ARMA 14-7055

Lateral Reinforcement of Fully Grouted Roof Bolts in a FLAC3D Simulated Coal Mine Entry

Murphy, M.M.
National Institute for Occupational Safety and Health (NIOSH), Pittsburgh, PA, USA
Esterhuizen, G.S.
National Institute for Occupational Safety and Health (NIOSH), Pittsburgh, PA, USA
Tulu, I.B.
National Institute for Occupational Safety and Health (NIOSH), Pittsburgh, PA, USA

ABSTRACT: Fully grouted roof bolts increase the stability of a bedded mine roof by providing resistance to both vertical and horizontal displacements. The bolts provide suspension reinforcement from axial loads and lateral reinforcement from shear resistance effects. The lateral reinforcement provided by a roof bolt is difficult to observe in the field, but is often suggested by observation of roof failure cavities. However, these observations do not indicate whether bolt shear failure precedes or is a result of collapse. This paper highlights the use of a well-calibrated FLAC3D numerical model to investigate the shear resistance provided by a fully grouted bolt. The study first looks at analytical solutions to determine the necessary element size to obtain appropriate deflections of thin beams within FLAC3D. The study then compares different models demonstrating that the shear resistance provided by a fully grouted bolt has a limited impact on the overall stability of the mine roof. The model results indicate that the axial suspension effects of fully grouted bolts are more significant than the lateral reinforcement provided.

1. INTRODUCTION

The Office of Mine Safety and Health Research (OMSHR), part of the National Institute for Occupational Safety and Health (NIOSH), is interested in improving the safety of miners working underground. Installation of roof bolts to create substantial beams out of roof strata is a primary method of achieving required safety for underground excavations. Thus, the mechanics of bolt interaction with strata, especially at the onset of collapse, is of great interest. Immediately after development of an underground coal mine entry, vertical and horizontal displacements take place around the excavation [1]. Sedimentary roof beds start to sag, separate from each other, and interbed slip occurs (Figure 1).

Fully grouted roof bolts create reinforcement by providing resistance to both the vertical and horizontal displacements of the roof beds. Resistance to vertical displacement is created from the interlocking friction between the grout and rock interface. Resistance to interbed slip is created when the strata begin to slide horizontally against the bolt/grout system [2]. These two types of resistance provided by the fully grouted bolt prevent the beds from separating and reduce the shear movement along the interfaces. Axial resistance provided by the bolt is relatively easy to measure in situ, either by monitoring the roof sag or the axial load on the bolt. However, the shear resistance provided by the bolt is difficult to observe and measure in the field, other than by monitoring sliding between beds with a borescope. Underground observations of roof falls demonstrate that bending and shearing of roof bolts occur frequently along bedding planes or laminations. However, these observations are made after collapse has occurred, so it is difficult to know the degree of the shear resistance in a pre-collapse state.

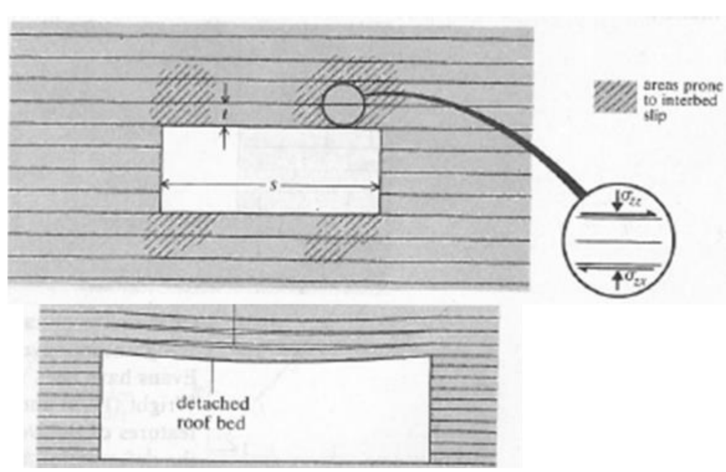

Fig. 1. Sagging, separation and slip along the roof beds (after Brady and Brown [1]).

The objective of this study is to use calibrated structural pile elements in a FLAC3D [3] simulated coal mine entry to investigate the degree of shear resistance provided by the roof bolt during initial deflection of the coal mine roof in a pre-collapsed state.

2. BACKGROUND

2.1. Laboratory Response of Bolt Reinforcement

Three different stages of load/displacement behavior have been reported during shearing of reinforced joint or bedding planes in laboratory experiments [4, 5]. The first stage corresponds to linear elastic behavior, where 50% [5] to 75% [4] of bolts' ultimate resistance was reached after 1- to 5-mm shear displacement across the joint. In the second stage, yielding of the steel took place with the formation of plastic hinges. McHugh and Signer explained the mechanics of hinge point formation based on an investigation of bending and axial strains on the bolt [6]. Initially one side of the bolt extends and the other side compresses near the joint surface where bending takes place. The extended side of the bolt reaches the yield strain very quickly and forms a plastic hinge point. In the third stage of load/displacement behavior, unconstrained plastic deformation of the bolt takes place. According to McHugh and Signer, after the formation of the hinge points, the compressed side of the bolt starts to extend and yield in tension until the bolt fails due to excess tensile strain (or stress) [6].

