A Preliminary Investigation on Stochastic Discrete Element Modeling for Pillar Strength Determination in Underground Limestone Mines from a Probabilistic Risk Analysis Approach

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ABSTRACT

Pillar strength determination has been one of the classic problems in underground mine design. A pillar is a load-bearing element left between excavations to provide global stability to the overall structure. Differing from other types of engineering structures, pillars are complex elements not only because of the stress fields that they are exposed but also because of their inherent anisotropy. Pillars are usually comprised of rock, specifically, in a rock mass scale where discontinuities are present. Therefore, their behavior not only depends on intact rock properties but also on the strength, distribution, and sizes of those discontinuities. Over the years, a series of analytical, empirical, observational, and numerical approaches have been proposed to estimate, determine, evaluate, and predict pillar strength and performance. However, many of these approaches do not consider site-specific conditions and generally consider "averaged" parameters in a deterministic way. This prevents mine operators from assessing the stability of the workings from a risk perspective. The integration of terrestrial laser scanning for rock mass characterization and stochastic discrete element modeling yields results that allow mining operators to predict site-specific rock fall hazards in underground operations. This work reviews existing pillar design approaches and focuses on those from a risk analysis basis. Additionally, a framework to estimate the pillar probability of failure based on the stochastic discrete element modeling approach is proposed.

INTRODUCTION

Pillar design has historically been one of the most challenging dilemmas in underground mine design. Despite the numerous research findings on this matter, pillar collapses continue to occur in underground mine environments. A series of pillar failures and collapses and their consequences have been reported in underground stone mines. Roberts et al. (2007) describe a domino-type pillar collapse that occurred in 1986 in a property adjacent to the Doe Run Mines with the potential of affecting their operation. Zipf (2001) references a series of pillar collapses that occurred in both coal and hard rock mines during the 1990s. In 2006, a pillar stability survey took place in 21 operating mines located in the central and eastern United States. This study yielded a total of nine pillar instability cases, where seven of these cases presented instability caused by geological structures or weak bedding planes, whereas the other two presented stress spalling and fracturing related instability (Esterhuizen et al., 2006). Even though major pillar collapses were not reported in this study, it was concluded that pillars intercepted by unfavorable geological structures and width-to-height ratios less than 1.5 are more prone to present instability. In 2011, the Mine Safety and Health Administration (MSHA) reported a massive pillar collapse that involved 19 pillars in the benched area of a portion of a mine that had been abandoned in the early 1990s. Fortunately, no injuries were reported during this event; however, there is a high risk for this collapse to continue to propagate to active areas of the mine (Phillipson, 2012). More recently in 2015, a cascade pillar failure was reported in a limestone mine in Pennsylvania, where a 3-hectare area encompassing 35 pillars collapsed. This collapse generated an airblast that seriously injured three mine workers (Esterhuizen, Tyrna, and Murphy, 2019).

For many years, a series of analytical, empirical, observational, and numerical approaches have been proposed for pillar design. However, there is no consensus on the most adequate solution for this problem. Even though pillar design has historically been based on empirical equations that are developed from specific case studies, a shift to numerical analysis calibrated and validated by observation and instrumentation has been the trend in more recent years. In addition to this shift, attention to probabilistic risk analysis approaches for pillar design has increased, but to a lesser extent (Walls, Mpunzi, and Joughin, 2015; Idris, Saiang, and Nordlund, 2015). This approach allows engineers to account for the effect of variability in the different parameters on the risk of failure during the design stage.

The objective of this work is to revise existing pillar design approaches and focus on those based on risk. This revision is considered in order to develop a framework to estimate the pillar probability of failure based on the stochastic discrete element modeling approach, which can be globally implemented by considering the site-specific conditions of each operation. This methodology can be applied in future work in an underground dipping deposit, where conventional design guidelines cannot be considered through conventional approaches (Monsalve et al., 2018). The "Pillar Failure Mechanisms" section describes and presents the two main failure mechanisms that can generate instability in pillars in underground stone mines. The "Analysis Methods for Pillar Design" section revises and describes the main pillar design methodologies that have been implemented in industry, including analytical, empirical, numerical, and observational approaches. Subsequently, a review of probabilistic risk analysis approaches proposed in an underground mine and pillar design is discussed in the "Risk Analysis Approaches in Pillar Design" section. The final section, "Proposed Methodology," describes in detail each of the stages of the above-mentioned methodology, which includes (1) site selection and data collection, (2) discrete element modeling for pillar strength estimation, (3) continuous modeling for pillar stress estimation, (4) stochastic analysis and probability distribution estimation, (5) probability of failure calculations, and (6) model validation.

