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A Frequency Domain Technique to De-Dopplerize the Acoustic Signal from a Moving Source of Sound

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The acoustic signal emitted by a moving source of sound and sampled by a stationary microphone is distorted in amplitude and frequency by the Doppler effect. For years, recovery of the original (undistorted) source signal has been done by means of a time domain de-Dopplerization technique. This technique requires the interpolation of every data point of the microphone signal in order to remove the Doppler effect. As a consequence, this technique is computationally intensive. This paper presents an alternative de-Dopplerization technique that performs all the computations in the frequency domain. This technique requires the linearization of the relation between emission time, i.e. the time at which the signal is emitted by the source, and reception time, i.e. the time at which the signal arrives to the microphone. Then, the linear, translation, and scaling properties of the Fourier transform are used to remove the Doppler effect. In essence, this technique computes the Fourier transform of the original (undistorted) source signal directly from the Dopplerized microphone signal. Since this frequency domain de-Dopplerization technique does not require interpolation of the microphone signal, it is computationally more efficient than the traditional time domain de-Dopplerization technique.

I. Introduction

Sources of sound can be categorized by their spatial location relative to the observer as fixed sources (time invariant location) or moving sources (time dependent location). Moving sources of sound can be found in several different categories such as airframe noise[1-3], railway noise[4], wind turbine noise[5], and coal cutting noise[6], just to mention a few. Furthermore, moving sources can have different types of motion, the most common being linear motion (i.e., sources moving along a linear trajectory) and rotating sources (i.e., sources moving along a circular trajectory).

The acoustic signal emitted by a moving source of sound and sampled by a fixed microphone is distorted in amplitude and frequency by the Doppler effect. Thus, in order to measure the correct amplitude and frequency content of the microphone signal, the Doppler effect needs to be removed.

For years, recovery of the original source signal has been done by means of a time domain de-Dopplerization technique[7,8]. However, this technique requires interpolation of every data point of the microphone signal in order to remove the Doppler effect, and therefore it is computationally intensive.

This paper presents an alternative de-Dopplerization technique that performs all the computations in the frequency domain. This technique requires the linearization of the relation between emission time, i.e. the time at which the signal is emitted by the source, and reception time, i.e. the time at which the signal arrives to the microphone. Then, the linear, translation, and scaling properties of the Fourier transform are used to remove the Doppler effect.

In what follows, the basic concept of the Doppler effect in the acoustic field of a monopole source is first presented. Then, the currently used time domain de-Dopplerization technique is described followed by an explanation of the new frequency domain de-Dopplerization technique. Finally, conclusions are drawn from comparison between both techniques.

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II. Basic Concepts

This section presents the derivation of the equation of the acoustic field of a fixed monopole source, and then the derivation is extended for a moving monopole source.

A. Acoustic Field of a Fixed Monopole Source

The derivation presented in this section is based on the work by Dowling and Ffowcs Williams[9]. The acoustic field produced by a distribution of fixed sources of sound is governed by the inhomogeneous wave equation,

$$\frac{1}{c^2}\frac{\partial^2 p\left(\vec{x},t\right)}{\partial t^2}-\nabla^2 p\left(\vec{x},t\right)=q\left(\vec{x},t\right) \tag{1}$$

where $p(\vec{x},t)$ is the acoustic pressure, $q(\vec{x},t)$ is the source function, $\vec{x}$ is the vector of spatial coordinates, t is the time variable, and c is the speed of sound. The coordinate system used here is Cartesian with the orthogonal directions referred by the indices 1, 2, and 3, respectively. If needed, the unit vectors are noted as $\vec{e}_\ell\ \ell=1,2,3$.

An expression for the acoustic pressure that satisfies equation (1) can be obtained using the Green's function method. This method allows for writing the solution of equation (1) in terms of a Green's function as

$$p\left(\vec{x},t\right)=\int_{-\infty}^{\infty}\int_V q(\vec{y},\tau)G\left(\vec{x},t\middle|\vec{y},t_s\right)d^3y\,d\tau \tag{2}$$

