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# Operations Research in Underground Mine Planning: A Review

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**Abstract.** At the time of this writing, the U.S. Geological Survey estimates that the average American-born human will need millions of pounds of fuels, minerals, and other extracted resources in his or her lifetime. Mining is a critical global industry, spanning all but one continent (Antarctica), with the highest-producing countries being China, the United States, Russia, Australia, and India. Increasingly, this demand is driving mining companies to explore and pursue deeper mineral deposits as near-surface deposits deplete. Correspondingly, there has been a significant rise in industry interest in applying operations research techniques to improve underground mine planning. Newman et al. [Newman AM, Rubio E, Caro R, Weintraub A, Eurek K (2010) A review of operations research in mine planning. *Interfaces* 40(3):222–245] present a review of such techniques, applied to both open pit and underground mining operations. We focus here on the advancements since that publication and concentrate on underground applications in metalliferous deposits, such as copper, iron, and gold.

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## Introduction

Mining is a critical industry. An average person in the United States uses approximately 39,000 pounds of minerals per annum (U.S. Geological Survey 2019). Mining is the process of extracting naturally occurring minerals from the earth through various methods and can be simply categorized as surface or underground mining, depending on whether the deposit is shallow or deeper lying, respectively. Both of these broad categorizations require a mine plan to extract ore profitably and safely. Open pit—namely, surface—mining operations have certain advantages over those related to underground mines, such as (i) faster access to the orebody, thereby creating revenue earlier in the mine’s life, and (ii) less up-front capital and shorter construction time to build the necessary infrastructure needed to reach the deposit. However, the declining discovery rate of large near-surface deposits, increasing environmental activism, and the need for a comprehensive social license to mine have been outweighing the benefits of open pit mining (Sparshott and Walls 2009, Jakubec 2016). Advancements in underground mining equipment, mineral processing, and safety technology have improved the economic margins of underground mineral deposits. The mining

industry has, in turn, called for new analytical methods and tools to improve the mine planning process. Newman et al. (2010) reviewed such operations research techniques applied to open pit and underground mining operations. At the time of their review, underground mine operations research techniques lagged significantly behind those applied to open pit operations. In the decade since, a number of researchers have addressed this shortcoming; we review these works here, confining ourselves to the areas of underground mine design and production scheduling, primarily in metalliferous deposits (e.g., copper, iron, and gold). Although we focus on literature that has appeared shortly before 2010 and more recently, we do mention earlier work, particularly if it (i) is seminal, (ii) does not appear in Newman et al. (2010), and/or (iii) provides historical context.

Mine planning determines strategic and tactical decisions for the exploitation of mineral reserves to maximize value while considering regulatory requirements (such as safety measures and social license to operate) and production scheduling constraints (such as precedence between extractable areas of an orebody and mining capacity). Open pit metalliferous deposits utilize modeling structures that are relatively consistent

in their operations and in the corresponding modeling decisions related to design and extraction. For example, an open pit mine must be extracted from the surface downward, whereas underground mines display more degrees of freedom with respect to mining direction. The mine plan adopted to economically exploit a mineral deposit can vary and is dependent on factors such as geology, the position of the orebody underground, and the strength of the host rock (O'Sullivan et al. 2015).

An underground mine consists of a series of openings needed to access minerals for economic benefit. The size, shape, location, and number of openings depend on the mining method, geological characteristics of the deposit, and operational requirements (Bullock 2011b). For example, Figure 1 depicts the layout of a sublevel stoping operation. The illustration is generic but indicates infrastructure common to underground mines. First, all underground mines need primary access to the underground deposit, which is established through permanent infrastructure such as a vertical shaft and/or decline (ramp) system. This permanent infrastructure, called primary development, is used to transport miners, materials, ore, and waste to and from the surface. Secondary development emanates from the shaft or decline to access the orebody—that is, the currently identified profitable extents of the mineral deposit. Examples of secondary development

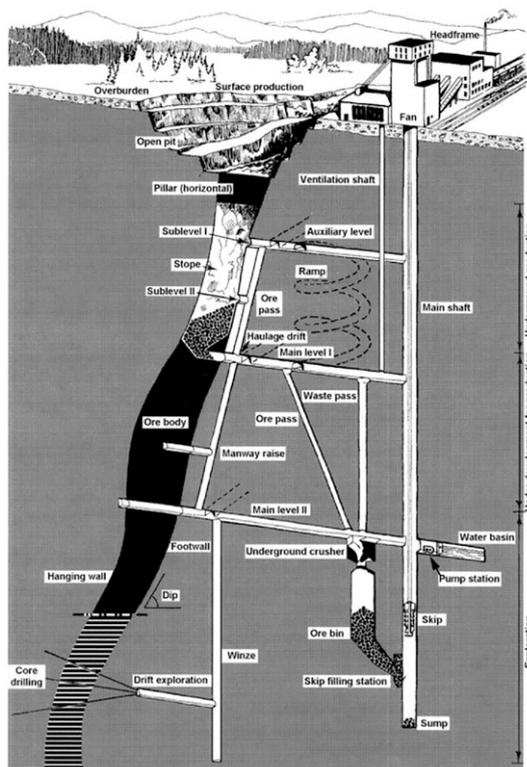
include production levels to access different areas of the orebody, ore passes to transport broken ore via gravity between levels, haulage levels to prepare and transport ore to the surface, and vertical raises to create a ventilation network. In addition to serving as roadways for equipment and miners, these developments carry essential utilities, such as electricity, water, and air (ventilation), to the working areas. Ore production occurs by mining stopes, which are profitable sections of the orebody identified during the design process. Once a stope has been mined, it may be back-filled with waste material to provide ground support or to facilitate the mining of adjacent stopes. Mined-out working areas—that is, parts of the orebody that have been exhausted of ore—and the respective secondary developments leading to them can be closed.

The scope of mine planning decisions moves from strategic to tactical as the mining project evolves. Prior to production, a mineral deposit is explored, and pre-feasibility and feasibility studies are conducted to determine the extent of the orebody, assess the economic potential and technical viability of extraction, and secure the capital investment necessary for initial infrastructure and development. The analysis specifies details regarding the implementation of an initial mine plan including (i) a geological model, (ii) mine design, and (iii) a production schedule.

Geological modeling, an initial step, discretizes the underground volume of interest into manageable units, mapping grade estimations to blocks according to sampling and statistical simulation and then organizing the result into a block model. Grade is defined as the amount or percentage of valuable commodity contained in a unit mass of rock. Nonuniform mineralization is a common feature in many metalliferous deposits and is a consequence of inherent geological variability.

In a subsequent step, a detailed mine design for (a) given mining method(s) is developed based on strategic inputs, such as production rate, cutoff grade, geotechnical data, and geological information (from the block model). *Production rate* is a measure of the mass or volume of material mined per time period and is dependent on the equipment employed at the operation. Mine planners distinguish ore from waste rock with a *cutoff grade*, which is determined by factors such as commodity price and mining and milling costs (Rendu 2014). Multiple cutoff grade values may be applied, depending on intended use (e.g., ore destined for stockpiling versus processing). Multiple mine designs may need to be evaluated based on the range of available, nondominated strategic inputs. Rock strength and geological structure information (geotechnical data) are used in simulation studies with finite element analysis to assess the stress behavior of the rock mass in the presence of underground excavations.

**Figure 1** Typical Underground Development Layout



Source: Hamrin (1998).

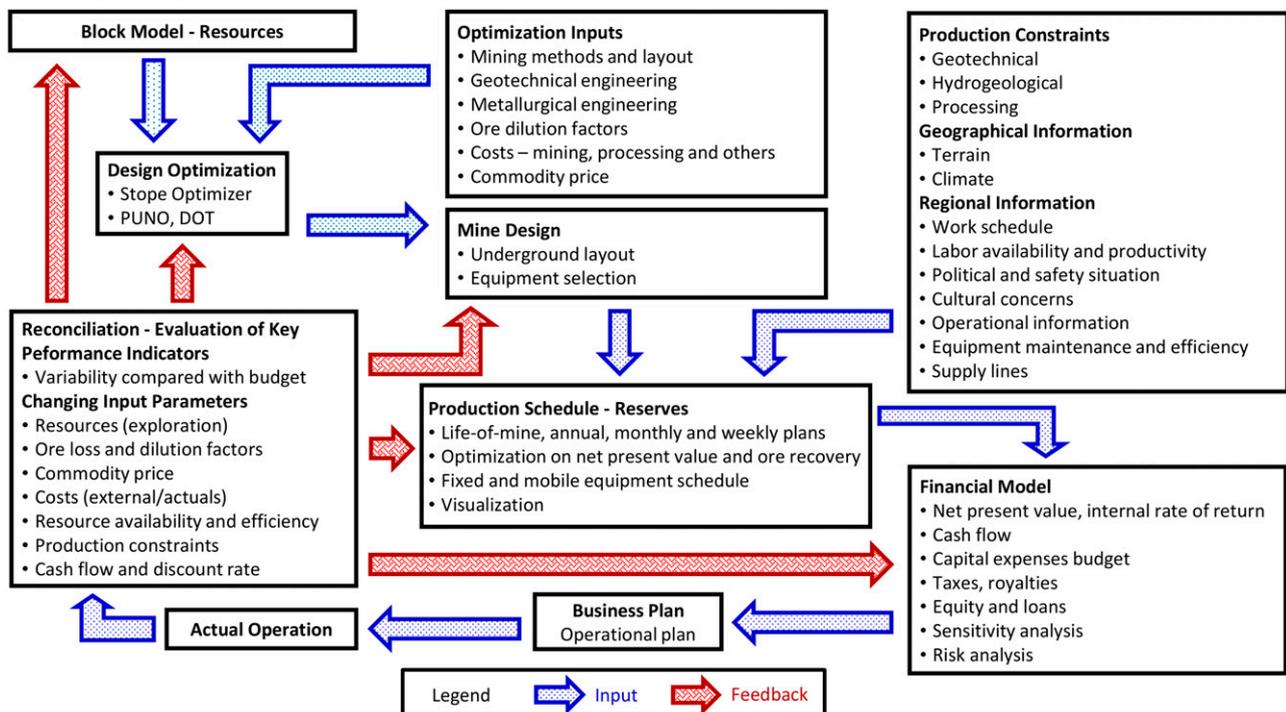
The last step consists of production scheduling, wherein the planner defines the extraction sequence of the orebody while considering *geological data* and adhering to technical rules associated with design and resource availability. Steps (ii) and (iii) lend themselves to operations research approaches, most commonly through optimization, although simulation and queuing theory are also employed. The mine design must enable the extraction of material from the mine to make a profit or to meet a contract while adhering to constraints on ground stability and safety. The myriad of factors that must be considered in the creation and evaluation of a production schedule include (i) economics, (ii) precedence constraints, and (iii) resource limitations. Economic factors encompass the (estimated or current) price of the commodity and may include fixed capital and variable operational costs associated with the removal of ore and waste. Precedence constraints preclude certain blocks of rock from being extracted before others and, in the underground mine planning setting, often include considerations involving preparation of an area to be mined and subsequently backfilled to create stability associated with a void (Kloppers et al. 2015). Resources are connected with the availability and capacity of equipment, such as haul trucks and infrastructure (e.g., processing plants) as well as with factors that help mitigate a hazardous work environment. Watson

(2009) emphasizes the importance of a proper mine design process for achieving production targets.