PILLAR FAILURE MECHANISMS

Pillars are load-bearing elements left between excavations to provide global stability to the overall structure (Brady and Brown, 1985). Different from other types of engineering structures, pillars are complex elements not only because of the stress fields that they are exposed to but also because of their inherent anisotropy. Pillars are usually constituted of rock, specifically, in a rock mass scale where discontinuities are present. Therefore, their behavior not only depends on intact rock properties but also on the strength, distribution, and sizes of those discontinuities. Numerical simulations and comparison with field observations have demonstrated that the presence of large discontinuities and weak bands can significantly reduce the strength of pillars in underground stone mines (Esterhuizen, 2000; Esterhuizen and Ellenberg, 2007).

Two main pillar failure mechanisms have been described by numerous authors (Zhang, 2014; Esterhuizen et al., 2011; Elmo, 2006; Lunder, 1994). The first mechanism is defined as a structurally controlled failure mechanism. It occurs because of the presence of discontinuities and structural features that offer weaker paths for the rock to fail. Figure 1A indicates different possible failure types occurring because of the presence of discontinuities in the rock mass, highlighting (a) rock block sliding, (b) throughgoing shear failure, (c) shear failure along transgressive joints, and (d) buckling. The other failure mechanism occurs in areas under high in situ stress and is referred to as stress-controlled instability. This failure mode has a progressive effect on the pillar stability and can be reflected by spalling from the pillar surfaces. Failure starts at the corners of the pillar and continues to propagate until substantial spalling is evident along with axial fractures. At earlier stages, the pillar core remains intact. This progressive pillar degradation progresses until the formation of the hourglass shape and ultimate failure, as shown in Figure 1B. Because the main failure mechanism observed in the case study mine is structurally controlled instability in pillars, this paper focuses on that type of failure (Monsalve et al., 2019). Furthermore, according to the National Institute for Occupational Safety and Health (NIOSH) reports, this appears to be the most prevalent failure mechanism in most of the underground limestone mines in the United States (Esterhuizen et al., 2006).

ANALYSIS METHODS FOR PILLAR DESIGN

Selecting the optimal pillar geometry that maximizes extraction without compromising safety and stability is not a trivial task. Especially, if the complexity of the materials conforming such pillars is considered. Traditionally, there are two main elements to consider in pillar design: the strength of the pillar and the stresses to which this pillar is exposed (Lunder, 1994). The ratio between the pillar strength and pillar stress allows practitioners to account for a factor of safety of such a system. Where if pillar stress exceeds the strength of the pillar, failure occurs in the system. This ratio is expressed in Equation 1. Different authors have stated that an adequate selection of a factor of safety allows one to account for uncertainty and variability inherent in the rock properties. However, the selection of these factors of safety can be arbitrary and dependent on experience; engineering judgment; and in the best-case scenario, on a statistical analysis of failed stable pillar cases (Salamon, 1970; Salamon and Munro, 1967).

$$F \cdot S = \frac{Pillar\ Strength}{Pillar\ Stress} = \frac{S_p}{\sigma_p} \tag{1}$$

Because mine pillars are composed of rocks on a rock mass scale, intact rock geomechanical properties are not representative of the mechanical behavior of these structural elements (Goodman, 1989). Because of this, a series of analytical, empirical, observational, and numerical approaches have been proposed to estimate the strength of pillars. Similarly, pillar stress estimation has been approached from analytical, observational, and numerical approaches. The following subsections describe the different methods that have been implemented in underground mine pillar design to estimate strength and stresses in pillars.

Analytical Methods

Theoretical approaches are derived from mathematical expressions to describe the performance of mine pillars subject to a load for a given set of input variables (Lunder, 1994). The most common analytical method used in pillar design is the tributary area for estimating pillar load. A limitation is that this calculation assumes flat-lying deposits and a flat surface above the deposit. The tributary area method only accounts for the component of the stress parallel of the axis of the pillar, which does not necessarily reflect the actual stress of the state on the pillar. This method also neglects other components of the pre-mining stress field, an assumption which in many cases is not tenable (Brady and Brown, 1985). Another famous analytical approach is the Wilson coal pillar strength equation, which considers pillars as a complex structure, with a non-uniform stress gradient, buildup confinement around high-stress core, and progressive failure (Wilson and Ashwin, 1972).

