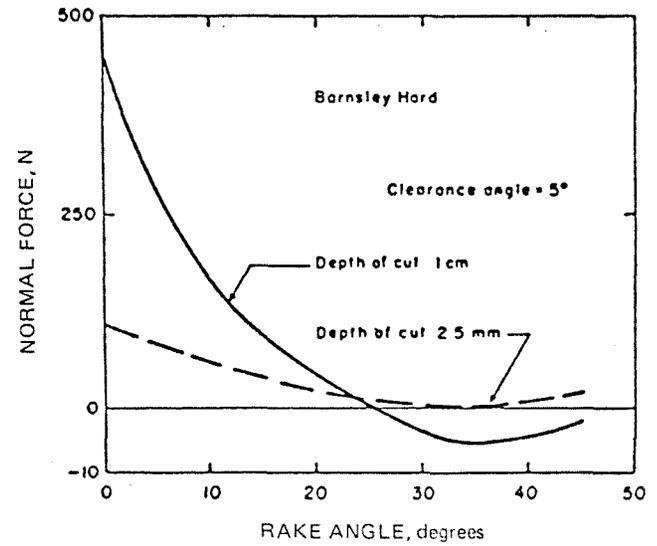
FIGURE D. Effect of rake angle on normal force.

FIGURE 2-2
FORCES ACTING ON TIP OF CHISEL BAR (2)

31
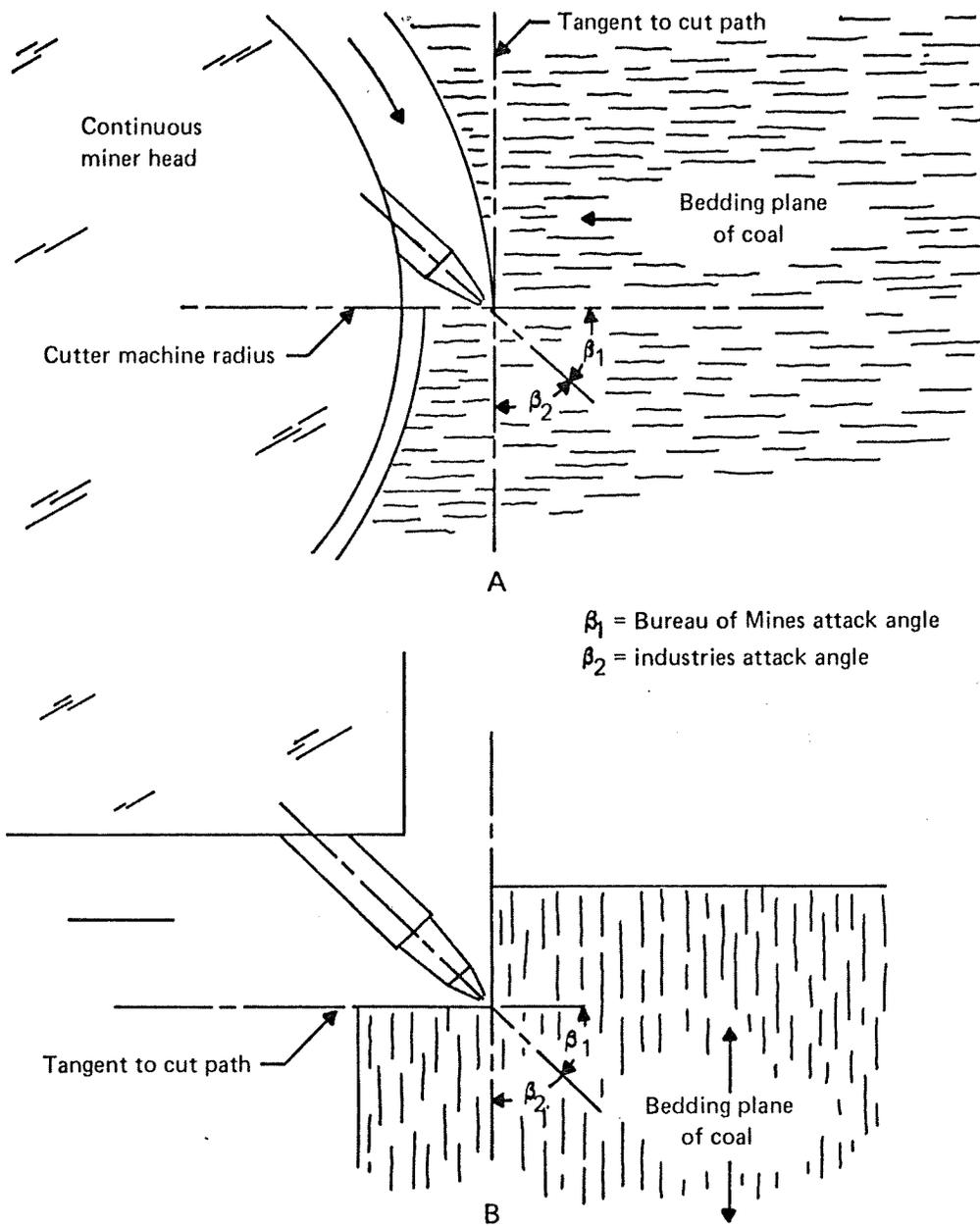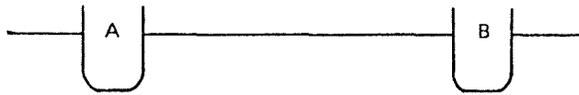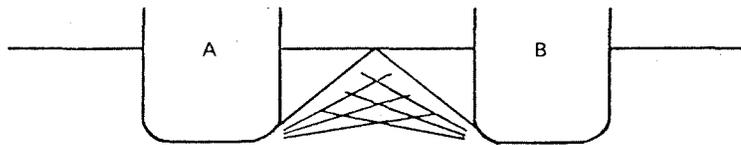
1037-15720

FIGURE 2-3
BIT ATTACK ANGLES:
A. Side view of cut with continuous mining machine;
B. Side view of linear test cut. (6)
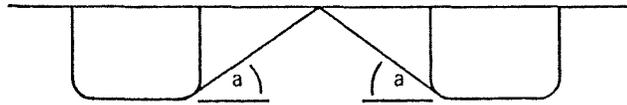
MBA

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32
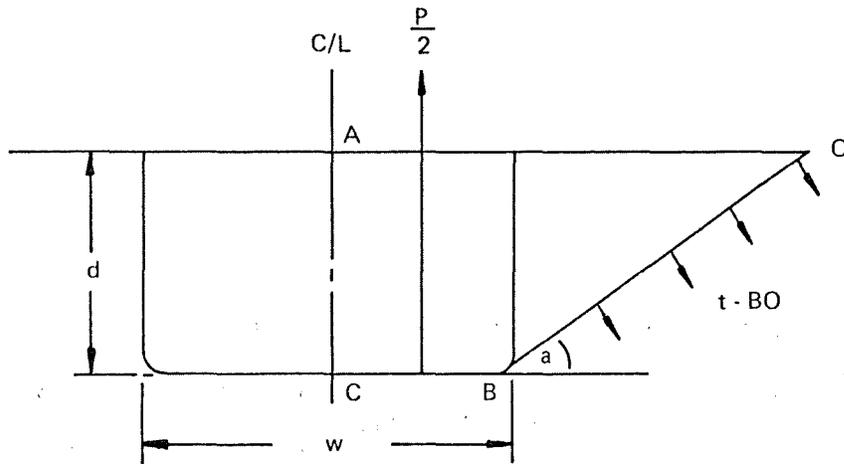
(a) Line Spacing Excessive

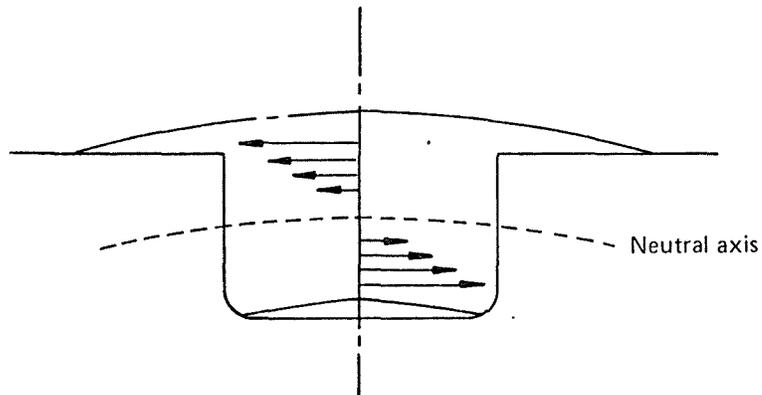(b) Picks at Optimum Line Spacing

FIGURE 2-4
INTERACTION OF PICKS

(a) Break-out Attributable to Side-splay
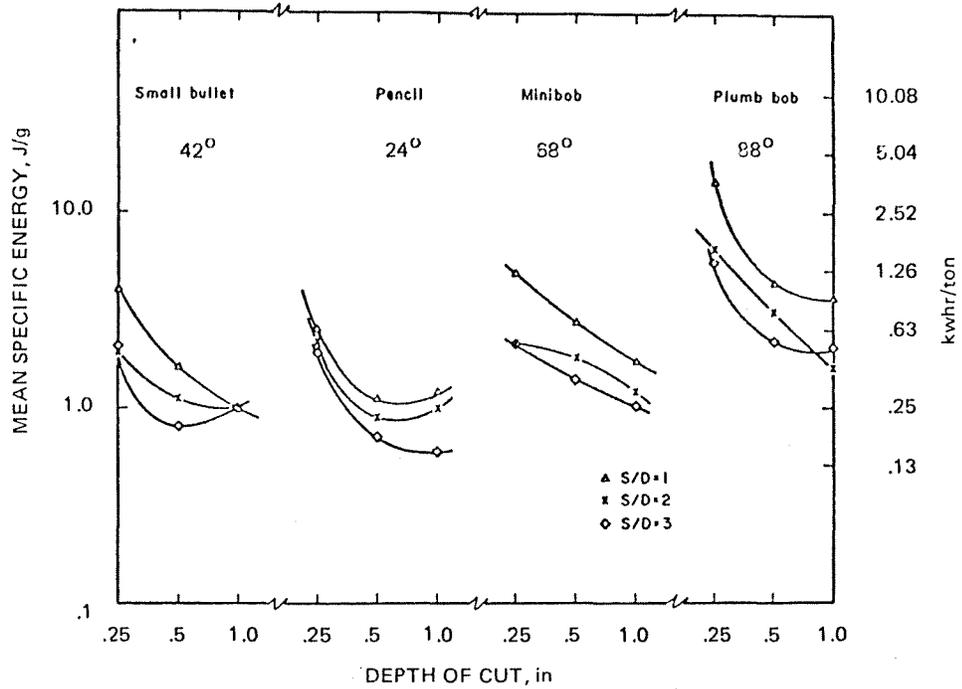

