

Roof wedge stability: a simple analytical model and numerical validation using deformable blocks

Mary M. MacLaughlin

Montana Tech of The University of Montana, Butte, Montana

ABSTRACT: Wedge-shaped roof falls are an extremely common mode of failure of underground openings in jointed rock. Although a number of sophisticated analytical solutions have been developed to determine the stability of roof wedges, a simple approach may also be used to provide an estimate of wedge stability [1]. The simple two-dimensional model assumes the wedge is symmetric in shape and the resultant force on the sides of the wedge is horizontal. It may be used to determine stability of a wedge given dip angles of the joints forming the sides of the wedge and the friction angle, assumed to be the same on both surfaces. It may also be used to calculate the friction angle for stability given the dip angles of the joint sets. Numerical analyses previously conducted using the distinct element method (Itasca's UDEC software) show excellent agreement with the simple analytical solution, for one specific set of conditions and a limited sensitivity analysis with rigid blocks [1, 2]. The results of a more in-depth sensitivity analysis with deformable blocks, presented here, show that the numerical results are quite insensitive to substantial variations in the Young's modulus and Poisson's ratio of the rock, orientation and stiffness of the joints, and *in situ* stresses. This suggests that the simple analytical model is applicable over a fairly wide range of conditions.

1. INTRODUCTION

Since wedge-shaped roof falls are such a common mode of failure in underground openings in jointed rock, a number of analytical solutions have been developed to determine the stability of roof wedges [3, 4, 5]. The solutions are quite sophisticated and account for the presence of *in situ* stresses in the rock, shear and normal stiffnesses of the joints, and both symmetric and asymmetric wedge shapes.

In practice, however, models must be as simple as possible; if a model is too complex, it will not be used. This is particularly true in the mining industry. A symmetric pattern identified in a previous numerical study [1] suggested a very simple model, in which roof wedge stability could be determined using only the joint dip angles and friction angle. Such a simple model could be easily used in practice, but it should not be used unless it is accurate and produces results that match real behavior of rock masses.

Although the simple analytical model was in excellent agreement with the distinct element analyses presented in [1], the range of conditions

investigated was fairly narrow. A sensitivity analysis was conducted to determine the effects of changing the values of the input parameters [2], but was limited to studies of rock masses containing rigid blocks. The objective of the current study was to conduct a more in-depth sensitivity analysis specifically focusing on cases with deformable blocks. The parameters investigated included joint orientation (dip angles), the Young's modulus and Poisson's ratio of the rock, as well as a range of *in situ* stress conditions. The previous rigid block study indicated that the numerical results were not very sensitive to the model assumptions regarding wedge symmetry and horizontal force resultant. The primary goal of this study was to determine if this was also the case for rock masses containing deformable blocks.

A similar rigid block numerical parametric study conducted for a site in Canada concluded that the results were sensitive to the joint normal and shear stiffness values used in the analyses [6]. A secondary objective of the current study was to quantify the influence of these parameters on the results of the cases investigated.

2. SIMPLE ANALYTICAL MODEL

A very simple analytical model for roof wedge stability was developed that incorporates a minimum number of factors [1]. As shown in Figure 1, the two-dimensional model assumes that a wedge-shaped block in the roof of an underground opening is formed by the intersection of two planar fractures. Assuming that the wedge-shaped block is symmetric, sliding will not occur if the resultant force on the sides of the wedge is within the "cone of friction" surrounding the normal vectors to both sides [7, 8]. The resultant is assumed to have a horizontal orientation, and thus will lie inside the friction cone of the normal if the normal is inclined no more than ϕ° above horizontal, where ϕ is the friction angle of the surface of contact between the wedge and the rock mass. Since the wedge is assumed to be symmetric, this condition is satisfied when $\alpha_{1/2}$, half of the apex angle of the wedge, is less than or equal to the friction angle. The wedge is predicted to be unstable when $\alpha_{1/2}$ is greater than the friction angle.

This rough approach can be taken one step further: to identify the friction angle required for stability of the wedge with a given apex angle. Ideally, the wedge would be symmetric, with each half of the apex angle identical to the other, and the friction angle required for stability equal to $\alpha_{1/2}$.