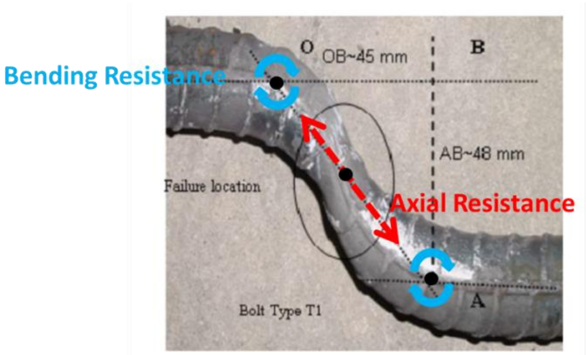

Fig. 2 - Resistance of the bolt (modified from Jallalifar and Aziz [5]).

In past research, much focus has been put on modeling the accurate axial resistance of a fully grouted bolt. This has come in the form of calibrating numerical models to axial bolt loads and roof deflections measured in the field. In order to capture realistic shearing resistance, Tulu et al. presented a systematic procedure to simulate the shear resistance of a fully grouted roof bolt undergoing a direct shear test in FLAC3D [7]. The research was able to successfully match laboratory results to model results.

2.2. Laboratory and Modeling of Lateral Displacement Between Beds

Panek published a report that summarized a series of laboratory tests that helped design systems of bolts to reinforce bedded materials [8]. Panek states that the reinforcing effect of a bolt consists of a component due to friction and a component due to suspension. The friction effect is the frictional resistance to bedding-plane slip, which arises from the clamping action of tensioned bolts. Tensioned rock bolts installed perpendicular to the bedding planes will compress the strata thus enhancing the shear strength between them and so reducing the flexure or bending stresses.

Haas et al. investigated the applicability of the results found by Panek for use in coal mines by using finite element modeling [9]. The authors indicated that no significant amount of beam building occurred due to large separation between the beams, and the size and preload on the bolts required to close the separation became impractical. Haas et al. also investigated the magnitudes of shear displacements that could be expected in an underground coal mine if the beams behave in an ideal elastic manner with no discontinuities. The study showed that an entry span of over 120 m (400 ft) is required for approximately 2.54 cm (1 inch) of shear displacement along a bed in the elastic zone. The authors concluded that the magnitude of the shear displacement between elastic beams was found to be small and that when there are large shear displacements in underground mines, they occur because of large discontinuous movements of blocks or slabs of rock, not because of slipping between beds.

Stephansson presented mathematical solutions of deflection for multilayered roofs in horizontally bedded rock, based on the theory of elastic beams on elastic supports [10]. The work showed the importance of abutment compression contributing to the deflection of the bedded roof. Since the bedded roof analyzed by Stephannson rests on a soft material, similar to a coal mine roof resting on a coal pillar, the analytical solutions are more suitable than in other works such as Panek.

Zhang and Peng discussed the effect of bedding planes on the stress distribution and failure modes with three-dimensional finite element modeling [11]. Model results showed that roof with bedding planes contains vertical displacements approximately 7-8 mm larger compared to massive strata, but there was no information about the amount of sliding that occurs between beds. The authors indicated that tensioned bolts are more effective when the immediate roof layers are weak but the horizontal stress is low.

Zhang et al. looked at how different bolting scenarios impacted a weak coal mine roof using numerical models [12]. Roof stability indicators included maximum yielding depth, maximum roof deflection, and plastic strains obtained from the models. The study included shear load magnitude values obtained from the model, which ranged from 32 to 65 kN. One of the major findings was that the primary support mechanism for very weak roof is suspension.

2.3. Field Observations of Lateral Displacement Between Beds

Mathews and Meek discussed rock reinforcement systems in cut and fill mining [13]. The authors mentioned that the shear resistance provided by fully grouted dowels was usually too low to prevent differential slip near the walls of the opening under conditions of high lateral stresses.

Gerdeen et al. prepared a report for the Bureau of Mines detailing a large number of roof failure mechanisms based on field surveys detailing geology, mining operations, field stresses, bolting patterns, and general roof control issues at the mine [14]. The main conclusion from this report was that the suspension mechanism was the main mechanism of support and that there was no indication of roof bolt resistance to sliding between the beds.

Peng gives numerous examples of underground observations of roof falls that demonstrate bending and shearing of roof bolts occurring frequently along bedding planes or laminations [15]. One example shows a black shale layer that appeared to be intact moving laterally approximately 15 cm (6 in). The bolt sheared and fell to the mine floor. Another example shows a black shale layer that was completely broken and able to expose a bent roof bolt. The black shale was estimated to have moved laterally 33 cm (13 in) between the hole location at the contact plane and the bolt head, but the bolt remained intact. These observations suggest that the bolts sheared due to the lateral movement of the strata, leading to collapse of the mine roof. However, since the observations were made after collapse had occurred, it is difficult to know the degree of the shear resistance in a pre-collapsed state.