(b) Forces Acting on Half-section of Cut

(c) Bending of Material before Breakage

FIGURE 2-5
GEOMETRY OF SIDE-SPLAY BREAKAGE

1027-15715

34

Specific energy as a function of depth of cut for Pittsburgh seam coal.
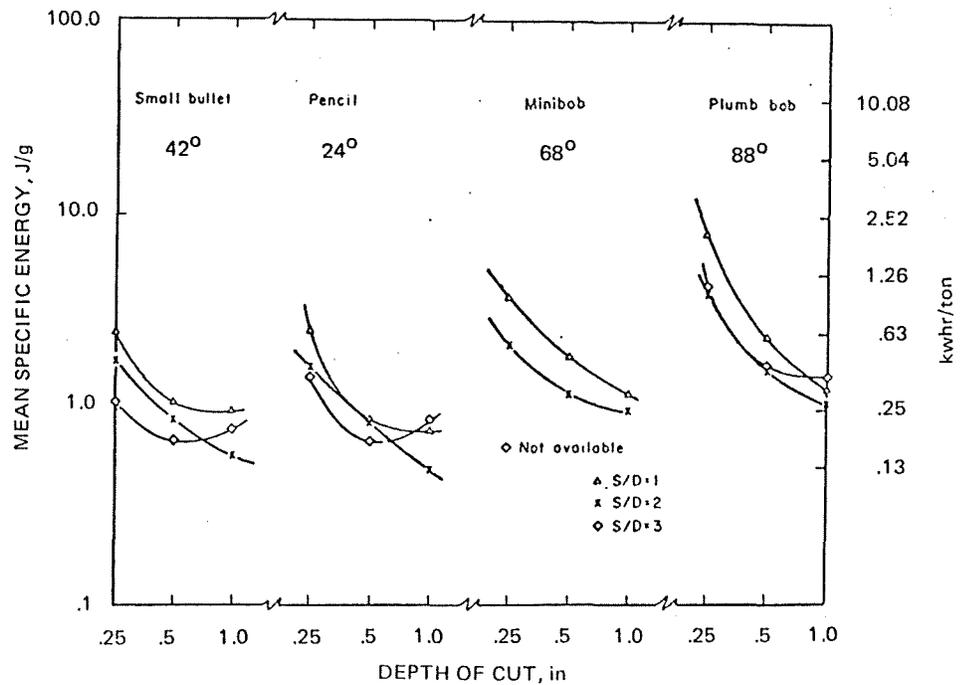
Specific energy as a function of depth of cut for Illinois No. 6 coal.

FIGURE 2-6
SPECIFIC ENERGY VERSUS DEPTY OF CUT
AND S/D RATIO FOR TWO COALS   (6)

1037-15722

35

MAX D AT CENTER OF CUT

SUMP

w

D ON EXIT = O

SHEAR

STARTS CUT @ O DEPTH

w

EXITS AT MAX D
EASY BREAK AWAY
FROM FACE

FIGURE 2-7
SUMP & SHEAR CUTTING CHARACTERISTICS

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36
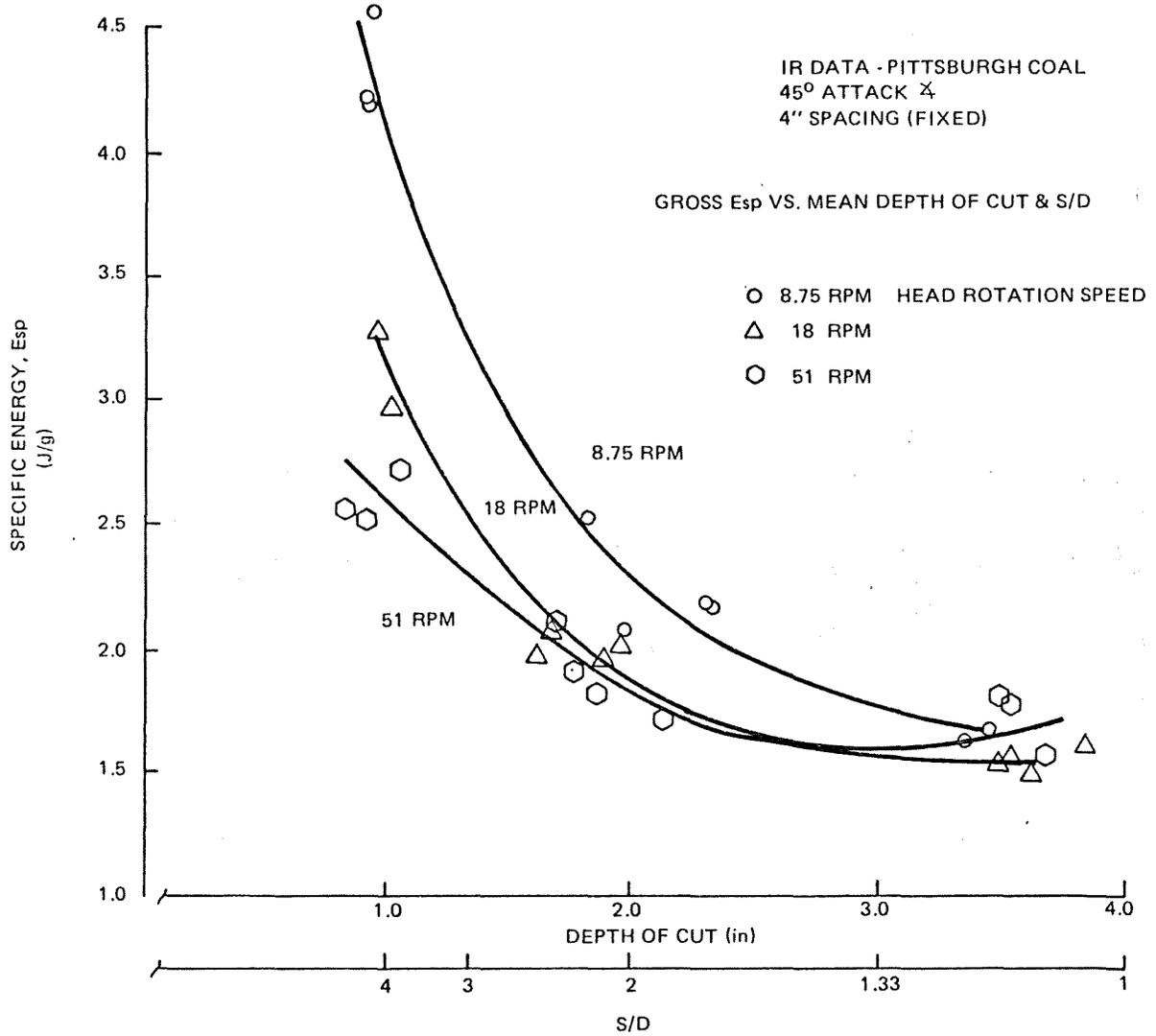
FIGURE 2-8
INGERSOLL-RAND DATA FOR SPECIFIC
ENERGY VERSUS DEPTH OF CUT

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IR DATA
PITTSBURGH COAL
45° ATTACK ⋏
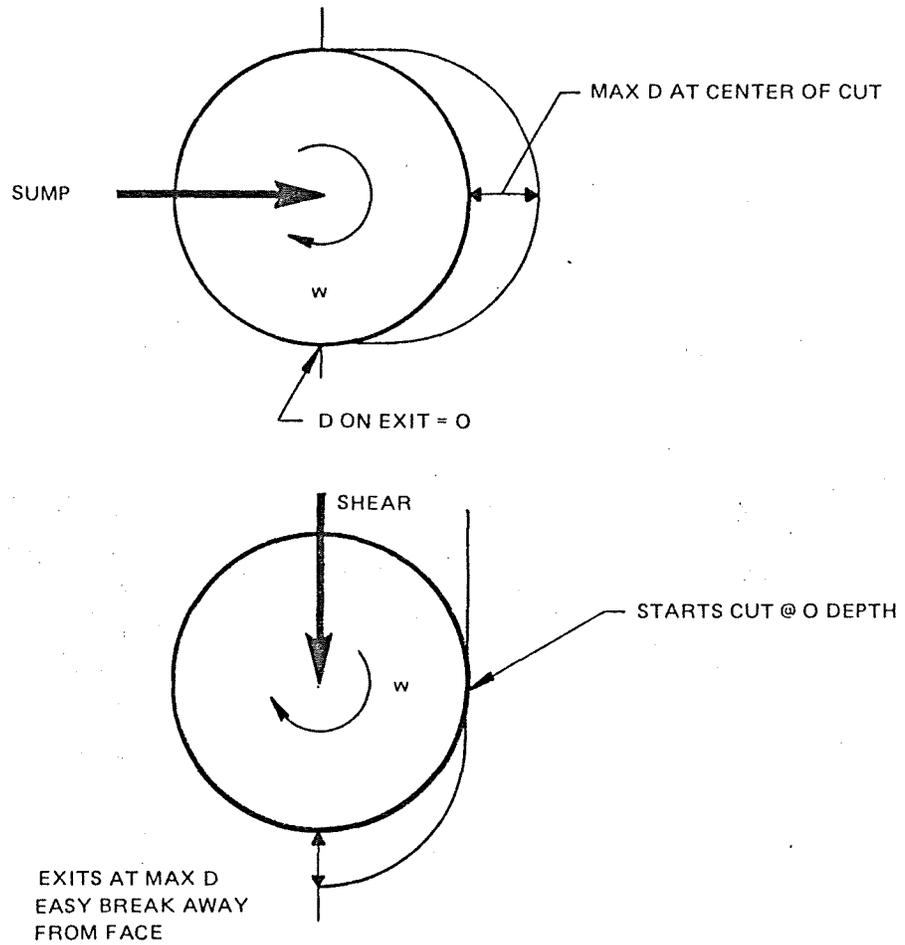4" SPACING (FIXED)
18 RPM

INTERMEDIATE SPEED SUMP & SHEAR Esp VS. DEPTH & S/D

SUMP

SHEAR

3.0

2.0

1.0

0

SPECIFIC ENERGY, Esp (J/g)

0   1.0   2.0   3.0   4.0
DEPTH OF CUT (in)

4   3   2   1.33   1
S/D

IR DATA
PITTSBURGH COAL
45° ATTACK ⋏
4" SPACING (FIXED)
8.75 RPM

SLOW SPEED SUMP & SHEAR Esp VS. DEPTH & S/D

SUMP

SHEAR

3.0

2.0

1.0

0

SPECIFIC ENERGY, Esp (J/g)

1.0   2.0   3.0   4.0
DEPTH OF CUT (in)