The inherent geologic variability of mineral deposits requires accounting for site-specific conditions, evaluating different operational scenarios, and identifying appropriate best practices. Decisions associated with developing a mine plan are difficult because of the plurality of feasible options (Bullock 2011a). Mine design and scheduling are part of the iterative and cyclical mine planning process (see Figure 2), in which multiple scenarios must be considered and evaluated. Feedback from previous decisions, completed operations, and observations can assist future decision making (Sebutsoe 2017).

A significant portion of the advancements in underground design and scheduling have arisen owing to advancements in optimization modeling and algorithmic capabilities. General-purpose linear and mixed-integer (nonlinear) solvers, such as CPLEX (IBM Corporation 2019) and Gurobi (Gurobi Optimization LLC 2019), have proved effective both in and of themselves and when embedded within another (e.g., decomposition) framework. In a related vein, less than a decade before the time of this writing, authors assumed that there was no single uniform way in which underground mine planning could be expressed as an integer program. Although underground operations are typically much more heterogeneous than their

Figure 2. (Color online) Iterative Nature of Underground Mine Planning



Source. Adapted from Fuykschot (2009).

Note. DOT, Decline Optimisation Tool; PUNO, Planar Underground Network Optimiser.

open pit counterparts (O’Sullivan et al. 2015), a general formulation applicable to many operations, regardless of method, exists. We use the notation that  $a \in \mathcal{A}$  denotes the set of activities to be executed under a given design, the predecessors to each activity  $a$  populate the set  $\mathcal{P}_a$ , and each activity is associated with a net present value  $c_{at}$  and a resource consumption  $q_{ra}$  whose maximum value for a given resource  $r$  in each time period  $t$  is  $\bar{r}_{rt}$ . We then determine whether activity  $a$  starts in time period  $t$ ,  $X_{at}$ , to maximize the value of the schedule of activities subject to resource and precedence constraints. Other considerations are the duration of each activity, the delay associated with a pair of activities, and resource constraints that may be aggregated over multiple time periods. We present this binary program, known as a *resource-constrained project scheduling problem* (RCPSP), in Appendix A, and note that it can be solved efficiently with the academic software OMP (Rivera et al. 2015), based on a specialized algorithm (Biestock and Zuckerberg 2010) that provides the linear-programming relaxation solution of the integer scheduling model. Rocher et al. (2011) formulate the underground mine production scheduling problem to consider both development advance and ore production so as to create a more robust schedule, implemented as part of the Underground Development Sequencer and Scheduler as a callable Python library linked to CPLEX and Gurobi. It is within the paradigm discussed in this paragraph that much of the literature found herein exists, though we also cite advancements related to stochastic methodologies, such as discrete-event simulation, Monte Carlo simulation, and other decision analysis tools, as appropriate.

### Underground Mine Design

Hall (2009) documents how traditional methods that include cost reduction policies to meet production targets do not achieve strategic goals such as maximizing net present value, maximizing return on

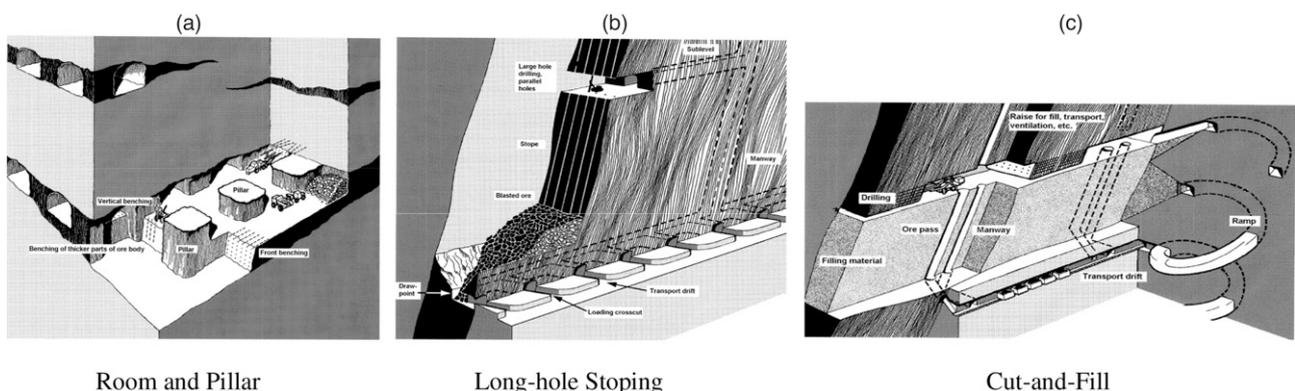
investment, or minimizing deviation from production goals. The author treats cutoff grade as time varying and accounts for development costs and production. Strategic decisions are typically associated with fundamental activities that affect the long-term value of a project, such as mining method, cutoff grade, production rate, and equipment fleet. These decisions have a long-term impact on the operation because they are often difficult and/or costly to modify or reverse (Elkington et al. 2010). The corresponding analysis considers insights from several models at a tactical level to create a global mine plan, which examines the interactions and influences between individual systems, omitting detail to maintain tractability (Little et al. 2012, Dowd et al. 2016). Many strategic decisions are made during the initial planning stages of a mining operation and are supported by a feasibility study (Rendu 2014). Five primary strategic decisions are (i) mining method selection, (ii) cutoff grade, (iii) layout and infrastructure, (iv) open pit-to-underground transition, and (v) equipment selection. We address each of these in turn.

### Mining Method Selection

The optimal mining method is the means by which to most profitably extract ore given geological, economic, and engineering conditions. There are numerous underground mining methods based on how the rock mass is supported during mining; broadly speaking, these are classified as (i) unsupported, such as, for example, room-and-pillar mining and sublevel stope mining; (ii) supported, such as, for example, cut-and-fill mining and longwall mining; and (iii) caving, such as, for example, sublevel caving and block caving (Hamrin 1998, Okubo and Yamatomi 2009).

For horizontally bedded deposits with a strong host rock, *room-and-pillar* mining (Figure 3(a)) extracts a series of rooms within the orebody and leaves pillars of

Figure 3. Common Underground Methods



Source: Hamrin (1998).

ore in place to support the overlying strata. This results in a repeating pattern, with room and pillar sizes determined by the strength of the pillar material and depth of mining; lower-strength material and greater depth result in larger pillars and smaller rooms. It may be possible to extract a subset of the pillars if geotechnically feasible—that is, if the remaining pillars are able to sustain the increased vertical stresses. This is known as retreat mining and begins at the pillar furthest from the exit. Steeply dipping deposits, where the orebody is particularly thick and the host rock is strong, are suited to the large-scale and economically efficient *long-hole stoping* process (Figure 3(b)), in which a sublevel is developed below the ore to be excavated. The ore is then drilled and blasted, after which it falls to the sublevel, where a loader scoops up the broken rock and carries it to a truck or ore pass to be transported to the surface. Where the mine has poor host rock strength, *cut-and-fill*, also known as *drift-and-fill*, mining is used (Figure 3(c)), in which a corridor of ore, known as a drift, is removed from the orebody. The resultant void is then backfilled with either waste rock or processing tailings, both of which would be combined with a binding agent, such as cement, to provide structural support before mining an adjacent drift through the ore (Hamrin 2001). Sometimes, more than one method is employed to extract ore in a single mine (O’Sullivan and Newman 2014).

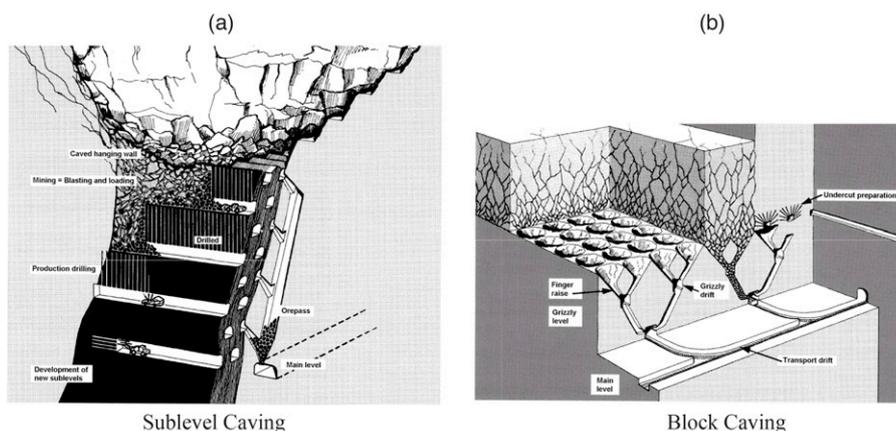
In *sublevel caving* (Figure 4(a)), parallel developments run along or across a steeply dipping orebody from the footwall—that is, the underlying side of the orebody. The ore is then drilled, blasted, and transported. Mining progresses toward the footwall and top-to-bottom along increasingly deeper levels, with the hanging wall (rock overlying the deposit) caving behind. By contrast, the *block caving method* (Figure 4(b)) is applied

to massive orebodies, whereas caving itself is used to effect ore breakage through a combination of internal rock stresses and weight. All development, consisting of drifts, drawbells, and haulage infrastructure, is located on a single extraction level below the orebody and is constructed prior to production. Once in place, the orebody is undercut—that is, a slice of the orebody supporting the overlying rock is extracted, thus allowing the orebody to cave. The broken ore is carefully drawn down to prevent the creation of voids in the caved material whose collapse can lead to hazardous conditions, such as an air blast, or an inrush of broken material into the working face (Abzalov 2016).

Mining method selection has evolved from being based on engineering intuition to various qualitative ranking systems. The *Nicholas method* (Nicholas 1981) represents the first truly quantitative ranking system. It uses input parameters, such as deposit geometry, grade distribution, geotechnical assessments, economics, and risk sources, to rank the applicability of different mining methods to a deposit. However, drawbacks to the Nicholas method and its derivatives include imprecise definitions of weights and only slight differentiation between favorable and unfavorable scores (Bitarafan and Ataei 2004).