Empirical Methods

Empirical approaches are equations derived from back-analysis of stable, failed, and unstable pillars and that consider parameters such as intact rock strength, height, and width of the pillar. These empirical relations are designed based on site-specific conditions, and all of them consider different geological settings, rock types, and mining conditions (Zhang, 2014). In the past 25 years, numerous authors have revised and studied pillar strength estimation empirical formulas (Lunder, 1994; Martin and Maybee, 2000; Malan and Napier, 2011; Oke and Kalenchuk, 2017). A general form of these empirical relations is presented in Equation 2,

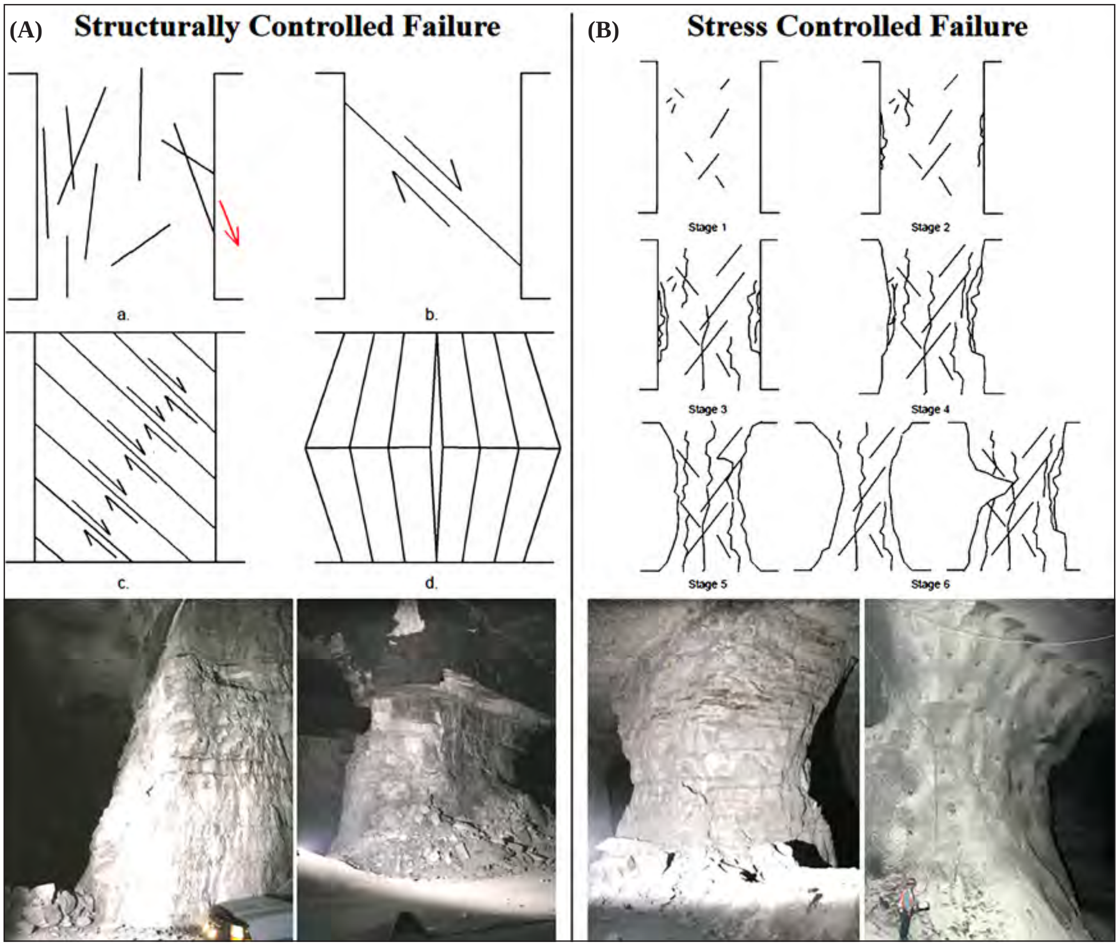

Figure 1. Main failure mechanisms in pillars in underground mines. A. Structurally controlled instability types and evidence of some of these mechanisms in underground limestone pillars. B. Stress-controlled pillar failure stages, hourglass shaping, and requirement of support to prevent further collapse (Adapted from Zhang, 2014; Esterhuizen et al., 2011).