In equation (2), G is the free-field Green's function, which is the solution of the wave equation for an impulsive point source located at point $\vec{y}$ that emits a sound pulse at time t_s. The spatial integration is over a volume *V*, enclosing the source. Thus, the Green's function satisfies the following equation:

$$\frac{1}{c^2}\frac{\partial^2 G\left(\vec{x},t\middle|\vec{y},t_s|\right)}{\partial t^2}-\nabla^2 G\left(\vec{x},t\middle|\vec{y},t_s|\right)=\delta(\vec{x}-\vec{y})\delta(t-t_s) \tag{3}$$

where δ is the Dirac delta function. Solution of equation (3) for the impulsive point source in an unbounded region yields the free-field Green's function[10],

$$G\left(\vec{x},t\middle|\vec{y},t_s\right)=\frac{\delta(t-t_s-r/c)}{4\pi r} \tag{4}$$

where r is the distance between the source and the observer locations given as

$$r=\left|\vec{x}-\vec{y}\right|. \tag{5}$$

Substitution of equation (4) into equation (2) yields

$$p(\vec{x},t)=\int_{-\infty}^{\infty}\int_V q(\vec{y},t_s)\frac{\delta\left(t-t_s-\left|\vec{x}-\vec{y}\right|/c\right)}{4\pi\left|\vec{x}-\vec{y}\right|}d^3\vec{y}dt_s \tag{6}$$

Evaluation of the time integral in equation (6) can be performed using the "sifting" property of δ functions, thus, obtaining an expression for the acoustic pressure induced by the source distribution, given by

$$p\left(\vec{x},t\right)=\int_V\frac{q\left(\vec{y},t-\left|\vec{x}-\vec{y}\right|/c\right)}{4\pi\left|\vec{x}-\vec{y}\right|}d^3\vec{y}=\int_V\frac{q\left(\vec{y},\tau\right)}{4\pi\left|\vec{x}-\vec{y}\right|}d^3\vec{y} \tag{7}$$

Figure 1 shows a schematic representation of a source field and the sound field it induces. As seen from this figure, the acoustic pressure perceived by an observer located at $\vec{x}$ is obtained by integration over the source field of the source function of an elemental source at $\vec{y}$ normalized by the distance from the elemental source to the observer. The variable τ represents the time at which the sound was emitted and is, thus, known as the emission time. It is given as

$$\tau = \frac{t - |\vec{x} - \vec{y}|}{c}. \tag{8}$$

Consequently, the difference $t - \tau = |\vec{x} - \vec{y}|/c$ accounts for the time required for the sound to travel from $\vec{y}$ to $\vec{x}$, and it is called the propagation time. Therefore, we have an emission time τ, arrival or reception time *t*, and propagation time $t - \tau = |\vec{x} - \vec{y}|/c$.

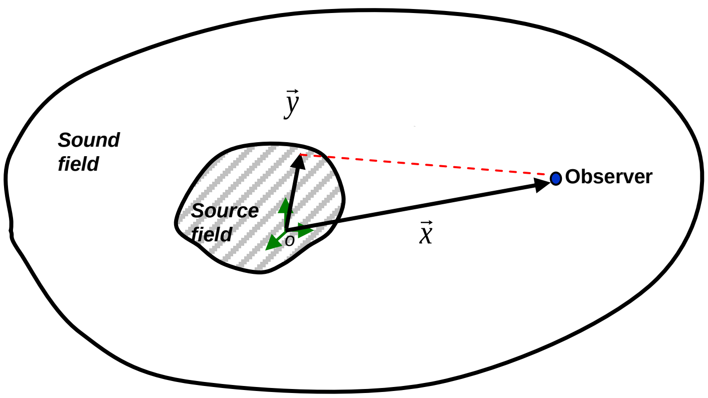

Figure 1. Schematic representation of a source field and the sound field induced by this source.

If the source is acoustically compact (i.e., source dimensions are small compared to the wavelength of the emitted sound), the source can be considered a point source, i.e., monopole source. In this case, the propagation times from different parts of the source to the observer are essentially the same. The source function for the compact source located at $\vec{x}_s$ becomes

$$q(\vec{x},t) = q(t)\,\delta(\vec{y} - \vec{x}_s) \tag{9}$$

where $q(t)$ is called the monopole source strength. Substitution of equation (9) into equation (7) yields

$$p(\vec{x},t) = \int_V \frac{q\left(t - |\vec{x} - \vec{y}|/c\right)\delta(\vec{y} - \vec{x}_s)}{4\pi|\vec{x} - \vec{y}|} d^3y\,. \tag{10}$$

Evaluation of the integral of equation (10) results in

$$p(\vec{x},t) = \frac{q\left(t - |\vec{x} - \vec{x}_s|/c\right)}{4\pi|\vec{x} - \vec{x}_s|}, \tag{11}$$

which is the expression for the acoustic pressure induced by a stationary monopole source in free field.

