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## **Design of an Underground Mine Layout**



**UNITED STATES DEPARTMENT OF THE INTERIOR**

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# **Design of an Underground Mine Layout**

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**UNITED STATES DEPARTMENT OF THE INTERIOR**

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# DESIGN OF AN UNDERGROUND MINE LAYOUT

by

William R. Sharp <sup>1</sup>

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## ABSTRACT

This Bureau of Mines report details a technique for aiding the engineer with the long-range design of an underground mine. The method is based upon the assumption that sufficient data are available to assign numeric values to some finite number of mineralized blocks; assignment of numerical values is problem dependent. By assigning appropriate weighting schemes, the algorithm developed in this report may be used to solve the optimal end-of-life underground mine layout, design least-cost electrical layout networks and efficient material handling networks, find least risk escape routes in the event of an accident, etc.

The method is based upon the assumption that sufficient cost and geologic data is available to build a detailed mineral block inventory of the mineral deposit. It is further assumed that the cost assigned to any minable unit is independent of the neighboring blocks. The basic approach relies upon concepts and ideas from the theory of networks.

To make the approach user oriented, a computer program is included and discussed. The computer code is written for an orthogonal, three-dimensional set of blocks. Computational costs and experience indicate that the program could be of great benefit in evaluating large complex mineral properties from the viewpoint of predesigning an underground mine layout.

## INTRODUCTION

The minerals industry in the United States is presently faced with the problem of meeting a rapidly increasing domestic and foreign demand for its mineral resources. This increased demand has added to the problems already plaguing the industry--increased land, labor, and material costs; increased foreign competition; more rigid Federal and State mining laws pertaining to health and safety; increased public awareness of air, stream, and land pollution; and increased public concern of mining practices and conditions within the mines. In addition, the industry must live with problems created by past

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<sup>1</sup>Mining engineer.

generations, such as abandoned strip mines, acid mine water drainage, and abandoned unmapped underground workings.

The "need" for a healthy, viable minerals industry within the United States has been stated many times over. In addition, it is desirable, if not a necessity, from the viewpoint of the consumer and the national economy that the cost of making available a mineral resource be minimal. The development and utilization of scientific mine planning will be an invaluable aid to the accomplishment of the design and operation of profitable, safe, healthy, and environmental-preserving underground mining operations.

Mine planning is usually divided into three areas: long range, short range, and production scheduling. A long-range plan defines the economic limit of mining at the termination of the deposit's mining life and serves as a guide for short-range plans.

In the process of winning a mineral resource, short-sighted goals can lead to costly development and possible loss of some resource to future exploitation. It is for this reason that this report will be primarily concerned with long-range planning.

A trend within the mineral industry, not unusual in other industries, is toward the development of large, complex physical plants in an effort to reduce unit costs. The trend is toward fewer mines employing more men. For example, in Colorado in 1960, there were 607 noncoal operating mines with a yearly employment of 5,220 men, compared with only 353 mines employing 6,660 men in 1970 as given in Colorado's yearly report by the Deputy Commissioner of Mines. This trend seems likely to continue.

Another factor that has had a profound effect in reducing the number of small operations has been stricter mining laws. For example, as a result of the Coal Mine Health and Safety Act of 1969,<sup>2</sup> it is stated that "In any coal mine opened after March 30, 1970, the entries used as intake and return air courses shall be separated from belt haulage entries...." This regulation has sharply increased mining costs for the small coal mine operator using belt haulage by forcing any egress into a coal property with a minimum of three interconnected parallel entries. In the past, the mine operator was able to operate with two entries, one for intake air and the other for exhaust air. The economic impact of this law has been significantly less for the large mine operator, who now usually operates with an excess of five parallel main entries.

With the trend towards larger, more complex mine plants comes the increased responsibility of the mining company to comply with the needs of society as a whole, thus increasing the needs for good scientific mine design.

Today, coal companies are looking at properties for exploitation that may contain multiple seams within a very complex geologic environment covering

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<sup>2</sup>Mandatory Safety Standards, Underground Coal Mines. Nov. 20, 1970, Federal Register, v. 35, No. 226, Sec. 75.326.

many square miles. Additional problems, such as land ownership, oil and/or gas wells intersecting the mining property, highways, surface buildings, powerlines, canals, water tables, and entrapped methane, present a mining company with many problems not encountered only a few years ago (3, 16).<sup>3</sup>

The need for sound mine planning within our complex sociological framework is well documented. The purpose of this report is to develop a tool for assisting the mine operator to efficiently evaluate alternative approaches to mine planning.

The mining engineer is faced with the need to respond quickly to modify a mine design to satisfy rapidly changing economical, ecological, and sociological constraints. Heretofore the mining engineer could be content with comparing only a few alternative mine designs. Today, he must explore many alternative plans constrained by factors unknown only a few years ago (16).

In terms of long-range planning, the design engineer must be alert to the changes in the overall mine plan resulting from changing land, material, and labor costs or the introduction of innovative mining concepts. Possibly the design engineer will be requested to make a quick evaluation as to whether or not a company should extend a lease on an unexploited piece of land. In terms of short-range planning, an overall mine plan is also extremely important to the engineer in charge of scheduling mine development and production to meet demand commitments. The end-of-life or ultimate mine layout is also of concern to the engineer in charge of designing efficient ventilation, haulage, drainage, and electrical networks.

The objective of this report will be to develop a method for defining an ultimate underground mine layout consistent with economic, safety, and conservation constraints. Knowing the ultimate mining configuration is vital to the efficient layout of the mine development, mine development is here defined as a network of primary accesses through which the bulk of the material is moved to the surface mine facilities from the mineral producing areas.

To achieve this objective, the following four criteria were used as guidelines in developing an algorithm for delineating the economic limits of a mineralized property:

- (1) Mine layout should be everywhere accessible to the surface.
- (2) Algorithm should be applicable to the general mineral-type deposit.
- (3) Algorithm should be capable of solving realistic size problems.
- (4) Technique should be user oriented.

Criteria 1, 3, and 4 above should be self-explanatory. What is intended in criterion 2 by the term "general mineral-type deposit" is that the

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<sup>3</sup>Underlined numbers in parentheses refer to items in the list of references preceding the appendixes.

distribution of economic worth may be freely dispersed throughout some three-dimensional rock mass. In other words, the distribution of mineral context need not be rigorously prescribed by some simplistic mathematical function. The problem of forecasting the distribution of mineralization within the earth's crust is recognized. The problems of delineating a mineral deposit are referred to in Becker and Hazen (1).

#### ACKNOWLEDGMENTS

The work described in this report was sponsored by the U.S. Bureau of Mines in partial fulfillment of the requirements for the degree of Master of Science at the Colorado School of Mines. I am also indebted to the many people at the Bureau of Mines in Denver who have contributed much to the detail and thought contained in this work. I am especially grateful to Dr. Thys B. Johnson, operations research analyst, U.S. Bureau of Mines, for his help in clearly defining the general problem area.

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#### GENERAL UNDERGROUND MINING PROBLEM

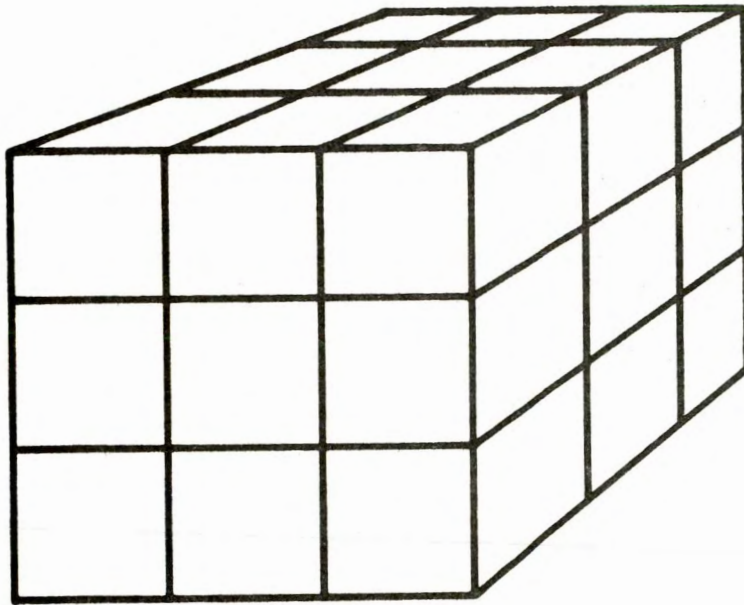
In view of the increasing complexity of the general mining problem, mining companies are now beginning to rely upon mathematical techniques and tools to predesign the layout of a mining operation. The following sections contain a brief review of literature, detail a means for expressing the general ultimate underground mine layout as a network, and define a general formulation of the problem.

#### Literature Review

Perhaps one of the best single sources of operations research techniques applied to mining problems is a recent book published in Hungary (17). This book describes some techniques for such problems as the location of a shaft, determination of level spacing in a dipping deposit, and the "optimal" location of crosscuts. This book does give a wide variety of simple applications for a wide variety of facility location type problems. The term "optimal" used here and discussed later in this paper will be used in a strict mathematical sense.

In the area of ultimate underground mine layout there has been a notable lack of published papers. The problem of designing an ultimate open pit has generated much more interest in the past several years than has the subsurface problem. The ultimate open pit problem has been approached by Lerch and Grossmann (14) using a dynamic programming technique, Johnson (12) employing both a dynamic programming technique and a network-flow model, and Ganthier and Gray (9) using a heuristic approach. An interesting and computationally practical approach, based upon the dynamic programming of Lerch and Grossmann, is given by Johnson and Sharp (13). All of these approaches use what is referred

to as the "block concept," which is simply a method for subdividing a mineral deposit into discrete minable units formed by a set of more or less orthogonally intersecting planes. Discrete block units appear to be a reasonable basis for a subsurface excavation model for the following reasons:



*A*

(1) Block concept is accepted and used within the minerals industry for ore reserve and delineation studies.

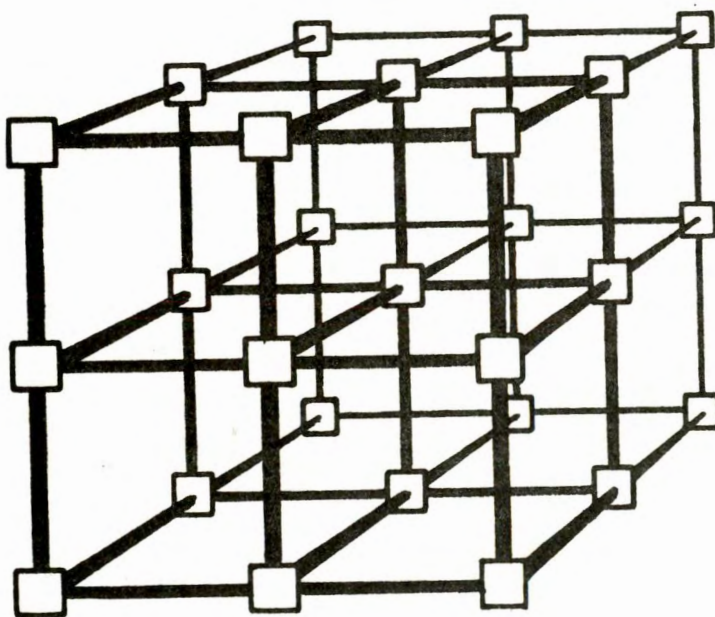
(2) Blocks can be made to conform with most mining methods used within the industry today.

(3) Block inventory is easily stored in a digital computer.

With respect to the ultimate pit problem, the blocks themselves help to describe a pit wall. In comparison, the subsurface problem need only imply connectivity of blocks; that is, existence of a passage from any set of minable blocks to all others in the set to be mined together with a passage to the surface.

#### Definitions and Sample Problem

The ultimate underground problem becomes more amenable to analysis if a weight is assigned to an individual block equivalent to its worth without regard for its neighboring blocks. In terms of utility, the block worth might represent the least-cost or potential profit that a mining concern



*B*

FIGURE 1. - Graphical representation (*B*) of an orthogonal set of blocks (*A*).

would reap given the fact that any particular block is physically mined. Dependent upon the problem, all block weights might represent a true cost (negative weight), for example, a tunneling operation. In a tunneling problem the contractor or mining company is only concerned with finding a least-cost passage between two points. Therefore, in terms of the block model, it would be a connected sequence of blocks between two points in which the individual blocks would be equivalent to a small segment of the tunnel.

Figure 1-A gives a graphic illustration of how the block might be interpreted in three-dimensional space where the length, width, and height of each block would be determined by the user. Figure 1-B is a graphical illustration of the set of blocks in figure 1-A above. Each block has been pulled apart,

figure 1-B, but remains connected to its adjacent neighbors by a line. For purposes of discussion, the lines will be referred to as arcs and the squares (dots or circles) as nodes. These terms are consistent with those used in network theory.

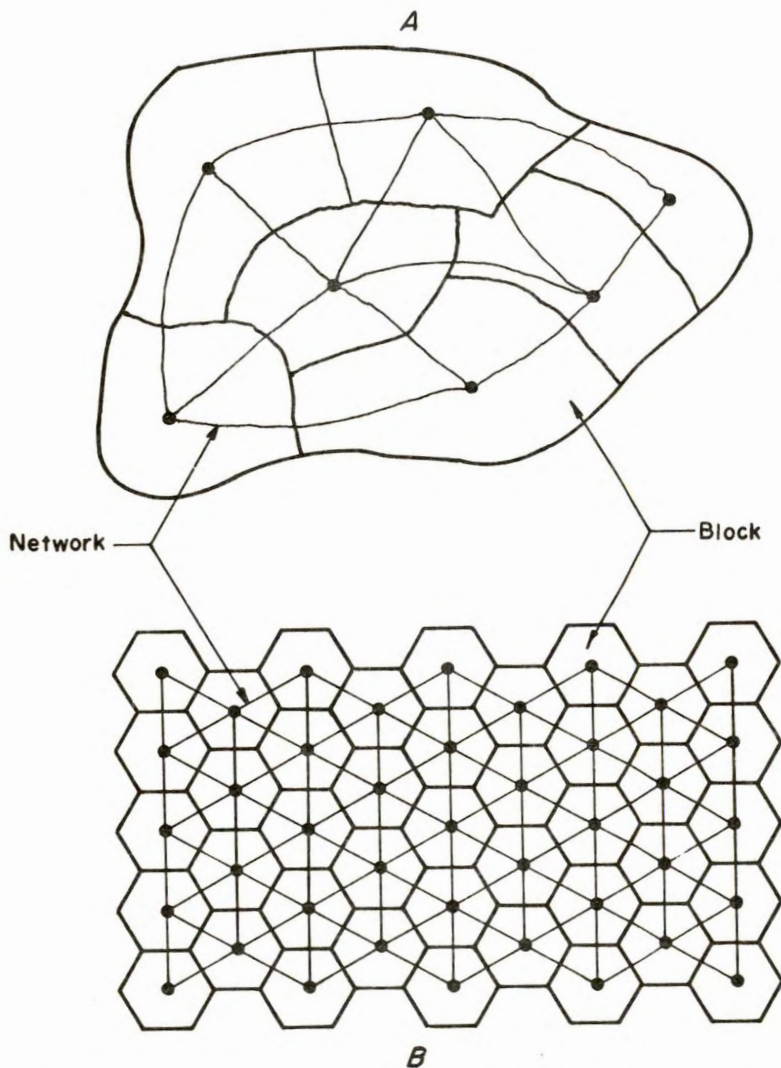


FIGURE 2. - Block and graphic illustration for (A) general two-dimensional block configuration and (B) systematic two-dimensional hexagonal block.