Subsequently, researchers have attempted to alleviate these concerns by applying multicriteria decision-making methods. For example, Musingwini and Minnitt (2008) use the *analytic hierarchy process* to compare four mining methods for the UG2 Reef, a geological formation in South Africa containing platinum-group metals, where the selected mining method provides the best trade-off between factors such as extraction ratio of ore and waste, production rate, and productivity while simultaneously minimizing dilution, capital costs, and operating costs. Bogdanovic et al. (2012)

Figure 4. Underground Caving Methods



Sources. Hamrin (1998).

Note. Caving methods are generally used for larger-scale, deep, vertical operations and are based on fracturing the mineralized and surrounding waste rock under controlled conditions.

and Gupta and Kumar (2012) also use the analytic hierarchy process to determine weights of individual selection criteria. The former also implement the preference ranking organization method for enrichment evaluation (PROMETHEE) to determine a ranking based on all mining methods, whereas the latter use the weights specifically for stope mining method variants. PROMETHEE offers a prescriptive ranking for complex multicriteria problems that are characterized by expert judgment (Brans and Mareschal 2005). Ataei et al. (2013) add Monte Carlo simulation to determine the confidence level of each alternative with respect to the variance of different decision makers' preferences; the conventional analytical hierarchy process determines the ranking of the individual mining methods.

Yavuz (2009) determines an underground mining method for the Kutahya-Sabuncupinar chromite orebody using *Yager's algorithm*, a process of select-prioritize-rank for a finite number of alternatives (Franceschini et al. 2016). Kabwe (2017) similarly uses this method, as well as a numerical ranking and the analytical hierarchy process, to determine that sublevel caving (over cut-and-fill) is optimal for a copper mine in Zambia. Nieto (2011) uses indicators that quantify deposit characteristics and map the advantages and disadvantages of individual mining methods to favorable deposit indicators; the author presents a simple ranking approach and provides a corresponding analysis. Javanshirgiv and Safari (2017) use multicriteria decision making (specifically, fuzzy logic) to determine which variant of stoping to employ at a fluorine mine, considering its geometry and grade distribution. Peskens (2013) determines a layout and design (including required rock support) for a copper mine in Finland that employs stoping using the methods espoused both by Kabwe (2017) (analytical hierarchy process) and by Javanshirgiv and Safari (2017), as well as the more classical Nicholas method (Nicholas 1981), to generate millions of dollars more profit than with the default method. Other authors either treat the problem more generally (e.g., Kenzap and Kazakidis 2013 and Balusa and Gorai 2019) or use established methods (Dehghani et al. 2017). See Iphar and Alpay (2019) for a review of traditional (qualitative) mining method selection processes and multicriteria decision-making models. With these ranking systems, the science of determining the best mining method has progressed from subjective analysis to a more objective approach, leveraging decision analysis tools based on expert elicitation that consider previous success in similar geological circumstances. Although these tools can eliminate dominated choices, they still sometimes fail to distinguish quantitatively between suitable options and instead rely on organizational familiarity for final selection. Table B.1 in Appendix B summarizes the relevant papers.

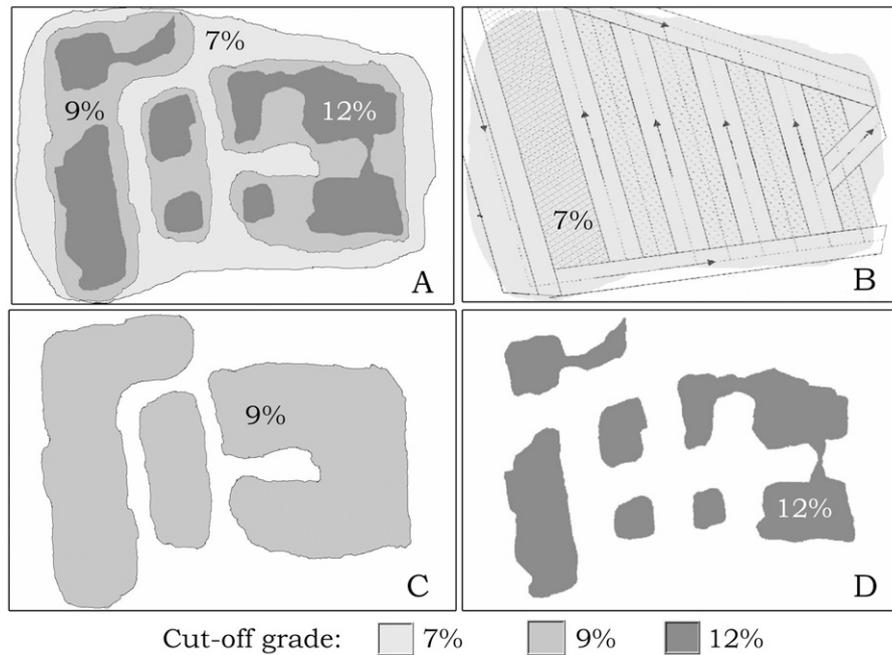
## Cutoff Grade

Cutoff grade is one attribute used to determine the extractable shapes of ore (Figure 5). A lower cutoff grade treats more rock as ore, which increases tonnage extracted at a lower average grade. Additional development may be required for lower cutoff grades, which can be time consuming and expensive. A higher cutoff grade yields less tonnage extracted and at a higher average grade and therefore fewer total ounces of metal produced; the reduced extraction effort may improve overall operational economics. An extremely high cutoff grade may not produce enough metal to justify the capital outlays.

Seminal work on cutoff grade selection lies in open pit mining (Lane 1988), which considers mining as a three-stage operation consisting of extraction, processing, and refining. Each stage's operating capacity determines a possible cutoff grade, with one of these representing the optimum contingent on the status of the operation. Hall (2014) describes the importance of cutoff grade in modern strategic mine planning and demonstrates commonly used methods in its determination (e.g., breakeven analysis, Mortimer's definition, and Lane's methodology). McIsaac (2008) develops a Monte Carlo process to set a commodity price combined with a production rate and cutoff grade to create realizations of profitability and recommends a robust long-term strategic plan. Elkington et al. (2010) compare simulation-based strategies for representing grade and cost uncertainty in selecting a cutoff grade. Gu et al. (2010) use dynamic programming to refine cutoff grades for a given mining method and corresponding design to maximize net present value. The authors assume a sequence of predetermined mineable sections to find the optimal final state and associated cutoff grade policy for each section. To the same end, Roberts and Bloss (2014) modify an optimization model configured for open pit mine design to accommodate stope layout for a given set of enumerated characteristics, for example, cutoff grade and mine development rate; the resulting schedules, although they do not necessarily yield a global optimum, provide strategic planning guidance.

Modern mine planning software allows operators to consider different mine designs and to evaluate a number of possible options. Alford and Hall (2009) identify tools to create thousands of stopes in a matter of seconds for any cutoff grade, eliminating the tedious task of manual drawing. Therefore, the mining industry is examining more cutoff grades than ever. However, mine planners are limited in their ability to analyze the net present value of a mine schedule for each cutoff grade for the following reasons: (i) underground production schedules are difficult to generate even for a fixed cutoff grade, with many mines still relying on heuristics (O'Sullivan and Newman 2015);

Figure 5. Effect of Cut-off Grades on Extractable Ore



Source: O'Sullivan and Newman (2014).

Note. Box A shows a representative composite of ore grades, a low cut-off grade (Box B) yields a large area classified as ore, whereas increasingly higher cutoff grades produce less extractable material (e.g., Boxes C and D).

(ii) studies are often performed in isolation and/or by examining a subset of cutoff grades; and (iii) the sheer number of cutoff grades that can exist in an underground mine precludes a thorough manual analysis. It is possible to use a mixed-integer program to improve the productivity of a mine complex by selecting a cutoff grade for each mine (Collard 2013). King and Newman (2018) leverage the work of Alford et al. (2007) to dynamically create a stope layout for a set of viable cutoff grades and evaluate the design for a fixed set of cutoff grades by determining an associated schedule, thereby linking decisions usually made in isolation; their ability to solve large instances is possible through the exploitation of the underlying RCPSP structure of their model. Table B.2 in Appendix B summarizes the relevant papers.

### Layout and Infrastructure

Underground layout and infrastructure decisions include determining the number and placement of developments and production areas with respect to the orebody. These decisions often follow the selection of a mining method. For stope mining methods such as cut-and-fill and long-hole stoping, the size and location of stopes must be determined to ensure a sufficiently high production rate—that is, the percentage of mineable reserves extracted—and a geotechnically safe working environment. Analogous constructs in other mining methods are (i) the size of pillars in room-and-pillar mining and (ii) drawbell

and drawpoint placement in block caving and sublevel caving operations, respectively. The primary criteria influencing room-and-pillar layout decisions are geotechnical—that is, whether the pillars can withstand the vertical stresses exerted by the overlying strata. The critical design element in block cave layouts is drawpoint spacing because it has a significant impact on production, dilution, and extraction (Ugarte et al. 2017).

**Stope Layout.** Bootsma et al. (2014) contrast the prevalent approach to prescribing stope shapes from a geological block model—that is, using an estimated resource model and fixed cutoff grade—with a conditional Gaussian simulation model so as to incorporate grade uncertainty into the stope design process. They leverage the Mineable Shape Optimizer (Alford 1995, Alford Mining Systems 2019) to determine different stope shapes at varying confidence intervals to reduce economic risk. Bai et al. (2013) present a *maximum flow algorithm* in which blocks (represented by nodes), or some subset thereof, constitute a single stope. The blocks chosen are subjected to constraints on stope wall angles, stope width, and stope height with respect to a single raise (i.e., vertical excavation). An optimally sized stope possesses maximum value. Bai et al. (2014) extend the previous work to also include the placement and length of vertical slots (i.e., raises) used to create a void to allow for the expansion of rock when blasted. The authors demonstrate the procedure on synthetic and real data. A

further extension eliminates unfavorable stopes based on additional geotechnical and shape considerations (Nelis et al. 2016). Nikbin et al. (2020) combine dynamic programming with a greedy algorithm to improve on more conventional methods—that is, *floating stope* and *maximum value neighborhood*—for determining stope boundaries. Their methodology yields a range of profits between more than twice as much for the former method to less than a percentage better than the latter algorithm for a silver-zinc-lead deposit containing more than a million blocks.