B. Acoustic Field of a Moving Monopole Source

To find an expression for the acoustic pressure induced by a moving monopole source with time dependent position $\vec{x}_s(t)$, the source function can be written as

$$q\left(\vec{x},t\right)=q\left(t\right)\delta\left(\vec{y}-\vec{x}_s\left(t\right)\right) \quad (12)$$

Substitution of equation (12) into equation (6) yields an expression for the acoustic pressure induced by this moving monopole source, as follows:

$$p\left(\vec{x},t\right)=\int_{-\infty}^{\infty}\int_V \frac{q(t_s)\delta\left(\vec{y}-\vec{x}_s\left(t_s\right)\right)\delta\left(t-t_s-\left|\vec{x}-\vec{y}\right|/c\right)}{4\pi\left|\vec{x}-\vec{y}\right|}d^3\vec{y}dt_s \quad (13)$$

.

Evaluation of the spatial integral yields

$$p\left(\vec{x},t\right)=\int_{-\infty}^{\infty}\frac{q(t_s)\delta\left(t-t_s-\left|\vec{x}-\vec{x}_s\left(t_s\right)\right|/c\right)}{4\pi\left|\vec{x}-\vec{x}_s\left(t_s\right)\right|}dt_s . \quad (14)$$

Since the argument of the Delta function is a function of t_s, the evaluation of the time integral is now more complicated. To this end, the Delta function must be re-expressed as

$$\delta\left[g\left(t_s\right)\right]=\sum_i \frac{\delta\left(t_s-\tau_i\right)}{\left|\frac{dg\left(t_s\right)}{dt_s}\right|_{t_s=\tau_i}} \quad (15)$$

where τ_i are the roots of the argument[11], e.g. $g\left(\tau_i\right)=0$. The summation is over the number of physically meaningful roots, e.g. assuming there is more than one non-spurious root. Here the function $g(t_s)$ is

$$g\left(t_s\right)=t-t_s-\frac{\left|\vec{x}-\vec{x}_s\left(t_s\right)\right|}{c} \quad (16)$$

The derivative of equation (16) with respect to t_s is given by

$$\left|\frac{dg\left(t_s\right)}{dt_s}\right|=\left|1-M_{so}\left(t_s\right)\right| \quad (17)$$

where M_{so} is the component of the source velocity $\vec{v}$ (in Mach) in the source-to-observer direction. That is,

$$M_{so}\left(t_s\right)=\frac{1}{c}\frac{\left(\vec{x}-\vec{x}_s\left(t_s\right)\right)}{\left|\vec{x}-\vec{x}_s\left(t_s\right)\right|}\cdot\vec{v} \quad (18)$$

Substitution of equations (16) and (17) expressed in the form of equation (15) into equation (14) yields

$$p(\vec{x},t)=\int_{-\infty}^{\infty}\frac{q(t_s)}{4\pi\left|\vec{x}-\vec{x}_s(t_s)\right|}\sum_i\frac{\delta(t_s-\tau_i)}{\left|1-M_{so}(t_s)\right|}dt_s\,. \tag{19}$$

Evaluation of the integral in equation (14) is now straightforward, yielding

$$p(\vec{x},t)=\sum_i\frac{q(\tau_i)}{4\pi\left|\vec{x}-\vec{x}_s(\tau_i)\right|\left|1-M_{so}(\tau_i)\right|}\,, \tag{20}$$

where τ_i is obtained as the solution of

$$t-\tau_i=\left|\vec{x}-\vec{x}_s(\tau_i)\right|/c\,; \tag{21}$$

that is, the roots of equation (21). Note that the summation in equation (20) should only include physically meaningful roots. For subsonic motion, the term $\left|1-M_{so}(\tau_i)\right|$ in equation (20) represents an amplification factor of the sound produced by the source motion. This effect is known as the Doppler amplification. In this equation, the source strength, $q(\tau_i)$, and source position, $\vec{x}_s(\tau_i)$, are defined in terms of the emission time , τ , while the pressure at the observer location is given in the reception time, t .