Zhang et al. measured the roof sliding in a low seam mine with horizontal stress four times the vertical stress [16]. The authors reported that roof sliding ranged from 0.51 to 1.3 mm (0.02 in to 0.05 in) and the direction of sliding was parallel with the horizontal stress or towards the open diagonal span of the intersections. The horizontal shifting was assumed to be mostly caused by horizontal stress, due to the agreement with the orientation of the stress and the sliding direction. The authors also indicated that roof falls were likely to occur when dense roof separations developed high in the roof and the fractures from the corner of the rib propagated to a certain height.

3. NUMBERICAL MODELING SETUP IN FLAC3D

A numerical modeling procedure for coal mine entries that provides realistic estimates of stresses, displacements, and stability during extraction has been developed and utilized for this study [17, 18]. In FLAC3D, a coal mine entry with thin beams in the immediate roof was modeled and is shown in Figure 3. The model simulated a 6-m-wide (20-ft-wide) coal mine entry that contained four beams in the immediate roof, each with a thickness of 0.45 m (1.5 ft) and held together with 1.8-m (6-ft) fully grouted bolts. The bolt length was made to equal the thickness of the four beams. The bolts were modeled using the built-in structural elements in FLAC3D and the node spacing and normal stiffness values were specified in accordance with Tulu et al. [7].

The model simulates a vertical slice through the coal mine entry and the thickness of the slice is equal to the support row spacing of 1.2 m (4 ft). The stratum layering is modeled with explicit interfaces between each bed. The material is representative of a 30-MPa (4350-psi) shale with a friction angle of 28 degrees and a tensile strength of 17.4 MPa (2520 psi). A major horizontal stress equal to 2.0 times the vertical stress was applied in the plane perpendicular to entry development. In subsequent analyses, the major stress was rotated by 45° in the vertical plane to induce asymmetrical failure in the model. The minor horizontal stress was equal to the vertical stress.

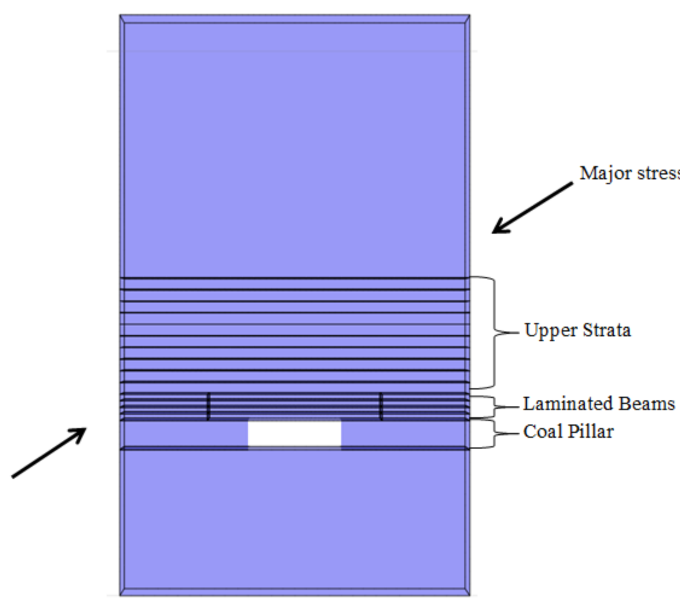

Fig. 3 - Simplified FLAC3D model that shows the four beams in the immediate roof.

Prior to investigating the shear resistance provided by the bolts in the coal mine entry model, a study was conducted to examine how to accurately model the beams and verify that the resulting rock matrix element size was appropriate for modeling the shear resistance provided by the bolts.

3.1. Determination of the Number of Elements in a FLAC3D Beam

The number of elements to model accurate beam bending deflections needed to be determined. Stephannson developed analytical equations for the deflection of single-beam and double-beam roofs resting on a soft material [10]. Stephannson's method considers the Young's modulus of the supporting pillar, the Young's moduli of the two beams (for the double-beam roofs), and the thicknesses of the beams (for the double-beam roofs). The single-beam and double-beam roofs were modeled in FLAC3D to determine the number of elements required in the beam to match the beam deflection that Stephannson's solutions predicted. Multiple combinations of Young's moduli and thicknesses of the beams were modeled and verified. One example is shown below in Figure 4 with input properties and results given in Table 1. It was found that a minimum number of five elements in the vertical direction were required to obtain deflections that matched well with a variety of Stephannson's roof configurations. Cube-shaped elements were used in all the models. If more than five rows of elements were used to create a beam, the deflection measurement became more accurate; however, the number of required elements increased dramatically, which resulted in model run times that were not practical. The 45-cm-thick (1.5-ft-thick) beams in the immediate roof of the entry (Figure 1) were therefore modeled using 9.0-cm (3.6-inch) cube elements.

Table 1 - Inputs and results for the comparison between the analytical and FLAC3D model solutions.