4   3   2   1.33   1
S/D

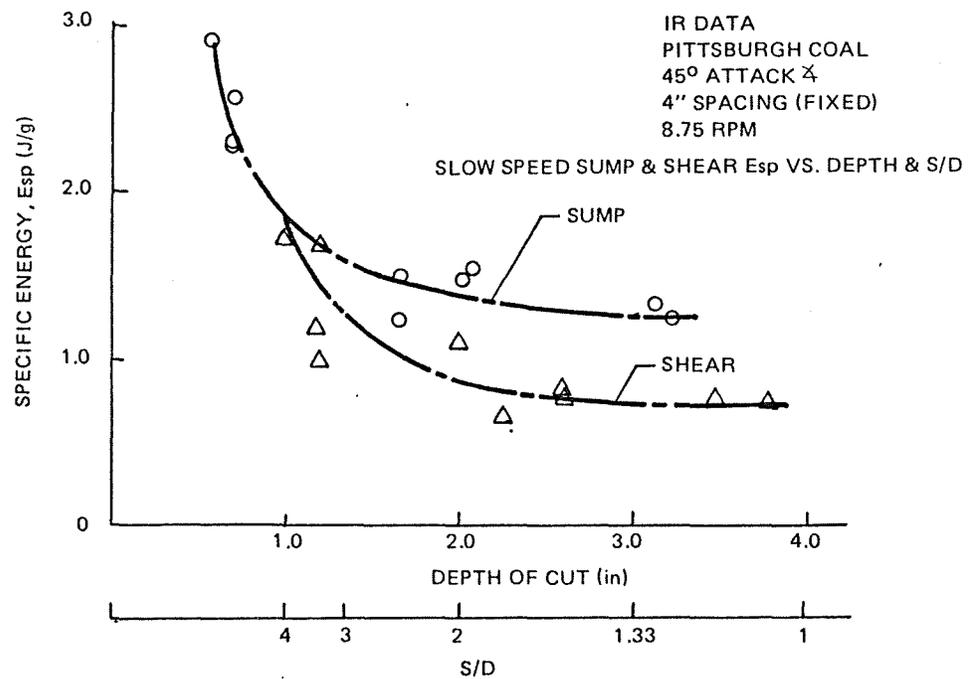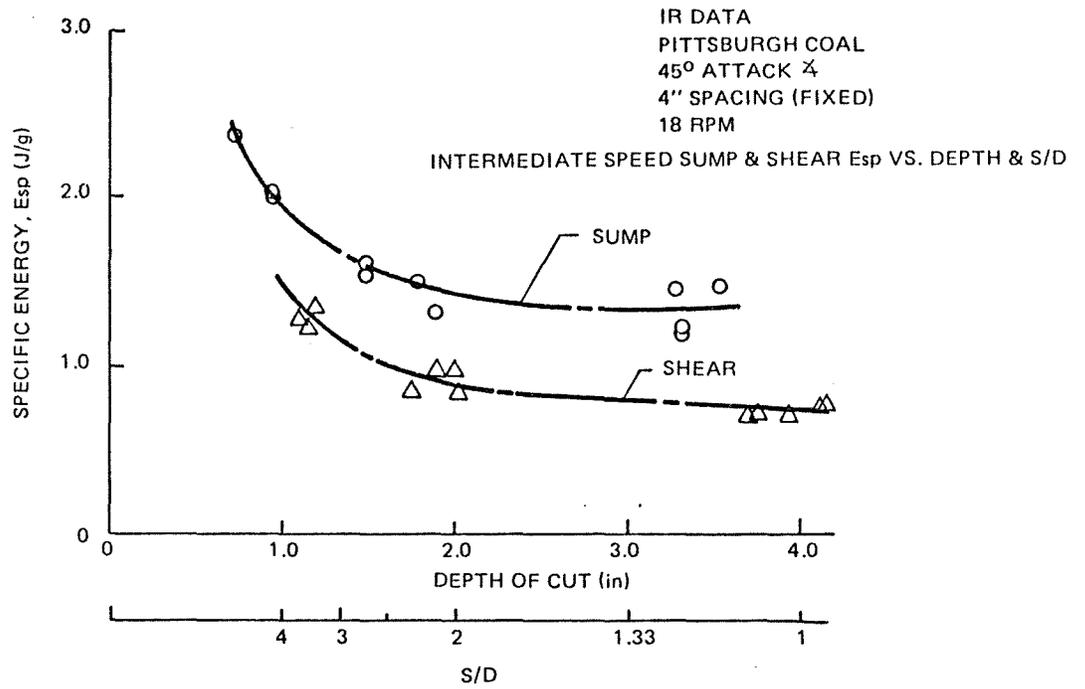FIGURE 2-9
INGERSOLL-RAND SUMP AND SHEAR
SPECIFIC ENERGIES VERSUS DEPTH OF CUT
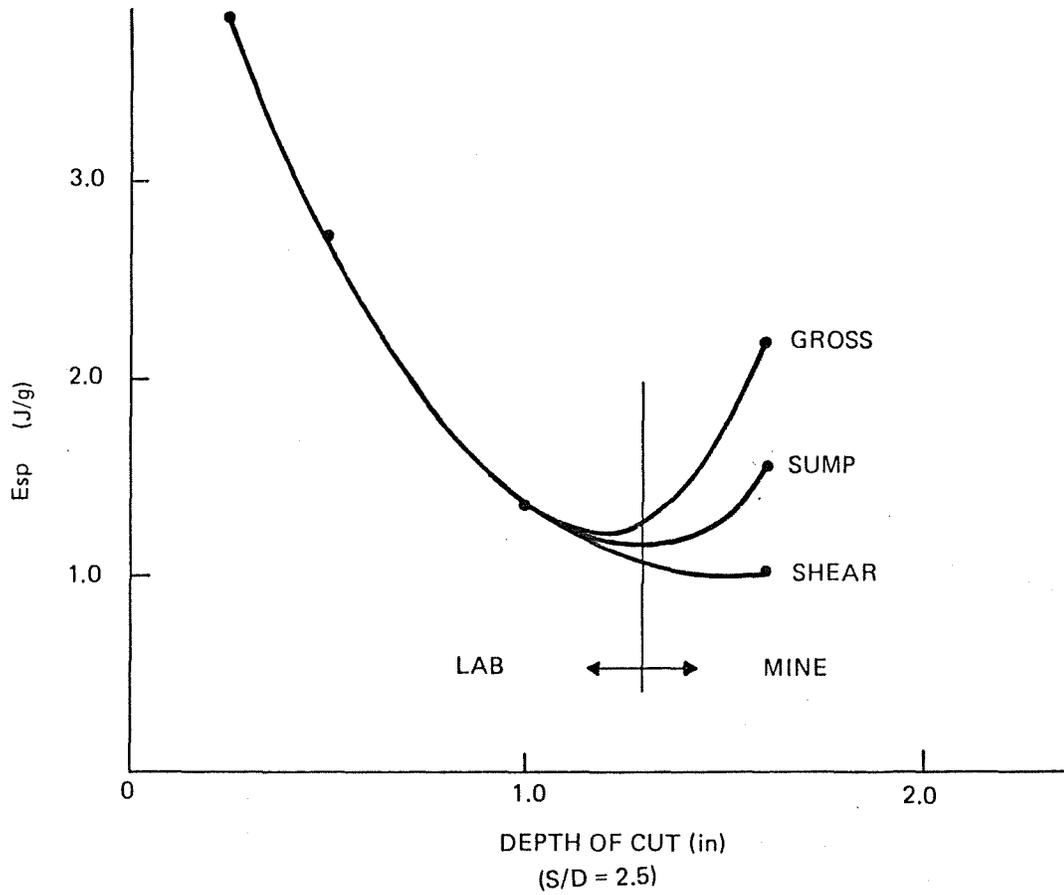
MBA

1037-15725

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FIGURE 2-10
SPECIFIC ENERGY VERSUS DEPTH OF CUT
FOR IN-LAB & IN-MINE TESTS

1037-15726

3.0 THE ANALYSIS OF COAL CUTTING

3.1 Introduction and Basic Assumptions

In the preceding sections of this report we reviewed the theoretical models used to study material removal operations and then summarized coal cutting research. Despite the activity in both areas, it is apparent that our fundamental understanding of the basic mechanism by which coal is cut is extremely limited.

The modelling of coal cutting under the complex stress field produced by moving picks or chisels requires first some basic assumptions about the mechanical behavior of coal under typical cutting conditions. As discussed earlier, coal breaks by tensile failure even while under normally compressive loads. The most successful approach to date in studying removal of coal by chipping is that of Evans[1], which is based on a critical tensile stress approach to fracture predictions. These and other observations strongly support the idealization of coal as a brittle solid in which material is removed by both localized "crushing" under the bit and by "chippings." The latter process occurs by the propagation and intersections of cracks ahead of and around the bit. For a simplified analysis of these processes, certain assumptions have to be made. The anisotropy of coal with its bedding and cleat planes will be ignored. It will be treated as an isotropic brittle solid with an inherent flaw distribution and strength that represent an average for the material. Since cutting and crushing involve fracture in all directions, such an idealization should be adequate in developing analyses for cutting forces and energy requirements based on average or typical material properties. In addition, it will be necessary to make assumptions about the size and shape of the chip broken from the coal during cutting and about the crushing characteristics under an indenting bit. We will indicate how certain predictions can be developed on this basis. However, a complete treatment would require critical experiments and a more detailed interplay between theory and experiment than is possible in this limited preliminary study.

In the cutting of coal one can isolate at least three phases. An understanding of two of these is essential for any analytical treatment of the removal process. At first, with very shallow cuts and sharp tools it is possible for "ductile grooving" to occur. Material may be removed in a pseudo-ductile manner due to the increased probability of flow rather than fracture in small regions. Also, cracking may occur due to residual tensile stresses produced by plastic deformation.[2] These processes while important in such applications as separating miniature electronic circuit components from a wafer of single crystal silicon appear to be irrelevant in considering bulk removal of coal. Next, in shallow cuts, crushing may occur due to the combined indenting and tangential action of a tool. This process is perhaps the hardest to describe analytically, but is important in an overall evaluation of coal

cutting. It appears to be the major contribution to dust formation, tool wear, and the energy requirement for coal removal. Finally, in deeper cuts, removal by "chipping" or "brittle cracking" occurs. Side splay takes place and the interaction of adjacent cuts becomes important. While pure brittle cracking would be the ideal method to cut coal from the point of view of efficiency and dust production, all present methods involve alternate "crushing" and "cracking." As we will attempt to show, the present cutting efficiencies lie between those expected for these two extremes. Figure 3-1 shows the effect of the transition between the two processes on volume removal and cutting force.

3.2   The Applicability of Prior Work on Removal Processes

Before discussing approaches which appear to have potential for the analysis of coal cutting, we will comment briefly on the relevance of the work on removal processes reviewed in Section 1.0.

Due to the importance of ductile metals in the past century, the cutting of these materials has dominated studies of material removal operations. Even with this attention, as we have seen, our present understanding of metal cutting is far from complete. It appears that the application of metal cutting analyses to coal and rock was made because no other analytical models were available. Since the behavior of brittle solids in removal operations is so different from that of ductile metals we are forced to conclude that metal cutting analyses are unlikely to be profitable in understanding coal cutting. For example, in eroding a surface by a stream of solid particles the variation of removal rate with angle of impingement is dramatically different for ductile and brittle solids. Methods of analysis which recognize the essential feature of brittle solids, that of fracture due to tensile stresses are much more likely to be useful. Our starting point in suggesting analytical models is based on the work of Evans and his colleagues, discussed in Section 1.5. For this reason, we review this work in some detail in the next section.

The proballistic aspects of the fracture of brittle solids, discussed in Section 1.6 are important when small regions are being fractured and may shed light on the crushing and comminution of coal. However, at this stage of our understanding of coal cutting, it appears more useful to treat crushing on an empirical basis. As bulk removal processes involving chipping become involved, the proballistic considerations become less important than an understanding of the factors governing crack propagation. Finally, an energy approach, as mentioned in Section 1.7 is unlikely to shed any light on the mechanisms of material removal. However, a comparison of the specific energies predicted by various analyses with experimental values should provide a useful check on the validity of analytical models.

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3.3   Tensile Theory of Breakage

On the basis of observations made on the cutting of coal by wedge action, a possible mode of breakage has been suggested by Evans.[1]   Figure 3-2 illustrates the action of a wedge in cutting a coal block while Figure 3-3 shows a similar diagram describing the assumptions of the tensile breakage theory.   Although this theory relates to broad faced chisels, a modification will be used to estimate the size and shape of the coal chip formed by point attack picks. Although the force for initial entry of the bit into the sample is related primarily to the compressive strength of the coal, most of the coal, by weight, is obtained from the seam by a breaking action involving the tensile strength.   After the initial penetration, cracks attributed to tensile breakage radiate from the tip of the bit.   The growth of these cracks to the surface and subsequent fracture of the buttock is the basis of the tensile breakage theory.   Our subsequent discussion of this theory is taken from the work of Evans.[1]

Referring to Figure 3-3, it is assumed that breakage is caused by tearing of the coal along a curve cd.   Evans[1] has shown that the most "potent" cracks are initiated from the wedge in the direction of the resultant force.   The crack, assumed to be a circular arc of radius r and angle 2α, is thus tangent to the direction of wedge penetration at point c.   For the preliminary theory it is also assumed that friction at the coal cutter interface plays no role and that the wedge is sharp.