III. Time Domain de-Dopplerization Technique

The objective of this technique is to compute the original source signal at emission times, $q(\tau)$. Therefore, the sampling time t_{sp} is treated as emission time τ ; in other words, $t_{sp}=\tau$. The source strength signal can be obtained from equation (20) where the microphone signals are in terms of reception times, $t_j,\ j=1,2,3,...,J$. That is,

$$q(\tau_j)=p(\vec{x},t_j)4\pi\left|\vec{x}-\vec{x}_s(\tau_j)\right|\left|1-M_{so}(\tau_j)\right|. \tag{22}$$

Figure 2 shows a schematic of this technique. As seen from this figure, the actual sampled microphone signals are in terms of emission time $p(\vec{x},\tau_j)$. In order to obtain the microphone signal in terms of reception time, the following calculations are required. First, the reception time t_j corresponding to the emission time τ_j is computed using equation (21) as (step 1 in Figure 2a):

$$t_j=\tau_j+\frac{\left|\vec{x}-\vec{x}_s(\tau_j)\right|}{c}\,. \tag{23}$$

Then, the microphone signal required for equation (22) is obtained by shifting the sampled microphone signals by a delay equal to $\left|\vec{x}-\vec{x}_s(\tau_j)\right|/c$, i.e. $p\left(\vec{x},\tau_j+\left|\vec{x}-\vec{x}_s(\tau_j)\right|/c\right)$. It is generally the case that the microphone signal at the required reception time t_j will not coincide with a sampled microphone signal data point, as shown in Figure 2b. Therefore, $p\left(\vec{x},\tau_j+\left|\vec{x}-\vec{x}_s(\tau_j)\right|/c\right)$ needs to be approximated by interpolation using the most immediate sample points as $p\left(\vec{x},\tau_j+\left|\vec{x}-\vec{x}_s(\tau_j)\right|/c\right)\approx\tilde{p}(\vec{x},t_j)$, where the "~" denotes that the microphone signal was obtained using interpolation. Figure 2 shows the sampled data points marked with an "x", whereas the interpolated values are marked with an "o".

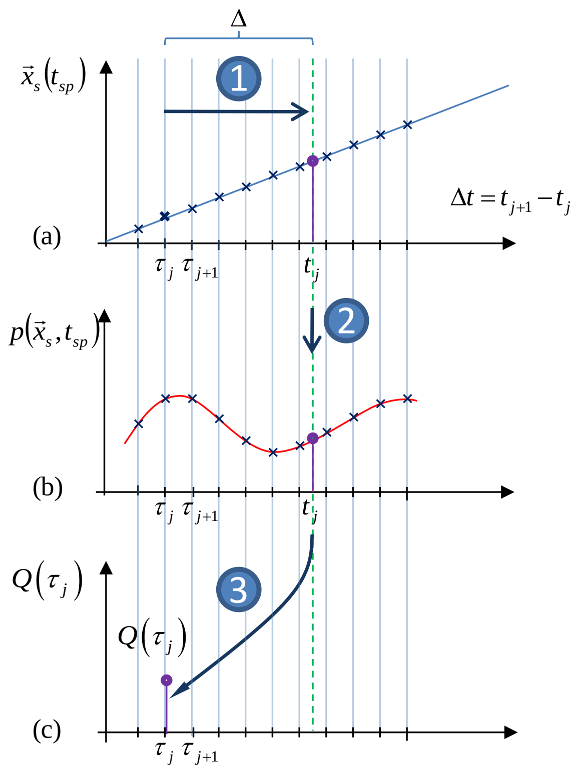

Figure 2. Steps required by the time domain de-Dopplerization technique: (a) Computation of the reception time t_j corresponding to the emission time τ_j, (b) computation of the microphone signal P at t_j, and (c) computation of the source strength Q at τ_j.

Knowing the microphone signal at reception time $\tilde{p}(\vec{x}, t_j)$, the source strength can then be obtained as (see Figure 2c)

$$q(\tau_j) \approx \tilde{p}(\vec{x}, t_j)\, 4\pi \left|\vec{x} - \vec{x}_s(\tau_j)\right| \left|1 - M_{so}(\tau_j)\right| \tag{24}$$

Figure 2a through Figure 2c shows the steps required to de-Dopplerize the microphone signal for data point $t_{sp} = \tau_j$ assuming the sampled data are given in terms of emission time. This process has to be repeated for every data point of the microphone signal.

IV. Frequency Domain de-Dopplerization Technique

The technique proposed in this paper uses the linear, translation, and scaling properties of the Fourier transform to de-Dopplerize the microphone signal. Therefore, an overview of these properties is first presented.

Figure 3 shows a signal, $p(t)$, with a finite duration time, T. This signal is then multiplied by a constant κ, translated by α seconds, and time scaled at a linear rate of β to result in a distorted signal, $p_d(t) = \kappa p(\alpha + \beta t)$. The time window where the distorted signal does not vanish is thus βT. For reasons that will become clear later, the center point of the time window is used here as the reference to identify the translation, i.e., the center of the window is at $t=0$.