Sandanayake et al. (2015) implement an enumerative heuristic, respecting physical mining and geotechnical constraints, to determine from a subset of possible stope layouts that which maximizes the value of an underground copper project composed of 47,052 blocks. Erdogan et al. (2017) apply to an existing mine two industry-practiced stope boundary optimization techniques, *floating stope* and *maximum value neighborhood*, and two heuristics introduced by Topal and Sens (2010) and Sandanayake et al. (2015); the authors compare the profits associated with the stope layouts to that given by Bootsma et al. (2014) and conclude that none of the resulting layouts yields as high a profit, although all approaches may yield infeasibilities. Sipeki et al. (2020) consider a related mining method, top-down open-stoping, but one in which pillars, rather than backfill, provide structural stability. This implies that an important consideration is to minimize the amount of ore left in situ while still satisfying geotechnical stability constraints. The authors use a particular, relaxed integer-programming formulation to enhance model tractability.

**Room-and-Pillar Layout.** The design emphasizes determining the pillar size and room span. Finite element analysis is commonly used to assess and size pillars, given geotechnical parameters such as rock strength, depth of mining, and discontinuities present in the host rock. The location of pillars is defined by a uniform pattern depending on regions with similar geotechnical properties. Anani (2016) determines an optimal sequence of room-and-pillar development using integer programming, respecting operational, quality, and precedence constraints according to objectives of (i) maximizing net present value and (ii) minimizing the discounted risk to each mining unit. This method is validated on a room-and-pillar mine in a lead deposit containing 4,722 blocks, each assigned geologic attributes including grade and geotechnical risk. Anani et al. (2017) investigate the impact of panel width on the cost and productivity of a room-and-pillar operation through discrete-event simulation. Gligorić et al. (2019) consider uncertainty in metal price, the value of extracted ore, and resources required for extraction in scheduling a room-and-pillar mine using a fuzzy optimization model in which their variables represent

whether a specific collection of blocks (i.e., a mining cut) is extracted in a given time period. They argue that the use of multiple instantiations of the objective to capture uncertainty provides a means to determine a schedule under uncertainty, and they illustrate their argument using a small case study.

**Caving Layout.** Caving layout decisions rely on specialized geotechnical models to ensure long-term stability of infrastructure under high stress stemming from consistent, long-term drawdown of caving material. Accurately predicting crack propagation properties of the orebody is critical in the development of an effective ore drawdown strategy. Gomes et al. (2013) provide a geotechnical risk assessment for the design of developments at the Chuquicamata copper mine. They use a Monte Carlo simulation model to account for risks such as seismic activity, groundwater infiltration, and geologic discontinuities impacting the safety and effectiveness of construction and mining operations to recommend a probabilistically robust set of extraction and support measures. Rafee et al. (2018) combine the *bonded particle model* for intact rock and *discrete fracture network simulation* to investigate the influence of seven rock qualities, changing the value of one parameter while fixing the other six. Zarate et al. (2018) inform drawpoint-layout and extraction-level decisions for a block cave mine design under grade uncertainty through enumeration of equally probable ore concentration realizations; they demonstrate the robustness of this approach over the traditional planning method that considers only a single realization.

**General Infrastructure Layout.** The following authors examine methods for optimally locating shafts and declines, which could be applicable to any underground mining method. Zuo et al. (2013) use multiobjective optimization for underground mine design considering interactions between production, safety, and environmental impacts in order to appropriately size infrastructure and meet strategic goals. They introduce an evolutionary algorithm to recommend a strategic design for an operational lead and zinc mine. Brazil et al. (2014) demonstrate a procedure for optimally placing a decline juncture, allowing for multiple concurrent faces to be developed, primarily for haulage operations, in order to maximize net present value. The authors use the Decline Optimization Tool, cast as a dynamic program, to choose between different decline configurations with maximum and minimum design considerations (e.g., gradient, turning radius); the Planar Underground Network Optimizer tool designs the secondary network of drives needed to connect the primary access point to the ore drives, using a weighted Steiner tree with the objective of determining the

least-cost design. In a similar vein with respect to the operations and objective function, Sirinanda et al. (2014, 2016) use a *gradient-constrained discounted Steiner point* algorithm to identify infrastructure junctures and to connect multiple sections of an orebody through declines. Carpentier et al. (2016) seek a robust cutoff grade for a cluster of underground nickel mines that use the same labor and material resources; their two-stage stochastic program includes mine opening and closure, as well as precedence and elastic constraints on mining operations (e.g., development and extraction), and it maximizes net present value and minimizes deviation from target production and geological risk. Grossman et al. (2018) develop a recursive algorithm and determine its theoretical complexity, to maximize the discounted value of extracted ore by representing the model as one of traversing nodes in a tree on a graph. The problem then is to determine which nodes to visit, representing which areas of the orebody to extract for a profit, and in what order. The single resource constraint precludes more than one mining activity from being executed at a time. Table B.3 in Appendix B summarizes the relevant research in underground mine layout.

### Open Pit-to-Underground Transition

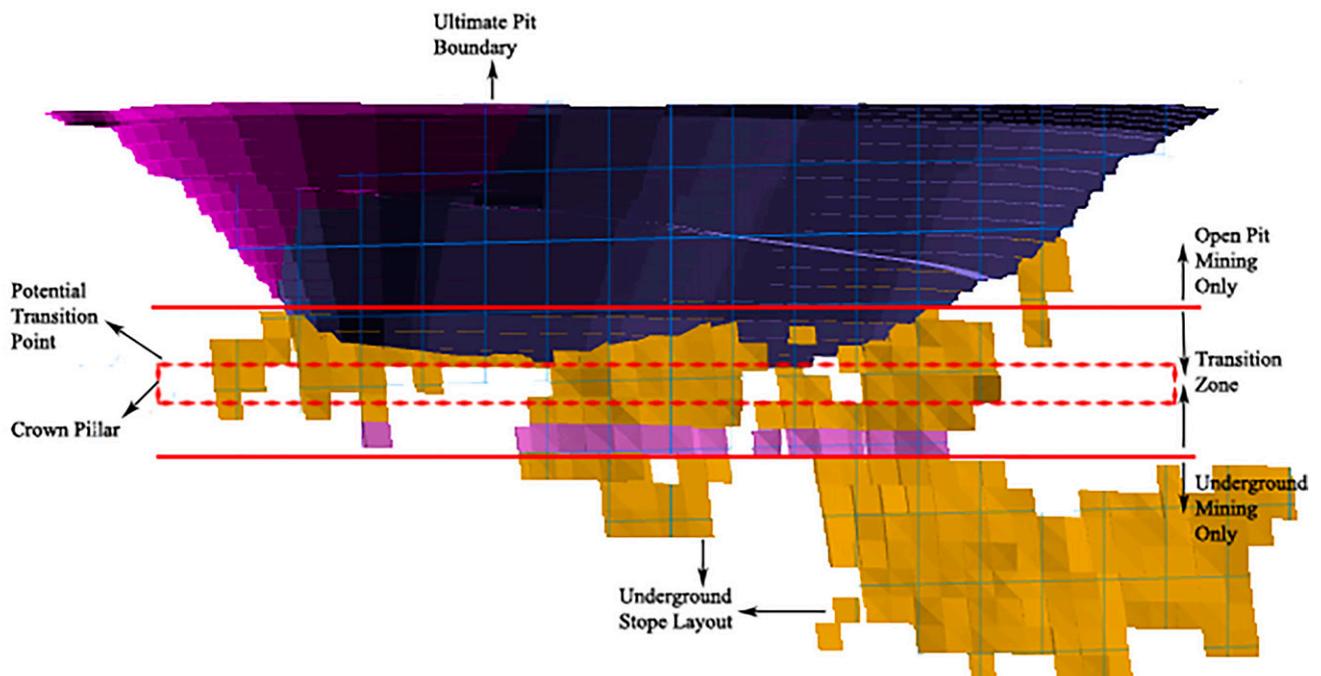
In addition to operations that consider a single broadly categorized mining method, many mines at the time of this writing are considering the transition

between open pit and underground as the depth of the open pit mine becomes too deep to economically and/or safely sustain. Or, alternatively, a prefeasibility or feasibility study may find certain cases in which the orebody limits are amiable to both open pit and underground mining (at different depths), leading to the decision to select a design using both methods with a deliberate transition (see Figure 6). In this case, there is a distinct portion of the orebody that is extracted via surface methods, and another part is extracted via one or more underground methods. A *crown pillar* separates and provides structural stability between these two areas. Eventually, the crown pillar may be mined out as well.

The Cripple Creek and Victor gold district in Colorado, United States, is an example of an expansive deposit extracted via both open pit and underground methods. Around the turn of last century, miners sought high-grade, narrow veins through underground mining, which was made economically possible by a high price per ounce of gold at that time. After World War II, with a fixed gold price and inflation, underground mining became economically infeasible (Sprague 1953). However, in the late 1970s, advancement in processing technologies allowed low-grade ore near the surface to be profitably extracted and processed, which induced economic feasibility of the open pit mining operation.

The decisions of when and how deep to install the underground infrastructure and the integration of

Figure 6. (Color online) Transitioning from Open Pit Mining to Underground Mining



Source: King et al. (2017).