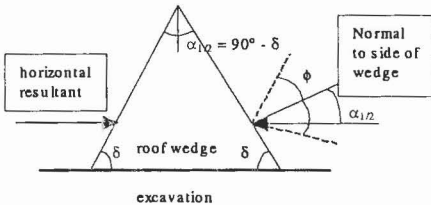
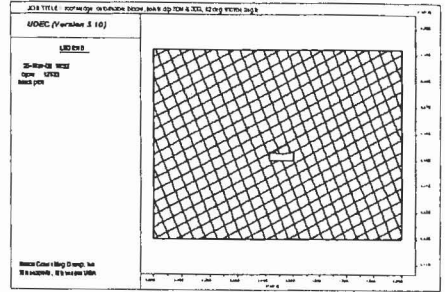


Figure 1. Schematic drawing of roof wedge. δ = dip of joints, ϕ = friction angle of joints, $\alpha_{1/2}$ = half of apex angle.

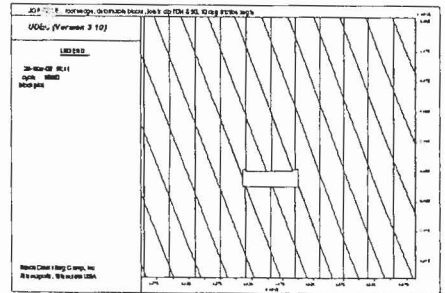
3. NUMERICAL ANALYSES

To investigate the ability of the simple model to predict roof wedge stability under a range of conditions, a suite of two-dimensional numerical analyses was conducted using the distinct element method, specifically Itasca's UDEC software [9]. The particular conditions modeled were a 9m wide by 3m high rectangular excavation in blocky rock with two perfectly persistent joint sets. One joint set was assumed to dip 70° N, and the dip angle of the other joint set was varied in 15° increments. The joint spacing for both sets was 2 m. The *in situ* stress values used were $\sigma_1 = 24$ MPa horizontal, and

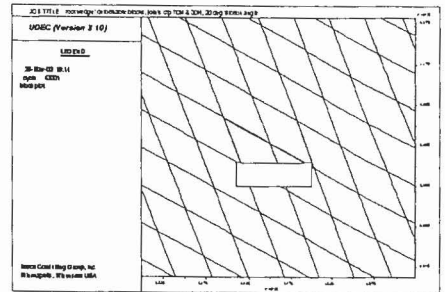
$\sigma_3 = 12$ MPa vertical, with $\sigma_2 = 18$ MPa out-of-plane; these values were based on a stress measurement made at one particular site [10, 11], and correspond to a depth of approximately 550 m below the ground surface. Figure 2 shows several of the different model configurations used.



(a)



(b)



(c)

Figure 2. Model geometries. (a) Case with JS #1 dipping 70° N (right) and JS #2 dipping 30° S (left), view of the whole model, (b) closeup of the area surrounding the excavation for case with JS #2 vertical, (c) closeup of the excavation for the case with JS #2 dipping 30° N.

All models were two-dimensional, with fixed boundaries and perfectly persistent joints spaced 2 m apart. The friction angle was varied, but assumed to be the same on both joint sets. Model parameters included a block corner rounding coefficient of 0.1 m, fixed boundaries (zero velocity), and joint normal and shear stiffnesses of 100 GPa and 50 GPa, respectively (10 GPa and 1 GPa in the rigid models). The modeling area consisted of 10 excavation widths in the x-direction and 24 excavation heights in the y-direction, resulting in an area 90 m wide by 72 m high. This large area was necessary due to the enlargement of the

“excavation” in the vertical direction caused by separation of the roof wedge from the adjacent rock.

Results generated using rigid blocks indicated that the simple analytical model was in complete agreement with the numerical solutions, over a wide range of joint dip angle values [1, 2]. Block deformability was incorporated into the analyses performed for the current study. The blocks were assumed to be elastic, with Young’s modulus of 120 GPa and Poisson’s ratio of 0.3. As shown in Table 1 and Figure 3, the results are essentially identical, with only two exceptions in which the results differed by one or two degrees.