Three forces are acting on the buttock of coal to be removed.   First, there is a force R' normal to the wedge face forcing the chip up.   The second force, T, is the resultant of the tensile force in the coal along arc cd.   In order to obtain force equilibrium as the chip is rotated about point d, a third force, S, must act through point d. The tensile resultant T may be calcuated if we assume plane strain, and we consider equilibrium of a slice of coal of unit thickness. Depending on the assumed stress distribution along line cd, T may vary.   However, it is assumed that since the crack initiates at c, the tensile strength, t, is reached there and decreases in some manner to zero at d.   Such a stress distribution may be of the form;

$$\sigma = t \left(\frac{x}{\ell} \right)^{n'}$$

where   x = the distance along cd from d.

ℓ = the length of cd

n' = constant.

Then, taking moments about d we get;

$$R' \ell \cos (\alpha+\theta) = t \int_0^\ell (x/\ell)^{n'} x\, dx = \frac{t\ell^2}{n'+2} \; .$$

Imposing the auxiliary geometric constraints,

$$r \sin \alpha = \frac{d}{2 \sin \alpha} = \frac{\ell}{2} \; ,$$

and equating to the total cutting force,

$$F_c = 2R' \sin \theta = \frac{2t\ell \sin \theta}{(n'+2) \cos (\alpha+\theta)}$$

$$F_c = \frac{2td}{(n'+2)} \; \frac{\sin \theta}{\sin \alpha \cos (\alpha+\theta)}$$

The assumption is now made that α will be such as to make F_c a minimum, i.e. $dF_c/d\alpha = 0$. Thus,

$$\cos \alpha \cos (\alpha+\theta) - \sin \alpha \sin (\alpha+\theta) = 0$$

$$\cos (\theta+2\alpha) = 0$$

$$\alpha = 1/2 \, (\pi/2 - \theta) \tag{2}$$

Hence,

$$F_c = \frac{2td}{(n'+2)} \; \frac{\sin \theta}{1 - \sin \theta} \tag{3}$$

The effect of adding the friction of the coal cutter interface to this analysis is to change θ to $\theta + \emptyset$ where $\emptyset$ is the angle of friction between the coal and the wedge. When the line of attack is skewed in this idealized study, so as to simulate the rake and back clearance angle of the actual cutting tool, it is found that the theory predicts a more rapid increase in cutting force with wedge angle than is experienced in practice. While the reason for this is not readily obvious, the effect is to nullify the increase in cutting force with increase of asymmetry that is predicted on the theory of pure tensile breakage. This leaves the theory of symmetrical attack still applicable - even for the asymmetrical approach. [3]

3.4 Surface Energy Considerations

The tensile theory of breakage, developed in Section 3.3, results in a force requirement to remove coal by wedge action. It does not, however, specify the energy which must be supplied to the face in order to break out a given amount of coal. As a first step in understanding the energy requirement, a surface energy approach is considered.

The specific energy to cut coal has been obtained for various types of bits and kinds of coals. In general [3-6], the lower limit is about 1 J/g. Assuming that all this energy goes into creating new surfaces, an expected average chip size can be calculated. Taking the coal surface energy, G_c, as approximately 10 J/m^2, based on preliminary tests we have made, and a specific gravity of 1.5 g/cm^2, the total surface area per unit volume is,

$$(1 \text{ J/g}) \times \left(\frac{1.5g}{cm^3}\right) \times \left(\frac{10^{-3}J}{cm^2}\right)^{-1} = 1.5 \times 10^3 \text{ cm}^2/\text{cm}^3$$

For the sake of simplicity, we now assume that the chips are cubical in shape with side dimension d. Then the average size is;

$$\left(\frac{1}{d^3}\right)\left(\frac{6d^2}{1}\right) = 1.5 \times 10^3 \text{ cm}^2/\text{cm}^3$$

$$d = 4 \times 10^{-3} \text{ cm}$$

The chip size, obtained in this manner, is about 2 orders of magnitude smaller than the average size found in practice. While the particle size data [3,7] shows extremes from less than .05 cm to greater than 10 cm the average chip sized based on surface area is about .3 cm. Part of this discrepancy is due to the model assumptions and averaging approximations. However, the bulk of the difference is probably a reflection of the inefficiency of the coal cutter interface (i.e. friction and dust formation) and the manner in which coal is broken from the seam.

3.5 A Brittle Cracking Model

Coal cutting in present machines occurs by progressive crushing and chipping. The crushing phase, while consuming much energy, does not involve the removal of large amounts of coal. The chipping or cracking mode of coal removal makes the use of the low tensile strengths and poor bonding between bedding and cleat planes. This process removes a volume of coal considerably larger than the area swept out by the cutting tool. To model this chipping process a modified form of Veldkamp and Wassink's [8] brittle grinding theory will be used.

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As in the tensile breakage theory, we assume that the cracks initiate at the bit tip where the stresses are tensile. This will result in a number of cracks which merely dead end, however, we will only look at the crack which results in breaking out a surface chip. Initiation of the crack will generally be from a defect in the material. It is to be expected then, that with an increase in the depth of cut the tensile stress zone increases in size such that more defects are encountered resulting in more cracks. Using a sharp conical pick with an included angle 2θ, the pick advancement is taken to be at a constant speed and at a given depth of cut, d. At a certain location a material defect comes into the region of sufficient tensile stress adjacent to the tool resulting in a "partial spherical cap" shaped chip, Figure 3-4. Figure 3-5 shows the geometry of the chip and the pick interaction.

Once a crack is started, there is a sudden decrease in the required tangential force on the pick. Continued pick advance proceeds to open the crack. The crack opening is thought to be the result of forcing the chip to slide up the cone as the pick advances a small distance. Additional assumptions are made that there is no crushing at the coal cutter interface*, and that the crack grows along a logarithmic curve. The forces acting at the pick tip are shown in Figure 3-6. With these requirement so imposed, a work-energy balance can be made for a pick advance of dx;

$$F_t \, dx = \mathcal{G}_c \, (\text{area of crack advance})$$

$$F_t \, dx = \mathcal{G}_c \, (r\phi/\cos \delta) \, dr$$

where F_t = horizontal force on pick,

and $\mathcal{G}_c$ = energy required to create fresh crack area.

At the pick tip, the crack opening, s, will be related to the pick advance by;

$$s = \text{crack opening} = x \tan^{-1} \gamma ,$$

thus leading to the crack advance relation, (Figure 3-7)

$$\frac{1}{2} \, x \tan^{-1} \gamma = \frac{N'}{E} \, \ell n \, \frac{r + R \cos \delta}{R \cos \delta}$$

N' = constant relating to crack opening size, friction,

and crack density.

---

* Although this is not likely to be the case, the crushing aspect of coal cutting will be handled separately in the next section.

$$E = \text{Young's Modulus}$$

$$R = \text{Constant distance}$$

or,
$$dx = \frac{2\, N \tan \gamma}{E} \left(\frac{dr}{r + R \cos \delta} \right) .$$

In general $r \gg R \cos \delta$ so that this equation reduces to;

$$dx = \frac{2N' \tan \gamma}{E} \frac{dr}{r} \qquad (2)$$

Combining Figures 1 and 2 yields;

$$r = \left[\frac{F_t (2N' \tan \gamma \cos \delta)}{\Phi E \mathcal{G}_c} \right]^{1/2} \qquad (3)$$

This expression relates the crack size r to the material constants and assumed crack configuration. Referring now to the model developed for the tensile breakage theory, we impose the geometrical constraint on r and δ ;

$$r = R_T (1 - \cos^2 \delta)^{1/2} . \qquad (4)$$

where R_T is the crack radius obtained from the tensile theory. Combining Eqs. (3) and (4) and solving for $\cos \delta$ yields,

$$\cos \delta = - \frac{F_t N' \tan \gamma}{\Phi E \mathcal{G}_c R_t^2} \pm \frac{\sqrt{\left(\frac{2 F_t N' \tan \gamma}{\Phi E \mathcal{G}_c} \right)^2 + 4 R_t^4}}{2 R_t^2} . \qquad (5)$$

For brittle cracking, the crack propagation energy, U_c, can be written,

$$U_c = \mathcal{G}_c (\text{Area}) = \mathcal{G}_c \int_o^r \frac{\Phi r dr}{\cos \delta} ,$$

or
$$= \mathcal{G}_c \int_o^\delta \Phi R_t \sin \delta (R_t d \delta).$$

Hence,
$$U_c = -R_t^2 \Phi \mathcal{G}_c \cos \delta . \qquad (6)$$

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Using the expression derived for $\cos \delta$, (Eq. (5)), in the energy equation, the result is the desired relationship;

$$U_c = \frac{F_t N' \tan \gamma}{E} \pm \frac{1}{2} \Phi \mathcal{G}_c \sqrt{\left(\frac{2 F_t N' \tan \gamma}{\Phi E \mathcal{G}_c}\right)^2 + 4 R_t^4} \quad . \quad (7)$$

Once again referring to the tensile breakage theory and specimen geometry, the cut depth, d, is;

$$d = 2 R_t \sin^2(\Delta/2) \qquad (8)$$

where $\Delta = (\frac{\Pi}{2} - \theta) = $ full angle of chip ($= 2\alpha$ from tensile theory)

and $\quad \theta = 1/2$ cone angle.

The force F_t is equal to an "average compressive tangential" stress on the pick times the pick area at the moment that the transition from crushing to brittle cracking takes place. Therefore,

$$F_t = \sigma_c d^2 \tan \theta \qquad (9)$$

Using these last two equations to simplify the energy relation once again, we obtain,

$$U_c = d^2 \left\{ \frac{N' \sigma_c \tan \theta \tan \delta}{E} \pm \frac{1}{2} \Phi \mathcal{G}_c \left[\left(\frac{2 N' \sigma_c \tan \theta \tan \gamma}{\Phi E \mathcal{G}_c}\right)^2 + 4 \left(\frac{1}{2 \sin^2(\Delta/2)}\right)^4 \right]^{1/2} \right\}$$

Since the energy required for chipping must be positive, only the positive root (of the $\pm$) is applicable. As expected, the energy is related to the chip area giving rise to the d^2 term.

Since we are interested in the energy required to remove a unit volume (or mass) we must look at the volume removed between defects or chips. Looking only at the chip removed, (ignoring crushing between chips) the volume, V, is,

$$V = (1/12) \, \Phi \, d \, (3R_t^2 \, \sin^2\!\Delta + d^2)$$

$$= (d^3/12) \, \Phi \left(\frac{3 \, \sin^2\!\Delta}{4 \, \sin^4 (\Delta/2)} + 1 \right) \tag{10}$$

The specific energy, the energy required to chip away a unit volume, can now be calculated,

$$E_{sp} = \frac{U_c}{V} = \frac{12}{d\Phi A} \left\{ \frac{N'\sigma_c \, \tan\theta \, \tan\gamma}{E} + \frac{1}{2} \, \Phi \, G_c \left[\left(\frac{2N'\sigma_c \, \tan\theta \, \tan\gamma}{\Phi E G_c} \right)^2 + \right. \right.$$

$$\left. \left. 4 \left(\frac{1}{2 \, \sin^2(\Delta/2)} \right)^4 \right]^{1/2} \right\} \tag{11}$$

where
$$A = \frac{3 \, \sin^2\!\Delta}{4 \, \sin^4(\Delta/2)} + 1 \,.$$

This expression for the specific energy shows the desired and expected result that the energy decreases as the depth of cut increases. While this derivation contains many assumptions and simplifications as well as a constant N' which should be determined from experiment, it is instructive to solve for the specific energy making the best "guesses" for values of the terms in the equation.