A block need not be defined as a cube. Figure 2 illustrates two different examples of a basic two-dimensional block configuration. Of interest is what may be represented by these block configurations again as a graph--similar to what was done in figure 1. The nodes represent the blocks and an arc indicates that two nodes are adjacent geometrically to one another.

Figure 3-A illustrates a two-dimensional graphic representation of a set of blocks. A value or weight which implies the worth or cost of mining that particular block has been assigned to each node (block). The path given in figure 3-A as a heavy line is the least-cost path connecting nodes A, B, and C. The problem of finding a connected subset of maximum total weight from a

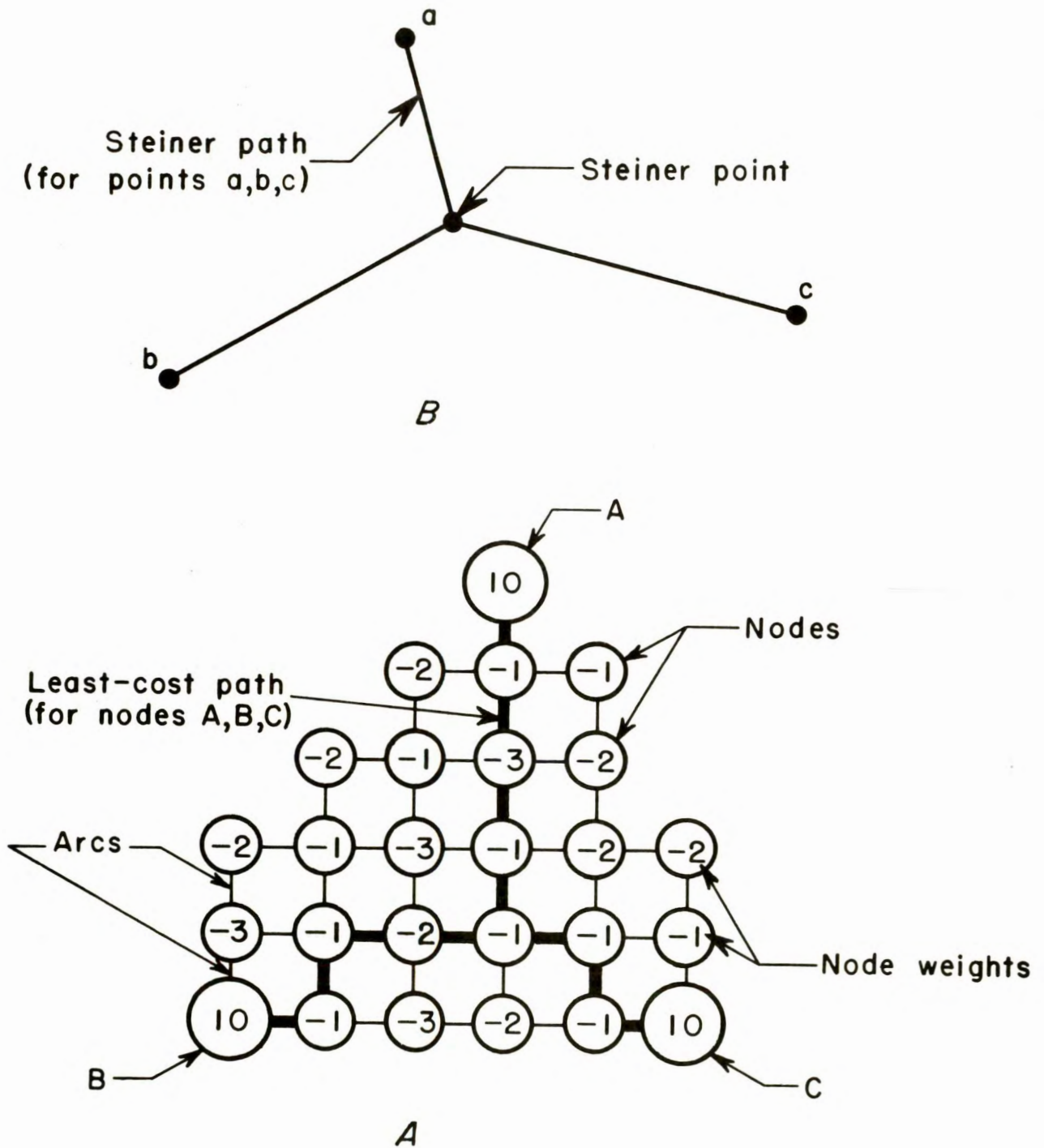


FIGURE 3. - Graphic (A) and geometric (B) illustration of the Steiner problem.

network is an often-pursued problem in network theory; however, there is no standard algorithm available to solve the problem. Approaching the mining problem in terms of a network gives flexibility of using any combination of

block configurations and ease of problem expression and manipulation with existing computational techniques.

Of particular interest in the mining problem is that there may exist a node belonging to the set of negatively weighted nodes, which, if forced into the solution set, will yield a better solution by acting as a common junction between three or more positively weighted node groups. Because this node is similar in many respects to a Steiner point defined in the classical Steiner problem (10), this type node will be referred to as a Steiner node. In brief, the Steiner problem may be stated as: Find the path of minimal geometric length connecting  $m$  points in a plane. Figure 3-B illustrates a least-length or Steiner path for three points in a plane. Figure 3-A is analogous to the general Steiner problem defined on a plane. The general Steiner problem was not pursued because (1) generally, an ore zone may be difficult and misleading to be represented as a geometric point, (2) generally, ore deposits do not occur in planes, and (3) Cockayne (4) states that available computer codes are limited to problems of 20 to 30 points.

Some additional terms are needed to clearly define the mining problem totally in terms of a network (7-8). A graph  $G = G(V, \Gamma)$  may be simply defined as consisting of a set of nodes,  $V = \{v_1, v_2, \dots, v_m\}$ , and a set of arcs,  $\Gamma = \{b_1, b_2, \dots, b_n\}$ . In addition, it is sufficient to define a set of weights,  $\{w(v_i)\}$ , such that  $w(v_1) \dots w(v_g)$  are  $\leq 0$  and  $w(v_{g+1}) \dots w(v_m) > 0$ . An arc,  $b_i$ , denotes a line segment connecting some node,  $v_k$ , to another node,  $v_j$ , or more simply as  $(k, j)$ . In terms of the mining problem, there is no need to describe a directed graph, therefore,  $(i, j) = (j, i)$ . Any two nodes will be called adjacent if they are joined by a single arc. A path,  $\pi_i^j$ , may then be defined as a sequence of ordered nodes and arcs as  $(v_1, (i, s), v_s, \dots, (t, j), v_j)$ . A path as used in this paper does not contain any cycles; that is, for the sequence of nodes in the path, there is no repetition of any node. A connected graph is defined as a graph where at least one path exists from any arbitrary set of nodes to all others in the graph. Finally, a tree is defined as a connected graph having no cycles. The nearest neighbor to some node,  $i$ , as used in this report, will be a single node,  $j$ , from a set of possible nodes having the least-cost path from node  $i$ .

#### Problem Formulation

The general subsurface mining problem defined on a set of blocks may be simply stated as find

$$\begin{aligned} \max \sum_i w_i X_i, \\ \text{such that } X_i - \sum_{k,j} \pi_{kj} X_j \leq 0, \end{aligned}$$

where  $\prod_j X_j$  represents the product of a sequence of geometrically adjacent blocks leading from block  $i$  to the surface and  $\sum_k$  represents the sum of all possible sequences,

and  $X_i = \begin{cases} 1 & \text{if block } i \text{ is mined} \\ 0 & \text{if block } i \text{ is not mined} \end{cases}$

and  $w_i$  = weight assigned to block  $i$ .

Mathematically this set of equations defines the general subsurface problem of concern in this paper; however, practically it would be extremely difficult to formulate or solve a problem in this form--even for small problems.

Considerable time and effort was taken to model this relationship by some standard mathematical programming techniques. Attempts to formulate the problem as both a linear and integer problem were not truly successful from the viewpoint of the general problem just defined. Difficulties arise in defining a set of linear constraints. It is a well-known fact that for practical-size problems with nonlinear constraints finding computationally efficient computer codes is a problem that must be considered. The need for an integer solution plus the need for comparing alternative block sequences make this problem extremely difficult to formulate in general terms. Computational efficiency of known integer codes decreases at a rate significantly faster than a comparable increase in the size of the problem.

Because no efficient mathematical programming technique was found, a network approach was pursued. In terms of a network, the ultimate underground mining plan may be simply stated as: Find a connected subgraph, call it  $G_0$ , such that the weights belonging to  $G_0$  is a maximum. It is also a generally recognized fact that there are many computationally efficient computer codes available to solve large network related problems; for example, the shortest route, the maximum spanning tree, and the maximum flow.

#### SOLUTION ALGORITHM

An algorithm for finding the ultimate subsurface mining configuration is given in this section and followed by a detailed description of the least-cost path algorithm which is a fundamental part of the solution algorithm. The least-cost path problem is comparable to the shortest route problem. Much work has been done in the development of algorithms for the shortest route problem. The algorithm for the mining problem is in two parts, phases 1 and 2.

Phase 1 of the solution algorithm begins by collecting all of the positively weighted nodes (blocks), which are adjacent into a single node having as its weight the sum of the weights of all its individual members. If no positive nodes exist or if only one is found, the algorithm terminates. If two or more positive nodes are found, phase 1 then proceeds by taking the maximum from the newly found disjoint positive nodes and finding its nearest positive neighbor using a least-cost path algorithm. If the weight of the two disjoint positively weighted nodes plus the least-cost path is strictly greater than the starting node weight, the algorithm proceeds by finding the

least-cost path to a positive neighbor from the combined subgraph consisting of the two positive nodes plus the least-cost path. The phase 1 algorithm continues until no additional nodes from the set of positive nodes can be brought into a connected set of nodes. Phase 1 will give a good solution; however, in general, it may not be optimal. The necessary conditions for a mathematically optimal solution will be discussed in detail under "Algorithm Structure." For phase 1, a computer code was developed and is discussed under "Computational Experience."

An upper bound on the value of the ultimate mine layout would be the sum of all the positive nodes and a lower bound would be zero, which implies the absence of any positive blocks. These facts could be useful in deciding when to terminate the algorithm. For example, if the upper bound is not sufficiently large to meet corporate objectives, the project could be abandoned immediately without further study.

Phase 2 describes a procedure by which a user may try to improve upon the solution resulting from phase 1 through the introduction of Steiner nodes. A Steiner node may be simply thought of as a node, which, if mined, would result in a negative return; however, if by mining that particular node it is possible to use this node as a common intersection of development entries (drifts) for three or more disjoint potential ore zones, then the overall mining costs would be reduced.

The solution algorithm assumes the problem to be defined as a general network as previously described. The mining problem was defined in terms of a general network so that a general algorithm could be found that will work for any number of different block configurations.

### General Algorithm

In this section the algorithm used to solve the general ultimate subsurface mine problem is amplified. For definition of terms, see appendix A.

Phase 1:

Step 0 Set  $AMAX = 0$ , set  $ATEMP = 0$ , set  $S = 0$ .

Step 1 Collect all vertices from the set  $\{(v_{g+1}) \dots (v_m)\}$ , which are adjacent, into  $P$  disjoint sets and form the set  $\{(\bar{v}_j) \mid j = 1 \dots P\}$ . Set  $T_0 = \Omega$ . If  $P = 0$ , stop; the set  $\{(\bar{v}_j) \text{ for all } j\}$  empty. If  $P = 1$ , stop; the optimal solution is  $w(\bar{v}_1)$ . Otherwise continue. Set  $L_j = 0$  for  $j = 1 \dots P$ .

Step 2 Set  $TEMP = 0$ , set  $i = 1$ .

Step 3 If  $L_j = \infty$ , for all  $j$ , go to phase 2. Find  $q$  such that  $w(\bar{v}_q) = \max \{w(\bar{v}_j) \mid L_j = 0, j = 1 \dots P\} = TEMP$ . Set  $L_q = \infty$ ,  $T_1 = \bar{v}_q$  and  $T = \Omega$ .

- Step 4 Find least-cost path  $l(\pi_s^j)$  from  $T_i + \bar{T}$  to nearest positive disjoint neighbor  $k$  such that node  $s$  is adjacent to tree,  $T_i + \bar{T}$ , and node  $j$  is adjacent to node  $k$ . If  $|l(\pi_s^j)| \geq \text{TEMP}$ , go to 7.
- Step 5 If  $\text{TEMP} + l(\pi_s^j) + w(\bar{v}_k) > \text{TEMP}$ , set  $\text{TEMP} = \text{TEMP} + l(\pi_s^j) + w(\bar{v}_k)$  and set  $L_k = \infty$  and go to step 6. Otherwise set  $\bar{T} = \bar{T} + \pi_s^k$  and go back to step 4.
- Step 6 Increment  $i$ ;  $i = i + 1$ . Set  $T_i = T_{i-1} + \pi_s^j + v_k$ . If  $L_j = \infty$  for all  $j$ , go to step 7. Otherwise set  $\bar{T} = \Omega$  and go back to step 4.
- Step 7 If  $\text{TEMP} \geq \text{ATEMP}$ , set  $\text{ATEMP} = \text{TEMP}$  and set  $T_0 = T_i$ . Go to step 2.

Phase 2 allows the user to force any node not present in the solution found in phase 1 into the solution set.