Table 1. Comparison of friction angle required for stability as a function of the dip angle of Joint Set #2, as predicted using the simple analytical model, and as determined using UDEC models using rigid and deformable blocks. Joint Set #1 dips 70° N. The two cases in which the rigid and deformable blocks produced slightly different results are designated with bold font.

Dip of Joint set #2	Friction angle of joints required for stability		
	Analytical	UDEC results: rigid blocks	UDEC results: deformable blocks
15° S	47.5°	48° unstable, 49° stable	48° unstable, 49° stable
30° S	40°	40° unstable, 41° stable	40° unstable, 41° stable
45° S	32.5°	32° unstable, 33° stable	32° unstable, 33° stable
60° S	25°	25° unstable, 26° stable	25° unstable, 26° stable
75° S	17.5°	17° unstable, 18° stable	17° unstable, 18° stable
90°	10°	10° unstable, 11° stable	10° unstable, 11° stable
75° N	2.5°	2° unstable, 3° stable	4° unstable, 5° stable
60° N	5°	5° unstable, 6° stable	5° unstable, 6° stable
45° N	12.5°	12° unstable, 13° stable	11° unstable, 12° stable
30° N	20°	20° unstable, 21° stable	20° unstable, 21° stable
15° N	27.5°	27° unstable, 28° stable	27° unstable, 28° stable

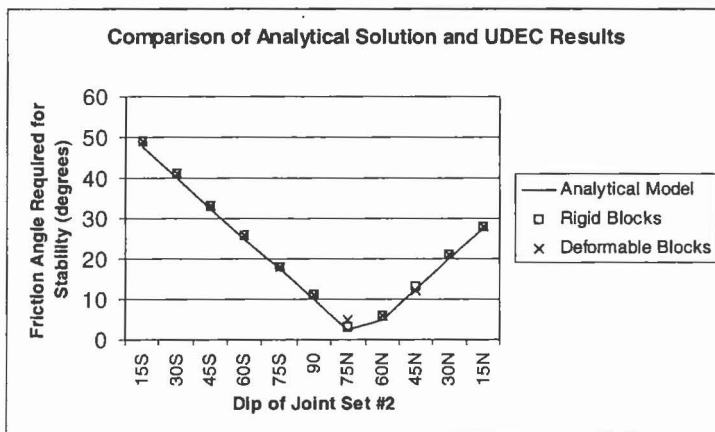


Figure 3. Comparison of the friction angle required for stability of the roof wedge, as predicted by the analytical solution (solid line) and calculated with UDEC (point data). Joint Set #1 dips 70° N. Data points correspond to different dip angles of Joint Set #2.

The Young's modulus and Poisson's ratio values of the rock were varied over a substantial range to determine the sensitivity of the results to these parameters. As shown in Figure 4, varying the Young's modulus from 4.8 GPa to 600 GPa had very little influence on the friction angle required for stability of the roof wedge. Varying the Poisson's ratio from 0.15 to 0.45 did not produce any change in the results.

The rigid block sensitivity analysis in [2] showed, quite surprisingly, that the results were fairly insensitive to drastic changes in the *in situ* stress levels. However, specifying the vertical stress larger than the horizontal stress did cause substantial changes in the friction angle required for stability for wedges with apex angles greater than 60°. One possible explanation for this is that the large vertical stress violates the model assumption of a horizontal force resultant on the sides of the wedge.

The results of the deformable block analyses investigating sensitivity to horizontal and vertical *in situ* stresses are displayed in Figure 5, and are very similar to the rigid block results. The baseline stress values were 24 MPa horizontal and 12 MPa vertical, corresponding to a stress ratio $K = \sigma_h / \sigma_v = 2.0$. The horizontal stress was reduced to 6 MPa and increased to 48 MPa, corresponding to K

values ranging from 0.5 to 4. The vertical stress was increased to 48 MPa and reduced to 2 MPa, corresponding to K values ranging from 0.5 to 12. Most of the cases showed no sensitivity to changes in the *in situ* stresses, the exceptions being cases with the largest wedge apex angle (80°) and horizontal stress less than vertical stress ($K < 1.0$).