Taking,
$$\sigma_c = 60 \, MN/m^2$$

$$G_c = 10 \, J/m^2$$

$$E = 4 \, GN/m^2$$

$$\Phi = 120^\circ = 2\pi/3$$

$$\gamma = 22.5 = \pi/8$$

$$\Delta = (\pi/2 - \gamma) = 3\pi/8$$

$$\theta = 22.5^\circ = \pi/8 \,,$$

then;

$$E_{sp} = 1/d \left\{ 1.9 \times 10^{-3} \, N' + 7.8 \, (6.1 \times 10^{-8} \, N' + 27.5)^{1/2} \right\} \, J/m^3$$

for   d in meters and

   N' in N/m.

Although N' is not known, the two extreme values for N can be assumed. First, if N' is small, for instance less than 100 N/m, then,

$$E_{sp} \approx \frac{40}{d} \, J/m^3 \approx \frac{3 \times 10^{-5}}{d} \, J/g$$

where d is taken to be in meters.

For a cut depth of 1 cm (.01 m) the specific energy is about .003 J/g - quite low considering values obtained in practice!

In case 2, for $N' > 10^5$, then the specific energy equation (12) reduces to;

$$E_{sp} \approx 3.8 \times 10^{-3} \left(\frac{N'}{d} \right) \, J/m^3 \approx 2.5 \times 10^{-9} \left(\frac{N'}{d} \right) \, J/g$$

where once again d is in meters and N' has units of N/m. Since N' relates to the crack opening distance, eq. (2), a reasonable estimation of its value can be made. Taking values of crack size and pick advancement compatible with this analysis, an N' of about 4×10^5 N/m is plausible. The resulting E_{sp} is thus,

$$E_{sp} = \frac{10^{-2}}{d} \, J/g$$

Taking a depth of cut similar to case I, ie. 1 cm, the resulting specific energy is 1 J/g.

Acknowledging that the specific energy obtained in this manner can be low compared to practice, it is important to understand the limitations of such a theory. As mentioned at the onset of this analysis we only considered the crack which surfaced causing a chip to be removed. The propagation of defects into dead end cracks may change the energy values by as much as an order of magnitude. Although this energy must be supplied to the coal in order to propagate the desired crack, the dead end cracks result in no additional volume removed. Along these lines, it is also noted that the chip is not removed in a continuous spherical cap, however, may be removed in

a number of pieces simulating this shape. The energy used to break apart this chip must also be considered wasted energy.

In all "in-mine" tests a pick similar to a plumb bob or minibob was used. The assumption of a sharp cone in these cases is not accurate. The tip of a sharp cone would be more efficient due to the manner in which the chip is formed and thus lead to lower energy results for similar depths of cut.

In the development of this theory we drew from the analysis of the tensile breakage theory for the chip size (i. e. , R_t and $\triangle$). While the tensile theory was developed using a wedge type action with a different geometry chip, the forces acting on the pick and the coal provide at least a basis by which to develop the pick chipping theory. An improvement to the chipping theory presented would be to establish a tensile breakage theory relating to the pick action on spherical caps in coal. Additionally, the tensile breakage theory does not account for the change in tangential force as the wedge attack angle (or in the chipping theory the pick angle of attack) is changed. To add this degree of complexity to the analysis requires an understanding of the conditions existing at the tip of the pick.

3.6   A Crushing Model

While chipping is taking place on the coal face there is an equally important but much less efficient process of crushing also at work. As some defect forms into a crack the force on the pick drops suddenly. Continued movement of the tool to open up the crack requires the material below the crack to be removed by crushing. Since the area of this coal in contact with the pick is initially small, the process of grooving is most applicable. Taking a simple model of this process, we can calculate the energy requirement for the crushing phase of coal cutting.

In a crushing mode the tangential force on the pick is related to the compressive or indenting strength of the coal, as discussed earlier relating to the transition from brittle chipping to crushing. The volume removed by such action is simply the submerged area of the pick times the distance traveled. The specific energy of crushing coal is then;

$$E_{sp} = \frac{F_t \cdot x}{volume} = \frac{\sigma_c (Area)(x)}{(Area)(x)}$$

$$E_{sp} = \sigma_c .$$

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If we take the compressive strength of a typical sample coal to be 60 MN/m^2, the specific energy is then 40 J/g. This value represents the upper limit to inefficient coal cutting. Since any pick action involves the combined action of crushing and cracking, at least to some degree, values of specific energy this high (or as low as predicted by the brittle chipping theory) would not be expected in practice. The use of the coal compressive strength in place of the average tangential crushing force or penetration resistance is based on the observations of Evans and Pomeroy. The various methods used to determine the compressive strength of a coal sample generally result in an approximate range from which likely average values can be assumed. The distinction between the penetration resistance, average tangential crushing force and compressive strength for a particular size sample is unimportant beyond knowing that the values are of the same order.

3.7 The Combination of Crushing and Cracking

At this point we have developed two models which represent the extremes by which coal cutting can take place. In reality we know that both processes are operating simultaneously and thus some average of the two should give more reasonable values of the true energy to remove coal. For a purely cracking mode, a specific energy of 1 J/g was calculated in Section 3.5 on a 1 cm depth of cut. This energy acts, however, only to remove a chip whose volume can be calculated using eq. 10;

$$V = \frac{d^3}{12} \, \Phi \left(\frac{3 \sin^2 \Delta}{4 \sin^4 (\Delta /2)} + 1 \right)$$

$$= 1.35 \text{ cm}^3$$

where

d = 1 cm depth

$\Phi = 2\pi/3$ = arc of spherical cap chip.

$\Delta = 3\pi/8$ from tensile theory.

The pick must now advance by crushing the coal under the chip until a new chip can be formed and the cycle repeated. The 40 J/g specific energy developed in Section 3-6 must therefore act on a volume in which all crushing takes place. The shaded section of Figure 3-5 shows the area to be crushed. The volume swept out by pick motion through such an area is:

$$\text{Vol} = \int_0^{R_t \sin\Delta} (y)\,(y \tan\theta)\,dx$$

where $\qquad y$ = depth of pick in the crushed material.

$$= R_t\,(1 - \cos\delta)$$

$$\theta = 1/2 \text{ pick body angle}$$

Reducing by geometrical constraints gives;

$$dx = R_t \cos\delta\,d\delta$$

$$\text{Vol} = \int_0^\Delta R_t^3\,(1 - \cos\delta)^2 \cos\delta \tan\theta\,d\delta$$

$$\text{Vol} = R_t^3 \tan\theta \int_0^\Delta (\cos\delta - 2\cos^2\delta + \cos^3\delta)\,d\delta$$

$$\text{Vol} = R_t^3 \tan\theta \left[\sin\delta - 2\left(\frac{\delta}{2} + \frac{\sin 2\delta}{4}\right) + \frac{\sin\delta}{3}(\cos^2\delta + 2)\right]_0^\Delta$$

Using the same size constraints imposed for the chipping model on a 1 cm cut depth and pick half angle 22.5^0, the volume removed by crushing is $.10 \text{ cm}^3$.

A specific energy for the combined crushing and chipping action can now be established based on the weighted average of volume removed by the two processes. The total specific energy is then;

$$E_{sp(Ave)} = \frac{E_{sp(chip}\,V_{chip)} + E_{sp(crush)}\,V_{crush}}{V_{chip} + V_{crush}}$$

$$= \frac{(1)\,(1.35) + (40)\,(.10)}{(1.35) + (.10)} \text{ J/g}$$

$$= 3.7 \text{ J/g}$$

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The value obtained in this manner corresponds more closely with the range of values obtained in the laboratory and in-mine tests for shallow depths of cut, i. e., 1 cm. While the close correspondence of values obtained by theory and those from actual test is encouraging, we must be fully aware that the final value obtained is a result of numerous simplification and assumptions.

3. 8   Some Fracture Mechanics Considerations in
            Coal Testing and Cutting

In the past twenty years there has been a dramatic breakthrough in our understanding of fracture. Most of the theoretical and experimental work has dealt with materials that exhibit essentially linear elastic behavior before and during fracture. This field of linear elastic fracture mechanics (LEFM) had its roots in the classic work of Griffith in the 1920's. However, it was not until fracture problems appeared in jet aircraft, turbo-generator rotors, and missile casings in the 1950's that a need was felt for an analytical approach to fracture prediction beyond that provided by "Transition Temperature" methods. Griffith's original energy approach, as modified by Irwin in the early 1950's, in essence relates the load carrying capacity of a part to the size of the crack it contains. An alternative approach, pioneered in the Soviet Union for geological problems, based fracture prediction on the critical value of the stress intensity factor for a cracked body. The demonstration by Irwin in 1958 that the energy and stress intensity factor approaches were equivalent stimulated great interest in LEFM. By now this field has been explored in detail, although some aspects such as crack stability, crack arrest, and dynamic fracture problems still require attention. Activity, at least for those working with metals, has switched largely to the prediction of fracture beyond the linear elastic range. However, in the fields of ceramics and rock mechanics the techniques of LEFM have only recently become of general interest. It has become apparent that an understanding of fracture properties is essential for an analysis of say rock blasting or rock removal processes. The ASTM Committee (E-24) responsible for codifying fracture test techniques for metal has now deleted "for Metals" from its title and has a subcommittee studying recommended procedures for fracture testing of rocks.

The procedures for plane strain fracture toughness testing of metals are described in ASTM standard E-399 and have been applied to a wide variety of materials (metals, ceramics, rocks, wood, ice, polymers, composites). Since we are unable to locate fracture toughness data for coal, two sets were made using one of the ASTM Standard configurations (3 point bend). Cracks were inserted using a 0. 009 inch jewelers saw. Our experience in testing rocks suggests that the sharpness of the crack is not critical in measuring the fracture toughness of materials such as coal which contains an extensive network of inherent cracks. Since the tests were made on specimens cut from a block that had served as a paperweight for an undetermined period,

it would be unwise to place any emphasis on the numerical values. However, the tests did demonstrate that it is possible to make fracture toughness measurements on coal. Figure 3-8 shows the configuration of the specimen and the orientation of the crack relative to the bedding plane. Case A, propagating the crack across the bedding planes, as might be expected showed a much higher toughness than Case B in which the bedding planes "divide" the crack front. Presumably, the third possibility, which was not tested, in which the crack propagates along a bedding plane would lead to even lower values of fracture toughness.

	Case A	Case B
Critical value of Stress intensity Factor K_c	$0.39 \ MN/m^{3/2}$	$0.18 \ MN/m^{3/2}$
Energy required to form unit crack area $G_c = K_c^2/E$ $E = 4 GN/m^2$ (assumed)	$38 \ J/m^2$	$8 \ J/m^2$

Conversion of values of K_c to G_c was made using the relation $K_c^2/E = G_c$. The factor of $(1-\nu^2)$ which is usually included to account for plane strain is near unity and was neglected because of the approximate nature of the test and the uncertainity in the value of Young's modulus E. Because of the low modulus of coal the values of G_c corresponding to a given K_c are surprisingly high. The direct measurement of G_c using "work of fracture" measurements under stable cracking conditions would be a worthwhile measurement in future investigations.