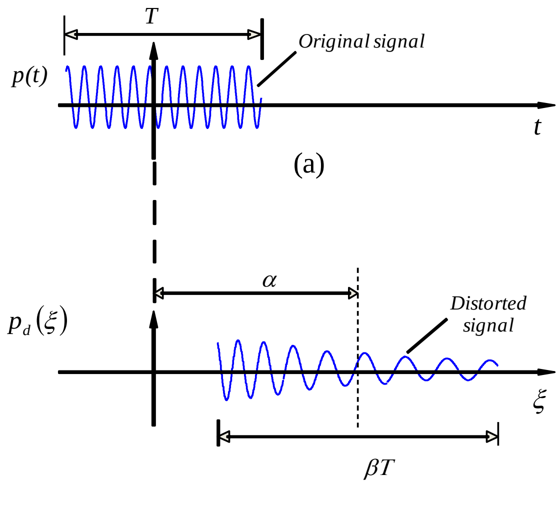

Figure 3. (a) Original and (b) translated and scaled signals.

The Fourier transform of the distorted signal is

$$P_d(f) = \int_{-\infty}^{\infty} p_d(\xi) e^{-j2\pi f \xi} d\xi = \int_{-\infty}^{\infty} p(\alpha + \beta t) e^{-j2\pi ft} dt \tag{25}$$

To solve equation (25) the following change of variables is used:

$$\xi = \alpha + \beta t \tag{26}$$

to express the FT of the distorted signal in terms of the original signal. Replacing equation (26) into equation (25) leads to

$$P_d(f) = \kappa \int_{-\infty}^{\infty} p(\xi) e^{-j2\pi f\left(\frac{\alpha-\xi}{\beta}\right)} \frac{d\xi}{\beta} \tag{27}$$

or

$$\begin{aligned} P_d(f) &= \kappa \int_{-\infty}^{\infty} p(\xi) e^{-j2\pi f\left(\frac{\alpha-\xi}{\beta}\right)} \frac{d\xi}{\beta} \\ &= \kappa \frac{e^{-j2\pi f\left(\alpha/\beta\right)}}{\beta} \int_{-\infty}^{\infty} p(\xi) e^{j2\pi \frac{f}{\beta}\xi} d\xi \\ &= \kappa \frac{e^{-j2\pi \hat{f}\alpha}}{\beta} P\left(\hat{f}\right) \end{aligned} \tag{28}$$

where $\hat{f} = f/\beta$.

This approach implies that the spectrum of the original signal can be obtained from the FT of the amplified, translated, and scaled signal. The spectrum needs to be corrected by adjusting the frequency, phase, and amplitude using the factors $\hat{f} = f/\beta$, $e^{j2\pi \hat{f}\alpha}$, and both κ and β, respectively. That is,

$$P\left(\hat{f}\right)=\frac{\beta}{\kappa}e^{j2\pi\hat{f}\alpha}P_d(f) \tag{29}$$

Note that in the computation of the FT in equation (27), the integral limits must be sufficiently long to encompass both the original and distorted signals. The lower and upper limits can be easily computed as $t_{lower}=-T/2$ and $t_{upper}=\alpha+\beta T/2$, respectively.

In the proposed de-Dopplerization method here, the FT of the distorted signal will be computed and used to estimate the original signal. For convenience, the FT will be carried out only over the window in which the signal does not vanish. That is,

$$P_d(f)=\int_{\alpha-\beta T/2}^{\alpha+\beta T/2} p_d(\xi)e^{-j2\pi f\xi}d\xi \tag{30}$$

In this case, the phase correction term induced by the translation must not be included in the computation of the FT of the original signal. Therefore, the FT becomes simply

$$P\left(\hat{f}\right)=\frac{\beta}{\kappa}P_d(f) \tag{31}$$

under the condition that $P_d(f)$ is computed using equation (30).

The Fourier transform properties will now be used to de-Dopplerize the microphone signal. As shown in Figure 4, consider a source moving along the 1-direction whose position is defined by the vector $\vec{x}_s(\tau)$. The source emits sound with source strength, $q(\vec{x}_s(\tau),\tau)$, at emission time τ. The sound is sensed with a fixed microphone placed at $\vec{x}_n$. The signal observed by the microphone is $p_n(t)$ at the reception time *t*. The source position and microphone signals are recorded simultaneously at the same sampling frequency.