Note. A crown pillar is left in place at the optimal transition point to provide structural stability.

extraction schedules above and below ground have a sizable impact on profits (Newman et al. 2013). Finch and Elkington (2011) provide an introduction to the open pit-to-underground transition problem and factors associated with determining a good transition point. The author also lists three manual approaches for determining the transition point, considering (i) the biggest economic pit, (ii) incremental undiscounted cash flow, and (iii) automated scenario analysis. Opoku and Musingwini (2013) determine the feasibility of an open pit-to-underground mine transition; they develop mine designs from different realizations of resource distributions based on block models and establish the validity of a transition based on the following indicators: (i) stripping ratio, (ii) average run-of-mill grade, (iii) profit margin per unit of mineral commodity, (iv) net present value of the operation, and (v) production rate. They apply this methodology to five international mineral deposits. Ben-Awuah et al. (2016) examine a mixed-integer program that considers both open pit and underground mining methods in concert. The authors include detailed mathematical stockpiling and blending constraints but only summarize geospatial requirements pertaining to infrastructure and precedence. The scale of the model is not specified, but instances can be solved as monoliths. The conclusion suggests that open pit and underground methods should be considered in concert to take advantage of the relative economic benefits of both. Chung et al. (2016) determine an optimal mining strategy for an orebody, weighing the extraction options of open pit only, open pit to underground (considering stoping or block caving with an arbitrary transition point), and underground only, using an integer programming formulation. MacNeil and Dimitrakopoulos (2017) calculate the optimal production schedule at each depth of the transition between open pit and underground operations under geologic uncertainty; this produces tactical schedules that reduce a mining project's susceptibility to geological risk. Whittle et al. (2018) consider the transition from open pit to underground as a maximum closure problem and include a decision to determine the shape of the crown pillar from a set of prescribed shapes. Dagdelen and Traore (2018) present an iterative procedure to determine the location of a transition using available software (e.g., Whittle, OptiMine, Studio 5D). They consider various interconnected factors such as the geology, production sequencing, cost of mining for open pit and underground, cost of processing, mining rate, discount rate, and revenue. Finally, King et al. (2017) provide a mixed-integer program that, when reformulated as an RCPSP, determines solutions that contain not only the depth of the transition but also schedules at the block and stope levels (approximately 1,300 open pit bin-block combinations

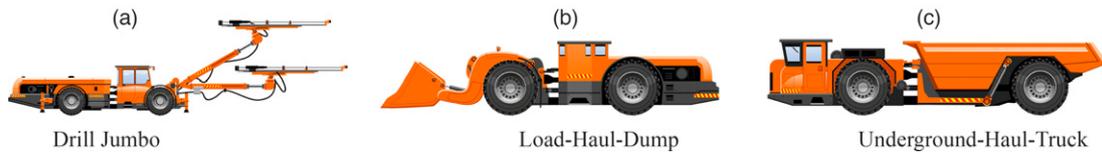
and 2,600 underground stoping—or stope-related—activities, respectively). These solutions are provable to within about 5% of optimality and demonstrate that a hybrid open pit–underground strategy is most economically beneficial relative to a strategy that does not combine mining methods. Afum et al. (2020) consider a model similar to that of King et al. (2017) but account for more detailed financial analysis and blending constraints, particularly as they affect mill operation. However, the authors do not employ as sophisticated a solution methodology, and the corresponding instances they solve are much smaller. Over the past decade, the problem of determining the transition between open pit and underground mining has helped plan open pit operations such that the resulting revenue partially offsets the cost of the initial development of underground operations. Further research will help to determine greater synergies from the feasibility stage onward, promoting less reactive policies. Table B.4 in Appendix B summarizes the relevant references.

### Equipment

Mining equipment is required to be rugged, specialized, and reliable, thereby constituting a significant portion of the strategic capital outlay at a proposed site. In mining, rock breakage is accomplished by either blasting, that is, drilling holes in the rock, loading them with explosives, and blasting or mechanical means, that is, breaking rock using a mechanical cutter continuous miner. The broken rock is then loaded onto haulage equipment by excavators for further transport. Open pit mines commonly use a combination of surface drill rigs, excavators, and trucks to mine both ore and waste rock. Underground mines, on the other hand, require a more varied set of equipment, each suited to a specific type of activity. For example, underground developments are usually excavated in two- to three-meter increments, with a multiboom jumbo (Figure 7(a)) being used to drill the blastholes. Vertical or near-vertical stopes require the use of production drills, which can advance up to 100 meters (Sandvik 2020). (In cut-and-fill mining, multiboom jumbos are used both for development and for production drilling.) A load-haul-dump, or LHD (Figure 7(b)), is a front-end loader that transports broken rock over short distances to the next step in the haulage chain (e.g., an underground haul truck, a conveyor, or an ore pass). An underground haul truck (Figure 7(c)) is an articulated low-profile truck to transport ore, waste rock, and backfill over long distances, potentially to and from the surface.

The design (i.e., width, height, and length) of mineable shapes is bounded by minimum and maximum values based on the extraction equipment

**Figure 7.** (Color online) Specialized Heavy Equipment for Underground Mines (Representative Illustrations)



Sources: Antropov (n.d.).

used, geotechnical criteria, the economics of the operation, and the orebody characteristics (McCarthy 2011). Anjomshoa et al. (2012) minimize the time required to achieve production targets by regulating vehicle departures from the active working area and by placing truck passing bays in locations determined by a mixed-integer program. Operational analysis is based on mine design, available resources, and production goals. Identifying differences between actual and projected production gives insight into improving operational efficiency and refines tactical modeling assumptions (Song et al. 2013). The most important factors for mine equipment selection are (i) site conditions; (ii) correct pass-matching between loaders and trucks; and (iii) proper machine configuration for optimal safety, reliability, and maintenance (Carter 2014).

Fioroni et al. (2014) use simulation to evaluate a transportation system in a generic underground gold mine. Their work considers four different layout options, with different shaft configurations and truck fleet compositions, and considers each year throughout the mine life separately, where different years correspond to specific production goals; the layout with the smallest transportation capacity achieves the desired production capacities in each year. Botin et al. (2014) also use simulation; this work assesses the financial risk associated with various equipment fleets in the pre-production stage of a block caving operation to guide investment decisions. Salama et al. (2014a) contrast two differently sized haulage units, paired with a single LHD. The authors use discrete event simulation to determine the feasibility of meeting production targets and evaluate operational conditions—that is, truck utilization and cycle time—based on tramming distances, mining depth, and an increased number of trucks. Salama et al. (2014b) utilize a similar approach to compare two loader-truck combinations and to determine each fleet’s ability to achieve planned production levels and present a comparison based on energy utilization of four haulage systems: diesel trucks, electric trucks, shaft haulage, and conveyor belts. Energy costs are determined using discrete event simulation, after which a mixed-integer program yields schedules. Each instance possesses a

gap of less than 5% after 10 hours of computation time. Research supporting the decisions concerning reliability assumes a fleet configuration and uses a genetic algorithm to suggest a maintenance and replacement schedule (Peng and Vayenas 2014).

Ozfirat et al. (2015) use a multiobjective fuzzy goal program to select critical equipment. Simulation software—for example, General Purpose Simulation System and Arena—improves estimates of equipment production rates by approximating operational processes under typical underground mining conditions (Musingwini 2016). Gransell (2016) also uses simulation to determine the appropriate equipment to employ in an underground mine. However, this work uses Monte Carlo methods to examine the effects that various types of drills have on drilling costs and on safety and environmental impacts.

Park et al. (2016) simulate a loader-truck haulage system using cycle time as a metric and investigate allocation options of trucks to working areas to improve daily production and to shorten average delay. Pérez et al. (2017) investigate fleet configuration through an iterative methodology, producing schedules with integer programming and evaluating resulting equipment performance via simulation. Consideration of equipment maintenance is often done after acquisition. Bouffard et al. (2018) apply discrete event simulation to evaluate potential designs for a potash mine as part of a prefeasibility study. Trade-off analysis compares expenditures and production values for different strategies of mine haulage, equipment maintenance, and automation to improve capital outlay.

Selecting a design requires many strategic decisions to achieve goals, often considering multiple criteria such as economics, safety, production, and public perception. No published model has yet demonstrated a comprehensive treatment. There is little evidence that the mine design process can be successfully decomposed to allow for an optimal solution via a series of short-sighted goals. There is likely a need to increase the scope of current strategic tools to identify and inform operations within the framework of an optimal strategic plan. Table B.5 in Appendix B summarizes the relevant literature.

### Combination of Design and Scheduling

Some models possess a sufficiently wide scope that decisions affecting both mine design and production

scheduling are made in concert. For example, Nehring et al. (2010, 2012) identify potential scheduling infeasibilities for a case study involving 30 stopes. The results show that the net present value only increases marginally for an approach that considers both strategic and tactical decisions, but the authors contend that their method is still superior in that it identifies infeasibilities in tactical schedules.

Epstein et al. (2012) present a supply chain model for Codelco. The authors evaluate underground and open pit production schedules in tandem with downstream mineral processing operations, using a capacitated multi-commodity network flow model with inputs derived from upstream production schedules. The linear programming relaxations of the production scheduling models are solved, and a rounding heuristic is used to find integer-feasible solutions. This unified approach has been implemented at multiple Codelco operations to improve individual mine production schedules and enable planners to better evaluate strategic decisions such as the simultaneous planning of underground and open pit operations.

Copland and Nehring (2016) determine a sublevel stoping layout and associated production schedule using an integer program that maximizes net present value subject to mining capacities, mill requirements, stope shape geotechnics, and stope sequencing constraints. Their case study using synthetic data demonstrates a reduction in solution time and a more detailed answer than that found manually. Basiri and Pourrahimian (2018) develop two different integer programming models to determine stope design and scheduling in concert, one of which considers blocks by level and the other, blocks in aggregate; the layout accounts for logical considerations addressing stope overlap, whereas the scheduling model considers precedence, mining capacity, and grade. A straightforward implementation of the monolith yields optimal layouts and corresponding production schedules over several decades for a silver mine containing five levels and approximately 100 stopes. Foroughi et al. (2019) use a multi-objective integer-programming formulation to simultaneously determine stope boundaries and the production schedule; the authors use a *nondominated sorting genetic algorithm* to obtain a solution. Martinelli et al. (2020) formulate a mixed-integer program to generate cutoff grade production scheduling decisions for a long-hole stoping and cut-and-fill mining operation. Their model possesses additional decisions associated with capital investment for development of a given mining area, which must then be closed when operation ceases. They construct a variant of a fix-and-relax heuristic to generate solutions that are within approximately 15% of optimality relative to the bound from the monolith. Nesbitt et al. (2020) employ an integer program that accounts for variation in orebody characteristics and

selects the optimal set of extraction methods (including leaving ore in situ) for each section and corresponding schedule to maximize net present value for instances based on data from an operational mine. Table B.6 in Appendix B summarizes.