where S_p is the pillar strength; K represents the strength of unit cube of the rock material forming the pillar; W and H are the pillar width and height respectively; A and B are empirically derived constants; and α and β are empirically derived power coefficients.

$$S_p = K\left(A + B\frac{w^{\alpha}}{H^{\beta}}\right) \quad (2)$$

It has been broadly acknowledged that even though these equations have been successfully implemented in numerous cases (even in locations out of the scope of such equations), each of them rely on a series of assumptions that must be considered during their implementation. Unfortunately, as stated by Malan and Napier (2011), it is common in underground design to turn interim solutions and initial assumptions into widespread practice. This becomes a potential risk during the design stages (Suorineni, 2014). If the designer does not consider site-specific conditions and initial assumptions considered in the development of the selected equation, this can trigger a possible pillar collapse in the future. Another drawback of these empirical equations is that particular pillar failure mechanisms in different rock masses are not considered. In addition, the effect of discrete discontinuities on the stability of the pillar is not accounted for (Esterhuizen and Ellenberg, 2007; Esterhuizen, 2000).

Observational Methods

Observational methods, more than a design approach is a verification method in which field engineers can verify if the condition of the pillar is in accordance with the design (Stille and Holmberg, 2008). In the past, a series of visual rating systems were proposed to evaluate the conditions of mine pillars. Esterhuizen et al. (2006) summarized the work of various authors into two visual rating systems: one to account for instability related to stress-controlled failure and the other related to structurally controlled failure. They used these charts to evaluate the condition of pillars in 21 different underground stone mines in central and eastern United States. Not only have observational methods been used to evaluate the condition of pillars, but also they have been used to calibrate numerical modeling. These methods also consider instrumentation in underground excavations. Instrumentation methods, such as stress meters, extensometers, and/or geophones, have been implemented in underground pillar mining to determine stress redistribution after the excavation takes place around the pillar system and measure deformations in the roof and the pillar (Gangrade et al., 2019; Slaker, Murphy, and Wifield, 2019; Esterhuizen et al., 2019). The most important use for monitoring results with instrumentation is to validate and calibrate numerical models so those are accurately simulating the phenomena of interest. In addition, laser scanning has been implemented to evaluate possible deformation on the pillar surface (Slaker et al., 2013) and to validate results from discrete element numerical models (Monsalve et al., 2019; Fekete and Diedrichs, 2013).

Numerical Methods

Numerical methods are computational simulation techniques to solve complex problems. This approach discretizes a continuous system with infinite degrees of freedom into a finite number of small elements whose behavior can be approximated by simple mathematical descriptions with finite degrees of freedom (Jing and Stephansson, 2007). Each of these elements must satisfy the governing equations of the model, such as the equations of motion for systems of rigid or deformable bodies. Problems related to stress and deformation of bodies subjected to either static or dynamic loads can be solved using this approach. This technique has been used in underground mine pillar design. Numerical modeling in pillar design has two main uses: estimate the stresses acting on the pillars and evaluate failure mechanisms in the pillar given a constitutive model to predict the material failure (Lunder, 1994). Recently, multiple authors have proven that numerical modeling techniques are a reliable method to estimate the strength of pillars in underground mines (Esterhuizen and Ellenberg, 2007; Esterhuizen, Dolinar, and Ellenberger, 2007; Rafiei-Renani and Martin, 2018; Jessu and Spearing, 2018). Not only have they been able to estimate pillar strengths, but they have also reproduced complex failure mechanisms that can occur when discontinuities are present in the rock mass (Esterhuizen, 2000; Elmo and Stead, 2010; Zhang, Stead, and Elmo, 2015).