Property	**Value**
Roof span	15 m
Beam 1 thickness	3 m
Beam 2 thickness	1 m
Mining height	5 m
Rock density	2.6 g/cm^2
Young's Modulus (Pillar)	1x10^5 kg/cm^2
Young's Modulus (Beam 1)	5x10^5 kg/cm^2
Young's Modulus (Beam 2)	5x10^5 kg/cm^2
Deflection (theoretical, Beam 1)	0.0400 cm
Deflection (model, Beam 1)	0.0456 cm
Deflection (theoretical, Beam 2)	0.1300 cm
Deflection (model, Beam 2)	0.1450 cm

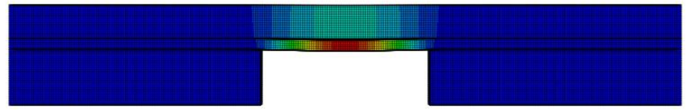

Fig. 4 - FLAC3D model solution of Stephannson's (1971) double layer roof configuration.

3.2. Verification of Bolt Modeling Accuracy

The 9.0-cm (3.6-inch) rock matrix element size is larger than the 2.5-cm (1-inch) element size used by Tulu et al. [7]. A brief study was conducted to determine whether the larger element size would affect the response of the bolts within the model.

The same model that Tulu et al. created was used for this study and an example is shown in Figure 5. The model represents a top block and a bottom block of rock, with a fully grouted bolt embedded in the two rock blocks. The blocks are cubes with 87-cm edge dimensions. The fully grouted bolt was modeled with 1-cm node spacing and grout normal stiffness of 1x109 N/m/m as recommended by Tulu et al. The top block is displaced as shown in Figure 4 which creates a shear resistance in the bolt. The shear displacement and bolt contribution to shear resistance is recorded. After the blocks have approximately 50 mm of shear displacement, the model is stopped.

The model tests were repeated for rock matrix element sizes that varied from 87 cm down to 2.9 cm. The effect of element size on bolt shear resistance can be seen in Figure 6. The results show that the bolt response is not sensitive to element size within the range modeled. From these results, it is expected that the bolt in the coal mine entry model will give similar shear resistance results as Tulu et al. found using much smaller elements.

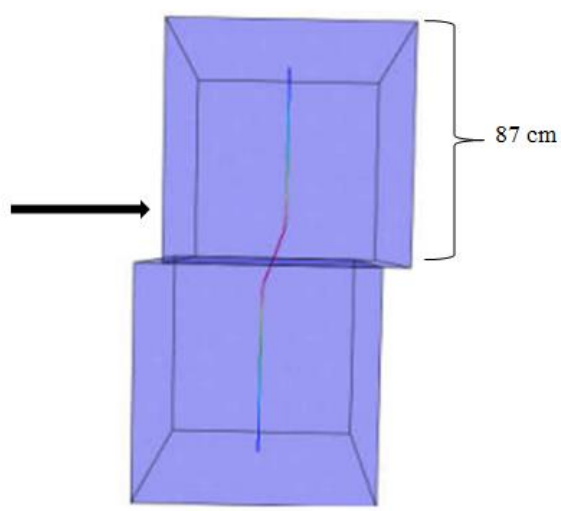

Fig. 5 - Model used to determine the effect of element size on bolt shear resistance.

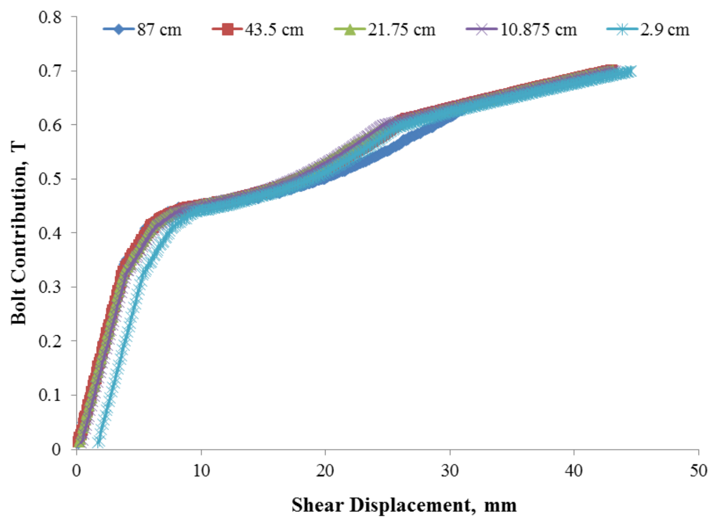

Fig. 6 - Effect on shear resistance response by changing the rock matrix element size.

4. NUMERICAL MODELING OF SHEAR RESISTANCE IN A COAL MINE ENTRY IN FLAC3D

4.1. Elastic Model

An elastic model with parameters given at the beginning of Section 2 was solved. The relative shear displacement along the bedding planes and horizontal force on the bolt were measured. For the elastic models, relative horizontal displacements between the beams in the immediate roof were in the range of 10-5 to 10-6 m. Due to the minimal displacements, no significant shear resistance by the bolt could be observed. Referencing work conducted by Haas [9] and given the span in the coal mine roof and number of beds used in this study, these low horizontal displacements were expected for an elastic model. A large number of models were created with different depths of cover and pretentions on the bolts, but due to the minimal horizontal displacements, there was no significant shear resistance that could be observed in the bolts. For an elastic model, Haas found that an entry span of over 120 m (400 ft) was required to have a relative horizontal displacement between bedding planes of 2.54 cm (1 in). Since these spans are significantly larger than what is encountered in a coal mine, it was concluded that for significant displacements along bedding planes to occur, the model needed to include failure of rock.