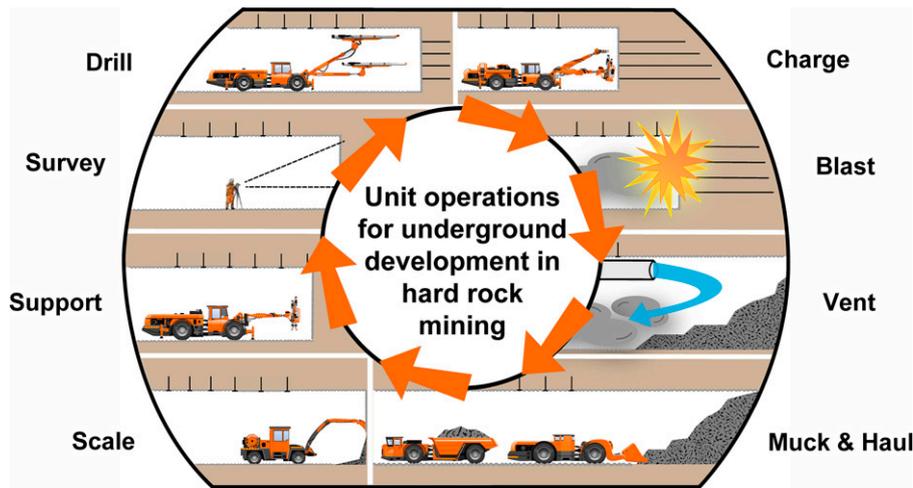
Some work cited in both the *Open Pit-to-Underground Transition* and *Cutoff Grade* sections combine design and scheduling; see King et al. (2017), MacNeil and Dimitrakopoulos (2017), and King and Newman (2018). Whereas MacNeil and Dimitrakopoulos (2017) use enumeration without a special algorithm to solve the resulting model, both King et al. (2017) and King and Newman (2018) exploit mathematical structure, leading to efficient solutions to large-scale models. Because underground mining decisions often prove interdependent, the ability to solve large-scale, integrated, design-and-scheduling models allows for a comparison of tactical and strategic options. However, at the time of this writing, the application of algorithms—for example, Rivera et al. (2015) and Bienstock and Zuckerberg (2010)—that can be used to exploit (underlying) mathematical structure is sparse. More commonly appearing are models that determine dynamic decisions by taking the design as fixed and performing production scheduling in isolation. We discuss these models next.

## Production Scheduling

Underground mines differ from their open pit counterparts in that the nature of the corresponding data is much less homogeneous. As mining progresses, the open pit expands outward and goes deeper; in underground mining, contingent on geotechnical considerations and the selected mining method, decisions have to be made regarding which section of the orebody to exploit next, and these decisions offer greater degrees of freedom. Moreover, the extractable shapes may differ greatly. Conversely, intricate and specific precedence constraints exist between activities. For example, drilling is followed by charging blastholes, which, in turn, is followed by blasting, and backfilling follows ore extraction (see Figure 8).

Strategic schedules in mining inform life-of-mine production targets by determining annual mining and processing capacities, the high-level feasibility of the operation, and the management of multi-mine complexes. Tactical schedules define an extraction sequence for a three- to five-year time horizon at monthly or quarterly fidelity. At the tactical level, a mine planner focuses on guiding operations to meet overall strategic goals and seeks to determine activity start dates, typically to meet an economic objective, while adhering to restrictions regarding geospatial precedences on the mining activities and resource restrictions that govern the extent to which groups of activities can be executed within any given time period. Planners commonly

Figure 8. (Color online) A Typical Underground Development Cycle



Notes. This cycle usually includes a combination of the following unit operations: (8) surveying and marking drill holes on the face, (1) collaring and drilling blast holes, (2) charging the blast holes with explosives, (3) blasting at the end of a shift, (4) ventilating the blasted face, (5) mucking blasted rock, (6) scaling the walls to remove loose pieces of rock, and (7) installing roof support. The cycle may be abbreviated or expanded to suit the operation's needs. Similarly, the production cycle may consist of a set of operations such as (i) drilling, (ii) charging, (iii) blasting, (iv) ventilating, (v) mucking, and (vi) backfilling.

attempt to create schedules to maximize the net present value of the operation, to minimize deviation of produced ore tons from those promised on long-term contracts, or to optimize a combination of other key performance indicators for the operation. The activities are derived from the mine design, with each individual activity tied to an excavation or mining task required to conduct safe operations. Resources consist of personnel with requisite skills, equipment necessary to provide the capability of executing mining activities, and availability of ventilation and refrigeration to preserve a safe operating environment. Mathematically, posing the *production scheduling problem* as an integer program requires durations for each activity and any delays between pairs of activities that directly follow each other. The remaining mathematical structure is unchanged from that of a basic open pit production scheduling problem without stockpiling—that is, the constrained pit limit, or CPIT, problem described in Espinoza et al. (2013). As of less than a decade before the time of this writing, researchers did not recognize the existence of a general framework for underground production scheduling. As a result, they tended to simplify the model to reduce problem size or relied on heuristics tailored to a specific instance. Improved tactical scheduling also offers insight into strategic decision making and informs shift-to-shift decision making.

### Stoping Methods

Fava et al. (2011) describe a genetic algorithm-based scheduling approach, implemented as the Schedule Optimization Tool, to evaluate different strategic mining scenarios considering cutoff grades, financial

models, and operational resource constraints. Maybee and Fava (2011) and Sharma (2015) build on this work by considering risk as a result of price uncertainty and geotechnical instability. The Kittilä gold mine uses a long-hole stoping method with backfilling and serves as a research platform for the following three papers. Song et al. (2015b) model underground mine scheduling as a flow shop problem to provide a tactical schedule with a set of heuristics to assign three machine sets to 35 working areas, each consisting of seven unit operations. Song et al. (2015a) then consider uncertainty in activity duration for a fixed production schedule and reallocate resources in a simulation to minimize the expected critical path. Åstrand (2018) solves a similarly defined flow shop problem with 10 faces and two cycles at the Kittilä mine using constraint programming and a relaxation-based search heuristic.

Terblanche and Bley (2015) describe a formulation, in RCPSp format, for a notional underground stoping mine to maximize net present value while adhering to precedence and resource constraints. By using the concept of earliest and latest activity start times, as well as both spatial and temporal aggregation, they expedite solutions. Brickey (2015) exploits the RCPSp structure present in an underground stoping mine to maximize net present value subject to precedence and resource constraints, where the latter include ventilation requirements. Results from a real-world instance demonstrate a gain in net present value even when considering ventilation requirements, relative to the *modus operandi*. Lopes (2017) formulates a detailed production scheduling model for an underground

stopping mine as an RCPSP and determines the effectiveness of using a sliding time window heuristic to expedite solutions for large (e.g., 31,000 activities and 730 time periods) real-world instances of the problem. Wang et al. (2018) formulate a multi-objective optimization model that determines grades at which to extract and process ore to maximize both profit and efficient resource utilization for a stopping mine; constraints bound the ore grades. The resulting Pareto-optimal solutions are found using a genetic algorithm. Nesbitt et al. (2021) characterize uncertainty in duration and economic value of underground mining activities corresponding to an instance of an operational hardrock mine containing 15,773 activities at daily fidelity. Their stochastic integer program, using a tailored methodology, can produce implementable schedules in an operationally feasible amount of time.

### Cut-and-Fill Methods

Given a design for a cut-and-fill mine, Huang et al. (2020) formulate a mixed-integer program to determine the timing of extraction and backfilling decisions to maximize net present value subject to a series of operational constraints including precedence, resource capacity, and blending. Using case studies containing 12 time periods and between 40 and 120 extractable units (stopes), the authors show, on average, more than 10% improvement over the net present value resulting from current operations. Manríquez et al. (2020) consider a bench-and-fill mine for which they use simulation optimization to determine short-term production planning decisions. Their iterative approach reduces deviation in the predicted tons extracted from an original estimation to one that better captures operational uncertainties associated with equipment, such as failures and rates of execution. Brickey et al. (2021) present an application of the RCPSP in determining five-year, deterministic tactical schedules at daily fidelity for Barrick's Turquoise Ridge cut-and-fill mine. These models incorporate greater operational details than their predecessors from a decade or more earlier, allowing for the creation of more realistic and adoptable schedules; this achievement arises from improved formulations and solution techniques.

### Caving Methods

Block caving operations differ significantly from other underground mining methods, in that most development activities are completed prior to full-scale production commencement. At block and panel cave operations, scheduling consists of dividing the ore columns directly above each drawpoint into slices and determining the sequence of extraction for individual slices (Rubio and Diering 2004). Typically, the schedules are strategic and serve to minimize dilution (waste mixed with ore, which reduces value) and to maintain the desired draw profile. Geovia's Personal

Computer Block Caving has been the industry's design and scheduling software standard for caving operations over the past two decades, incorporating different ore mixing models and grade reconciliation tools to determine effective drawdown rates (Diering et al. 2010). Pourrahimian et al. (2013) address production scheduling in block caving operations using a mixed-integer program, modeling the ore column above each drawpoint as a collection of slices and then determining the sequence of extraction of the individual slices. Khodayari and Pourrahimian (2015) present a review of optimization applications in block-cave scheduling, identifying common mine design considerations and generally describing the scheduling problem as the selection and extraction of columns of ore over time, considering geomechanical, operational, economic, and environmental constraints. Nezhadshahmohammad et al. (2018) develop a mixed-integer linear program to optimize the extraction sequence of drawpoints with respect to mining capacity, continuous extraction, production grade, limits on active drawpoints, and precedence between them while demonstrating results with an instance of 325 drawpoints and 15 time periods. Alonso-Ayuso et al. (2014) incorporate uncertainty of commodity price in the block caving scheduling problem with a stochastic mixed-integer program that considers block clustering and precedence, flow constraints that limit the processing stream, and extraction capacity to maximize expected net present value. They compare their results to a deterministic approach that uses the expected value of the uncertain parameters in a case study modeled after the El Teniente copper mine in Chile. Malaki et al. (2017) use an optimization model, combined with sequential Gaussian simulation, to generate scenarios on which model instances are based, so as to determine a production schedule for a block caving mine under grade uncertainty. The model maximizes net present value subject to constraints on mining capacity, average extracted grade, extraction rate, and precedence. Reus et al. (2018) consider operational disruptions resulting from equipment and infrastructure failures in creating extraction schedules for the Chuquicamata underground mine. The authors first utilize a deterministic mixed-integer linear program to create schedules; they then extend the model to consider capacity uncertainty of the overland conveyor and allow the mine planner to choose an appropriate profit target given the risk associated of achieving that target. Khodayari and Pourrahimian (2019) present a stochastic mixed-integer program that introduces uncertainty in ore mixing among adjacent drawpoint columns, showing up to a 4% improvement in net present value over that provided by standard industry software for an operational block caving operation of 424 drawpoints. Dirx et al. (2019)

account for uncertainty in grade and drawdown rate in determining the feasibility of strategic production targets for a potential mineral deposit using block cave mining. The authors employ stochastic mixed-integer programming to maximize the net present value and minimize production target deviation with respect to mining capacity, continuous extraction, production grade, inter-drawpoint precedence, and milling operations. Nezhadshahmohammad et al. (2019) determine a production schedule composed of a sequence of drawpoints to maximize net present value while considering the production capacity, average grade, number of simultaneously active drawpoints, precedence, geotechnical, draw rate, and conditional draw rate constraints. The authors test their methodology on an instance containing 325 drawpoints and 15 time periods. Olivares (2019) formulates a similar model but uses tailored stochastic methodologies to capture geological and price uncertainty that involves Lagrangian relaxation to handle nonanticipativity constraints; the modeling and solution procedures are tested on a copper mine in Chile.