RISK ANALYSIS APPROACHES IN PILLAR DESIGN

Risk can be understood as the expectation of an adverse outcome that can generate uncertainty on defined objectives (Baecher and Christian, 2003). Risk analysis is the process through which an understanding of all the possible risks that can arise during a particular stage of a process is developed. A risk analysis process provides inputs during decision-making stages in an engineering project to define which of the many possible risks that can possibly occur should be addressed. This process considers the source of the risk, its consequences, and the likelihood of those consequences occurring. Even tough, probabilistic risk analysis (PRA) has been widely used in the last 30 years in some applications, such as the analysis of critical facilities, risk assessment for dams, and construction and project management; its practice has just been recently implemented in underground mine design. PRA consists of assessing probability density functions (PDFs) of design parameters, such as loads and capacity for a certain system, and computing from these estimates the probability of failure (Brown, 2012).

In 1995, Hoek and colleagues discussed risk-based design in underground excavations, including probabilistic design methods. They highlighted the difficulty of implementing probability analysis into underground excavation design, especially in those problems involving stress-driven instability (Hoek, Kaiser, and Bawden, 1995). Contrary to what they described, numerous authors have been able to implement stochastic design approaches for both stress-driven instability and structurally controlled instability. These advances have been possible in part because of great advances in computational power, numerical modeling software, rock mass characterization techniques, and data analysis tools. The following paragraph summarizes many papers where reliability-based design and PRA approaches have been implemented in underground mine design.

Griffiths, Fenton, and Lemons (2002) assessed the influence of spatially varying strength in the stability of underground mine pillars via numerical modeling. Their models combined random field theory with an elastoplastic finite element algorithm in a Monte Carlo framework. Even though their approach may lack practical applicability, they highlighted the importance of reinterpreting traditional approaches based on factors of safety into a probability of failure approach founded on reliability theory. Nomikos and Sofianos (2011) discuss and describe reliability theory and its applications in common stability problems in underground mine design. They use analytical solutions to estimate the probability of pillar failure and roof collapse in underground mines by considering distribution functions for the input parameters from each case. Idris, Saiang, and Nordlund (2015) used artificial neural network (ANN) analysis to generate a relationship between horizontal in situ stress, rock mass deformation modulus, and pillar axial strain, considering variability of the inputs (deformation modulus and stresses). Results from these analyses were used to evaluate the probability of pillar failure and reliability index (1 – probability of failure), considering axial strain results from the ANN model and the maximum strain that the pillar can withstand. Their results indicated that the thickness of the overburden and pillar dimensions have a substantial effect on the probability of failure and reliability index (Idris, Saiang, and Nordlund, 2015). Son and Yang (2018) evaluated the reliability of pillar case studies presented by Van Der Merwe and Mathey (2013) by performing a probability analysis using the updated Salamon and Munro strength formula and the Mathey strength formula. They concluded that stable pillars were observed to have reliability values greater than 83%, whereas failed pillars presented values slightly larger than 50%. Monsalve et al. (2019) used 3DEC to generate a stochastic discrete element model to predict the probability of rock block falls in an underground limestone mine drift. It was concluded that results from this approach can be implemented in risk management systems, allowing engineers and operators to have greater control over possible block failures, reduce ground control related accidents, and improve the safety of these operations. Walls, Mpunzi, and Joughin (2015) performed a pillar failure risk analysis in an underground platinum narrow vein mine, where pillar strength was determined using the Headley and Grant (1972) empirical formula, and average pillar stress was estimated using continuous elastic numerical modeling. The risk of failure for selected areas was estimated based on the proportion of number of pillars with a factor of safety below 1. Additionally, Monte Carlo simulations were carried out to evaluate the impact of pillar size variability on the design (Walls, Mpunzi, and Joughin, 2015). Joughin et al. (2012) proposed and implemented a risk evaluation model to quantify the expected injuries and economic losses resulting from rock falls in underground mines in South Africa. They analyzed different case study mines and quantified the risk of a rock fall event occurring. To quantify the risk, different parameters were considered, such as time exposure of personnel, spatial coincidence of the event, expected frequency of injuries, and severity of injuries and fatalities. They did not use a numerical modeling approach to estimate the probability of rock falls but considered an analytical approach that allowed them to estimate stochastically the probability of a rock fall (Joughin et al., 2012).