Phase 2:

- Step 1 If  $S = 0$ , go to step 2. Otherwise set  $\text{ATEMP} = \text{ATEMP} - M * S$ . If  $\text{ATEMP} \geq \text{AMAX}$ , set  $\text{AMAX} = \text{ATEMP}$  and  $T = T_0$ .
- Step 2 Option to stop at this iteration.
- Step 3 Out of the set of nodes, such that  $w(v_t) \leq 0$ , select a set of Steiner nodes  $t = 1 \dots S$ ,  $S \leq P - 2$ . Set  $w(v_t) = w(v_t) + M$ ,  $t = 1 \dots S$ , and add  $v_t$  to the set of positive nodes. Go to phase 1, step 2.

Using good perceptual judgment at this point can speed convergence of a realistic problem by helping to reduce unnecessary computational time which could be introduced by an enumerative selection of Steiner nodes.

#### Least-Cost Path Algorithm

A detailed labeling algorithm is given for finding the least-cost path from any node,  $q$ , to every other node,  $t$ , in a network, given that all node weights are negative and weights on the arcs are all zero. The algorithm is essentially a dynamic programming approach very similar to that discussed by Dreyfus (6). The major difference is the fact that the weights are defined exclusively on the nodes as opposed to the arcs. This, in fact, simplifies the general approach suggested by Dreyfus. In terms of the general algorithm, a set of nodes  $\{q\}$  would represent a set of nodes in a tree.

Algorithm:

- Step 1 Label a set of source nodes  $\{q\}$  as  $L(-,0)$ .
- Step 2 Find an unlabeled node  $t$  and a labeled node  $s$  such that  $l(\pi_q^t) = \min \{l(\pi_q^s) + w(v_t)\}$  for all nodes  $s$  and  $t$  such that node  $t$  is unlabeled and adjacent to a labeled node.
- Step 3 Label node  $t$  as  $L(s, l(\pi_q^t))$ .
- Step 4 Does an unlabeled node exist? If yes, go to step 2. Otherwise, stop.

NOTE:  $q$  and  $s$  are indexes for labeled nodes and  $t$  is for the unlabeled. Also, for the label,  $L(s, l(\pi_q^t))$ ,  $s$  denotes the adjacent node from which node  $t$  is labeled and  $l(\pi_q^t)$  denotes the best path from node  $t$  to node  $q$ .

#### DISCUSSION OF SOLUTION ALGORITHM

##### Sample Problem

The following problems were constructed to illustrate how the algorithm works and under what conditions the algorithm can lead to nonoptimal solutions. The illustrations are overly simplistic and in most cases the optimal solution is obvious. In a realistic problem, however, the optimal solution will not be obvious.

Figure 4-A gives a network equivalent to a mining problem having three disjoint ore pods (A,B,C) which would yield a payoff of 10 units each if they could be mined. In addition, node G is adjacent to node C and yields a value of one unit. For example, if node G were exposed to the surface, then nodes C and G could be mined at a combined payoff of 11 units because they are adjacent. In the winning of nodes A and/or B, it is possible to proceed from C by three different paths--all of which contain negatively weighted nodes. Obviously then, the user wishes to pick the path that yields the least-cost for the potential gain. For figure 4-A the optimal solution is, obviously, equivalent to the node blocks A through G having a total weight of 25 units.

For the problem in figure 4-A the algorithm would proceed as follows beginning at phase 1: Step 0, set  $AMAX$ ,  $A_{TEMP}$ , and  $S = 0$  and move to step 1. For step 1 combine the node weights C and G and form a single node. Call it node C. Figure 4-B will then become equivalent to the network in figure 4-A where no two adjacent positively weighted nodes exist.  $P$  is equal to 3 or, in this problem, let  $j = A, B,$  and  $C$ . Set  $L_A, L_B, L_C = 0$ ,  $i = 1$ , and  $TEMP = 0$  in step 2. In step 3,  $q = C$  because the maximum of  $\{w(v_A), w(v_B), w(v_C)\}$  is  $w(v_C)$ ; therefore, set  $TEMP = 11$ . Now set  $L_C = \infty$ . In step 4 the least-cost path from node C to either node A or B would result in choosing node E, which yields a net of 18 units. The nearest neighbor,  $k$ , then is node B. Since  $|l(\pi_E^E)| = 3$ , which is less than 11. Go to step 5. In step 5 because  $18 > TEMP$  (which was set = 11 in step 3), set  $L_B = \infty$ , and go to step 6. Increment  $i$  ( $i = 1 + 1 = 2$ ) and set  $T_2 = \{C, E, B,\}$  and go to step 4. The least-cost path from  $T_2$  to the remaining disjoint positively weighted node

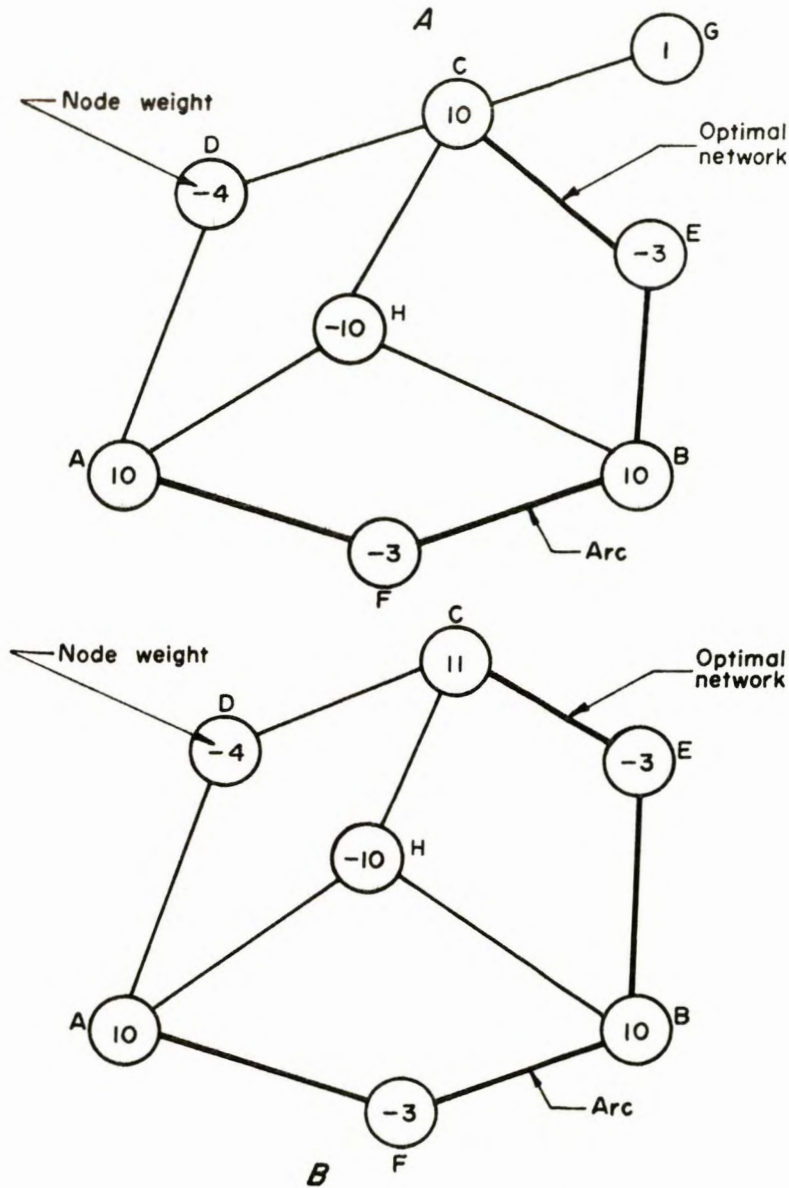


FIGURE 4. - Graphic illustration for problem (A) and (B).

ing a weight of 25; however, for this problem, the network  $\{H, A, B, C\}$  yields 26 units, which is  $>25$ . By adding a large positive weight,  $M$ , to the node weight of  $H$  ( $-5 + M$ ) the phase 1 algorithm would yield  $26 + M$ , less  $M$  is 26, which is  $>25$ . Thus the optimal graph  $G_0 = \{H, A, B, C\}$  has a total weight of 26. Figure 5-B illustrates a situation where the positive disjoint nodes are not of sufficient weight to allow the algorithm to yield an optimal answer without forcing the Steiner node into the solution. Of interest is that for the phase 1 algorithm not to yield an optimal solution, the optimal graph  $G_0$  must contain a Steiner node. Detection of Steiner nodes is left to the user in phase 2. However, the author feels that in a real problem it should not be

( $k = A$ ) would be the node  $\{A\}$ , which yields a net of  $18 + (10 - 3) = 25$ , which is greater than TEMP (18). Therefore in step 5 let  $L_A = \infty$  and proceed to step 6, to step 7, to step 2 and finally to step 3 where  $L_A, L_B, L_C = \infty$ , which terminates phase 1 part of the algorithm. If it were desirable to force the solution through the Steiner node  $H$  (phase 2), a large positive value,  $M$ , would be added to the node weight of  $H$ , which in this instance would be  $-10 + M$ . Returning to step 1, phase 1, and passing through the phase 1 algorithm again will yield a net of  $21 + M$ . Subtracting  $M$  from  $21 + M$  gives an answer of 21, which would be  $<25$ . Thus for this problem, figure 4-B, because there is only one possible Steiner node, the solution,  $\{C, E, B, F, A\}$ , is optimal.

To illustrate the two possible alternative kinds of things that can cause the phase 1 to yield a mathematically nonoptimal answer, refer to figure 5. Without the introduction of a Steiner node, the problem in figure 5-A would result in a solution, similar to that found in figure 4-B, yield-

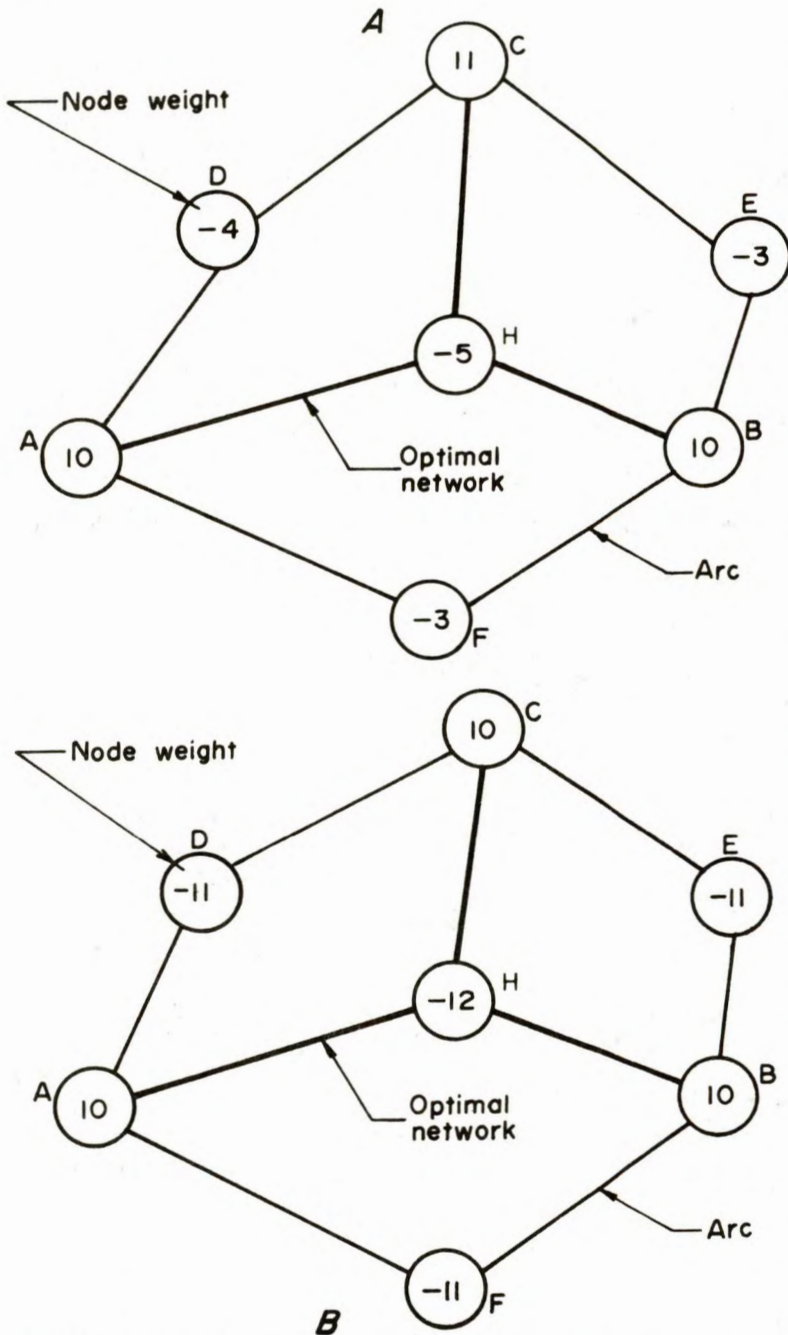


FIGURE 5. - Graphic illustration for problem (A) and (B).

too difficult to locate the most obvious location of Steiner nodes. Mathematically, then, the described algorithm will yield an optimal given that no Steiner-type nodes need be introduced.

Algorithm Structure

Given that we have an optimal graph,  $G_0$ , to the ultimate mining layout previously stated on a network, there are several characteristics that the solution will have. They are as follows:

Statement 1--All adjacent positive nodes  $i$  and  $j$  will either be completely within the optimal solution or completely outside.

Statement 2--Considering only the positive disjoint nodes,  $\bar{v}_i$ , such that  $i=1 \dots P$  and an absence of Steiner nodes, the optimal path connecting these  $P$  disjoint nodes must be a maximum spanning tree.

Statement 3--There will be, at most,  $P-2$  Steiner nodes for a problem with  $P$  positive disjoint nodes  $\bar{v}_i$ .

Statement 1 may easily be shown to be true. If we assume that two adjacent positive weighted nodes are not both in the optimal

solution, the solution is obviously not optimal because a better solution could be found by simply including the adjacent node not presently in the solution set.