Finally, two papers address the sublevel caving method: Martinez and Newman (2011) use a mixed-integer program to minimize deviations between contractual obligations and scheduled production for an iron ore mine in Kiruna, Sweden. Their tailored solution methodology solves real instances whose solutions adhere to mining capacity, precedence constraints, and other operational considerations. Shekhar et al. (2019) examine an ore draw control strategy in sublevel caving operations. Their optimization model takes as inputs estimated grades and average operational mining costs to determine the time at which to cease loading at a drawpoint; solutions improve ore recovery and minimize dilution while ensuring that operational constraints are met, and they show promise at two iron ore mines in Sweden.

### Combined Methods

O'Sullivan and Newman (2014) use optimization-based heuristics to develop production schedules for an underground lead and zinc mine that uses three different mining methods in Ireland. Their optimization-based heuristic yields production schedules that maintain feasibility and that can be evaluated for several end-of-life-of-mine scenarios. Campeau and Gamache (2020) maximize discounted ore tonnes extracted from a Canadian mine using cut-and-fill and long-hole stoping. The authors determine the start of an activity and the corresponding tonnes extracted while accounting for unit operations and geotechnical requirements. They produce a near-optimal schedule with weekly fidelity for half a year using a straightforward implementation of their model in a commercial solver (after some standard preprocessing techniques) that prioritizes ore processing over development.

There are many systems that impact an underground mining schedule that are either not incorporated or not considered in concert at the time of this writing. For example, ground control systems and ventilation networks are notably absent, but the lack of accurate models that can be coordinated within an optimization framework, combined with complex mathematical structures, currently precludes this level of sophistication. A complete summary of production scheduling research with respect to the articles we have cited can be found in Tables B.7 and B.8 in the appendix.

### Conclusions

Over the past decade, a number of advances in operations research techniques have enhanced underground mine plans. Data-driven methodologies increasingly supplement expert judgment. Improved representation of operational procedures, and the inclusion of interdependent decisions, have allowed for objective analysis of trade-offs between mine design and scheduling decisions. Additionally, integrated models may determine fleet configuration decisions and maintenance and capital expenditures; they could also simultaneously inform designs for multimine complexes.

Nonetheless, mathematical tools available at the time of this writing may require significant adjustments or modifications to obtain results for specific cases, such as multi-method operations. The lack of a single, universal underground mine planning model (which might admittedly include variants) and a corresponding solver have caused underground mine planning (including design) to lag behind its open pit counterpart. In this sense, the corresponding models are smaller and can omit the detail necessary for strategic planning or for implementation dictated by the more complicated nature of underground operations such as geotechnical stability and ventilation requirements. Although some models—for example, for equipment selection and operation—do incorporate uncertainty via simulation and other stochastic techniques, few models that address a decision space as rich as that found in many optimization models for underground mine planning account for probabilistic scenarios. This is, in part, due to the intractability of these models with current modeling paradigms, software, and hardware and in part because of the inability of practitioners to enable mine operators to interpret the solutions in a meaningful manner. These considerations also inhibit widespread use of stochastic models (e.g., queuing, Markov chains) to determine strategies for various practical settings.

More must also be done to combine corporate strategic initiatives with the day-to-day realities facing mine operators. In this sense, multi-objective programming that considers not only the net present value of an operation but also a smooth production

schedule might prove particularly relevant. Furthermore, mining operations rely on the advice of external consultants, who use proprietary software and other confidential methodologies that do not appear in the open literature and are therefore not easily extended in a mathematical sense or widely disseminated in industry, particularly to smaller mining operations that lack the financial means to hire such consultants.

In order to remove obstacles of progress, there is need for greater collaboration across the disciplines of mining, geology, operations research, statistics, and computer science. Active areas of research include addressing the volatility of real-time operations through more robust schedules and integrating design and scheduling—including improved geotechnical modeling and supply chain management. Commercial software platforms will continue to define data standards and facilitate technology transfer among academia and industry, expediting the adoption of modeling techniques. However, this transfer could be facilitated by more transparency between industry and academia regarding not only existing models but also corresponding relevant improvements to them based on industry trends. Data management may also improve confidence in decision making, allowing for a reduction in perceived economic and operational risk, thus easing the implementation of underground mine plans (Benndorf and Jansen 2017).

### Acknowledgments

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### Appendix A. Formulation

An RCPSP possesses two main constraint types: (i) precedence (the majority) and (ii) knapsacks (the minority). Although usually used for longer-term planning, such formulations can be tailored to short-term use by modifying the detail contained in each activity, the time fidelity and horizon of the model, and the types of resource constraints considered, for example. These problems possess a special structure whose linear-programming relaxation can be solved using a tailored algorithm (Bienstock and Zuckerberg 2010) and a rounding heuristic (e.g., TopoSort) to obtain an integer-feasible solution (Chicoisne et al. 2012, Brickey 2015). The latter is a list-ordering heuristic that uses the “expected” completion time of an activity given by the linear programming solution. An early-start algorithm (Lambert et al. 2014) can reduce the number of activity–time period pairs. This formulation is taken from Brickey et al. (2021). The formulation is as follows (notation is defined in Table A.1):

$$(\text{RCPSP}) \quad \max \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} \delta_t c_a (X_{at} - X_{a,t-1}) \quad (\text{A.1a})$$

$$\text{s.t.} \quad X_{a,t-1} \leq X_{at} \quad \forall a \in \mathcal{A}, t \in \mathcal{T}, \quad (\text{A.1b})$$

$$X_{at} \leq X_{\bar{a},t-d_a} \quad \forall a \in \mathcal{A}, \bar{a} \in \mathcal{P}_a, t \in \mathcal{T}, \quad (\text{A.1c})$$

$$\sum_{a \in \mathcal{A}} \frac{q_{ra}}{d_a} (X_{at} - X_{a,t-d_a}) \leq \bar{r}_{rt} \quad \forall r \in \mathcal{R}, t \in \mathcal{T}, \quad (\text{A.1d})$$

$$\sum_{t \in \hat{\mathcal{T}}_m} \sum_{a \in \mathcal{A}} \frac{q_{ra}}{d_a} (X_{at} - X_{a,t-d_a}) \leq \hat{r}_{rm} \quad \forall r \in \hat{\mathcal{R}}, m \in \mathcal{M}, \quad (\text{A.1e})$$

$$X_{\bar{a}1} = 1 \quad \forall \bar{a} \in \tilde{\mathcal{A}}, \quad (\text{A.1f})$$

$$X_{at} \text{ binary} \quad \forall a \in \mathcal{A}, t \in \mathcal{T}. \quad (\text{A.1g})$$

The objective (A.1a) maximizes net present value, which is a discounted function of the monetary value associated

**Table A.1.** Notation

Variable	Definition
<b>Indices and sets</b>	
$a \in \mathcal{A}$	An activity within the set of all activities
$\bar{a} \in \tilde{\mathcal{A}} \subset \mathcal{A}$	An activity within the set of activities whose start dates have been predetermined
$\bar{a} \in \mathcal{P}_a$	An activity $\bar{a}$ within the set of predecessor activities to activity $a$
$r \in \mathcal{R}$	A resource within the set of resources, such as production and development capacity, whose limits are enforced on a daily basis
$r \in \hat{\mathcal{R}} \subset \mathcal{R}$	A resource within the set of resources, such as production and development capacity, whose limits are enforced on a monthly basis
$t \in \mathcal{T}$	A day within the set of daily time periods
$m \in \mathcal{M}$	A month within the set of monthly time periods
$t \in \hat{\mathcal{T}}_m$	A day within the set of days contained in month $m$
<b>Parameters</b>	
$c_a$	Monetary value associated with completing activity $a$ (in \$)
$q_{ra}$	Consumption of resource $r$ associated with completing activity $a$ (in tonnes, meters)
$\bar{r}_{rt}$	Maximum amount of resource $r$ available on day $t$ (in tonnes, meters)
$\hat{r}_{rm}$	Maximum amount of resource $r$ available in month $m$ (in tonnes, meters)
$d_a$	Duration of activity $a$ (in days)
$d_{\bar{a}}$	Duration (including mandatory delay) of activity $a$ (in days)
$\delta_t$	Discount factor for period $t$ (in fraction)
<b>Decision variable</b>	
$X_{at}$	Equal to 1 if activity $a$ is completed by the end of time $t$ and 0 otherwise

with the (on-time) completion of activity  $a$  and the time at which said activity is completed. We express the latter as the difference of two variables corresponding to the time by which an activity is completed (Lambert et al. 2014). Although not stated here, the model can easily be adapted to accommodate other objectives such as minimizing makespan (under the assumption that all activities must be completed by the end of the horizon) or minimizing “tardiness” (under the assumption that activities have due dates); the former variant corresponds to the classical RCSP. Constraints (A.1b) ensure that once an activity is completed at time  $t - 1$ , it remains completed for all future time periods  $t, \dots, |T|$ . Constraints (A.1c) enforce pre-

cedence between an activity  $a$  and its predecessors  $\bar{a}$ , such that  $a$  cannot start unless  $\bar{a}$  starts sufficiently early that, when accounting for its duration, it is finished by the time  $a$  starts. Constraints (A.1d) constitute knapsacks and ensure that the amount of resource of a particular type consumed by all activities on any given day cannot exceed the availability of said resource. Constraints (A.1e) do the same for a subset of the resources whose consumption must be restricted on a monthly basis. Activities whose start dates have been previously determined to coincide with the beginning of our time horizon must be inserted into the schedule per Constraints (A.1f). All variables are required to be binary by Constraints (A.1g).