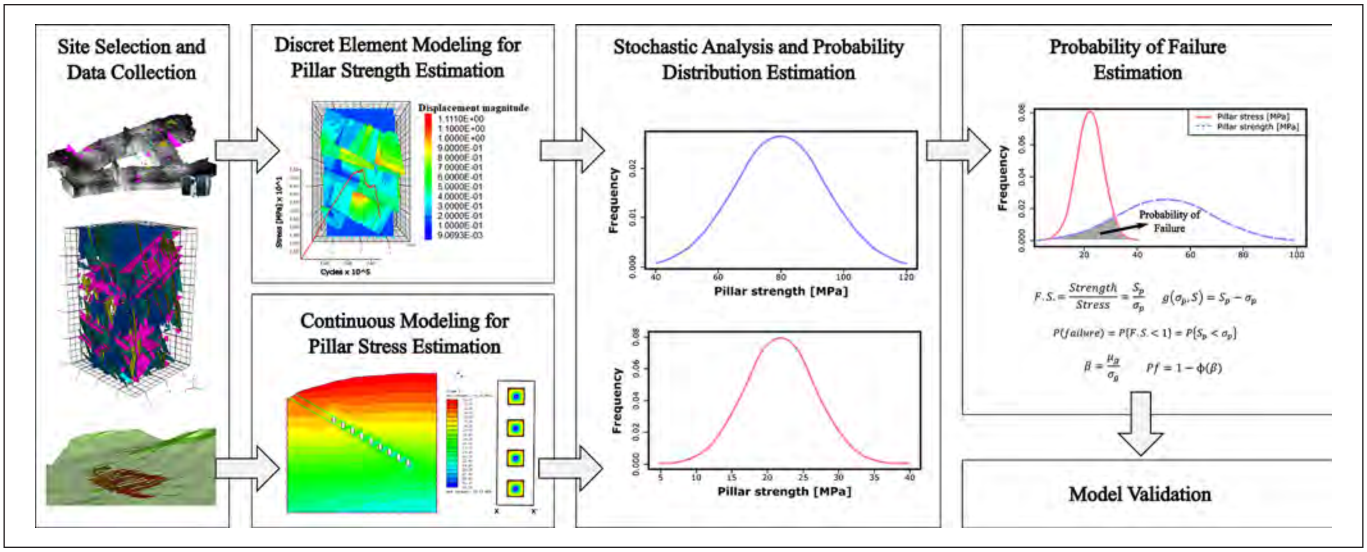

Figure 2. Proposed workflow for the integration of laser scanning and 3DEC modeling in the case study mine.

PROPOSED METHODOLOGY

Figure 2 presents the proposed workflow to estimate the probability of failure of pillars considering PDFs of the pillar strength and the pillar stress obtained from stochastic continuous and discontinuous numerical modeling. This methodology is divided into six phases: site selection and data collection, discrete element modeling for pillar strength estimation, continuous modeling for pillar stress estimation, stochastic analysis and probability distributions estimation, probability of failure calculation, and model validation. Following are the details of these stages.

Site Selection and Data Collection

The area of interest of the case study mine is selected for analysis. Once the area is selected, information regarding the mine design, surface topography, geotechnical properties of the intact rock, and the discontinuities and structural geology is collected. This information is used as inputs for the following numerical modeling stages.

Discrete Element Modeling for Pillar Strength Estimation

Discontinuity properties are used to generate a discrete fracture network, which is used to generate a fractured rock mass model in 3DEC. A fractured pillar, considering the design dimensions, is tested by applying velocity boundaries on the pillar. Applied stress is measured as the simulation progresses in order to generate stress versus time step plots. The pillar strength is defined at the point in which the pillar loses its capacity to support the load. Figure 3 displays a 5- by 5- by 10-m preliminary pillar model in 3DEC. The estimated pillar strength indicates 55 MPa and a maximum block displacement of 1.1 m. It is possible to observe that the pillar presents multiple structurally controlled failure mechanisms. Throughgoing shear failure, as well as the toppling of some blocks on the left side of the pillar, can be observed. Considering that a direct comparison with the real pillar strength requires the actual failure of a pillar, a comparison with conventional pillar strength estimation equations takes place to evaluate the discrepancy between the numerical model and empirical approaches.

Continuous Modeling for Pillar Stress Estimation

Pillar stresses are determined with a FLAC3D model. The topography of the study area and the location of the excavations with respect to the surface are considered in the stress analysis. A simplified mine design is considered, and the progressive excavation is simulated. Pillars are left in different levels similar to the mining method in the case study mine. The average vertical stress in the middle of each pillar on each level is measured. Figure 4A presents an isometric view of the topography and of the stopes and crosscut as designed in the case study mine. The section X–X′ in Figure 4B shows a two-dimensional section of the maximum principal stress distribution. One can visualize how applied stresses increases with depth.