The second statement implies that the optimal solution, given no Steiner nodes, is a maximal spanning tree. First, the optimal must be a tree because

if it were not, at least one node (and its incident arcs) having a negative weight could be dropped without destroying the connectivity of the nodes in the set  $\{\bar{v}_1\}$ , thus a better solution is possible. A spanning tree is here defined as the tree consisting of only the negatively weighted nodes connecting all of the nodes belonging to the set  $\{\bar{v}_1\}$ . Secondly, given an optimal spanning tree,  $G_0$ , assume a path not found in the optimal tree between some pair of nodes in the set  $\{\bar{v}_1\}$ , which is better than some path in  $G_0$ . This assumed path together with some set of arcs and nodes in  $G_0$  will form a cycle. The length of the assumed path must be less negative than those paths connecting paired nodes in the set  $\{\bar{v}_1\}$  if it is to lead to an improvement. Because this path is not in  $G_0$ , it must be strictly greater than the other possible paths. Therefore, by contradiction of the assumption,  $G_0$  must be a maximal spanning tree.

Repeated usage of the least-cost algorithm constructs a stage-wise spanning tree for the set of disjoint positively weighted nodes. However, it is possible that the general algorithm will construct a forest (more than one tree) in which case the maximum graph,  $G_0$ , will occur within some subset of  $\bar{v}_1$ -type nodes. In other words, the general algorithm terminates whenever the spanning tree for some subset results in a negative contribution. In this situation the algorithm simply picks the maximum from the subset of spanning trees.

In terms of the general problem, the algorithm itself may introduce Steiner nodes. Therefore, statement 2 is not a strong statement. The usefulness of this statement--given that all of the Steiner type nodes can be defined--is that the algorithm herein (phase 1) will yield an optimal solution.

The third statement can also be easily shown to be true because the number of arcs incident to Steiner nodes must be at least three. Any more than three will reduce the number of possible Steiner nodes for a fixed number of  $P$  disjoint positively weighted nodes. The total number of Steiner nodes  $S$  must be strictly less than  $S^*$ , which will be here defined as the maximum number for a fixed number  $P$  of positively weighted nodes. Therefore, using the basic properties of a tree the total number of arcs may be given as  $\frac{3S^* + P}{2}$ , which is equivalent to  $S^* + P - 1$  nodes. Setting these equal to each other and solving for  $S^*$  in terms of  $P$  yields  $S^* = P - 2$ ; therefore, in general the number of possible Steiner nodes  $S$  in any problem must be less than or equal  $P - 2$ . This completes the proof.

The least-cost path algorithm, upon which the ultimate underground problem is based, is similar to the labeling algorithm first described by Dijkstra, Whiting, and Hillier as quoted by Dreyfus (6). The primary difference between what is written by Dreyfus and least-cost path algorithm described herein is how the weights are defined. For the mining problem all weights are defined on the nodes as opposed to the arcs for the standard shortest route algorithm. In addition, Dreyfus states the shortest route problem is a minimization technique compared with a maximization for the mining problem. He, also, defines positively weighted arcs (it is sufficient to have no negative cycles) as essentially a set of negatively weighted nodes in the mining problem.

The labeling algorithm, described by Dreyfus, works with two mutually exclusive and totally exhaustive sets of nodes, either permanently labeled or temporarily labeled nodes. At each iteration of the labeling algorithm, one node is transferred to the permanently labeled set and the algorithm terminates when there are no more nodes left in the temporary set. The algorithm thus constructs a maximal spanning tree.

Proof of the least-cost labeling algorithm, as used in this paper, may be made by induction. It is necessary to remember that the least-cost algorithm is defined completely on a set of negatively weighted nodes. At any stage in the least-cost algorithm the nodes may be classified into two sets--one containing the labeled set of nodes and another containing all the unlabeled sets

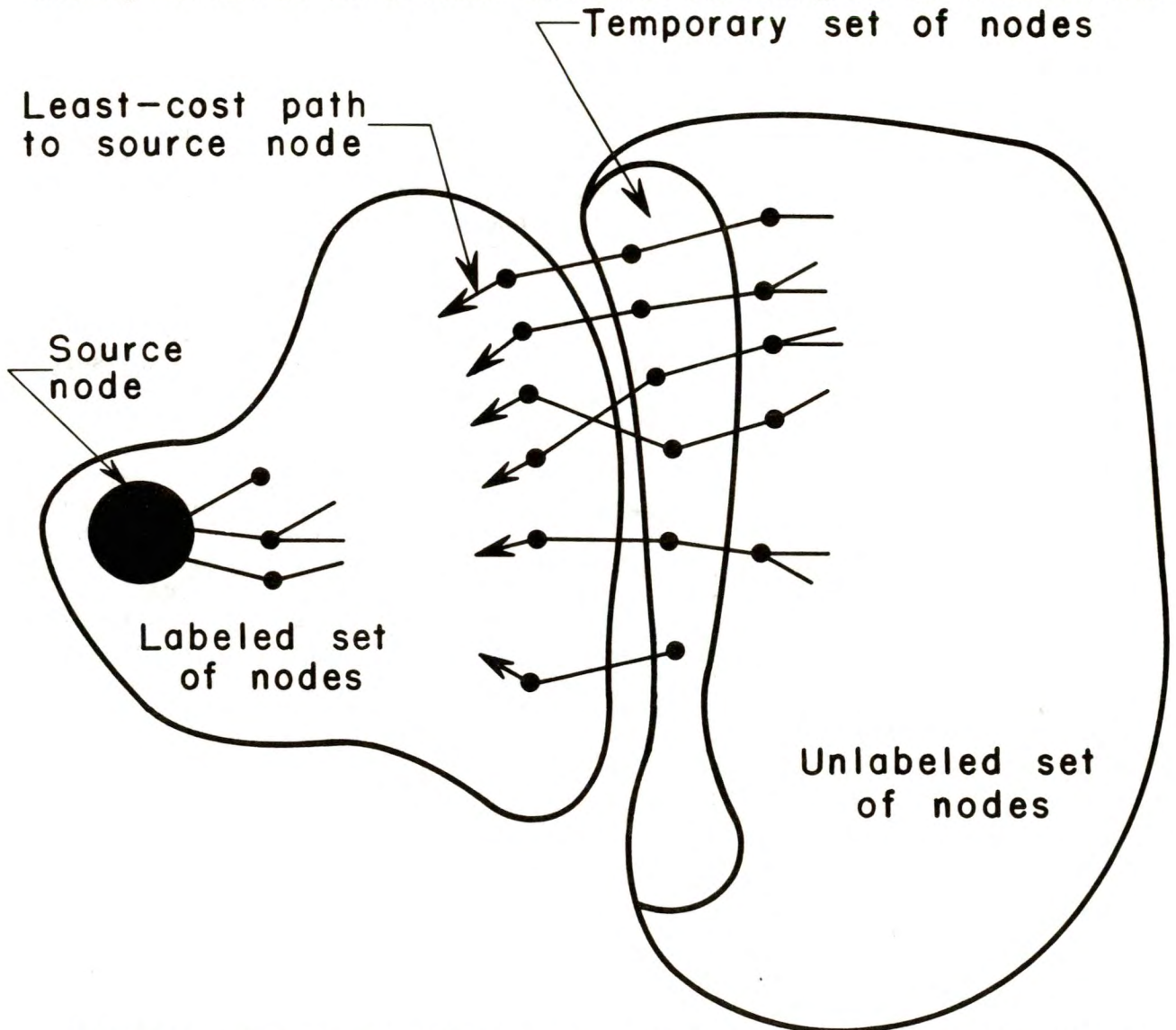


FIGURE 6. - Illustration of proof of least-cost path algorithm for only negatively weighted nodes.

of nodes (fig. 6). Labeled nodes are those for which a least-cost has been defined to a source node. For all those nodes in the unlabeled set adjacent to the labeled set, it is possible to find the least-cost distance to the source node such that all nodes in the least-cost path will be in the labeled set except one which will be in the unlabeled set. This distance is only temporary. Obviously, then the node in the unlabeled set which has the minimum temporary distance may be transferred to the labeled set. For if it is assumed that there exists a better less costly path, it would have to exist as a path in the unlabeled set. Because all of the nodes are negatively weighted, this path does not exist.

## IMPLEMENTATION

### Application to a Coal Property

Setting up a problem to be solved using this basic approach demands a certain amount of innovation on behalf of the user. For example, a user might wish to allocate to a block having length, width, and depth a portion of the unmined block for ground support. In terms of the network equivalent, this will be represented by a node weight. In the event of an uneconomic block (negative weight), the user might choose to consider only the cost of driving development entries through the block. On the other hand, a user might wish to define a block as a minable unit equivalent to net worth or a mine support unit and assign a large negative cost. The block weight need not be a cost; it could be the risk of roof fall in a particular block and the design engineer wishes to find the best development entry layout between two or more points having the least chance of a roof fall.

An illustrative problem is given in figure 7 for a hypothetical coal mine. For this illustration the problem is contained entirely within the coal seam for clarity of presentation; however, a three-dimensional model might be much more realistic. The square is a realistic block configuration, as a coal seam is normally mined consistent with a rectilinear movement. For a coal seam the coordinate directions would, more than likely, be structured so that they coincide with butts and cleats (planes of weakness) in the coal seam, but not necessarily. This would be dependent upon ground control problems. The block size has been taken equal to 800 by 800 feet with block weight equivalent to the profit or loss to the mining company. Block weights might be entirely different for another company. A block in which the net worth is less than zero for this problem indicates that as a minimum requirement, an access (set of entries) be driven through the block. This assumes that the cost of mining in either direction is the same, whereas, in reality this might not be precisely true.

If there does not exist a natural access to the surface, as is the situation for many mineral deposits, it would be necessary to assign some subset of surface blocks a large positive weight. This would force whatever material that could be economically mined to be connected to the surface. In addition, if it were desirable for some reason to force the mine through a particular set of blocks, this could be accomplished by adding an artificial block weight to those blocks, which would be subtracted later. To force access to the

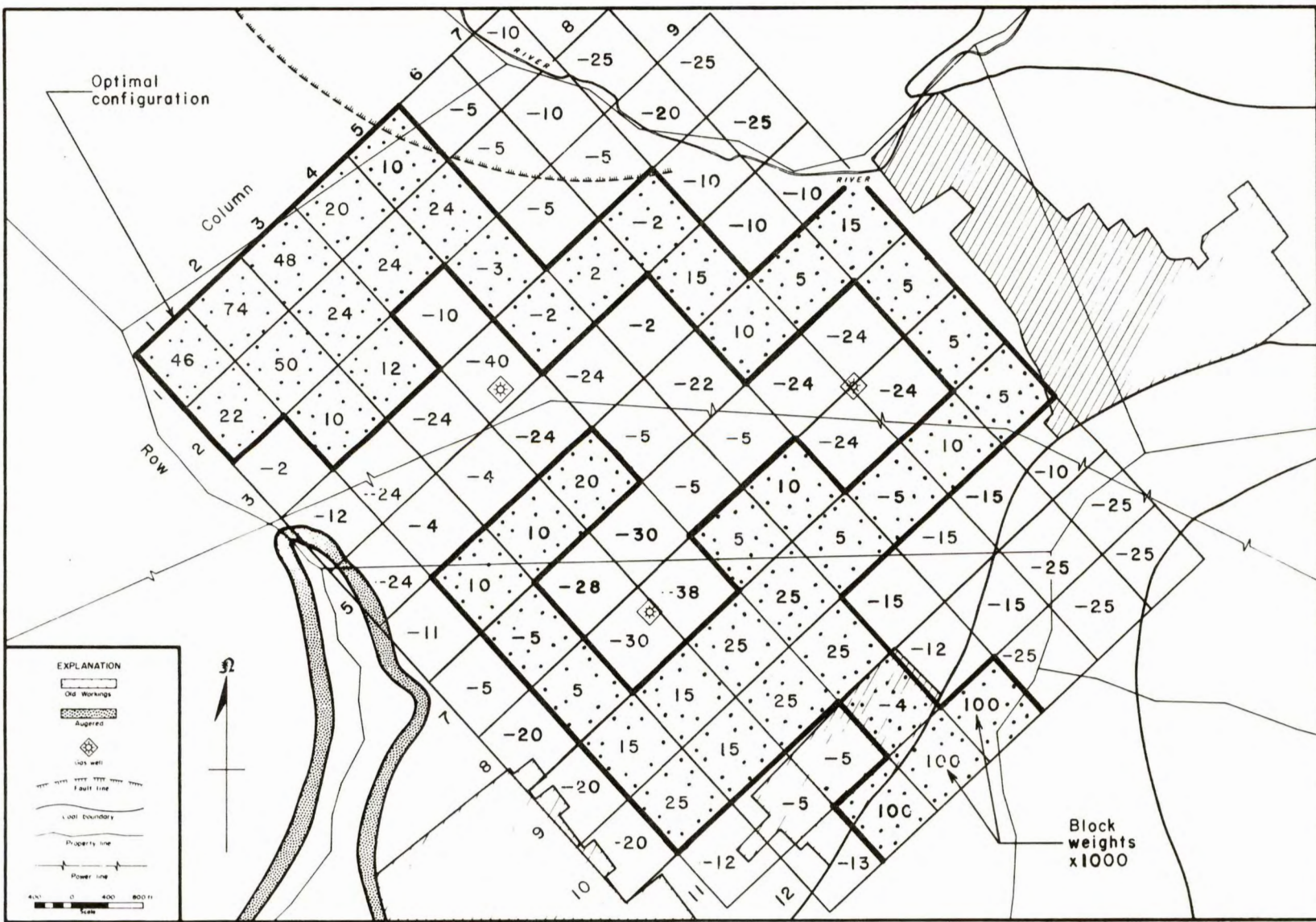


FIGURE 7. - Two-dimensional application for a coal property.

surface in this problem, blocks (columns 4 through 6 in row 12) were given an artificial weight equal to \$100,000 each. From a practical viewpoint these blocks might be considered as potential sites of a mine plant or mine portal. For this problem three oil wells are located in the coal seam of interest. The cost of mining through these blocks considered such costs as plugging the wells or the uncertainty of the well location within the block. The Coal Mine Safety Act of 1969<sup>4</sup> specifies that a barrier of 300 feet must be maintained around an active or inactive oil or gas well. This fact is reflected in the weights of the blocks within the neighborhood of oil wells. The physical and chemical properties of the coal, ownership, nearness to powerlines, location of old mine workings, chemical and physical properties of the coal, etc., all had a bearing on the assigned block weight. Results of this particular problem are given in appendix B.

Of interest in this particular problem is that the results would not be consistent with one's intuitive concepts of mine layout. However, by strategically reassigning block weights and repetitive use of algorithm herein described, the user could study many alternative plans at a minimal cost. Varying the block size would allow the user to compare different methods of mining.