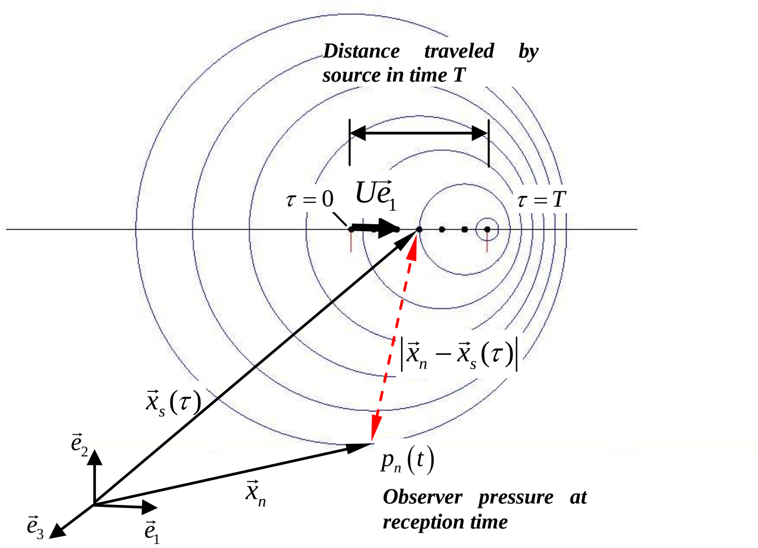

Figure 4. Sound source moving along the 1-direction, sensed by a fixed microphone located at $\vec{x}_n$.

Note that the source position is known at the emission times while the microphone signal is recorded at the reception times. Since the signal recorded by the microphone is non-stationary, we consider the motion of the source over a short duration of time T. For convenience, this time window is selected such that $-T/2 \le \tau \le T/2$, e.g. at $\tau = 0$ the source is at the center of the window. For a given sampling frequency f_s, the number of sampled points in the window is $N_s = f_s T$. Thus, the sampled emission times are $\tau_k = k\Delta t$ for $k = 0,1,\cdots,N_s - 1$.

The microphone time history is given as

$$p_n(t) = \frac{q(\tau)}{4\pi\left|\vec{x}_n - \vec{x}_s(\tau)\right|\left|1 - M_{so}(\tau)\right|} \tag{32}$$

where the emission and reception times are related as

$$t = h_n(\tau) = \tau + \frac{\left|\vec{x}_n - \vec{x}_s(\tau)\right|}{c}. \tag{33}$$

To implement the Fourier transform properties here, the microphone signal plays the role of the distorted signal while the source strength is analogous to the original signal in Figure 3, i.e. $p_n(t) \leftrightarrow p_d(\xi)$ and $q(\tau) \leftrightarrow p(t)$.

This implies that the sound emitted by the source over time T can be related to the time window that the sound is observed by the microphone, $T_{reception}$, as shown in Figure 5. The reception times at the beginning and end of the time window are simply $t_{start} = h_n(-T/2)$ and $t_{end} = h_n(T/2)$. Also, note that the time window at reception times is different from the time window at the emission times, e.g. $T_{reception} = t_{end} - t_{start} \ne T$. In other words, the number of samples within this reception time window is $N_{s-reception} = T_{reception}/\Delta t$ that is different from the number of samples over time T.

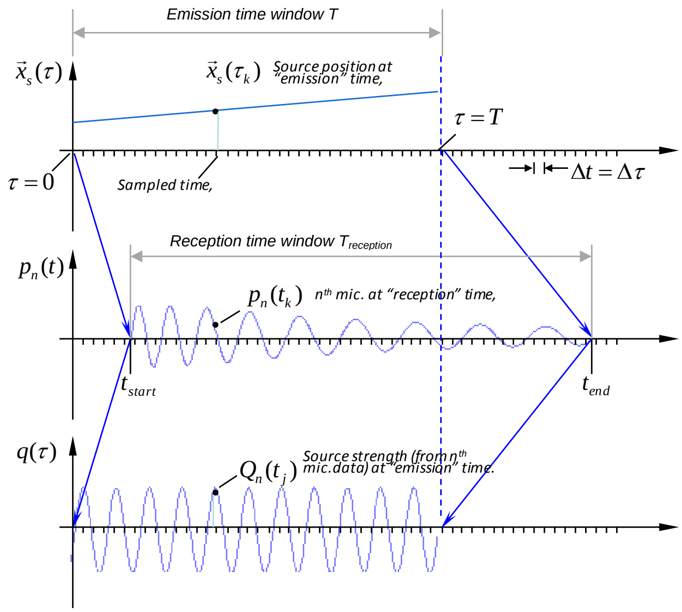

Figure 5. Emission time window and its correspondent reception time window over a short time period.