Stochastic Analysis and Probability Distribution Estimation

A stochastic scheme can be applied for both calibrated models, pillar strength (3DEC), and pillar stress estimation (FLAC3D). For each case, multiple models are run by varying the random seed number. The strength estimation model allows variation of the orientation, size, and frequency of the discontinuities based on the parameters extracted from laser scanning and virtual discontinuity mapping, as it was done in previous work (Monsalve et al., 2019). The ultimate pillar strength is extracted from each iteration, and a probability density function is defined for this variable. For the stress estimation model, however, variation is allowed in the deformational properties of the rock mass based on the laboratory test data on the rock elastic properties. In this model, average pillar stress is measured for each pillar at each level for each iteration, and a probability density function at each pillar depth is estimated.

Probability of Failure Calculation

Resulting PDFs are used to estimate the pillar probability of failure by considering a reliability theory approach, where pillar strength is defined as capacity, and pillar average stress is defined as demand. A limit state function is defined by subtracting the applied stress to the pillar strength, and failure is defined for values less than 0, as shown in Figure 2. A reliability index is estimated as the expected

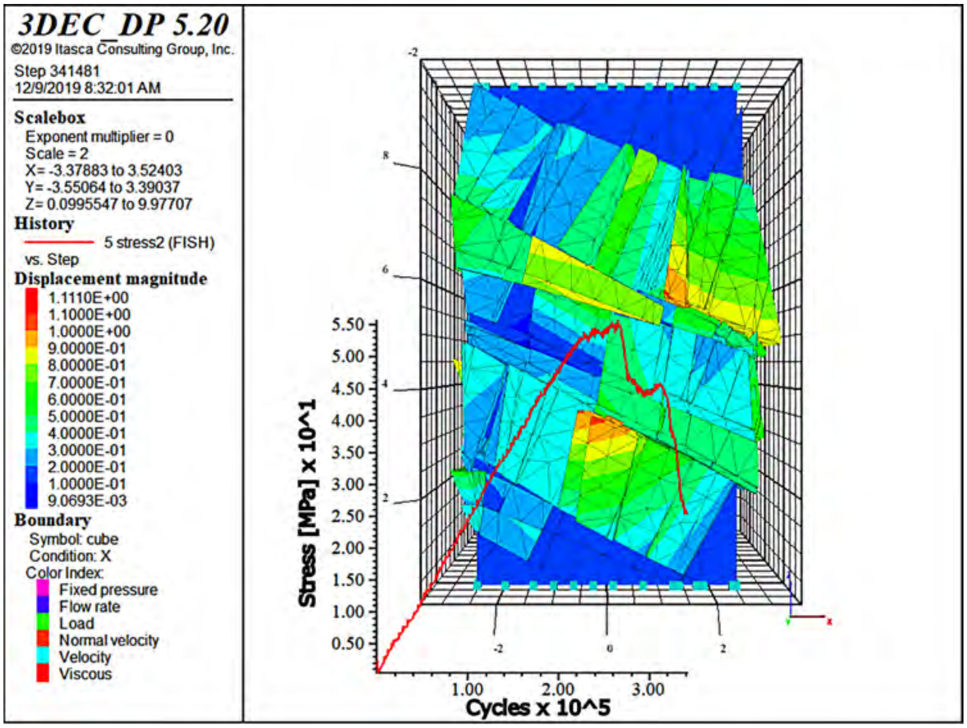

Figure 3. 3DEC pillar strength estimation test.