4.2. Mohr-Coulomb Model

The Mohr-Coulomb model setup was similar to Figure 3, except the rock matrix elements representing the four beams in the immediate roof were assigned elastic-perfectly plastic behavior. The elements representing the upper strata, coal pillar, and floor remained elastic to prevent complex failure mechanisms from occurring. A model was evaluated for a 30-MPa shale geology; the major principal stress was 2.0 times the vertical stress and rotated by 45 degrees. The depth of cover was 350 m. The rotated stress and depth of cover were expected to cause failure of the beam material, sliding between the beams and inducing shear resistance in the bolts. The bolting pattern only included three bolts per row to allow more significant sliding along the bedding and potential shear failure planes. Figure 7 shows an example of the results in which the roof was critically loaded prior to the onset of complete collapse. In the figure, the rock matrix elements that are colored blue are de-stressed and the rock matrix elements colored red are highly stressed. The gray elements within the bolted horizon represent a rock element that has failed. The gray elements in the upper strata, coal pillar, and floor are elastic and cannot fail. The displacements in the model have been magnified to show the effect of the failure in the rock matrix elements.

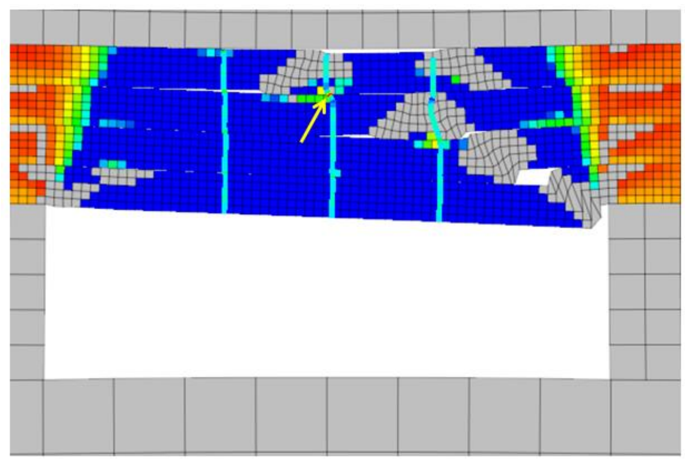

Fig. 7 - Critically loaded weak shale roof at 350 m depth with a major horizontal stress 2.0 times the vertical, with the degree of lateral force on the bolt indicated.

A large shear beginning on the right and moving up and diagonal is the main mode of roof failure. The model indicates a high horizontal reaction force in the middle bolt of approximately 50 kN, shown by the red nodes in the bolt pointed out by the arrow. This value is in the same range as another numerical modeling study of weak coal mine roof stability [12]. At the same bolt node, an axial force of approximately 50 kN is present. For this specific location, the model indicates that the bolt is resisting both lateral and vertical displacements an equal amount. However, the axial forces on other sections of the bolt, mainly the middle and right side of the bolt, are very close to the ultimate bolt strength of 190 kN, indicated by the red color on the bolt nodes in Figure 8. Zhang also found that, on average, the axial force magnitude was much greater than the horizontal load magnitude. Overall, the model indicates that stability is primarily achieved through axial suspension.

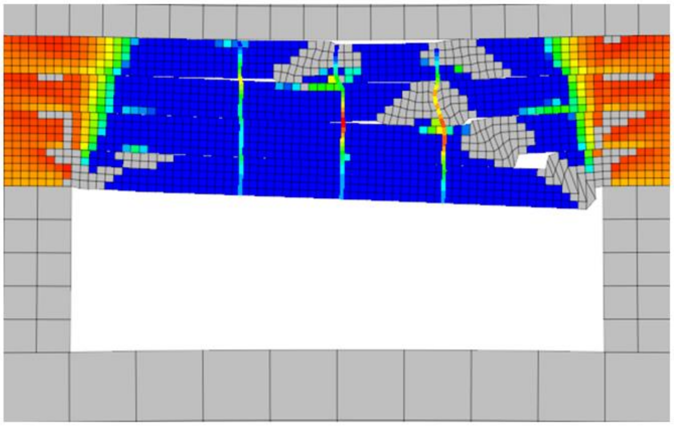

Fig. 8 - Critically loaded weak shale roof at 350 m depth with a major horizontal stress 2.0 times the vertical, with the degree of axial force on the bolt indicated.

To investigate the degree that the lateral resistance had on the overall stability of the rock mass, a second model was run where the grout strength was reduced by a factor of 10. By reducing the grout strength, the bolt's ability to resist lateral movements diminished. The result is shown in Figure 9.