#### Computational Experience

A computer program was developed in Fortran IV for a Bureau of Mines Burroughs 5500 in Denver. A copy of the computer program is included in appendix C. The computer program is written to handle a problem with a maximum of 6,000 blocks. The upper bound on the problem size is only dependent upon the size of core available. The processing time was found to be primarily dependent upon the total number of blocks, and the number and distribution of disjoint positive blocks. Table 1 gives the results for four different size problems.

TABLE 1. - Computation results for four different size problems

Problem size			Number disjoint blocks	Computational times (sec)	
I	J	K		Processor	I. O.
6	6	1	3	3	14
12	9	1	5	7	14
5	20	3	17	38	16
10	10	6	2	33	14

Input requirements consist primarily of the block weights, problem parameter, problem title, and the output needs. Input must be prepared in the following sequence and contain the following information:

---

<sup>4</sup>Mandatory Safety Standards, Underground Coal Mines. Nov. 20, 1970, Federal Register, v. 35, No. 226, Sec. 75.1700.

Card 1--Option card:

<u>Column</u>	<u>Variable name</u>	<u>Description</u>
1 - 5	LIST	1 ⇒ List block weights
6 - 10	PLOT	1 ⇒ Plot disjoint positive blocks and negative path in solution

Card 2--Problem name:

<u>Column</u>	<u>Variable name</u>	<u>Description</u>
1 - 78	FMT	Problem title (13A6)

Card 3--Data format:

Format of data on cards (left justify and include brackets).

Card 4--Parameter card:

<u>Column</u>	<u>Variable name</u>	<u>Description</u>
1 - 5	IJK (1) - IMAX	Number rows
6 - 10	IJK (2) - JMAX	Number columns
11 - 15	IJK (3) - KMAX	Number sections

Card 4--Data . . .

Data [(I, J, K), I = 1, IMAX, J = 1, JMAX, K = 1, KMAX)].

Program stores data in a one-dimensional array, block (I).

Output is under program control to the extent that the block weights may be printed or suppressed as can the plot of the solution. Included in appendix B is a copy of the output for the sample problem given in figure 6.

## CONCLUSIONS AND EXTENSIONS

The algorithm defined in this report offers the mining engineer a simple technique for long-range mine planning. In addition through the use of a computer program, the technique was demonstrated to be programable for solving an orthogonal, three-dimensional set of blocks and reasonably efficient for those problems tested. In view of the lack of comparable studies, it is exceedingly difficult to fairly evaluate this work; however, the author is extremely optimistic as to the potential of this study.

Such factors as time value of money can be taken into account by making allowance in the calculation of block weights. This forces the individual to design for the long term consistent with conservation of resources. In mining, development costs generally are higher than production costs, and, therefore, the faster a mine can be brought into production, the greater will be its present worth. However, forcing a mine into early production may have the effect of increasing future development costs. In fact, favoring production at the expense of development may lead to the loss of a potential resource.

The general procedure given in this paper will have application to a wide range of subsurface mining-type problems. Interpretation of block weights could be other than a dollar cost; for example, it could represent a risk or some estimate of hazard to be encountered in a particular block. Thereby, the algorithm could be used to study possible escape routes when a disaster has occurred at some place in the mine network.

This general algorithm is constrained in that no account is made for the direction in which a block is to be extracted. For example, it would be less costly to extract a block from the bottom as opposed to extracting it from the top or side. In fact, mining costs might be closely related to local fracture or joint patterns. The problem of mining cost might be best accomplished by assigning arc weights in addition to the node weights, allowing the arc weights to represent the mining cost of mining blocks  $j$  from  $i$  and the node weight, the block mineral worth.

A problem presented by the orthogonal network as developed herein for the computer program is that the alternatives for moving from block to block is constrained to an orthogonal movement (forwards and back, left or right, up or down). Without the loss of generality--specifically with respect to the computer program--the basic network could be modified to allow a user to move diagonally across blocks or possibly even skip particular blocks if the strategy for block jumping could be generalized. Use of other basic geometric configurations would also give the user more flexibility. Use of hexagons in a two-dimensional space, for example, allows the user six degrees of freedom to advance from a particular block compared with only four in a plane for which the computer program was written. Figure 2 and the book by Cole and King (5) indicate other possible geometries. The importance of using an orthogonal set of blocks lies in the fact that because of the highly structured resulting network, there is no need to store an elaborate arc incident matrix.

Much can be done in defining a strategy for designing a production scheduling sequence for a given ore zone or a safe pillar configuration for a given mine by modifying the basic orthogonal network presented in this paper.

The reader should be alerted that the mining problem should be investigated as a total system as the ultimate layout is but one small problem; however, due to the complexity of the underground problem, and the usual limited data base available on a property prior to mining, a piecemeal approach does seem appropriate dependent upon design requirements. For example, facility location and scheduling problems might or might not be of particular importance depending upon the particular property. This approach gives the

minerals industry a tool for studying alternatives to mine layout; it does not specifically answer to the facility location (15), or to the general problem of mine layout with respect to development and production faces, nor does it answer to the problem of scheduling a property to meet production demands and milling constraints (2).

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## APPENDIX A.--TABLE OF SYMBOLS

<u>Symbol</u>	<u>Description</u>
A, B, C, D, E, F, G..	Nodes (used in illustrations)
AMAX.....	Maximum mine plan for a given problem
ATEMP.....	Temporary value (used in algorithm)
$b_i$ .....	An arc $i$
G.....	A graph consisting of a set of nodes $V$ and arcs
$G_o$ .....	Optimal subgraph
$i, k, q, s, t$ .....	Indexes 1, 2 . . . , etc.
$l(-\frac{j}{i})$ .....	Length of path from node $i$ to node $j$
$L(i)$ .....	A label for disjoint positive blocks 1 . . . $P$
$L(s, l(-\frac{t}{q}))$ .....	A label for node $t$ labeled from node $s$ where $l(-\frac{t}{q})$ is the least-cost path from node $t$ to node $q$
M.....	Large positive number
$m$ .....	Total number of vertices
$n$ .....	Total number of arcs
P.....	Number of positive disjoint blocks (does not include Steiner nodes)
S.....	Number of Steiner nodes
$T_o$ .....	Optimal tree (for a given set of node weights)
$T_i$ .....	Tree $i$
$\bar{T}$ .....	Temporary tree
TEMP.....	Temporary value (used in algorithm)
$w_i$ .....	Weight of block $i$
$w(v_i)$ .....	Weight assigned to node $v_i$
V.....	Set of nodes
$v_i$ .....	Node $i$

<u>Symbol</u>	<u>Description</u>
$\bar{v}_i$ .....	Node i such that $v_j$ belongs to $\bar{v}_i$ for some set of j
$X_i$ .....	Block i
$\bar{\cdot}$ .....	Set of arcs
$\prod$ .....	Product over all j
$\Pi_s^t$ .....	Path from node s to node t inclusive
$\sum_j$ .....	Summation over all j
$ $ .....	Such that
$\Omega$ .....	Empty set





POSITIVE DISJOINT TREES \* INDEX = I + (J-1) \* MAX(J) + (K-1) \* MAX(K)

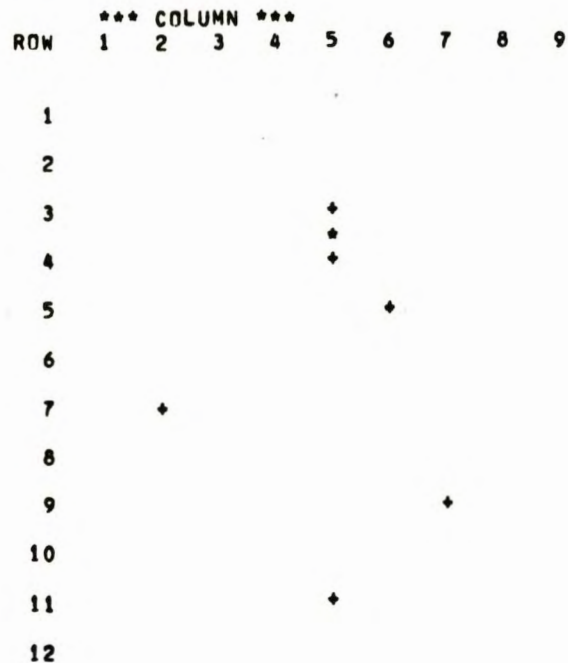
TREE	VALUE	INDEX	TREE BOUNDARIES						** CENTER OF GRAVITY **			** CENTER OF UTILITY **		
			MIN I	MAX I	MIN J	MAX J	MIN K	MAX K	I	J	K	I	J	K
1	364.0	1	1	3	1	5	1	1	2	3	1	2	3	1
2	40.0	18	6	6	2	4	1	1	6	3	1	6	3	1
3	195.0	20	8	10	2	6	1	1	9	4	1	9	4	1
4	102.0	48	12	12	4	6	1	1	12	5	1	12	6	1
5	2.0	64	4	4	6	6	1	1	4	6	1	4	6	1
6	-70.0	77	5	9	7	9	1	1	7	8	1	7	8	1

ITERATION	TREE	VALUE
1	1	364.0
1	6	429.0
1	3	619.0
1	4	717.0
1	2	752.0

PLOT OF POSITIVE NODES IN ULTIMATE PLAN

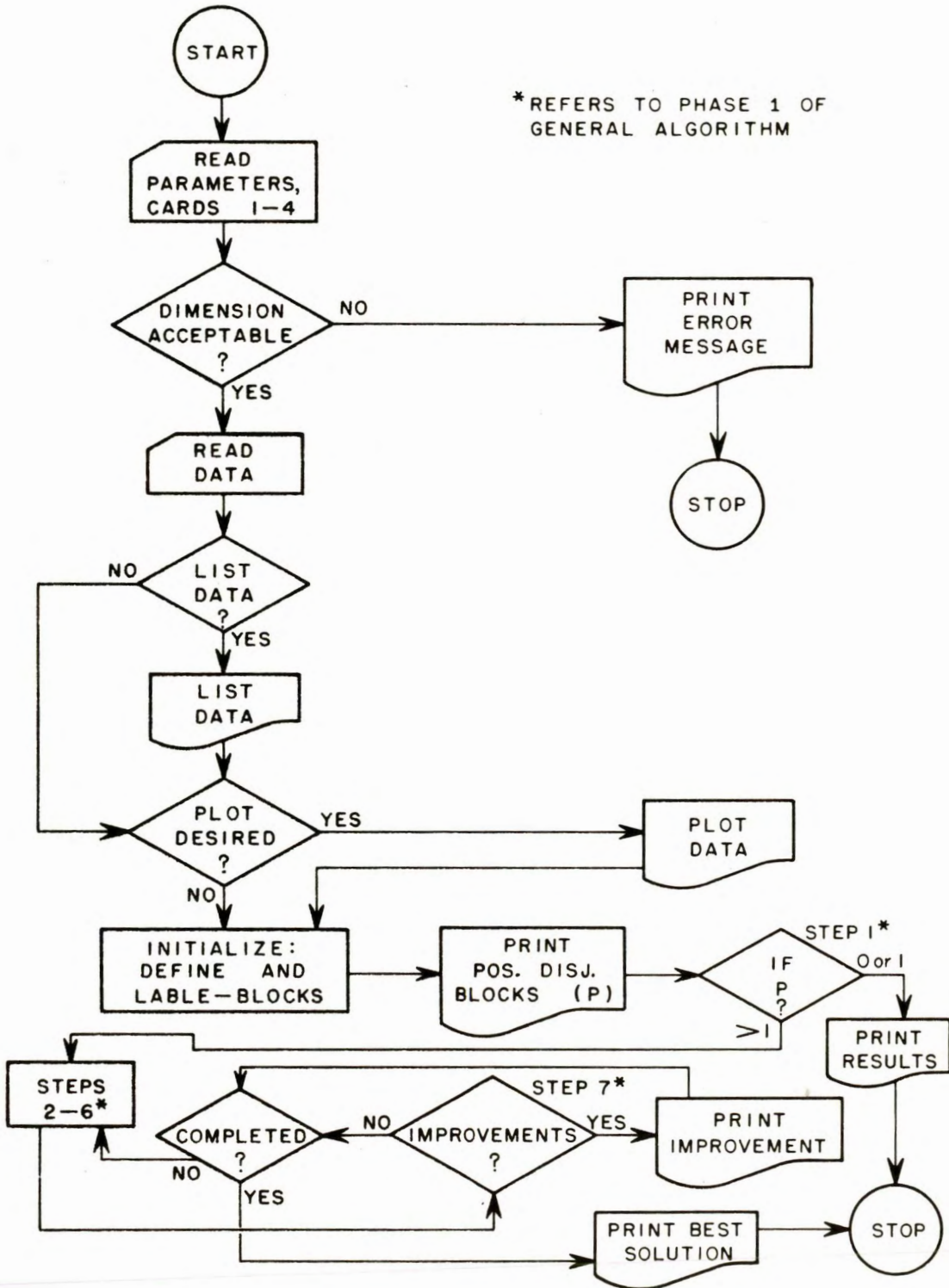
TABLE OF SYMBOLS	
SYMBOL	EXPLANATION
+	POSITIVE NODE
*	CONNECTIVITY IN I = J PLANE
U	CONNECTIVITY WITH K-1 PLANE
D	CONNECTIVITY WITH K+1 PLANE
B	CONNECTIVITY WITH ADJACENT PLANES