For the linear motion assumed here and assuming the coordinate system to be located at the source at time $\tau = 0$, $h_n(\tau)$ has a closed form expression given by

$$t = h_n(\tau) = \tau + \frac{\sqrt{(x_{1n} - U\tau)^2 + x_{2n}^2 + x_{3n}^2}}{c} \tag{34}$$

where $\vec{x}_n = x_{1n}\vec{e}_1 + x_{2n}\vec{e}_2 + x_{3n}\vec{e}_3$.

The translation and rescaling properties of the FT can be used if the reception/emission time relationship $t = h_n(\tau)$ is linear. Thus, using Taylor's series expansion around τ_o, this relationship can be written as a linear function as

$$t = h_n(\tau) \approx h_{no} + \frac{dh_n}{d\tau}\tau = h_{no} + h_{n\tau}\tau \tag{35}$$

For $h_n(\tau)$ defined as in equation (34) and the expansion taken at the mid-point of the time window T or $\tau_o = 0$ results in a closed form expansion as

$$t \approx \frac{|\vec{x}_n|}{c} + \left(1 - M\frac{x_{1n}}{|\vec{x}_n|}\right)\tau, \quad -T/2 \le \tau \le T/2 \tag{36}$$

with *M=U/c.*

The linear term, $h_{no} = |\vec{x}_n|/c$ of the expansion is just the delay time from the source to the microphone at $\tau = 0$. The scaling term, $h_{n\tau} = 1 - M\, x_{1n}/|\vec{x}_n|$, is related to the projection of the source velocity in the source-microphone direction, $M_{sn}(0)$ at the expansion time $\tau = 0$. Note that when the microphone is directly underneath the source $h_{n\tau} = 1$ then the Doppler amplification factor vanishes.

Thus, the goal is to estimate the FT of the stationary source strength signal from the FT of the Dopplerized (non-stationary) microphone signal using the linear, translation, and scaling properties of the FT. This computation is performed by relating the source strength signal and the microphone signal to the original signal and the scaled signals shown in Figure 3, respectively. Furthermore, as the signal propagates from the source to the microphone its amplitude decreases due to spherical spreading by $1/4\pi|\vec{x}_n - \vec{x}_s|$. Therefore, the original source strength signal and the Dopplerized microphone signal are related as

$$q(\tau) = p_n(t) 4\pi |\vec{x}_n - \vec{x}_s(\tau)| \tag{37}$$

where the range for the emission and reception times are

$$-T/2 \le \tau \le T/2$$
$$-T_{reception}/2 \le t \le T_{reception}/2 \, .$$

We can now approximate the source strength signal in terms of the emission time using equation (35). That is,

$$p_n(t) \approx \left(\frac{1}{4\pi|\vec{x}_n - \vec{x}_s(\tau)|}\right) q(h_{no} + h_{n\tau}\tau) \, . \tag{38}$$

Note that in equation (38), the term $1/4\pi|\vec{x}_n - \vec{x}_s(\tau)|$ is the multiplying factor κ in equation (31).

To relate the FT of the source signal to the FT of the microphone signal, we need to take the FT of equation (38) over the reception time window $T_{reception}$. To this end, we need to extract from the total microphone signal only the part that is produced by the source over the time window *T* prior to taking the FT. This reception time window can be computed exactly using equation (34) rather than the approximation in equation (35). This process is schematically illustrated in Figure 6.

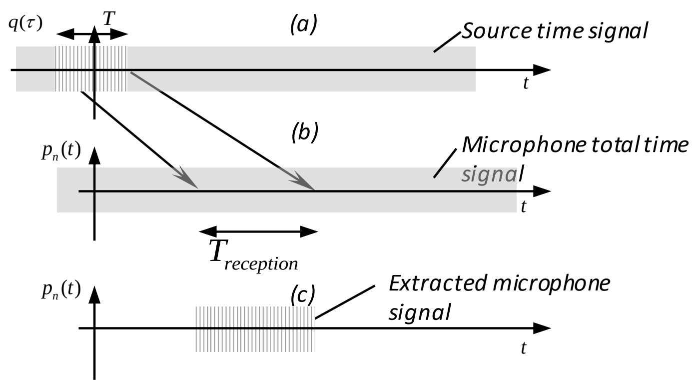

Figure 6. Extraction of the microphone signal that is associated to the emitted sound by the source during time window *T*.