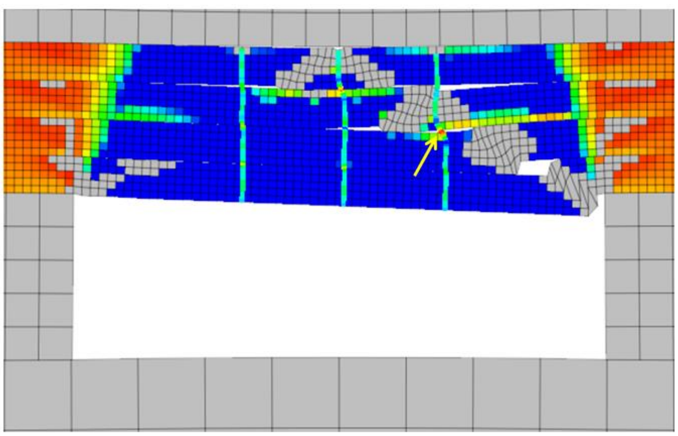

Fig. 9 - Critically loaded weak shale roof at 350 m depth with a major horizontal stress 2.0 times the vertical, with the grout strength reduced by a factor of 10.

The same large shear seen in the previous model beginning at the right side of the entry is the main mode of roof failure. Due to the shear failure plane, a high horizontal reaction force is generated in the right bolt, indicated by the red color on the bolt node. The horizontal reaction force has a magnitude of 13 kN, much lower than the previous model. In the same bolt nodes as the high horizontal reaction force, the axial force on the bolt has a magnitude of 166 kN, which is very close to the ultimate bolt strength of 190 kN. The bolt appears to provide some shear resistance, but the suspension effect provided by the bolt is much greater. The horizontal reaction force of 13 kN generated on the bolt was dependent on the grout strength, as grout around the bolt in this location was crushed. Although the bolt's ability to resist lateral movements was diminished, the overall failure mechanisms and vertical displacements were not substantially different than when the bolts had the ability to resist lateral movements.

Additional detailed models were run for beams consisting of a 60-MPa sandstone with the same horizontal stress condition as before, but at a depth of 500 m. No significant additional observations were made in relation to the bolts resisting relative horizontal displacements enough to provide further stability to the mine roof.

4.3. *Stability Factor Calculations for Weak vs. Strong Grout*

The strength reduction method [19] has been utilized to calculate a stability factor for supported coal mine entries using FLAC3D models [18]. The approach has been found to provide realistic results when verified against field monitoring studies and empirically determined stability factors for coal mine entries [20]. An advantage to using this method is that the impact of a single change to the mining condition or support design can be measured.

To further investigate the effect that a weak strength grout has on overall roof stability, a series of models were analyzed with varying geologies, depths, and stress conditions. Stability factors were calculated for the various mining conditions that included four 1.8-m (6-ft) fully grouted bolts with an entry spacing of 1.2 m (4 ft). The geologies were within a Coal Mine Roof Rating (CMRR) [21] range of 36 to 43, with depths between 100 m and 300 m and stress fields corresponding to conditions encountered in the Eastern and Western U.S. coal mines [22]. The rock matrix element sizes in the vicinity of the entry were 0.25 m (10 inches). The rock matrix elements were given ubiquitous joint properties to account for thinly bedded strata. A chart showing the stability factor comparisons between the strong grout and the weak grout for each mining condition is plotted in Figure 10. A 1:1 line is drawn through the results, and it can be seen that the points plot on or just below the line. This chart indicates that weakening the grout by a factor of 10 did not have a significant effect on the overall stability of the rock mass.

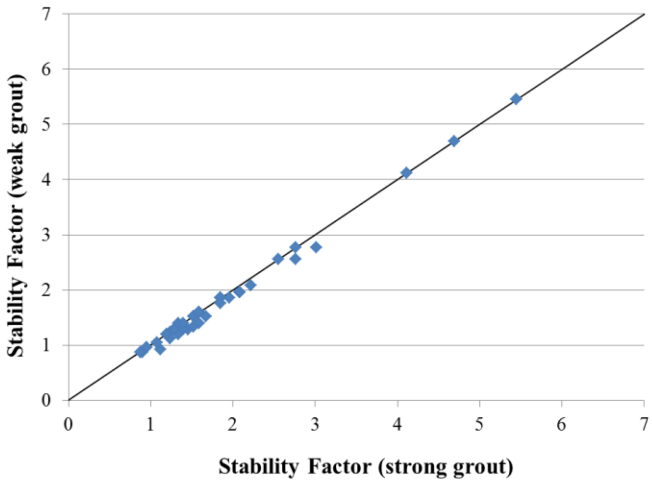

Fig. 10 - Stability factor comparison for strong and weak grout over a wide range of mining conditions.

A visual comparison between two of the models is shown in Figure 11. The mining condition for this model is a geology with 30-40 MPa shale within the bolted horizon and 60 MPa sandstone high above, a major horizontal stress two times the vertical, and a depth of 200 m. The top figure is representative of a support design with strong grout and the bottom figure is representative of a support design with weak grout. The model represents the conditions of the rock mass and support units at the onset of collapse. The heavily shaded areas within the rock matrix elements represent heavy damage to the strata, such as shear failures. The coloring in the bolts is indicative of the yield state of the normal coupling spring between the bolt and the rock, which represents whether the grout has failed or not. A dark blue color indicates no yield in the normal coupling spring and a light blue color represents yield in the normal coupling spring. The model for the support design with weak grout (bottom) indicates that in a pre-collapsed state, the grout has failed on all the bolts. However, the degree of the rock mass failure and displacement of the roof is not significantly different than when the grout is stronger and does not yield.