SECTION 1



APPENDIX C.--COMPUTER PROGRAM

C-1.--Flow Chart of the Computer Code





1005	FORMAT(1X,17HJ INDEX TOO SMALL )	00005600	R	0118
	GO TO 2000	00005700	R	0118
7	KMAX = IJK(3)	00005800	R	0119
	DO 8 I = 4,6	00005900	R	0120
A	IJK(I) = 1	00006000	R	0127
	PRINT 1004, (IJK(I),I=1,3)	00006100	R	0133
1004	FORMAT(10X,5HI = ,15,3X,5HJ = ,15,3X,5HK = 15)	00006200	R	0155
C		00006300	R	0155
	MAXJ = IJK(1) + IJK(2)	00006400	R	0155
	MAXK = MAXJ + IJK(3)	00006500	R	0159
	IF(IJK(3) .EQ. 0) MAXK = MAXJ	00006600	R	0163
	IF(IJK(1) .LE. ICON) GO TO 10	00006700	R	0168
	PRINT 1003	00006800	R	0173
1003	FORMAT(1X,25HTOO MANY BLOCK IN I ARRAY)	00006900	R	0177
	GO TO 2000	00007000	R	0177
10	INDEX = 0	00007100	R	0178
	IF(LIST .NE. 0) PRINT 1010	00007200	R	0178
1010	FORMAT(/ 6X, 5HINPUT/)	00007300	R	0184
	IF(MAXK .LT. ICON) GO TO 20	00007400	R	0184
	PRINT 1006	00007500	R	0188
1006	FORMAT(1X,15HPROBLEM TOO BIG )	00007600	R	0192
	GO TO 2000	00007700	R	0192
20	LMAX = MAXK	00007800	R	0193
	READ FMT, (BLOCK(J), J=1,LMAX)	00007900	R	0194
	IF(LIST .EQ. 0) GO TO 40	00008000	R	0215
	PRINT FMT, (BLOCK(J),J=1,LMAX)	00008100	R	0217
40	IF(IPLLOT .EQ. 0) GO TO 100	00008200	R	0237
C		00008300	R	0238
C	FIND ALL POSITIVE TREES	00008400	R	0238
C		00008500	R	0238
	PRINT 1111	00008510	R	0239
1111	FORMAT(1H1,1X,23HPLOT OF POSITIVE NODES //)	00008520	R	0243
50	CALL PLOT	00008600	R	0243
C		00008700	R	0243
100	CALL BEGIN(NSORT,MTREE,IMAX,JMAX,KMAX)	00008800	R	0244
	IF(MTREE = 1) 61,65,70	00008900	R	0245
61	PRINT 1012	00009000	R	0251
1012	FORMAT(1X,24HNO POSITIVE BLOCK EXISTS)	00009100	R	0255
	GO TO 2000	00009200	R	0255
65	PRINT 1013, SUM(1)	00009300	R	0256
1013	FORMAT(1X,45HONLY ONE POSITIVE SUBTREE EXISTS WHICH YIELDS,F12,2)	00009400	R	0267
	GO TO 2000	00009500	R	0267
70	CALL MAXTRE(MTREE,IMAX,JMAX,KMAX)	00009600	R	0268
2000	CALL EXIT	00009700	R	0270
	END	00009800	R	0271
		SEGMENT	1	IS 282 LONG

```

SUBROUTINE BEGIN(NSORT,MTREE,IMAX,JMAX,KMAX)
COMMON BLOCK(6000),ITREE(6000),LBK(6000),SORT(1020),ITAG(1020),
1NPR(101),NPL(30),IDA(6),JDA(6),KDA(6),IJK(6),SUM(30),MAXJ,MAXK,
2ICON,NPD(30),IT(30),KTAG(1020),ADD(120),JTAG(120),ARRAY(1020),
3IZAP(60),NPATH(1020),BARRAY(1020),KKTAG(1020),NBEST(1020),
4LABEL(30)
START OF SEGMENT ***** 2
00009900 R 0000
00010000 R 0000
00010100 R 0000
00010200 R 0000
00010300 R 0000
00010400 R 0000
00010500 R 0000
00010600 R 0000
00010700 R 0002
00010800 R 0005
00010900 R 0008
00011000 R 0010
00011100 R 0013
00011200 R 0016
00011300 R 0023
00011400 R 0029
00011500 R 0030
00011600 R 0030
00011700 R 0038
00011800 R 0043
00011900 R 0045
00012000 R 0049
00012010 R 0049
00012100 R 0049
M00012200 R 0049
00012300 R 0049
00012400 R 0049
00012500 R 0055
00012600 R 0061
00012700 R 0069
00012800 R 0070
00012900 R 0070
00013000 R 0070
00013100 R 0072
00013200 R 0073
00013300 R 0079
00013400 R 0081
00013500 R 0083
00013600 R 0085
00013700 R 0085
00013800 R 0086
00013900 R 0087
00014000 R 0088
00014100 R 0088
00014200 R 0089
00014300 R 0090
00014400 R 0091
00014500 R 0093
00014600 R 0094
00014700 R 0096
00014800 R 0097
00014900 R 0099
00015000 R 0100
00015100 R 0100
00015200 R 0102
00015300 R 0104

C
JDA(1) = 1
JDA(2) = IJK(1)
JDA(3) = MAXJ
JDA(4) = -1
JDA(5) = -IJK(1)
JDA(6) = -MAXJ
DO 9 I = 1,30
9 SUM(I) = 0,
NTREE = 0
MTREE = 0
DO 10 I = 1,MAXK
10 ITREF(I) = 0

C
20 PRINT 1002
1002 FORMAT(1H1/1X,69HPOSITIVE DISJOINT TREES * INDEX = I + (J-1) * MAX
1(J) + (K-1) * MAX(K)//4X,4HTREE,5X,5HVALUE,4X,
25HINDEX,3X,2H**,10X,18HTREE BOUNDARIES,10X,2H**,1X,17HCENTER OF
3 GRAVITY,1X,2H**,1X,17HCENTER OF UTILITY,1X,2H**,31X, 42H MIN Y
4AX I MIN J MAX J MIN K MAX K ,2(4X,1HI,6X,1HJ,6X,1HK,2X)/)
DO 30 INDEX = 1,MAXK
IF(BLOCK(INDEX) .LE. 0.) GO TO 30
IF(ITREE(INDEX) .NE. 0) GO TO 30
GO TO 97
30 CONTINUE
GO TO 160

C
91 KK = 7 - I
INDEX = INDEX + JDA(KK)
GO TO 100
97 NTREE = NTREE + 1000000
MTREE = MTREE + 1
ITEST = 0
SD = 0.
SI = 0.
SJ = 0.
SK = 0.
SWI = 0.
SWJ = 0.
SWK = 0.
MINX = IJK(1) + 1
MAXX = 0
MINY = IJK(2) + 1
MAXY = 0
MINZ = IJK(3) + 1
MAXZ = 0
IBEG = INDEX
GO TO 100
99 IF(NDEX .EQ. IBEG) GO TO 130
INDEX = NDEX

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100	DO 110 I = 1,6	00015400	R	0105
	NDEX = INDEX + JDA(I)	00015500	R	0110
	GO TO (101,102,103,104,105,106) , I	00015600	R	0116
101	IF(MOD(NDEX,IMAX)=1) 108,110,108	00015700	R	0126
102	IF(MOD(INDEX,MAXJ) .EQ. 0) GO TO 110	00015800	R	0130
	IF(NDEX/MAXJ = INDEX/MAXJ) 110,108,110	00015900	R	0134
103	IF(NDEX = MAXK) 108,108,110	00016000	R	0140
104	IF(MOD(NDEX,IMAX)) 108,110,108	00016100	R	0144
105	IF(NDEX .LE. 0) GO TO 110	00016200	R	0147
	IF(MOD(NDEX,MAXJ) .EQ. 0) GO TO 110	00016300	R	0149
	IF(NDEX/MAXJ = INDEX/MAXJ) 110,108,110	00016400	R	0153
106	IF(NDEX) 110,110,108	00016500	R	0159
108	IF(BLOCK(NDEX) .LE. 0,) GO TO 110	00016600	R	0162
	IF(ITREE(NDEX) .GT. 0) GO TO 110	00016700	R	0167
	ITREE(NDEX) = KDA(I) + NTREE	00016800	R	0175
109	SO = SO + 1	00016900	R	0186
	IVAL = MOD(NDEX,IMAX)	00017000	R	0187
	IF(IVAL .EQ. 0) IVAL = IMAX	00017100	R	0188
	IF(IVAL .GT. MAXX) MAXX = IVAL	00017200	R	0191
	IF(IVAL .LT. MINX) MINX = IVAL	00017300	R	0193
	SI = SI + IVAL	00017400	R	0195
	SWI = SWI + BLOCK(NDEX)*IVAL	00017500	R	0196
	KVAL = (NDEX + MAXJ - 1)/MAXJ	00017600	R	0201
	JVAL = (NDEX - MAXJ*(KVAL-1) + IMAX - 1)/IMAX	00017700	R	0206
	IF(JVAL .GT. MAXY) MAXY = JVAL	00017800	R	0210
	IF(JVAL .LT. MINY) MINY = JVAL	00017900	R	0213
	SJ = SJ + JVAL	00018000	R	0215
	SWJ = SWJ + BLOCK(NDEX)*JVAL	00018100	R	0216
	IF(KVAL .GT. MAXZ) MAXZ = KVAL	00018200	R	0221
	IF(KVAL .LT. MINZ) MINZ = KVAL	00018300	R	0223
	SK = SK + KVAL	00018400	R	0226
	SWK = SWK + BLOCK(NDEX)*KVAL	00018500	R	0227
	SUM(MTREE) = SUM(MTREE) + BLOCK(NDEX)	00018600	R	0232
	IF(ITEST .EQ. 1) GO TO 159	00018700	R	0247
	GO TO 99	00018800	R	0249
110	CONTINUE	00018900	R	0250
	IF(IBEG .EQ. INDEX) GO TO 151	00019000	R	0250
130	DO 150 I = 1,6	00019100	R	0253
	IF(I .GT. 3) GO TO 131	00019200	R	0258
	K = 7 - 2*I	00019300	R	0261
	GO TO 132	00019400	R	0262
131	K = 14 - 2*I	00019500	R	0264
132	ITEMP = ITREE(INDEX)/10**(K-1) = (ITREE(INDEX)/10**K)*10	00019600	R	0266
	IF(ITEMP .EQ. 0) GO TO 150	00019700	R	0282
	GO TO 91	00019800	R	0284
150	CONTINUE	00019900	R	0285
151	IF(SUM(MTREE) .GT. 0) GO TO 159	00020000	R	0286
	ITREE(INDEX) = NTREE	00020100	R	0293
	ITEST = 1	00020200	R	0298
	NDEX = INDEX	00020300	R	0299
	GO TO 109	00020400	R	0300
159	ISI = SI/SO + 0,5	00020500	R	0301
	ISJ = SJ/SO + 0,5	00020600	R	0305
	ISK = SK/SO + 0,5	00020700	R	0309
	ISWI = SWI/SUM(MTREE) + 0,5	00020800	R	0313
	ISWJ = SWJ/SUM(MTREE) + 0,5	00020900	R	0322
	ISWK = SWK/SUM(MTREE) + 0,5	00021000	R	0331

PRINT 1001, MTREE, SUM(MTREE), INDEX, MINX, MAXX, MINY, MAXY, MINZ, MAXZ,	00021100 R 0340
11SI, ISJ, ISK, ISWI, ISWJ, ISWK	00021200 R 0368
1001 FORMAT(4X, I3, 2X, F11.1, 2X, I4, 6X, 12(2X, I3, 2X))	00021300 R 0384
	SEGMENT 3 IS 126 LONG
GO TO 30	00021400 R 0384
C INITIALIZE SORT	00021500 R 0384
C	00021600 R 0384
C	00021700 R 0384
160 IF(MTREE .LE. 1) GO TO 280	00021800 R 0385
DO 161 I = 1, MAXK	00021900 R 0387
161 IF(BLOCK(I) .EQ. 0.) BLOCK(I) = - 1	00022000 R 0394
169 INDEX = 0	00022100 R 0404
N = 0	00022200 R 0404
DO 200 K = 1, KMAX	00022300 R 0405
IT1(3) = K	00022400 R 0410
IT1(6) = K	00022500 R 0412
DO 200 J = 1, JMAX	00022600 R 0414
IT1(2) = J	00022700 R 0420
IT1(5) = J	00022800 R 0422
DO 200 I = 1, IMAX	00022900 R 0424
IT1(1) = I	00023000 R 0430
IT1(4) = I	00023100 R 0432
INDEX = INDEX + 1	00023200 R 0434
IF(BLOCK(INDEX) .GT. 0) GO TO 200	00023300 R 0436
ISAVE = 0	00023400 R 0441
180 DO 189 II = 1, 6	00023500 R 0443
IF(IT1(II) .EQ. IJK(II)) GO TO 189	00023600 R 0448
NDEX = INDEX + JDA(II)	00023700 R 0460
IF(BLOCK(NDEX) .LE. 0.) GO TO 189	00023800 R 0466
NVAL = ITREE(NDEX)/1000000	00023900 R 0472
GO TO 190	00024000 R 0480
189 CONTINUE	00024100 R 0482
GO TO 200	00024200 R 0482
190 IF(ISAVE .EQ. 0) GO TO 192	00024300 R 0484
DO 191 KK = 1, ISAVE	00024400 R 0486
IF(KTAG(KK) .EQ. NVAL) GO TO 189	00024500 R 0491
191 CONTINUE	00024600 R 0499
192 ISAVE = ISAVE + 1	00024700 R 0500
KTAG(ISAVE) = NVAL	00024800 R 0501
N = N + 1	00024900 R 0507
SORT(N) = BLOCK(INDEX)	00025000 R 0509
ITAG(N) = INDEX + NVAL * 1000000	00025100 R 0518
GO TO 189	00025200 R 0527
200 CONTINUE	00025300 R 0528
NSORT = N	00025400 R 0529
205 K = 0	00025500 R 0531
DO 220 I = 2, N	00025600 R 0531
IF(SORT(I-1) .GE. SORT(I)) GO TO 220	00025700 R 0537
210 TEMP = SORT(I)	00025800 R 0551
ITEMP = ITAG(I)	00025900 R 0556
SORT(I) = SORT(I-1)	00026000 R 0562
ITAG(I) = ITAG(I-1)	00026100 R 0574
SORT(I-1) = TEMP	00026200 R 0586
ITAG(I-1) = ITEMP	00026300 R 0592
K = 1	00026400 R 0599
220 CONTINUE	00026500 R 0601
N = N - 1	00026600 R 0601

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IF(K .EQ. 1) GO TO 205
KOUNT = 0
NPOS(1) = 0
J = 1
DO 270 I = 1, MTREE
DO 260 K = 1, NSORT
IF(ITAG(K)/1000000 .NE. I) GO TO 260
KOUNT = KOUNT + 1
ARRAY(KOUNT) = SORT(K)
KTAG(KOUNT) = ITAG(K) - I + 1000000
ITAG(K) = KTAG(KOUNT)
260 CONTINUE
J = J + 1
NPOS(J) = KOUNT
270 CONTINUE
DO 275 I = 1, NSORT
ITAG(I) = KTAG(I)
275 SORT(I) = ARRAY(I)
1113 FORMAT(1X,2I12)
280 RETURN
END

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00026700 R 0602
00026800 R 0604
00026900 R 0605
00027000 R 0607
00027100 R 0608
00027200 R 0613
00027300 R 0619
00027400 R 0629
00027500 R 0630
00027600 R 0641
00027700 R 0655
00027800 R 0666
00027900 R 0666
00028000 R 0667
00028100 R 0674
00028200 R 0674
00028300 R 0679
00028400 R 0691
00028500 R 0702
00028600 R 0702
00028700 R 0705
SEGMENT 2 IS 725 LONG

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	START OF SEGMENT	*****	4
SUBROUTINE MAXTRE(MTREE,IMAX,JMAX,KMAX)	00028800	R	0000
COMMON BLOCK(6000),ITREE(6000),LBK(6000),SORT(1020),ITAG(1020),	00028900	R	0000
INPR(101),NPL(30),IDA(6),JDA(6),KDA(6),IJK(6),SUM(30),MAXJ,MAXK,	00029000	R	0000
2ICON,NPOS(30),IT1(30),KTAG(1020),ADD(120),JTAG(120),ARRAY(1020),	00029100	R	0000
3IZAP(60),NPATH(1020),BARRAY(1020),KKTAG(1020),NBEST(1020),	00029200	R	0000
4LABEL(30)	00029300	R	0000
KDA(1) = 1000	00029400	R	0000
KDA(2) = 10000	00029500	R	0002
KDA(3) = 100000	00029600	R	0005
KDA(4) = 1	00029700	R	0009
KDA(5) = 10	00029800	R	0011
KDA(6) = 100	00029900	R	0013
ISTOP = 0	00030000	R	0015
ITRE = 0	00030100	R	0016
TVAL = 0	00030200	R	0017
PRINT 5040	00030300	R	0017
5040 FORMAT(1H1,1X,16HITERATION TREE ,15X,5HVALUE/)	00030400	R	0022
DO 10 I = 1, MTREE	00030500	R	0022
10 LABEL(I) = 0	00030600	R	0028
C HAVE ALL POSITIVE DISJOINT TREES BEEN ATTEMPTED	00030700	R	0033
20 IF(ITRE, EQ, 0) GO TO 15	00030800	R	0035
DO 14 I = 1, MAXK	00030900	R	0037
IF(ITREE(I), GT, 0) GO TO 14	00031000	R	0043
ITREE(I) = 0	00031100	R	0051
14 CONTINUE	00031200	R	0057
DO 8 I=1,MAXK	00031300	R	0057
8 PRINT1188,ITREE(I),BLOCK(I)	00031400	R	0064
1188 FORMAT(10X,I12,F10.1)	00031500	R	0085
15 ITRE = ITRE + 1	00031600	R	0085
NODE = 0	00031700	R	0086
IZ = 0	00031800	R	0087
NP = 0	00031900	R	0087
AVAL = 0	00032000	R	0088
IT = 1	00032100	R	0089
ITP = IT	00032200	R	0090
DO 19 I = 1, MTREE	00032300	R	0090
IF(LABEL(I), NE, 0) GO TO 19	00032400	R	0096
IF(SUM(I), LE, AVAL) GO TO 19	00032500	R	0104
NODE = I	00032600	R	0111
IT1(IT) = NODE	00032700	R	0111
LABEL(NODE) = NODE	00032800	R	0117
AVAL = SUM(I)	00032900	R	0123
19 CONTINUE	00033000	R	0130
DELTA = AVAL	00033100	R	0130
IF(NODE, EQ, 0) GO TO 1000	00033200	R	0131