The out-of-plane stress, originally an intermediate principal stress of 18 MPa, was varied to a maximum stress of 48 MPa and a minimum of 6 MPa. The friction angle required for stability of the roof wedge was not influenced by these stress changes.

To investigate the effect of the joint normal (jkn) and shear (jks) stiffness values on the numerical results, the jkn and jks were simultaneously increased and reduced by one order of magnitude from their initial values of 400 GPa and 200 GPa, with the jkn/jks ratio held constant at 2.0. A second suite of analyses was performed with a jkn/jks ratio of 10.0, with jkn values of 4000, 400, and 40 GPa. The friction angle required for stability of the roof wedge was only slightly affected (maximum 2° change) by the joint stiffness values, and only in the cases with the highest stiffnesses and with the largest wedge apex values.

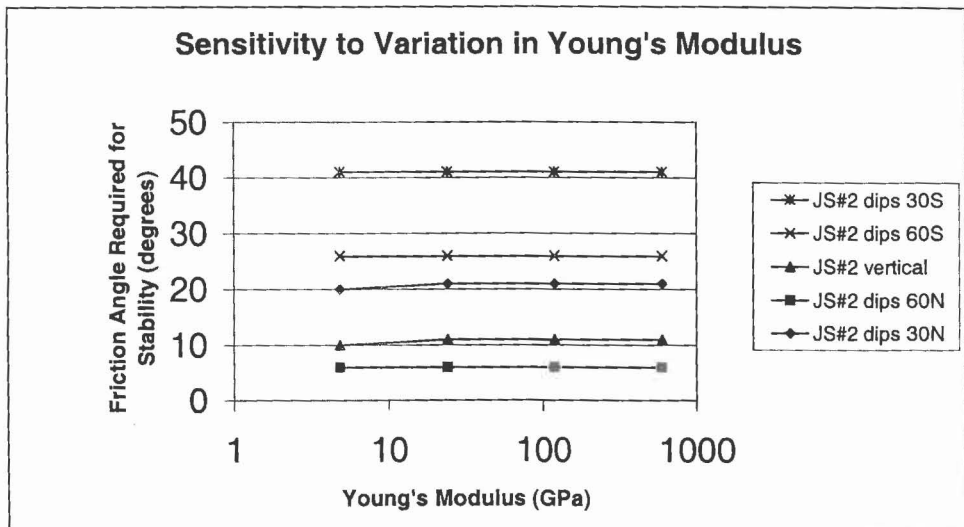
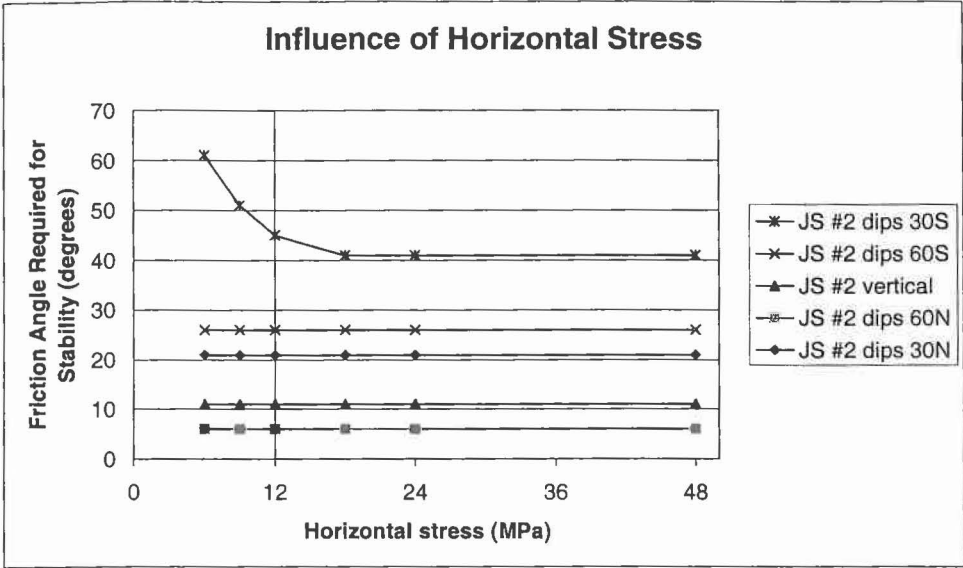
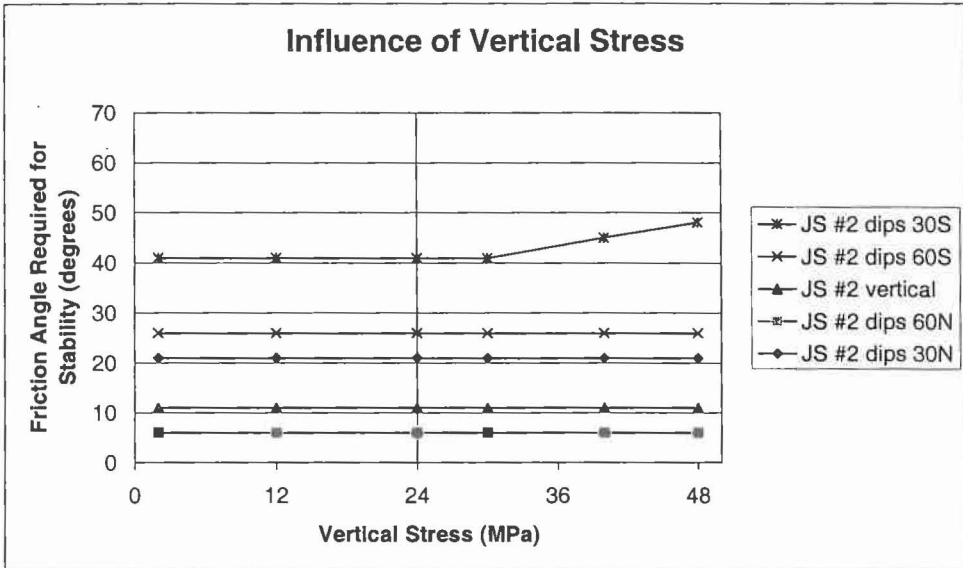