It is believed that the failed coal mine roof contains complex mechanisms that affect the axial and lateral forces that act on the bolt. The failed coal mine roof beds do not purely slide at their interfaces because the intact rock below or above the sliding plane may also be failing. In addition, as the beds slide along the interface they are also subject to separation. Based on the model results in this study, it is believed that the sliding that occurs between two beds in a coal mine roof is not easily compared to the sliding that occurs in a controlled laboratory direct shear test. Therefore, the shear force magnitude on a bolt could be significantly different in a coal mine roof setting for the same amount of shear displacement in a controlled laboratory shear test.

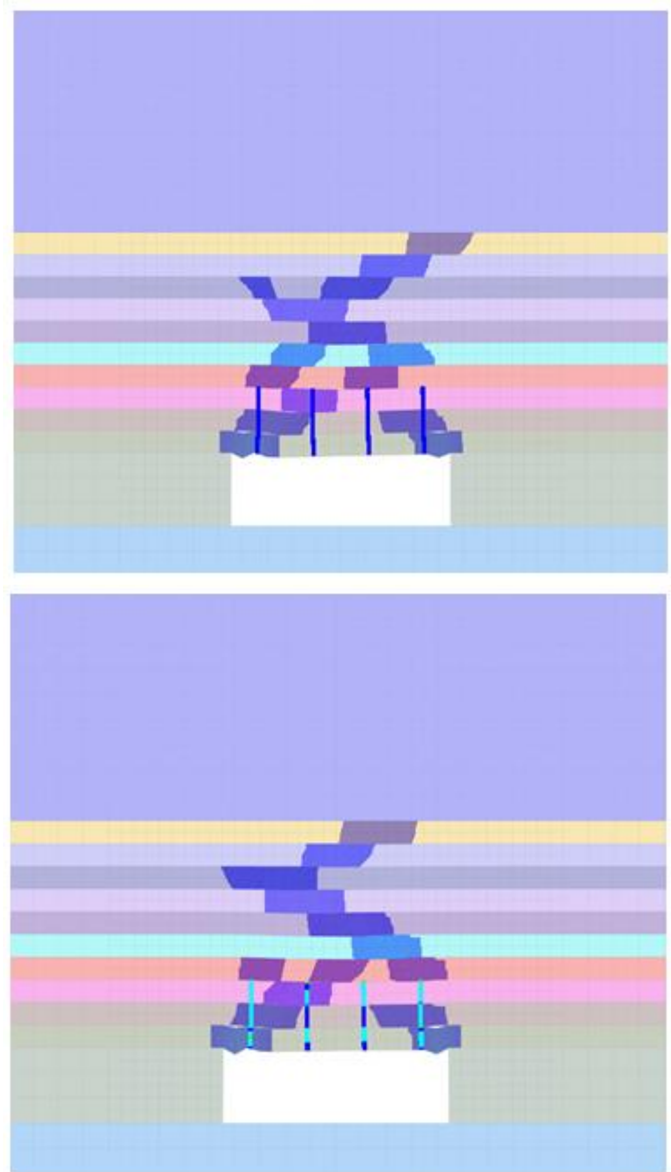
Fig. 11 - Comparison of rock mass and support conditions for a strong (top) and weak (bottom) grout.

5. CONCLUSION

Previous work showed that a fully grouted bolt could be modeled to resist shear displacements and measure similar horizontal forces that have been found in laboratory tests [7]. The work conducted by Tulu assumed pure sliding along an interface, which could cause significantly higher horizontal forces acting on the bolt. In a coal mine, as the beams slide across an interface, they could also separate as the rock is failing around the sliding location, which complicates and affects the amount of lateral reinforcement provided by the bolt. Since it is difficult to measure these forces in the field, numerical models were created to study the lateral reinforcement provided by a fully grouted bolt.

In an elastic model, since failure did not occur, the degree of horizontal movement caused insignificant lateral resistance in the bolts and had minimal impact on the roof deflection.

For a Mohr-Coulomb model, the beams deflected and major shear planes developed; however, the bolts provided a significantly larger axial resistance than lateral resistance. The lateral resistance was dependent on the grout strength used in the model, but the overall stability of the mine roof was not affected by different grout strengths. It is expected that if larger horizontal movements occurred in the model, larger shear resistances could have been measured. However, any further horizontal displacements between the beds would have caused complete collapse of the mine roof.

A series of analyses in which the strength reduction method was applied for coal mine entries in a variety mining conditions showed that the lateral resistance of the grouted bolts has little impact on the ultimate stability of the roof.

It is believed that the field observation of shear failures in bolts potentially happen during massive collapses, not from high shear loads in the pre-collapsed state. The model results indicate that the axial suspension effects of fully grouted bolts are more significant than the lateral reinforcement provided.

REFERENCES

1. Brady, B.H.G., and E. T. Brown. 2004. *Rock mechanics: for underground mining*.3rd Ed. London: Kluwer Academic Publishers.