C INITIALIZE ARRAY	00033300	R	0132
ILAST = NPOS(NODE + 1) - NPOS(NODE)	00033400	R	0133
IND = NPOS(NODE)	00033500	R	0144
NSORT = ILAST	00033600	R	0150
NI = NSORT	00033700	R	0151
DO 40 I = 1, ILAST	00033800	R	0152
IND = IND + 1	00033900	R	0157
ARRAY(I) = SORT(IND) + AVAL	00034000	R	0159
KTAG(I) = ITAG(IND)	00034100	R	0170
BARRAY(I) = ARRAY(IND)	00034200	R	0180
KKTAG(I) = ITAG(IND)	00034300	R	0191

	ITREE(ITAG(IND)) = -99	00034400 R 0202
40	CONTINUE	00034500 R 0214
	PRINT 5050, ITRE,NODE,SUM(NODE)	00034600 R 0214
C		00034700 R 0228
C	DYNAMIC = SHORTEST PATH ALGORITHM	00034800 R 0228
C		00034900 R 0228
99	KK = 0	00035000 R 0234
	DO 100 I = 1, 6	00035100 R 0234
	INDEX = KTAG(1) + JDA(I)	00035200 R 0240
1881	FORMAT(5X,3I12,F10.1)	00035300 R 0248
	GO TO (21,22,23,24,25,26),I	00035400 R 0248
21	IF(MOD(INDEX,IMAX) = 1)28,100,28	00035500 R 0257
22	IF(MOD(KTAG(1),MAXJ),EQ. 0) GO TO 100	00035600 R 0261
	IF(INDEX/MAXJ = KTAG(1)/MAXJ) 100,28,100	00035700 R 0266
23	IF(INDEX = MAXK) 28,28,100	00035800 R 0273
24	IF(MOD(INDEX,IMAX)) 28,100,28	00035900 R 0277
25	IF(INDEX ,LE, 0) GO TO 100	00036000 R 0280
	IF(MOD(INDEX,MAXJ),EQ. 0) GO TO 100	00036100 R 0282
	IF(INDEX/MAXJ = KTAG(1)/MAXJ) 100,28,100	00036200 R 0286
26	IF(INDEX) 100,100,28	00036300 R 0293
28	IF(ITREE(INDEX),LT. 0) GO TO 100	00036400 R 0296
	IF(BLOCK(INDEX),LE. 0) GO TO 50	00036500 R 0303
	IV = ITREE(INDEX)/1000000	00036600 R 0308
	DO 31 IX = 1,IT	00036700 R 0316
31	IF(IV ,EQ, IT1(IX)) GO TO 100	00036800 R 0323
C	THIS IS A BREAK THROUGH	00036900 R 0330
	MODE = IV	00037000 R 0331
	GO TO 199	00037100 R 0332
50	IF(ITREE(INDEX),EQ, -99) GO TO 100	00037200 R 0333
	KK = KK + 1	00037300 R 0340
	ADD(KK) = BLOCK(INDEX) + ARRAY(1)	00037400 R 0341
	JTAG(KK) = INDEX	00037500 R 0352
	ITREE(INDEX) = -KDA(I)	00037600 R 0358
100	CONTINUE	00037700 R 0370
C	UPDATE ARRAY	00037800 R 0370
1882	FORMAT(1X,6I10)	00037900 R 0371
	NSAVE = NSORT	00038000 R 0371
	NSORT = NSORT + KK - 1	00038100 R 0371
	IF(NSORT ,LE, 0) GO TO 20	00038200 R 0373
	N = KK	00038300 R 0375
	NLAST = NSORT	00038400 R 0376
	IF(KK = 1) 140,779,105	00038500 R 0377
105	K = 0	00038600 R 0383
	DO 120 I = 2, N	00038700 R 0383
	IF(ADD(I-1),GE, ADD(I)) GO TO 120	00038800 R 0389
	TEMP = ADD(I)	00038900 R 0402
	ITEMP = JTAG(I)	00039000 R 0408
	ADD(I) = ADD(I-1)	00039100 R 0414
	JTAG(I) = JTAG(I-1)	00039200 R 0426
	ADD(I-1) = TEMP	00039300 R 0438
	JTAG(I-1) = ITEMP	00039400 R 0444
	K = 1	00039500 R 0451
120	CONTINUE	00039600 R 0453
	N = N - 1	00039700 R 0453
	IF(K ,EQ, 1) GO TO 105	00039800 R 0454
150	NLAST = NSORT + 1	00039900 R 0457
	DO 160 I = 1, NSORT	00040000 R 0458

NLAST = NLAST - 1	00040100 R 0463
IF(KK ,EQ, 1) GO TO 179	00040200 R 0465
IF(NSAVE ,EQ, 0) GO TO 152	00040300 R 0467
IF(ARRAY(NSAVE) ,LT, ADD(KK)) GO TO 154	00040400 R 0469
152 ARRAY(NLAST) = ADD(KK)	00040500 R 0482
KTAG(NLAST) = JTAG(KK)	00040600 R 0492
KK = KK - 1	00040700 R 0503
GO TO 160	00040800 R 0504
154 ARRAY(NLAST) = ARRAY(NSAVE)	00040900 R 0506
KTAG(NLAST) = KTAG(NSAVE)	00041000 R 0516
NSAVE = NSAVE - 1	00041100 R 0527
160 CONTINUE	00041200 R 0529
GO TO 99	00041300 R 0529
140 DO 145 I = 1, NSORT	00041400 R 0530
II = I + 1	00041500 R 0535
ARRAY(I) = ARRAY(II)	00041600 R 0537
KTAG(I) = KTAG(II)	00041700 R 0547
145 CONTINUE	00041800 R 0559
GO TO 99	00041900 R 0559
179 NLAST = NLAST - 1	00042000 R 0560
DO 180 I = 1, NLAST	00042100 R 0561
IF(ARRAY(I+1) ,LT, ADD(1)) GO TO 181	00042200 R 0566
ARRAY(I) = ARRAY(I+1)	00042300 R 0576
180 KTAG(I) = KTAG(I+1)	00042400 R 0588
ARRAY(NLAST+1) = ADD(1)	00042500 R 0600
KTAG(NLAST + 1) = JTAG(1)	00042600 R 0607
GO TO 99	00042700 R 0615
181 ARRAY(I) = ADD(1)	00042800 R 0617
KTAG(I) = JTAG(1)	00042900 R 0623
1401 FORMAT(1X, 5(F6.0,15,I10))	00043000 R 0631
GO TO 99	00043100 R 0631
C IS THIS BREAKTHROUGH AN IMPROVEMENT	00043200 R 0631
199 IZ = IZ + 1	00043300 R 0632
IZAP(IZ) = MODE	00043400 R 0633
IZ = IZ + 1	00043500 R 0639
IZAP(IZ) = KTAG(1)	00043600 R 0641
LABEL(IV) = IV	00043700 R 0647
IT = IT + 1	00043800 R 0653
IF(IT ,EQ, MTREE) ISTOP = 1	00043900 R 0655
IT1(IT) = IV	00044000 R 0657
IF(ARRAY(1) + SUM(IV) ,GT, AVAL) GO TO 300	00044100 R 0663
C UPDATE ARRAY AND CONTINUE	00044200 R 0671
IADD = NPOS(MODE + 1) - NPOS(MODE) - 2	00044300 R 0672
DO 220 I = 1, NSORT	00044400 R 0684
II = NSORT - I + 1	00044500 R 0689
IP = II + IADD	00044600 R 0691
ARRAY(IP) = ARRAY(II)	00044700 R 0692
220 KTAG(IP) = KTAG(II)	00044800 R 0704
SAVE = ARRAY(1)	00044900 R 0715
NSAVE = NSORT	00045000 R 0716
NSORT = NSORT + IADD	00045100 R 0717
KK = NPOS(MODE) + 1	00045200 R 0718
MSAVE = JTAG(KK)	00045300 R 0725
NTEST = NPOS(MODE + 1)	00045400 R 0730
IADD=IADD + 1	00045500 R 0736
ICT = 1	00045600 R 0737
TEMP = SORT(KK) + SUM(MODE) + SAVE	00045700 R 0738

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L = 0
DO 240 I = 1, NSORT
IF(KK .GT. NTEST) GO TO 239
IF(ICT .GT. NSAVE) GO TO 232
IF(ARRAY(I + IADD) .GE. TEMP) GO TO 239
237 IF(ITAG(KK) .EQ. MSAVE) GO TO 236
IL = L + 1
IF(L .EQ. 0) GO TO 234
DO 233 IW = 1, L
233 IF(KTAG(IW) .EQ. ITAG(KK)) GO TO 236
234 DO 235 IW = IL, NSAVE
IY = IW + IADD
235 IF(KTAG(IY) .EQ. ITAG(KK)) GO TO 236
L = L + 1
ARRAY(L) = TEMP
KTAG(L) = ITAG(KK)
ITREE(KTAG(L)) = -99 - MODE + 1000000
IADD = IADD - 1
236 KK = KK + 1
TEMP = SORT(KK) + SUM(MODE) + SAVE
GO TO 240
239 L = L + 1
ARRAY(L) = ARRAY(L + IADD)
KTAG(L) = KTAG(L + IADD)
ICT = ICT + 1
240 CONTINUE
NSORT = NSORT - IADD
GO TO 99
C FIND WAY BACK TO BEST TREE
C ADD ADDITIONAL NODE TO ARRAY
300 NJ = 0
INDEX=KTAG(1)
AVAL = ARRAY(1) + SUM(MODE)
IT = ITP + 1
IT1(IT) = MODE
400 LAST = NPOS(MODE + 1)
NST = NPOS(MODE) + 1
DO 420 I = NST, LAST
DO 410 J = 1, NI
IF(ITAG(I) .EQ. KKTAG(J)) GO TO 420
410 CONTINUE
NI = NI + 1
KKTAG(NI) = ITAG(I)
NJ = NJ + 1
ADD(NJ) = AVAL + SORT(I)
JTAG(NJ) = ITAG(I)
420 CONTINUE
NSTART = NP + 1
301 IF(ITREE(INDEX) .LE. -1000000) GO TO 350
NP = NP + 1
NPATH(NP) = INDEX
IF(ITREE(INDEX) .EQ. -99) GO TO 316
DO 309 II = 1, 6
IF(-ITREF(INDEX) .NE. KDA(II)) GO TO 309

NDX = INDEX
00045800 R 0750
00045900 R 0751
00046000 R 0756
00046100 R 0759
00046200 R 0761
00046300 R 0769
00046400 R 0776
00046500 R 0777
00046600 R 0779
00046700 R 0785
00046800 R 0798
00046900 R 0803
00047000 R 0805
00047100 R 0817
00047200 R 0818
00047300 R 0824
00047400 R 0835
00047500 R 0850
00047600 R 0852
00047700 R 0853
00047800 R 0865
00047900 R 0866
00048000 R 0867
00048100 R 0879
00048200 R 0891
00048300 R 0893
00048400 R 0893
00048500 R 0894
00048600 R 0894
00048700 R 0894
00048800 R 0896
00048900 R 0896
00049000 R 0898
00049100 R 0905
00049200 R 0907
00049300 R 0913
00049400 R 0919
00049500 R 0926
00049600 R 0931
00049700 R 0937
00049800 R 0950
00049900 R 0950
00050000 R 0951
00050100 R 0962
00050200 R 0963
00050300 R 0975
00050400 R 0987
00050500 R 0987
00050600 R 0989
00050700 R 0998
00050800 R 0999
00050900 R 1005
00051000 R 1013
00051100 R 1018
SEGMENT 4 IS 1023 LONG
START OF SEGMENT ***** 5
00051200 R 0009

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IF(II .LE. 3) IY = II + 3	00051300 R 0010
IF(II .GT. 3) IY = II - 3	00051400 R 0013
INDEX = INDEX + JDA(IY)	00051500 R 0016
ITREE(NDX) = -98	00051600 R 0022
GO TO 301	00051700 R 0029
309 CONTINUE	00051800 R 0030
PRINT 3490	00051900 R 0031
3490 FORMAT(1X,6HSTOR 1)	00052000 R 0035
CALL EXIT.	00052100 R 0035
316 DO 349 IND = NSIART, NP	00052200 R 0037
307 DO 340 I = 1, 6	00052300 R 0043
NDEX = NPATH(IND) + JDA(I)	00052400 R 0048
GO TO (321,322,323,324,325,326) , I	00052500 R 0059
321 IF(MOD(NDEX,IMAX) = 1) 328,340,328	00052600 R 0069
322 IF(MOD(INDEX,MAXJ) .EQ. 0) GO TO 340	00052700 R 0073
IF(NDEX/MAXJ = INDEX/MAXJ) 340,328,340	00052800 R 0077
323 IF(NDEX = MAXK) 328,328,340	00052900 R 0083
324 IF(MOD(NDEX,IMAX)) 328,340,328	00053000 R 0087
325 IF(NDEX .LE. 0) GO TO 340	00053100 R 0090
IF(MOD(NDEX,MAXJ) .EQ. 0) GO TO 340	00053200 R 0092
IF(NDEX/MAXJ = INDEX/MAXJ) 340,328,340	00053300 R 0096
326 IF(NDEX) 340,340,328	00053400 R 0102
328 IF(BLOCK(NDEX) .GT. 0) GO TO 340	00053500 R 0105
IF(ITREE(NDEX) .EQ. -98) GO TO 340	00053600 R 0110
DO 330 J = 1, NI	00053700 R 0118
IF(NDEX .EQ. KKTAG(J)) GO TO 340	00053800 R 0123
330 CONTINUE	00053900 R 0131
NI = NI + 1	00054000 R 0131
KKTAG(NI) = NDEX	00054100 R 0132
NJ = NJ + 1	00054200 R 0138
ADD(NJ) = AVAL + BLOCK(NDEX)	00054300 R 0140
JTAG(NJ) = NDEX	00054400 R 0149
340 CONTINUE	00054500 R 0156
349 CONTINUE	00054600 R 0157
ITP = IT	00054700 R 0157
GO TO 359	00054800 R 0158
C BLOCK VALUE IS POSITIVE	00054900 R 0158
350 NP = NP + 1	00055000 R 0159
NPATH(NP) = INDEX	00055100 R 0160
IV = -ITREE(INDEX)/1000000	00055200 R 0166
IT = IT + 1	00055300 R 0174
IT1(IT) = IV	00055400 R 0176
IST = NPNS(IV) + 1	00055500 R 0181
ILT = NPNS(IV + 1)	00055600 R 0188
DO 354 I = IST, ILT	00055700 R 0194
DO 353 J = 1, NI	00055800 R 0199
IF(ITAG(I) .EQ. KKTAG(J)) GO TO 354	00055900 R 0205
353 CONTINUE	00056000 R 0218
NI = NI + 1	00056100 R 0218
KKTAG(NI) = ITAG(I)	00056200 R 0219
NJ = NJ + 1	00056300 R 0230
ADD(NJ) = AVAL + SORT(I)	00056400 R 0231
JTAG(NJ) = ITAG(I)	00056500 R 0243
354 CONTINUE	00056600 R 0255
IND = -1	00056700 R 0255
DO 355 I = 1, IZ	00056800 R 0256
IND = IND + 2	00056900 R 0261

	IF(IV ,EQ, IZAP(IND)) GO TO 358	00057000 R	0263
355	CONTINUE	00057100 R	0271
358	INDEX = IZAP(IND + 1)	00057200 R	0272
	GO TO 301	00057300 R	0278
C	MERGF ADD AND BARRY TO ARRAY	00057400 R	0278
359	DO 360 I = 1, NI	00057500 R	0279
360	BARRAY(I) = AVAL + BLOCK(KKTAG(I))	00057600 R	0285
	IF(NJ ,EQ, 0) GO TO 371	00057700 R	0300
	N = NJ	00057800 R	0302
361	K = 0	00057900 R	0304
	DO 369 I = 2, N	00058000 R	0304
	IF(ADD(I-1) ,GE, ADD(I)) GO TO 369	00058100 R	0310
	TEMP = ADD(I)	00058200 R	0323
	ITEMP = JTAG(I)	00058300 R	0329
	ADD(I) = ADD(I-1)	00058400 R	0335
	JTAG(I) = JTAG(I-1)	00058500 R	0347
	ADD(I-1) = TEMP	00058600 R	0359
	JTAG(I-1) = ITEMP	00058700 R	0365
	K = 1	00058800 R	0372
369	CONTINUE	00058900 R	0374
	N = N - 1	00059000 R	0374
	IF(K ,EQ, 1) GO TO 361	00059100 R	0375
371	II = NI - NJ	00059200 R	0378
	NLAST = NI + 1	00059300 R	0379
	DO 380 I = 1, NI	00059400 R	0380
	NLAST = NLAST - 1	00059500 R	0385
	IF(II ,EQ, 0) GO TO 372	00059600 R	0387
	IF(NJ ,EQ, 0) GO TO 374	00059700 R	0389
	IF(BARRAY(II) ,LT, ADD(NJ)) GO TO 374	00059800 R	0391
372	ARRAY(NLAST) = ADD(NJ)	00059900 R	0404
	KTAG(NLAST) = JTAG(NJ)	00060000 R	0414
	NJ = NJ - 1	00060100 R	0425
	GO TO 380	00060200 R	0426
374	ARRAY(NLAST) = BARRAY(II)	00060300 R	0428
	KTAG(NLAST) = KKTAG(II)	00060400 R	0438
	II = II - 1	00060500 R	0449
380	CONTINUE	00060600 R	0451
	NSORT = NI	00060700 R	0451
	DO 390 I = 1, NI	00060800 R	0452
	BARRAY(I) = ARRAY(I)	00060900 R	0457
	ITREF(KTAG(I)) # = 99	00061000 R	0468
390	KKTAG(I) = KTAG(I)	00061100 R	0480
C	CLEAR ARRAY	00061200 R	0485
	DO 449 I = 1, MAXK	00061300 R	0491
	IF(BLOCK(I) ,GT, 0) GO TO 449	00061400 R	0497
	IF(ITREE(I) ,EQ, -98 ,OR, ITREE(I) ,EQ, -99) GO TO 449	00061500 R	0503
	ITREF(I) = 0	00061600 R	0517
449	CONTINUE	00061700 R	0524
C	PRINT ARRAY	00061800 R	0524
500	IF(AVAL ,LT, TVAL) GO TO 510	00061900 R	0525
	TVAL = AVAL	00062000 R	0527
	IF(NP ,EQ, 0) GO TO 506	00062100 R	0528
	DO 505 I = 1, NP	00062200 R	0530
505	NBEST(I) = NPATH(I)	00062300 R	0536
506	NB = NP	00062400 R	0547
510	PRINT 5050, ITRE,MODE,AVAL	00062500 R	0548
5050	FORMAT(1X,2(I5,5X),10X,F10,1)	00062600 R	0562