The time dependence of the source-microphone distance complicates the computation of the FT. The simplest approach is to approximate the source-microphone distance by using time independent quantities such as taking its value at the mid-point of the trajectory over the time interval *T*. That is,

$$\left|\vec{x}_n - \vec{x}_s(\tau)\right| \approx \left|\vec{x}_n - \vec{x}_s(0)\right| . \tag{39}$$

Note that $\vec{x}_s(0) = 0$ since we selected to place the origin of the coordinate system at the source at time $\tau = 0$. Then, taking the FT of the microphone extracted signal (Figure 6c) over only the time window $T_{reception}$ leads to

$$FT\left\{p_n\left(t, T_{reception}\right)\right\} \approx \left(\frac{1}{4\pi\left|\vec{x}_n - \vec{x}_s(0)\right|}\right) FT\left\{q\left(h_{no} + h_{n\tau}\tau, T\right)\right\} \tag{40}$$

and using the linear, translation, and rescaling properties of the Fourier transform, the FT of the stationary source strength signal is obtained according to equation (29) as

$$Q\left(\frac{f}{h_{n\tau}}, T\right) \approx \frac{h_{n\tau}}{\left(\frac{1}{4\pi\left|\vec{x}_n - \vec{x}_s(0)\right|}\right)} P_n\left(f, T_{reception}\right) \tag{41}$$

Comparing equation (29) to equation (41), it is clear that the terms $1/4\pi\left|\vec{x}_n - \vec{x}_s(\tau)\right|$, h_{no}, and $h_{n\tau}$ in equation (41) correspond to κ, α, and β in equation (29), respectively. The significance of equation (41) is that it allows obtaining the Fourier transform of the original (undistorted) source signal directly from the Dopplerized microphone signal.

V. Remarks on the Alternative Frequency Domain de-Dopplerization Technique

It is important to mention that in equation (41), the value of the term $h_{n\tau} = \left(1 - M_{sn}\left(0\right)\right)$ is always positive. Furthermore, $h_{n\tau} < 1$ for the source approaching the microphone, $h_{n\tau} = 1$ when the source is immediately above the microphone, and $h_{n\tau} > 1$ for the source moving away from the microphone. More relevant is that this term is thus identical to the Doppler amplification factor in equation (32) evaluated at the center of the time window, i.e., $\left|1 - M_{sn}\left(0\right)\right|$. Thus, it can be observed that the proposed approach automatically removes the Doppler amplification factor at least in an approximated way.

In equation (41), the FT of the source strength is estimated from the microphone FT including the Doppler effect. However, using the translation and scaling properties of the FT the Doppler effect was removed without the need to de-Dopplerize the time signal. In other words, the de-Dopplerization was carried out in the frequency domain without the need to resample the data.

It is important to understand the advantages, approximations, and limitations in this formulation. They are:

a. Firstly, the shorter the emission time window, the closer the reception-emission relationship is to being linear. However, the frequency resolution for the DFT will be degraded. Thus, there is a trade-off between frequency resolution, accuracy, and computation effort.
b. Secondly, unlike in the time domain approach there is no interpolation of the data, which is the time consuming part of the time domain approach. However, the new method still requires the computation of the Fourier transform of the source strength signal for every point of the scanning grid.
c. Thirdly, a source moving at variable speed can be easily accounted for by taking the average speed in the time window. Finally, the obvious advantage is the computational efficiency of the frequency domain approach.

For convenience, the source was assumed to be a monopole in this derivation. However, directivity of the source can be easily included in the derivation by multiplying the source strength by the directivity index in the direction of the source-microphone, i.e., $d_{sn}(\tau)$. Once again the value of the directivity index can be taken to be constant at the mid-point of the time window. Generally, the directivity of the sought sources is unknown and thus a monopole assumption is typical.

VI. Conclusions

A new de-Dopplerization technique to remove the Doppler effect from acoustic signals of moving sources was developed. The approach used for this technique consists in breaking down the motion of the source in short time periods. Then, the short-time Fourier transform is used to transform the acoustic signals into the frequency domain. For each time period, the original (undistorted) source strength signal is estimated. The Doppler effect is accounted for using the linear, translation, and scaling properties of the Fourier transform. Therefore, the de-Dopplerization of the signal is performed entirely in the frequency domain. Since this technique does not require resampling and interpolation of the microphone signal, it is more computationally efficient that the currently used time domain de-Dopplerization technique.

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