## Appendix B. Tables

**Table B.1.** List of Works in Underground Mining Method Section

Reference	Model	Nature	Algorithm	Data	Implemented
Musingwini and Minnitt (2008)	AHP	Deterministic	Expert choice	Real world	Yes
Yavuz (2009)	Fuzzy MADM using Yager’s method	Stochastic	Enumeration	Real world	Unknown
Nieto (2011)	Analytical	Deterministic	Enumeration	Artificial	Unknown
Bogdanovic et al. (2012)	AHP, PROMETHEE	Deterministic	Enumeration	Real world	Yes
Gupta and Kumar (2012)	AHP	Deterministic	Enumeration	Real world	No
Ataei et al. (2013)	Monte Carlo AHP	Stochastic	Enumeration	Real world	Unknown
Peskens (2013)	Fuzzy AHP	Deterministic	MS Excel	Real world	Unknown
Dehghani et al. (2017)	Grey model, TODIM	Deterministic	MS Excel	Real world	Unknown
Kabwe (2017)	AHP, fuzzy MADM	Deterministic	Enumeration	Real world	Unknown
Javanshirgiv and Safari (2017)	Fuzzy TOPSIS	Deterministic	Enumeration	Real world	Unknown
Balusa and Gorai (2019)	TOPSIS, VIKOR, ELECTRE, PROMETHEE II, WPM	Deterministic	Enumeration	Real world	No

*Notes.* All papers consider a strategic decision horizon. AHP, Analytical Hierarchy Process; ELECTRE, ELimination Et Choix Traduisant la REalité; MADM, Multiple Attributes Decision Making; PROMETHEE, preference ranking organization method for enrichment evaluation; TODIM, Tomada de Decisão Interativa Multicriterio; TOPSIS, technique for ordering preference by similarity to ideal solution; VIKOR, visekriterijumska optimizacija i kompromisno resenje; WPM, weighted product method.

**Table B.2.** List of Works in Cutoff Grade Section

Reference	Model	Nature	Algorithm	Data	Implemented
McIsaac (2008)	Monte Carlo simulation	Stochastic	Steepest ascent method	Real world	No
Elkington et al. (2010)	Sequential Gaussian simulation	Stochastic	Enumeration	Artificial	No
Gu et al. (2010)	Dynamic program	Deterministic	Unknown	Real world	Unknown
Collard (2013)	Integer program	Deterministic	CPLEX	Real world	Unknown
Roberts and Bloss (2014)	Unknown	Deterministic	Blasor, enumeration	Real world	Unknown
King and Newman (2018)	Integer program	Deterministic	OMP solver, enumeration	Real world	Yes

*Note.* All papers consider a strategic decision horizon.

**Table B.3.** List of Works in Layout and Infrastructure Section

Reference	Model	Nature	Algorithm	Data	Implemented
Stope layout					
Bai et al. (2013)	Network	Deterministic	Genetic algorithm	Real world	No
Bai et al. (2014), Nelis et al. (2016)	Network	Deterministic	Maximum flow algorithm	Artificial	No
Bootsma et al. (2014)	Sequential Gaussian simulation	Stochastic	Enumeration	Real world	Yes
Sandanayake et al. (2015)	Dynamic program	Deterministic	Heuristic	Real world	No
Nikbin et al. (2020)	Dynamic program	Deterministic	Heuristic	Real world	No
Sipeki et al. (2020)	Integer program	Deterministic	Pillar selector (custom algorithm)	Real world	Yes
Room-and-pillar layout					
Anani (2016)	Integer program	Deterministic	Branch-and-cut	Real world	No
Anani et al. (2017)	Discrete event simulation	Stochastic	Arena	Real world	Yes
Gligorić et al. (2019)	Fuzzy linear program	Stochastic	Unknown	Artificial	No
Caving layout					
Gomes et al. (2013)	Monte Carlo simulation	Stochastic	Enumeration	Real world	Yes
Ugarte et al. (2017)	Sequential Gaussian simulation	Stochastic	Enumeration	Real world	No
Noriega et al. (2018)	Integer program	Deterministic	CPLEX	Real world	No
Rafiee et al. (2018)	Numerical simulation	Stochastic	Enumeration	Real world	Yes
Zarate et al. (2018)	Sequential Gaussian simulation	Stochastic	Enumeration	Real world	No
General infrastructure layout					
Zuo et al. (2013)	Multi-objective program	Deterministic	Genetic algorithm	Real world	Unknown
Brazil et al. (2014)	Dynamic program	Deterministic	Simulated annealing	Real world	Yes
Sirinanda et al. (2014, 2016)	Dynamic program	Deterministic	Custom algorithm	Real world	Unknown
Carpentier et al. (2016)	Stochastic integer program	Stochastic	CPLEX	Real world	No
Grossman et al. (2018)	Network	Deterministic	Recursive algorithm	Artificial	No

Note. All papers consider a strategic decision horizon.

**Table B.4.** List of Works in Open Pit-to-Underground Transition Section

Reference	Model	Nature	Algorithm	Data	Implemented
Finch and Elkington (2011)	Discounted cash flow	Deterministic	Enumeration	Artificial	No
Newman et al. (2013)	Network	Deterministic	CPLEX	Real world	Yes
Opoku and Musingwini (2013)	Conditional simulation	Stochastic	Enumeration	Real world	Unknown
Ben-Awuah et al. (2016)	Mixed integer program	Deterministic	Gurobi	Real world	Unknown
Chung et al. (2016)	Integer program	Deterministic	CPLEX	Artificial	Unknown
King et al. (2017)	Integer program	Deterministic	OMP solver, Toposort, enumeration	Real world	Yes
MacNeil and Dimitrakopoulos (2017)	Stochastic integer program	Stochastic	Tabu search	Artificial	Unknown
Afum et al. (2020)	Mixed integer program	Deterministic	CPLEX	Real world	No
Dagdelen and Traore (2018)	Mixed integer program	Deterministic	OptiMine, enumeration	Real world	No
Whittle et al. (2018)	Integer program	Deterministic	Manual, enumeration	Artificial	No

Note. All papers consider a strategic decision horizon.

**Table B.5.** List of Works in Equipment Section

Reference	Model	Nature	Algorithm	Data	Implemented
Botin et al. (2014)	Discrete event simulation	Stochastic	Enumeration	Real world	Unknown
Fioroni et al. (2014)	Discrete event simulation	Deterministic	Arena	Artificial	No
Peng and Vayenas (2014)	Simulation	Deterministic	GenRel	Real world	Unknown
Salama et al. (2014b)	Discrete event simulation, mixed integer program	Deterministic	GPSS/H, CPLEX	Artificial	No
Salama et al. (2014a)	Discrete event simulation	Deterministic	SimMine	Artificial	No
Ozfirat et al. (2015)	Fuzzy analytical hierarchy process, fuzzy goal program	Stochastic	Unknown	Artificial	No
Gransell (2016)	Monte Carlo simulation	Deterministic	@RISK	Artificial	No
Park et al. (2016)	Discrete event simulation	Deterministic	GPSS/H	Real world	Unknown
Pérez et al. (2017)	Simulation optimization	Stochastic	UDESS, DSIM	Artificial	No
Bouffard et al. (2018)	Discrete event simulation	Deterministic	Arena	Real world	Yes

Note. All papers consider a strategic decision horizon.

**Table B.6.** List of Works in Combined Design and Scheduling Section

Reference	Model	Decision horizon	Nature	Algorithm	Data	Implemented
Nehring et al. (2010, 2012)	Constraint program	Operational	Deterministic	Neighborhood search	Real world	Yes
Epstein et al. (2012)	Mixed integer program	Strategic	Deterministic	CPLEX	Real world	Yes
Basiri and Pourrahimian (2018)	Integer program	Strategic	Deterministic	CPLEX	Artificial	No
Martinelli et al. (2020)	Mixed integer program	Strategic	Deterministic	Custom heuristics	Real world	Yes
Nesbitt et al. (2020)	Integer program	Strategic	Stochastic	CPLEX	Real world	No

**Table B.7.** List of Works in Production Scheduling Section, Part I

Reference	Model	Decision horizon	Nature	Algorithm	Data	Implemented
Stopping methods						
Fava et al. (2011)	Integer program	Strategic	Deterministic	Genetic algorithm	Real world	Yes
Maybee and Fava (2011)	Integer program	Strategic	Deterministic	Genetic algorithm	Real world	Yes
Brickey (2015)	Integer program	Tactical	Deterministic	OMP Solver, Toposort	Real world	Yes
Sharma (2015)	Integer program	Strategic	Deterministic	Genetic algorithm	Real world	No
Song et al. (2015b)	Integer program	Operational	Deterministic	Heuristics	Real world	Unknown
Song et al. (2015a)	Simulation	Operational	Stochastic	Heuristic	Real world	Unknown
Terblanche and Bley (2015)	Mixed integer program	Tactical	Deterministic	CPLEX	Artificial	No
Lopes (2017)	Integer program	Tactical	Deterministic	OMP Solver, TopoSort	Real world	Yes
Åstrand (2018)	Constraint program	Operational	Deterministic	Neighborhood search	Real world	Yes
Wang et al. (2018)	Linear program	Strategic	Deterministic	Nondominated sorting genetic algorithm	Real world	No
Nesbitt et al. (2021)	Stochastic integer program	Strategic	Stochastic	CPLEX	Real world	No
Cut-and-fill methods						
Huang et al. (2020)	Mixed integer program	Tactical	Deterministic	CPLEX	Real world	Unknown
Brickey et al. (2021)	Integer program	Tactical	Deterministic	OMP solver, Toposort	Real world	Yes
Manríquez et al. (2020)	Mixed integer program	Operational	Deterministic	Simulated annealing, CPLEX	Artificial	No

**Table B.8.** List of Works in Production Scheduling Section, Part II

Reference	Model	Decision horizon	Nature	Algorithm	Data	Implemented
Caving methods						
Pourrahimian et al. (2013)	Mixed integer program	Tactical	Stochastic	CPLEX with clustering and sequencing	Real world	No
Alonso-Ayuso et al. (2014)	Mixed integer program	Tactical	Stochastic	CPLEX	Real world	Unknown
Malaki et al. (2017)	Mixed integer program, sequential Gaussian simulation	Strategic	Stochastic	CPLEX	Real world	No
Nezhadshahmohammad et al. (2018)	Mixed integer program	Strategic	Deterministic	Heuristics, CPLEX	Artificial	No
Reus et al. (2018)	Mixed integer program, stochastic integer program	Tactical	Stochastic	CPLEX	Real world	Yes
Dirkx et al. (2019)	Stochastic integer program	Strategic	Stochastic	CPLEX	Real world	Unknown
Khodayari and Pourrahimian (2019)	Mixed integer program	Strategic	Stochastic	CPLEX	Real world	No
Nezhadshahmohammad et al. (2019)	Mixed integer program	Tactical	Deterministic	CPLEX	Artificial	No
Olivares (2019)	Stochastic linear program	Strategic	Stochastic	Volume algorithm	Real world	Unknown
Shekhar et al. (2019)	Economic model	Operational	Deterministic	Manual	Real world	Yes
Combined methods						
O'Sullivan and Newman (2014)	Integer program	Tactical	Deterministic	CPLEX	Real world	Yes
Campeau and Gamache (2020)	Mixed integer program	Operational	Deterministic	CPLEX	Artificial	No

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