Turning now to coal-chipping models, calculations based on LEFM would be more realistic than assuming a tensile breakage theory but would require more extensive computations. As an illustration we consider the chipping geometry postulated in Figure 3-4 and the forces shown in Figure 3-6. The forces acting on the coal are equal and opposite to those shown in Figure 3-6 which act on the pick. From a fracture point of view, the force F_n is primarily responsible for detaching the chip. The force F_t which appears as a compressive force parallel to the initial crack at the tip of the pick influences the direction of the crack. A larger compressive force would be expected to force the crack to run horizontally for a longer distance before turning toward the free surface. In principle, it should be possible to predict the crack trajectory from LEFM. In practice such calculations would be extremely time consuming. For this reason we indicate only how the maximum value of F_n required to detach a chip can be estimated using the procedures of LEFM. We consider the wedge shaped section shown in Figure 3-9

and assume that the crack is propagating parallel to the surface. As it turns upwards the force required for crack propagation should decrease. If we neglect the area of the sides of the chip as small compared to the pie-shaped main crack, during stable crack propagation we may write:

$$G_c = \frac{P^2}{2(\Phi a)} \ (dc/da)$$

where c, the compliance, is the deflection in the direction of the resultant force P divided by the force. If the height of the chip is small compared to the crack length we can consider only bending deflections and approximate c by y/F_n and replace P by F_n. The rate of change of compliance with crack length depends only on the dimensions of the "beam" at the tip of the crack. From elementary beam theory it may be shown that:

$$\frac{dc}{da} = \frac{12}{E} \ \frac{a^2}{d^3 (a\,\Phi)}$$

Hence $F_n = \Phi \, (G_c \, E)^{1/2} / \sqrt{6} \ d^{3/2} = \Phi K_c d^{3/2} / \sqrt{6}$

where Φ is in radians. Assuming $\Phi \simeq \sqrt{6}$ we arrive at the simple approximation $F_n \simeq K_c d^{3/2}$.

This result is not immediately useful because the measured normal force, as seen from Figure 3-6 is $F_N - F_n$ with $F_N - F_n \simeq F_t$ in typical force records. Making the gross assumption that F_n and F_t are related by a coefficient of friction of about 0.5 we obtain $F_N \simeq 2F_t$ and $F_n \simeq F_t$. The specific energy is proportional to F_t and the volume removed for geometrically similar chips is proportional to d^3. Hence this greatly simplified model would predict a maximum tangential force of $K_c d^{3/2}$ and a specific energy proportional to $K_c/d^{3/2}$.

At this point in view of all the assumptions that have been made it does not appear worthwhile to carry this model any further. However, the demonstration indicates the type of predictions which could be attempted if more were known about the fracture properties of coal, the geometry of cutting, and the friction or crushing conditions at the tip of the pick.

3.9   Future Directions in Coal Cutting Research

From the material reviewed in this report it appears that our present understanding of the mechanisms of coal cutting is very limited. Laboratory experiments and actual coal cutting results can currently be compared, only on the basis of specific energy. This comparison in Section 2.5 of the report leads to encouraging results and suggests that laboratory tests may adequately simulate actual cutting processes. For a detailed comparison, instrumented picks would have to be used to measure forces occuring in actual cutting operations.

After reviewing the existing theories for material removal, we conclude that, despite its limitations, the Evan's theory of tensile breakage is the only one which can be applied without extensive additional research. The approach taken by Evans is for two dimensional wedges. We have attempted in Section 3.5 to extend this theory to cutting by picks, which requires additional assumptions. Modification would have to be made to this approach if large variation in cone angle or tip radius were to be studied. Also if the case shown in Figure 3-9 were to be treated by the tensile breakage model, the theory would have to be modified to account for a crack opening which is not dependent on the coal sliding over the body of the pick.

As pointed out, the tensile breakage model leads to much lower specific energies than obtained with present mining machines. We have explained this by pointing out that present operations involve "crushing" with high specific energies as well as "chipping" (tensile cracking). In Section 3.7 a method has been suggested in which combined effect of crushing and chipping could be treated. Manual coal cutting rates of 40 tons in an eight hour shift by a single miner (1/10 hp) have been reported. This corresponds to a specific energy of about 1/20 J/g or an order of magnitude better then present machines. In this case, tensile cracking which exposes planes of weakness and the near elimination of "crushing" must be involved. Both this result and our analysis show that considerable benefits in energy requirements could result by minimizing crushing during removal operation.

The fracture toughness of coal, along with other mechanical properties, would be expected to be highly anistropic. Our two preliminary tests showed a variation of almost 5:1 in G_c for the crack directions shown in Figure 3-8. Even lower values of G_c would be expected for cracks propagating along the bedding plane. As pointed out, a major limitation of the tensile breakage theory is that it requires the stresses to be prescribed along the fracture surface at the moment of fracture and treats the material as isotropic. Attempts should be made to remove these limitations using concepts from LEFM and possibly incorporating the ideas of Sikarskie, Altiero and Glenn (ref. 1-23, 1-24 and 1-25). In Section 3-8 we have shown, in a very simplified form, one approach based on LEFM to predicting forces and specific energy trends.

At this stage it does not appear worthwhile to attempt to develop more detailed analytical models. Rather, carefully conducted experiments in crushing, chipping and fracture are needed to guide theoretical developments.

With physically reasonable theoretical models and detailed experimental results, it should be possible to develop analyses which make realistic predictions of cutting forces and specific energies for various pick geometries and rake angles. With this information, and an improved understanding of pick wear, a more rationale design of coal removal equipment should be possible.

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Coal volume

Chipping

Groove length 1 m
Element width 1 cm

* Transition to chipping.

Volume swept by element

COAL REMOVED, g

GROOVE DEPTH, cm

Force per cm of cutter width

Cutting force versus
groove depth.

MEAN PEAK FORCE, N
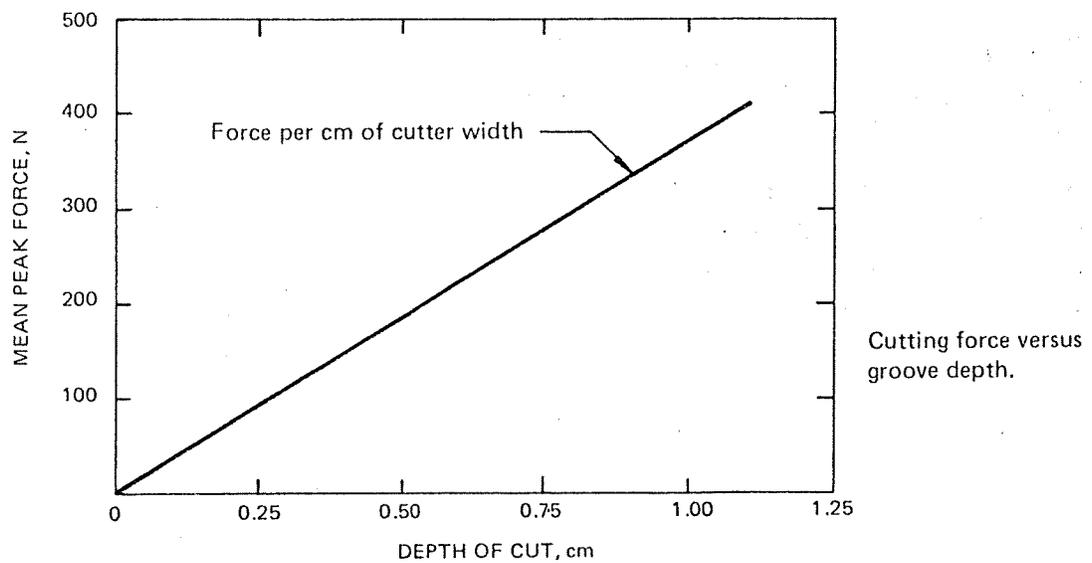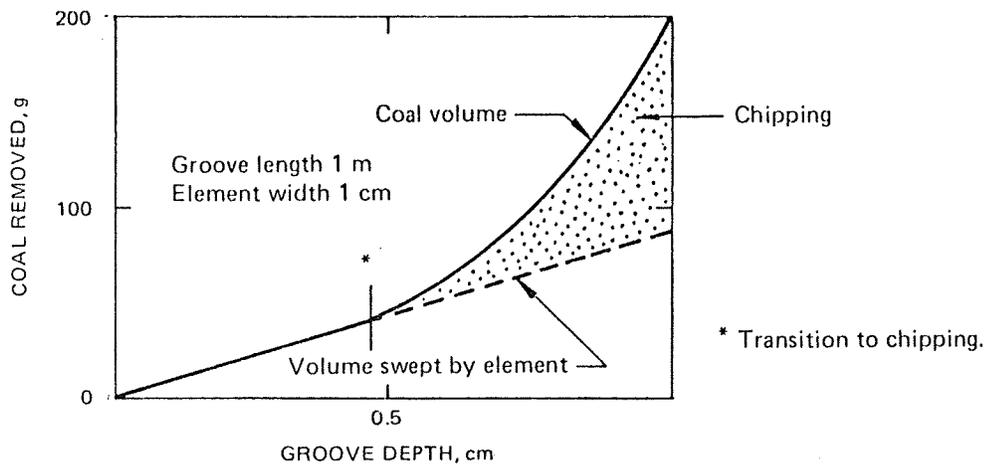
DEPTH OF CUT, cm

FIGURE 3-1
DEPTH OF CUT VERSUS COAL REMOVAL AND
PEAK CUTTING FORCE, SHOWING TRANSITION
TO CHIPPING   (4)
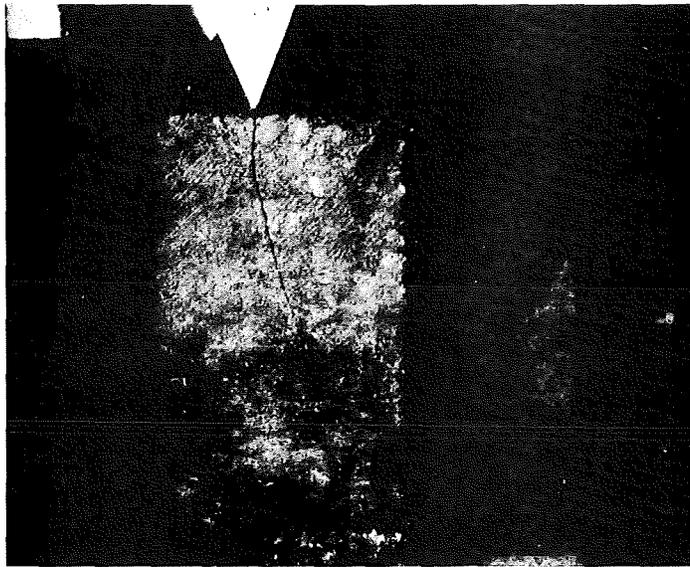
FIGURE 3-2. Illustrating the action of the wedge in cutting
a block of coal. (3)
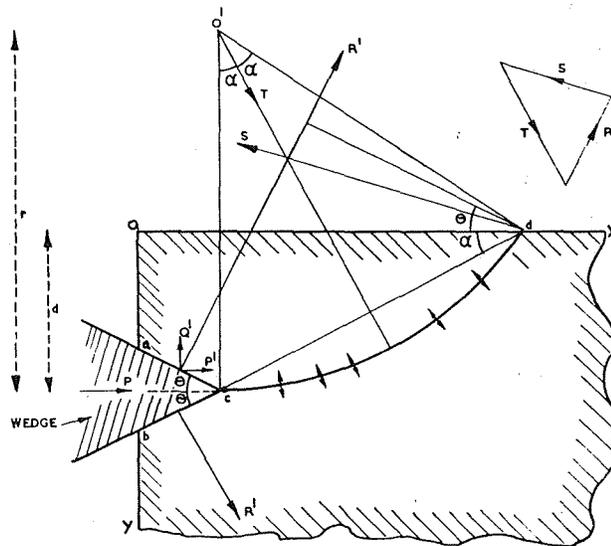
FIGURE 3-3. Illustrating assumptions of tensile breakage
theory. (3)