Figure 4. Friction angle required for stability as a function of the Young's modulus of the rock ($E = 4.8, 24, 120$ and 600 GPa), for 5 different joint dip angle combinations. Joint Set #1 dips 70°N, Joint Set #2 varies from 30°S to 30°N in 30° increments. The Poisson's ratio is 0.3. (Note log scale on the x-axis.)



(a)



(b)

Figure 5. Sensitivity to variation of horizontal and vertical *in situ* stresses. Friction angle required for stability as a function of the horizontal and vertical stress, for 5 different joint dip angle combinations. Joint Set #1 dips 70°N, Joint Set #2 varies from 30°S to 30°N in 30° increments. (a) With the vertical stress held constant at 12 MPa, the horizontal stress was varied from 6 to 48 MPa, corresponding to stress ratio K values of 0.5 to 4.0. (The vertical line on the graph crosses at $K=1.0$; K -values for data points to the left of the vertical line are < 1.0 , and for data points to the right are > 1.0 .); (b) with vertical stress held constant at 24 MPa, the horizontal stress was varied from 2 to 48 MPa, corresponding to stress ratio K values of 12 to 0.5. (The vertical line on the graph crosses at $K=1.0$; K -values for data points to the left of the line are > 1.0 , and for data points to the right are < 1.0 .)

4. DISCUSSION

The results of the suite of analyses performed using deformable blocks were almost identical to those of a previous study conducted using rigid blocks [2]. In an effort to investigate the influence of the geometry of the wedge, 11 different wedge configurations were modeled, none of which was perfectly symmetric. The numerical calculations of the friction angle required for stability of a roof wedge are in excellent agreement (within 1.5°) with the values predicted using the simple analytical model over a wide range of joint orientations. This suggests that the model assumption regarding wedge symmetry does not have a significant influence on the results. It should be noted, however, that a wider range of joint dip angle combinations was tested in [2]; in two of the cases with large wedge apex angles (greater than 90°), the numerical and analytical results were not in as close agreement. Further analyses will be done to determine if this is the case with deformable blocks as well.