```
IF(ISTOP .EQ. 1) GO TO 1000
IZ = 0
GO TO 99
1000 IF(NR .EQ. 0) GO TO 2000
DO 1100 I = 1, MAXK
1100 BLOCK(I) = 0
DO 1200 I = 1, NB
1200 BLOCK(NBFST(I)) = 1
PRINT 1111
1111 FORMAT(1H1,1X,40HPLOT OF POSITIVE NODES IN ULTIMATE PLAN //)
CALL PLOT
2000 RETURN
END
```

```
SEGMENT 6 IS 7 LONG
00062700 R 0562
00062800 R 0564
00062900 R 0565
00063000 R 0566
00063100 R 0568
00063200 R 0575
00063300 R 0579
00063400 R 0586
00063410 R 0595
00063420 R 0599
00063500 R 0599
00063600 R 0600
00063700 R 0603
SEGMENT 5 IS 622 LONG
```

		START OF SEGMENT *****	7
	SUBROUTINE PLOT	00063800 R	0000
	COMMON BLOCK(6000),ITREE(6000),LBK(6000),SORT(1020),ITAG(1020),	00063900 R	0000
	1NPR(101),NPL(30),IDA(6),JDA(6),KDA(6),IJK(6),SUM(30),MAXJ,MAXK,	00064000 R	0000
	2ICNN,NPOS(30),IT1(30),KTAG(1020),ADD(120),JTAG(120),ARRAY(1020),	00064100 R	0000
	3TZAP(60),NPATH(1020),BARRAY(1020),KKTAG(1020),NBEST(1020),	00064200 R	0000
	4LAREL(30)	00064300 R	0000
C	JCON = 24	00064400 R	0000
	ZERO = 0.	00064500 R	0000
	I4 = 4	00064600 R	0000
	I3 = -3	00064700 R	0001
	PRINT 1011	00064800 R	0002
	1011 FORMAT(10X,18HTABLE OF SYMBOLS/ 5X,6MSYMBOL, 7X, 11HEXPLANATION/	00065100 R	0003
	1 7X, 1H+, 7X, 13HPOSITIVE NODE/ 7X, 1H+, 7X, 27HCONNECTIVITY IN I	00065200 R	0007
	2= J PLANF/ 7X, 1HU, 7X, 27HCONNECTIVITY WITH K=1 PLANE/ 7X, 1HD,	00065300 R	0007
	3X, 27HCONNECTIVITY WITH K=1 PLANE/ 7X, 1HB, 7X, 33HCONNECTIVITY WI	700065400 R	0007
	4TH ADJACFNT PLANES//)	000065500 R	0007
		00065600 R	0007
C	I4 = MAXJ	00065700 R	0007
	ITFR = (IJK(2) + JCON - 1)/JCON	00065800 R	0007
	DO 100 K = 1, IJK(3)	00065900 R	0009
	IEND = 0	00066000 R	0012
	I4 = I4 + MAXJ	00066100 R	0018
	JVAL = 0	00066200 R	0019
	DO 100 JPLOT = 1, ITER	00066300 R	0021
	PRINT 1001, K	00066400 R	0022
	1001 FORMAT(/1X, 9HSECTION ,15/)	00066500 R	0027
	ISTART = IEND + 1	00066600 R	0038
	IEND = ISTART + JCON	00066700 R	0038
	IF(JPLOT .EQ. ITER) IEND = IJK(2)	00066800 R	0039
	PRINT 1002, (N,N=ISTART,IEND)	00066900 R	0040
	1002 FORMAT(10X,14H*** COLUMN ***/ 4X,3HROW,25I4)	00067000 R	0043
	PRINT 1012	00067100 R	0060
	1012 FORMAT(/	00067200 R	0060
	IE = (IEND - ISTART) + I4 + 1	00067300 R	0064
	INT = IEND - ISTART + 1	00067400 R	0064
	I2 = JVAL + I4 = IJK(1)	00067500 R	0066
	DO 49 I = 1, IJK(1)	00067600 R	0068
	INDEX = I2 + I	00067700 R	0070
	NO = I3	00067800 R	0077
	DO 2 II = 1, IE	00067900 R	0079
2	NPR(II) = 1H	00068000 R	0079
	DO 3 III = 1, INT	00068100 R	0086
3	NPL(III) = 1H	00068200 R	0094
	DO 4A J = ISTART, IEND	00068300 R	0100
	INDEX = INDEX + IJK(1)	00068400 R	0108
	NO = NO + I4	00068500 R	0113
	IF(BLOCK(INDEX) .LE. ZERO) GO TO 4B	00068600 R	0116
	NPR(NO) = 1H+	00068700 R	0117
9	IF(J .EQ. IJK(2)) GO TO 15	00068800 R	0122
	NDEX = INDEX + IJK(1)	00068900 R	0131
	IF(BLOCK(NDEX) .LE. ZERO) GO TO 15	00069000 R	0134
	DO 13 KK = 1, 3	00069100 R	0136
13	NPR(KK+NO) = 1H+	00069200 R	0142
15	IK = 0	00069300 R	0148
	IF(K .EQ. 1) GO TO 20	00069400 R	0157
		00069500 R	0157

```

NDEX = INDEX + MAXJ
IF(BLOCK(NDEX) .LE. ZERO) GO TO 20
NPR(NO) = 1HU
IK = 1
20 IF(K .EQ. IJK(3)) GO TO 30
NDEX = INDEX + MAXJ
IF(BLOCK(NDEX) .LE. ZERO) GO TO 30
NPR(NO) = 1HD
IF(IK .EQ. 1) NPR(NO) = 1HB
30 IF(I .EQ. IJK(1)) GO TO 48
NDEX = INDEX + 1
IF(BLOCK(NDEX) .LE. ZERO) GO TO 48
NN = J - ISTART + 1
NPL(NN) = 1H*
48 CONTINUE
JVAL = INDEX + IVAL
PRINT 1003, I, (NPR(L), L=1, IE)
1003 FORMAT(2X, I5, 3X, 101A1)

IF(I .EQ. IJK(1)) GO TO 49
PRINT 1004, (NPL(L), L=1, INT)
1004 FORMAT(10X, 24(A1, 3X))
49 CONTINUE
100 CONTINUE
RETURN
END

```

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00069600 R 0160
00069700 R 0162
00069800 R 0167
00069900 R 0175
00070000 R 0177
00070100 R 0180
00070200 R 0182
00070300 R 0188
00070400 R 0195
00070500 R 0205
00070600 R 0208
00070700 R 0209
00070800 R 0215
00070900 R 0216
00071000 R 0225
00071100 R 0225
00071200 R 0226
00071300 R 0250
SEGMENT 8 IS 125 LONG
00071400 R 0250
00071500 R 0253
00071600 R 0275
00071700 R 0275
00071800 R 0276
00071900 R 0277
00072000 R 0280
SEGMENT 7 IS 290 LONG

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