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FIGURE 3-4
Assumed chip shape for a
conical pick cutting into
a coal face
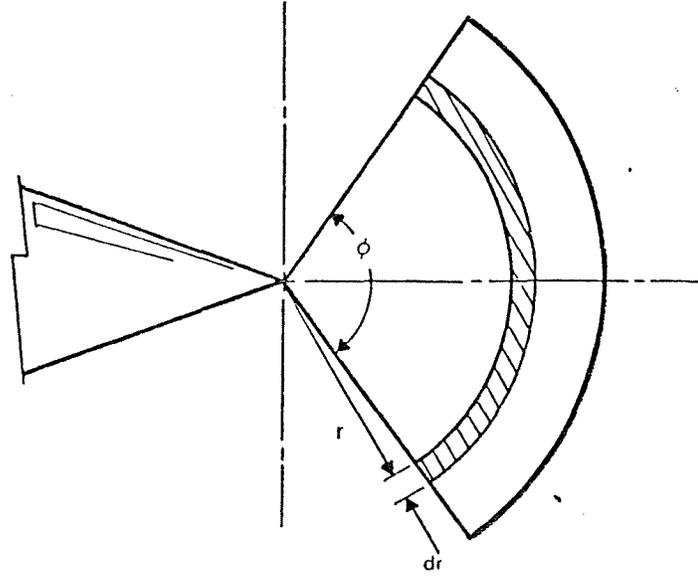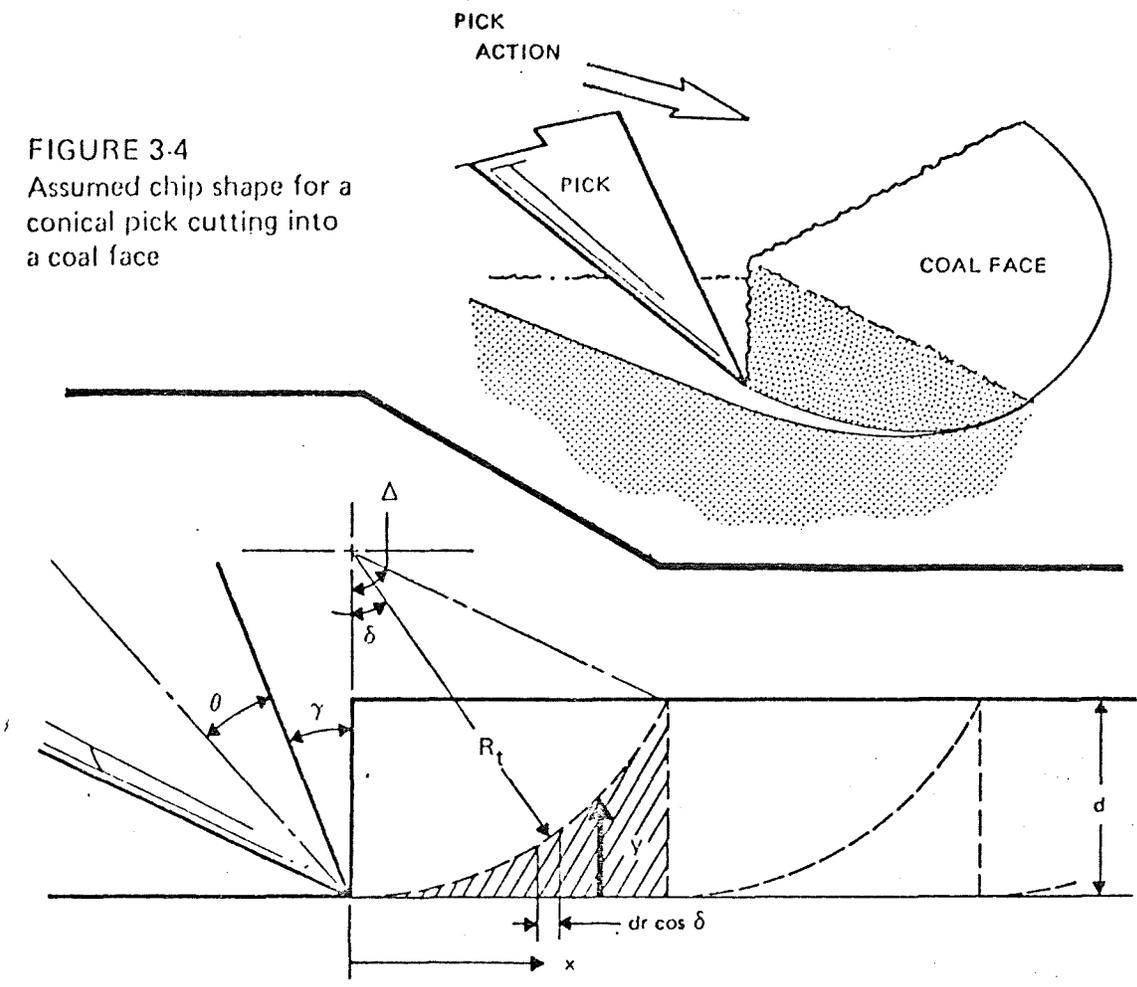
FIGURE 3-5
Chip geometry and pick
interaction.
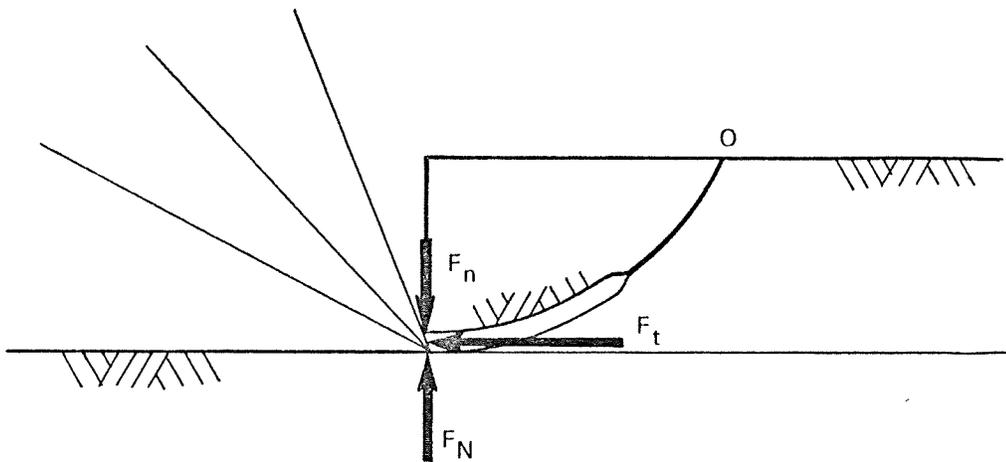
FIGURE 3-6
FORCES ACTING AT TIP OF PICK

$S = x \tan^{-1} \gamma$ at the pick tip
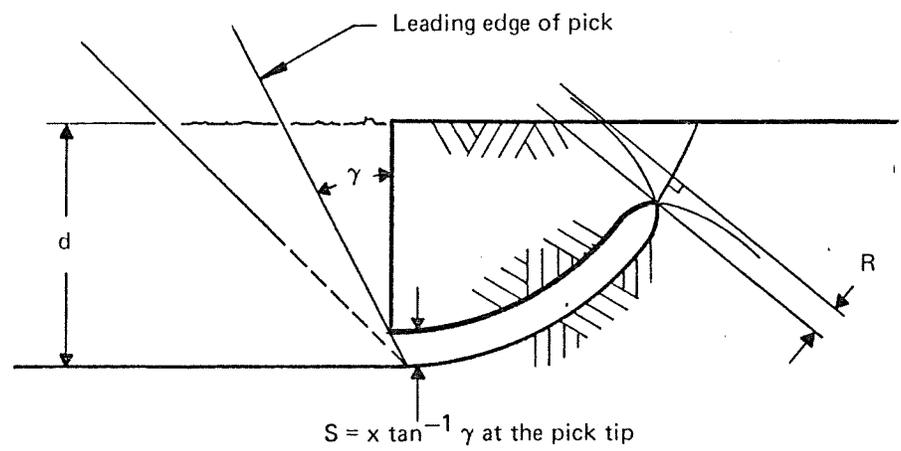
FIGURE 3-7
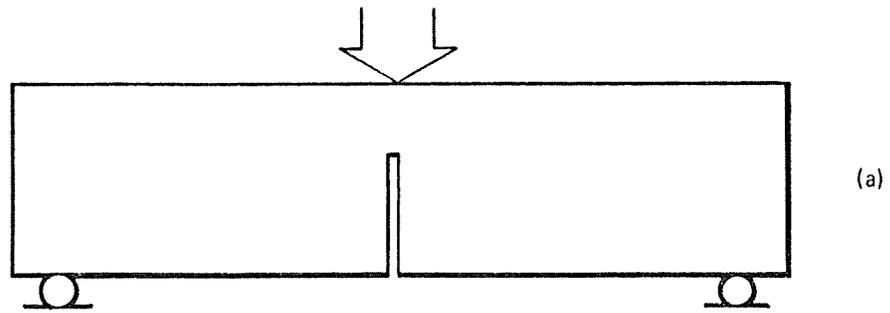CRACK OPENING DUE TO PICK ADVANCE