The Young's modulus of 120 GPa used in the initial analyses is extremely high, and was used only because it reflected the actual value at one particular site being studied. This value is sufficiently high that rigid block models may capture the observed behavior at the site. Since most rock materials have a lower value of Young's modulus, investigating the sensitivity of the results with respect to wide variations in this particular parameter is particularly justified. Interestingly, the results were essentially insensitive to a wide range of values of the elastic properties (Young's modulus 4, 24, 120 and 600 GPa, and Poisson's ratio 0.15, 0.30 and 0.45) of the rock material. The largest differences between the rigid and deformable block models occurred in cases with the lowest Young's moduli, but the magnitude of the difference was only 1° .

Variation of the *in situ* stresses had less of an effect on the results than anticipated, which indicates that the simple analytical model is fairly robust with respect to the assumption of the perfectly horizontal resultant force on the roof wedge. However, in cases with large wedge apex angles and horizontal stress less than the vertical stress, the numerical analyses indicated significantly higher friction angles required for stability than predicted using the simple analytical model.

The joint stiffness values had a slight influence on the friction angle required for stability. It appeared, however, that other aspects of the model behavior, for instance, solution time and volume of unstable

material in the roof, were much more sensitive to changes in the joint stiffnesses.

The effects of varying other parameters of the numerical model, particularly block corner rounding and boundary conditions, were not investigated. Joint sets were assumed to be perfectly persistent, and since most rock masses do not contain perfectly persistent joints, quantification of the influence of this feature of the numerical models would be useful. Most rock masses do not contain joint sets with equal values of joint friction angle, so modifying the model to account for varying friction angle on different joint sets could prove to be valuable, as well.

One of the most basic limitations of this study is that all of the analyses are two-dimensional. While some situations may be adequately modeled with 2D techniques, in general, rock stability problems are three-dimensional in nature. If the dominant joint sets in the rock mass being modeled do not have approximately parallel strike values, this simple model is probably not appropriate, and a more sophisticated three-dimensional model or tool, such as block theory [8], may be the best approach. The close agreement between the analytical and numerical solutions suggests that it may be worthwhile to investigate the possibility of extending the simple analytical model to the third dimension.

Most importantly, it should be noted that the close agreement between the analytical and numerical solutions does not guarantee accuracy with respect to real behavior of the rock mass. Crawford and Bray [3] mentioned that their physical model results indicated that the analytical solutions may overestimate the stability of the system, and these discrete element results, particularly since they were produced using rigid blocks, may do the same. Validation using actual field data is a critical component of the process, and has not yet been performed for this particular investigation.

5. CONCLUSIONS

A simple analytical model was developed for predicting the friction angle required for stability of a wedge-shaped block in the roof of an underground opening in fractured rock. The results were in excellent agreement with discrete element analyses performed using Itasca's UDEC software. A sensitivity analysis was conducted to determine both the range of conditions for which the model produced acceptable results, and the degree of influence of the various parameters.

The results were essentially insensitive to drastic changes in the Young's modulus and Poisson's ratio of the rock. Sensitivity to variation of the *in situ* stress levels was lower than expected, indicating that the simple analytical model is quite robust with regard to that particular assumption. The degree of asymmetry did not appear to have a significant influence on the results, as evidenced by the close agreement of the analytical and numerical results for a range of joint dip angles that produced very asymmetric wedges.

One aspect of this model that merits investigating further is incorporating different friction angles on different joint sets. The primary limitation of the simple analytical model is that it is only a two-dimensional model, and most rock mass stability problems are three-dimensional in nature. Furthermore, verification of the model by comparing the results to actual behavior of blocky rock masses is a key step that has not yet been accomplished.

6. ACKNOWLEDGMENTS

Initial funding for this project was provided by the National Institute for Occupational Safety and Health - Spokane Research Laboratory. Guidance, assistance and advice provided by NIOSH-SRL personnel, particularly Tom Brady, Jeff Whyatt, Mark Larsen, and Jeff Johnson were extremely helpful and greatly appreciated.