2. Peng, S.S. 2008. *Coal mine ground control*. 3rd ed. Morgantown WV.

3. Anon. 2012. *Fast Lagrangian Analysis of Continua in Three Dimensions (FLAC3D), User's Guide*. Minneapolis MN: Itasca Consulting Group.

4. Grasselli, G. 2005. 3D Behavior of bolted rock joints: experimental and numerical study. *Int. J. Rock Mech. Min Sci*. 42: 13–24.

5. Jalalifar, H. and N. Aziz. 2010. Experimental and 3D Numerical Simulation of Reinforced Shear Joints. *Rock Mech Rock Eng*. 43: 95–103

6. McHugh E. and S.P. Signer. 1999. Roof bolt response to shear stress: laboratory analysis. In *Proceedings of the 18th International Conference on Ground Control in Mining*. 232–238. Morgantown WV.

7. Tulu, I.B., G.S. Esterhuizen, and K.A. Heasley. 2012. Calibration of FLAC3D to Simulate the Shear Resistance of Fully Grouted Rock Bolts. In *Proceedings of the 46th US Rock Mechanics / Geomechanics Symposium, 24-27 June 2012*. Chicago IL.

8. Panek, L.A. 1964. Use of vertical roof bolts to reinforce an arbitrary sequence of beds. In *Proceedings of the International Symposium on Mining Research. Vol 2.* 499-508.

9. Haas, C.J., R.L. Davis, H.D. Keith, D. Junakkumar, W.C. Patrick and J.R. Strosnider Jr. 1976. *An Investigation of the Interaction of Rock and Types of Rock Bolts for Selected Loading Conditions. Third Annual Report*. Rock Mechanics and Explosives Research Center, University of Missouri-Rolla. USBM Contract Report (H0122110).

10. Stephansson, O. 1971. Stability of single openings in horizontally bedded rock. *Engineering Geology*. 5: 5-71.

11. Zhang, Y. and S.S. Peng. 2001. Effects of bedding plane sliding and separation and tensioned bolt in layered roof. In *Proceedings of the 20th International Conference on Ground Control in Mining*: 226-234. Morgantown WV.

12. Zhang, Y., P.L Meighen, M. Gadde, K. Morsy and S.S. Peng. 2007. Design considerations of roof bolting under very weak roof conditions. *Mining Engineering* 59(9): 67-72.

13. Mathews, K.E. and J.L. Meek. 1975. Modelling of Rock Reinforcement Systems in Cut and Fill Mining. *Second Australia-New Zealand Conference on Geomechanics*, Brisbane, The Australian Geomechanics Society.

14. Gerdeen, J.C., V.W. Snyder, G.L. Viegelahn, and J. Parker. 1977. *Design Criteria for Roof Bolting Plans Using Fully Resin-Grouted Nontensioned Bolts to Reinforce Bedded Mine Roof, Volume II, Field Survey*. Department of Mechanical Engineering-Engineering Mechanics, Michigan Technological University, Houghton, Michigan 49931.

15. Peng, S.S. 2007. *Ground Control Failures – A pictorial view of case studies.* Morgantown WV.

16. Zhang, P., S. Wade, E. Zeglen, S. Peterson, R. Lawrence, M. Mishra, R. Smith, G. Deemer and R. Bottegal. 2010. Roof bolting in low seam mining with thick "stack rock" roof under high horizontal stress. In *Proceedings of the 29th International Conference on Ground Control in Mining*. Morgantown WV.

17. Zipf, R.K. 2006. Numerical modeling procedures for practical coal mine design. In *Proceedings of the 41st U.S. Rock Mechanics Symposium 17-21 June 2006 Golden, Colorado*. 1-11. Alexandria, Virginia: American Rock Mechanics Association.

18. Esterhuizen, G.S. 2012. A Stability Factor for Supported Mine Entries Based on Numerical Model Analysis. In *Proceedings of the 31st International Conference on Ground Control in Mining July 31 – August 2, 2012,* Morgantown WV.

19. Zienkiewicz, O.C., C. Humpheson and R.W. Lewis. 1975. Associated and Non-Associated Visco-Plasticity and Plasticity in Soil Mechanics. *Geotechnique*. 25: 617–689.

20. Esterhuizen, G.S., T.S. Bajpayee, M.M. Murphy, and J.L. Ellenberger. 2013. Validation of entry stability factors determined by the strength reduction method against empirical methods. In *Proceedings of the 32nd International Conference on Ground Control in Mining, July 30-August 1, 2013, Morgantown, West Virginia*. 82-89. West Virginia University.

21. Molinda G.M., C. Mark. 1994. *Coal Mine Roof Rating (CMRR): A Practical Rock Mass Rating for Coal Mines*. Pittsburgh, PA: U.S. Department of the Interior, Bureau of Mines. IC 9387. 83 pp.

22. Murphy, M.M. and G.S. Esterhuizen. 2013. Quantifying the benefit of cable bolts as supplementary support in coal mines using the strength reduction method. In: *Proceedings 47th US Rock Mechanics / Geomechanics Symposium, San Francisco, CA*, eds. L.J Pyrak-Nolte, A. Chan, W. Dershowitz, J. Morris, J. Rostami. ARMA Paper 13-143.