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61
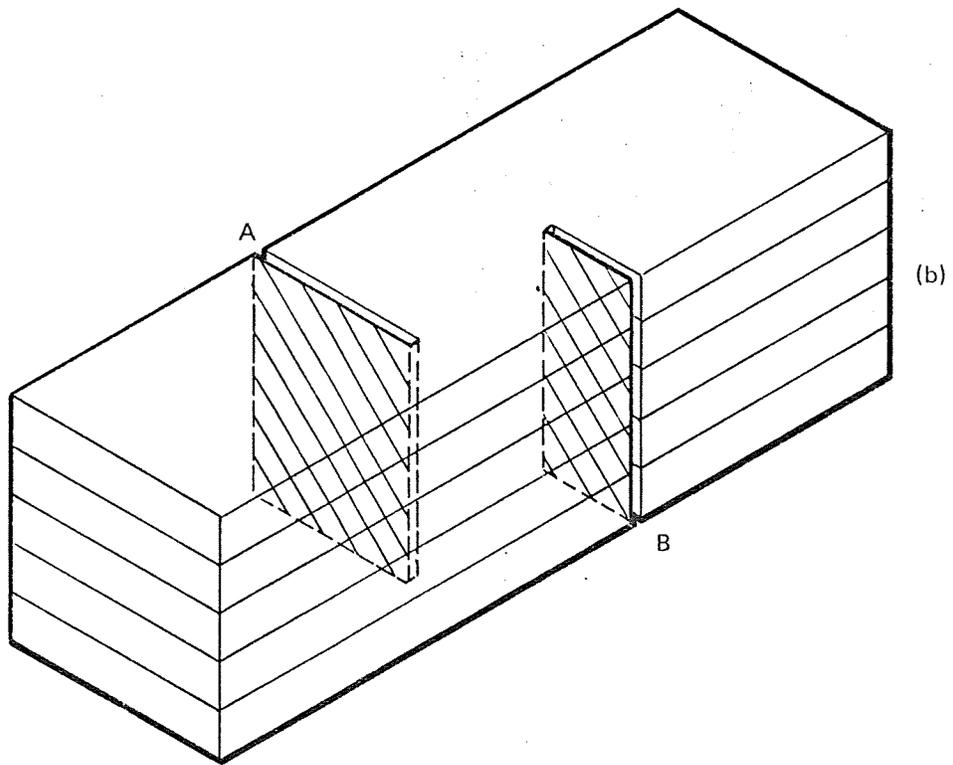
FIGURE 3-8
(a) SCHEMATIC OF ASTM THREE-POINT
    BEND SPECIMEN
(b) ORIENTATION OF SAW CUTS WITH RESPECT
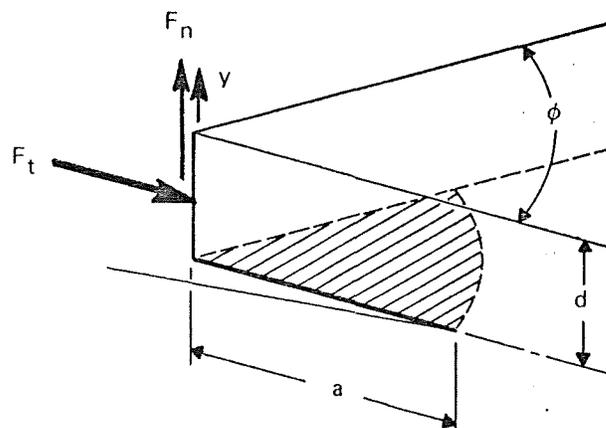   TO BEDDING PLANES FOR SPECIMENS A & B

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FIGURE 3-9
FORCES ON CHIP OF COAL BEING DETACHED

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4.0 SUMMARY OF CONCLUSIONS
AND RECOMMENDATIONS

4.1   Conclusions

4.1.1   Although metal cutting theories are well-developed, they do not adequately model the coal removal process due to coal's brittle nature and the maze of inherent flows.

4.1.2   Coal properties vary dramatically between various mines and geographic locations. Generalized coal properties are useful, however, since the mode of coal removal seems to be independent of the seam obtained.

4.1.3   Lab tests show that regardless of the pick shape (chisel type as used in Britain or conical point attack bits used in the United States), the mode of coal removal is one of continuous crushing and chipping, yielding specific energies of coal removal ranging from 1 J/g to 10 or more J/g.

4.1.4   Orientation of cutting tool with respect to the coal face and adjacent picks play an important role on the force and energy requirements of the mining operation, for example:

- Cutting force decreases as the rake angle increases.

- Mean specific energy decreases as the depth of cut increases.

- Mean specific energy generally decreases as the S/D increases to some optimum value.

4.1.5   Lab tests have shown encouraging results in predicting the energy range for actual in-mine continuous mining operation.

4.1.6   Evan's tensile breakage theory, which is somewhat elementary, reasonably predicts the cutting forces on chisel type picks.

4.1.7   The theory developed for continuous chipping and crushing, based on the tensile breakage theory and the surface energy

release rate, G, lead to results which show the decreased energy requirement as depth of cut is increased. Additionally, this theory predicts the range of energies expected in the mining operation.

4.1.8   Fracture mechanics considerations show the possibility of being useful in coal removal testing (i.e., pull out tests) and cutting operations.

4.2   <u>Recommendations</u>

4.2.1   Field measurements of cutter bit forces during actual mining operations should be accomplished to permit better correlation of all laboratory work done to date to the real environment under which coal must be cut.

4.2.2   Continuance of lab tests to show the fracture process. In order to give a more accurate description of the crushing and chipping action, the tests should be oriented to yield chip size shape as well as the quantity of crushed material.

4.2.3   A chip size analysis for the various modes of cutting would prove beneficial to understanding the crushing and chipping process.

4.2.4   Conduct multiple pick tests to investigate groove interaction and chip shape.

   Optimum S/D should be investigated to produce a minimum Esp for a given depth of cut.

4.2.5   Fracture toughness testing of coal samples would aid in the theoretical descriptions of the crushing and chipping process as well as seam testing.

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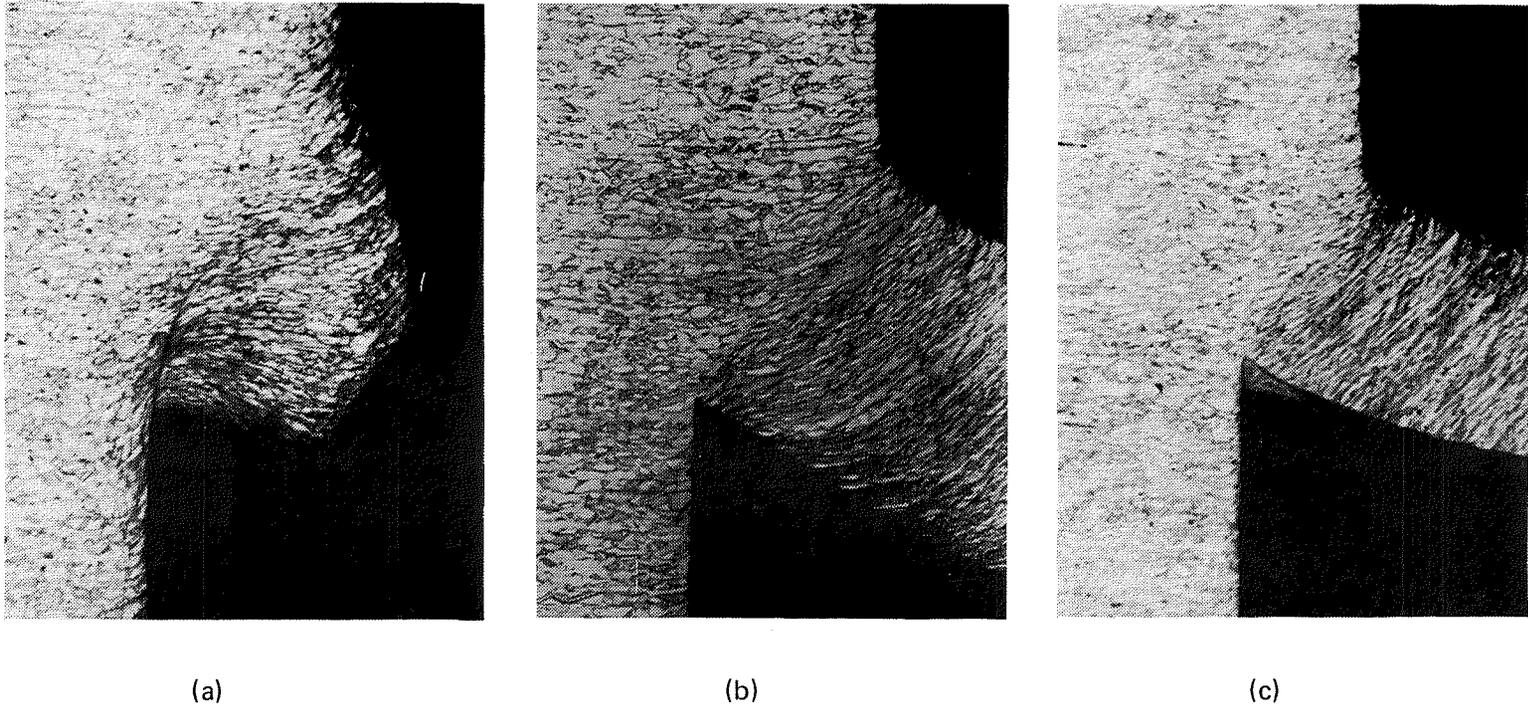
FIGURE 1-1

Photomicrographs of β brass being cut at 0.5 inches/min with no cutting fluid.

    (a) Discontinuous chip. Rake Angle 15°, Depth of cut 0.008 inches.

    (b) Continuous chip. Rake Angle 30°, Depth of cut 0.010 inches.

    (c) Continuous chip with built up edge. Rake Angle 15°, Depth of cut 0.007 inches.
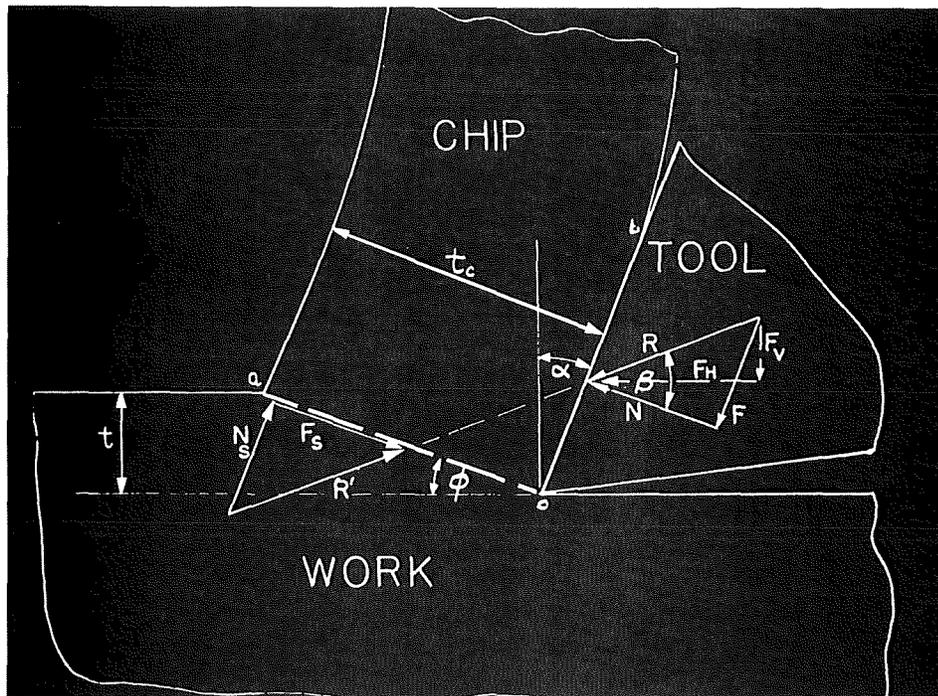
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FIGURE 1-2
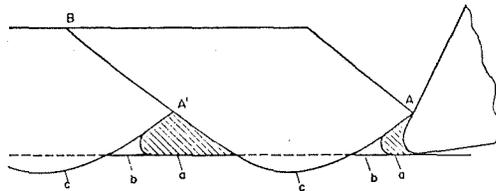Forces and Chip Geometry for Two-Dimensional Cutting.

FIGURE 1-3
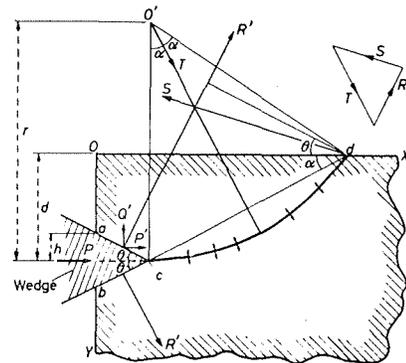An Illustration of the Failure Process
of Rock Cutting, Nishimatsu[17].

FIGURE 1-4
The Chipping Configuration
Studied by Evans[21].

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