REFERENCES

1. MacLaughlin, M.M., R.B. Langston, and T.M. Brady. 2001. Numerical Geomechanical Modeling for Stillwater Mine, Nye, Montana. In *Rock Mechanics in the National Interest, Proceedings of the 38th U.S. Rock Mechanics Symposium, Washington D.C., 7-10 July 2001*, eds. D. Elsworth et al, 423-432. Lisse: Balkema.
2. MacLaughlin, M.M. 2002. Roof Wedge Stability: A Simple Analytical Model and Numerical Validation. In *Proceedings of the 37th Symposium on Engineering Geology and Geotechnical Engineering, Boise, 27-29 March 2002*, ed. S. Sharma.
3. Crawford, A.M. and J.W. Bray. 1983. Influence of the *in situ* stress field and joint stiffness on rock wedge stability in underground openings. *Can. Geotech. J.* 20: 276-287.
4. Sofianos, A.I. 1986. Stability of rock wedges in tunnel roofs. *Int. J. Rock Mech. Min. Sci.* 23(2): 119-130.
5. Sofianos, A.I., P. Nomikos, and C.E. Tsoutrelis. 1999. Stability of symmetric wedge formed in the roof of a circular tunnel: nonhydrostatic natural stress field. *Int. J. Rock Mech. Min. Sci.* 36: 687-691.
6. Grenon, M., J. Hadjigeorgiou, J.P. Harrison, and R. Banas. 2000. An investigation of drift stability in moderately jointed rock mass. In *Pacific Rocks,*

Proceedings of the 4th North American Rock Mechanics Symposium, Seattle, 31 July - 3 August 2000, eds. J. Girard et al, 967-973. Rotterdam: Balkema.

7. Goodman, R.E. 1976. *Methods of Geological Engineering in Discontinuous Rocks*. St. Paul: West Publishing Company.
8. Goodman, R.E. 1989. *Introduction to Rock Mechanics, Second Edition*. New York: John Wiley & Sons.
9. Itasca. 2000. *UDEC User's Guide (Version 3.1)*. Minneapolis: Itasca Consulting Group Inc.
10. Langston, R.B. 1997. Report on *in-situ* stress test. Stillwater Mining Company internal memo (unpublished) 23 September 1997.
11. Johnson, J.C., T. Brady, M. Larson, R. Langston, and H. Kirsten. 1999. Use of strain-gauged rock bolts to measure rock mass strain during drift development. In *Rock Mechanics for Industry, Proceedings of the 37th US Rock Mechanics Symposium, Vail, 6-9 June 1999*, eds. B. Amadei et al, 497-502. Rotterdam: Balkema.

PROCEEDINGS OF THE 5TH NORTH AMERICAN ROCK MECHANICS SYMPOSIUM AND THE 17TH
TUNNELLING ASSOCIATION OF CANADA CONFERENCE: NARMS-TAC 2002
TORONTO/ONTARIO/CANADA/7 – 10 JULY 2002

NARMS-TAC 2002

“Mining and Tunnelling Innovation and Opportunity”

Edited by

Dr. Reginald Hammah

Lassonde Institute, University of Toronto, Toronto, Ontario, Canada

Rocscience Inc., Toronto, Ontario, Canada

Dr. Will Bawden

Lassonde Institute, University of Toronto, Toronto, Ontario, Canada

Dr. John Curran

Lassonde Institute, University of Toronto, Toronto, Ontario, Canada

Mark Telesnicki

Golder Associates Ltd., Mississauga, Ontario, Canada

VOLUME 1

University of Toronto/2002

Printed and bound by UNIVERSITY OF TORONTO PRESS, Toronto

Copyright © 2002 University of Toronto

The texts of the various papers in this volume were set individually by the authors concerned.

The contents of these proceedings are copyrighted to the University of Toronto. They may not be reproduced, stored or transmitted via means such as photocopying, electronic information storage and retrieval systems, without written permission from the University of Toronto.

For the complete set of two volumes: ISBN 0 7727 6708 4

For Volume 1: ISBN 0 7727 6706 8

For Volume 2: ISBN 0 7727 6707 6

Printed in Toronto, Ontario, Canada

1004234