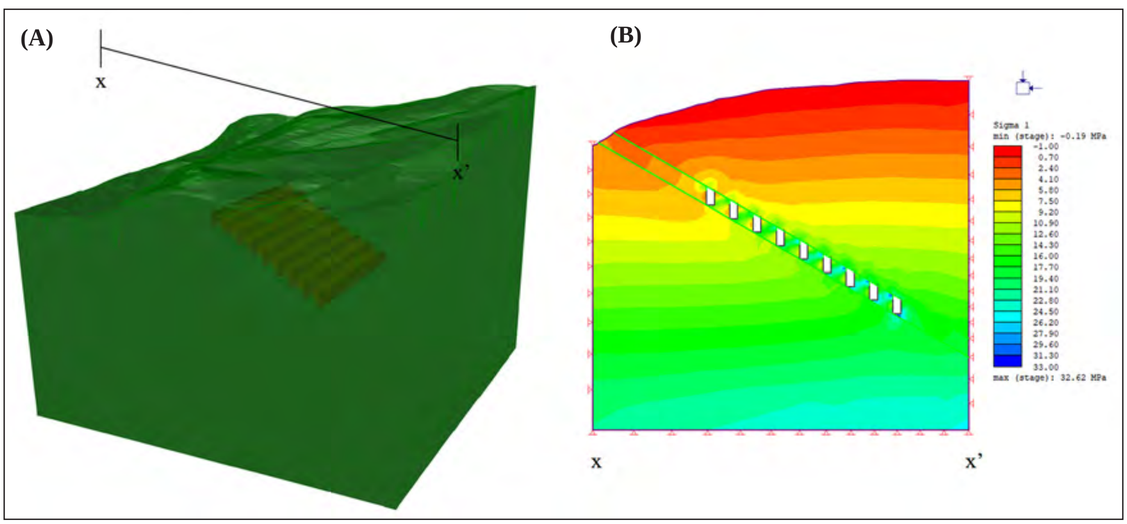

Figure 4. (A) Isometric view of the topography and the excavations to be simulated. (B) Two-dimensional stress analysis on the section X-X′ evaluating stress distribution on a series of pillars for different levels.

value of the limit state function divided by its standard deviation, and the probability of failure is defined as $P_f = 1 - \phi(\beta)$. This approach strongly depends on the distributions obtained from the numerical models. In addition, the potential correlation between pillar strength and stress should be explored and considered as this can strongly influence the results of the analysis.

Model Validation

Considering that no pillar failure has taken place in the case study mine, direct pillar strength validation cannot be considered in this case. However, the numerical model can be evaluated based on pillar performance under typical conditions. Once the average stresses have been determined throughout the study area, additional static models to evaluate the response of the fractured pillar to such average stress levels can be considered. The response of the pillars to these loading conditions can be compared with the observed conditions of the pillars in the field.

SUMMARY AND CONCLUSIONS

This paper has reviewed different methods that have been used over time to evaluate and design pillars in underground hard rock mines. The focus of this work centers in those cases where pillar failures or instability occurs because of the presence of discrete discontinuities affecting the strength of the pillar. Analytical, empirical, numerical, and observational methods have been discussed. In addition to this, risk analysis approaches that some authors have used to improve pillar design have been also discussed. The following are a series of conclusions that have arisen from this review.

- Two main failure mechanisms occur in pillars in underground hard rock mines: stress-controlled instability and structurally controlled instability. It is important to identify which is the most likely failure type to occur in each specific case because this indicates which is the most appropriate pillar design method to use.
- Empirical pillar design methods are only applicable in specific cases where the geological, geotechnical, and operational conditions are similar to those of the mines where each empirical relation was produced. These equations have been used routinely without considering all of the initial assumptions. It is the responsibility of the engineer and mine manager to recognize and understand such assumptions, because neglecting those could trigger a possible pillar collapse in the future.
- Numerical models have become a more reliable tool for simulating the behavior of pillars and estimating their strength. However, their results strongly depend on the reliability of the input parameters. Therefore, instrumentation for ground control monitoring have begun to be implemented with more frequency in underground mining projects because results from instrumentation can be used to calibrate and validate numerical models.
- Different authors have demonstrated that the state of the art of numerical modeling techniques offer an adequate tool not only to estimate the strength of a pillar but also to understand underlying mechanics associated with the collapse of pillars in underground hard rock mines.
- Even though discrete element models anzd the synthetic rock mass approach have been used to estimate the effect of multiple discontinuity sets in the strength of a pillar, an attempt to stochastically evaluate this effect has not been considered as of yet. These models are usually generated using discrete fracture networks that are generated from statistical distributions of parameters such as orientation, size, and frequency. Therefore, each iteration yields different values for strength. Results from these models should be validated by evaluating the performance of the modeled pillar under regular operational conditions with the behavior of actual pillars in the mine environment.
- A methodology that integrates stochastic discrete element modeling and finite difference modeling to estimate pillar stress and strength with a probability risk analysis approach based on reliability theory is presented.

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