Pulse Residence in Short Chromatographic Columns

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By a simple geometric construction the mean residence time in linear chromatography is shown to be independent of the degree with which equilibrium is attained. The general relationship is that if a constant input of *m* mol/s leads to a steady-state accumulation of *M* moles in the column, the mean residence time for a pulse input is *M*/*m* s. This leads to simple calculations of the effects of input and effluent boundary conditions on the mean residence time. Although an irreversible chemical reaction reduces the residence time, a procedure is available to determine the residence time in the absence of chemical reaction. The residence times following inputs into the mobile and stationary phases, respectively, differ by a constant.

The calculation of pulse residence times in short chromatographic columns would appear to be complex, for in a short column severe inequilibrium may be expected between the moving and stationary phases. Furthermore boundary conditions at the inlet and outlet would, in view of the shortness of the column, have a relatively greater effect. Yet for the case of linear chromatography, many of these difficulties are a mirage. It has been known for some time that the mean residence time in a linear system—a system in which the equilibrium and diffusion coefficients are independent of concentration—should be absolutely independent of any rate constants for mass transfer (*1*, *2*). Yet research reports have appeared in which it appeared that the mean residence time was affected by mass transfer effects (*3*–*5*). Here we will examine some specific examples of holdup in linear chromatographic systems, showing precisely how mass transfer affected the mean residence time. Finally a general procedure will be developed which permits the mean residence time to be determined without having to solve fully the partial differential equations governing mass transfer within chromatographic columns.

INEQUILIBRIUM BETWEEN MOVING AND STATIONARY PHASES

Young (*6*) gave an exact solution to the simplest possible model for interphase inequilibrium in a chromatographic column. His partial differential equation assumes that the mass transfer to and from the stationary phase is proportional to the concentrations in each phase, viz.

$$\frac{\partial F}{\partial x} = -\alpha\left(F - \frac{Q}{k'}\right) \qquad (1)$$

where F is the molar flux, defined as the moles per second flowing past the point "x" at a time "t" (mol/s), Q is the uptake of sorbate in stationary phase (mol/cm), α is the constant for mass transfer from mobile to stationary phase (cm^{-1}), and k' is the constant giving equilibrium uptake in the stationary phase per unit flux of sorbate in the mobile phase. k' has units of s/cm. By a mass balance the uptake of sorbate in the stationary phase must be

$$Q = -\int_0^t \frac{\partial F}{\partial x}\,\mathrm{d}t \qquad (2)$$

Assuming the boundary condition of a pulse input of m moles into the mobile phase at $t = 0$, $x = 0$, the solution for the above equations is

$$F = me^{-N}\delta(0) + \frac{mN}{(tt')^{1/2}}e^{-N(1+(t/t'))}I_1\left(2N\left(\frac{t}{t'}\right)^{1/2}\right) \qquad (3)$$

where $t' = k'L$, $N = \alpha L$, L is the length of column (cm), I_1 is the modified Bessel function of order 1, and $\delta(0)$ is the unit delta function at time $t = 0$. The purpose of giving the above equation is to illustrate the effects of interphase mass transfer under conditions of very poor mass transfer efficiency. In eq 3, the number of theoretical plates, N, describes the kinetics of mass transfer within the column. The greater the number of theoretical plates, the more efficient the mass transfer. This can be seen in Figure 1, which shows breakthrough curves calculated for various values of N. In eq 3, the factor $e^{-N}\delta(0)$ represents the fraction of the initial input that leaves the column before passing at least once into the stationary phase. Normally this equation would be applied to columns for which $N > 100$, and in these cases the fraction of the input that passes through the column without being adsorbed at least once is negligible. But suppose that N is equal to 0.5. In this case 61% of the input passes through the column unadsorbed. Yet, remarkably, the mean residence time, t_h, calculated from the integral

$$t_\mathrm{h} = \int_0^\infty tF\,\mathrm{d}t \Big/ \int_0^\infty F\,\mathrm{d}t \qquad (4)$$

remains unchanged and, regardless of the value for N, is precisely equal to t'. The importance of this calculation is that it shows that interphase mass transfer resistance cannot, ipso facto, alter the mean residence time. On first appearance it would seem that this conclusion is belied by eq 4 of Galan et al. (*4*), which predicts the mean residence time from an equation containing rate constants. But the two rate constants given are the forward and reverse rate constants which determine an equilibrium coefficient. And in fact the mean residence time depends on their ratio (or equilibrium coefficient) and not to the degree to which equilibrium was attained. This is just what Galan et al. want to demonstrate.

INTERPHASE DIFFUSION

Dispersion of an injected pulse occurs also from diffusion in the mobile phase as the sorbate is transported across the column. The high diffusion coefficients of sorbates in gases make this effect especially important in gas chromatography at low carrier gas velocities.

To determine the effect that dispersion in the mobile phase has on the mean residence time, we begin with the partial differential equation describing the combined effects of diffusion and convection on concentration

$$D\frac{\partial^2 C}{\partial x^2} - V\frac{\partial C}{\partial x} = (k+1)\frac{\partial C}{\partial t} \qquad (5)$$

where C is the concentration of sorbate (mol/cm^3), D is the diffusion coefficient for the sorbate in the carrier gas (cm^2/s), V is the interparticle carrier gas velocity (cm/s), equal to $v/\epsilon A$, v is the flow of carrier gas (cm^3/s), A is the cross-sectional area

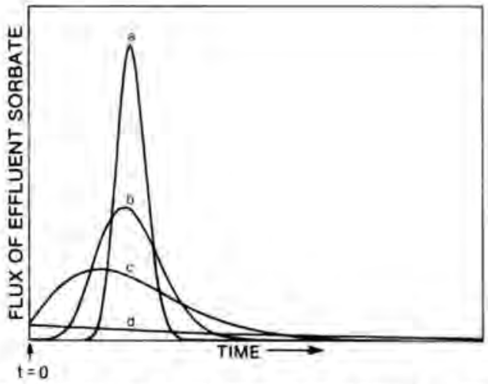

Figure 1. Breakthrough curves from columns having different mass transfer efficiencies. Curves a, b, c, and d show, respectively, breakthrough curves calculated from eq 3, with N, the number of theoretical plates, equal to 100, 20, 5, and 0.5, respectively. In the text it is shown that in spite of the effect that the value of N has on the shape of the breakthrough curve, the mean retention time is independent of the value of N.

of column (cm^2), ϵ is the fractional interparticle void volume, and k is the equilibrium partition coefficient between moving and stationary phases (dimensionless). The solution to this equation, for a pulse input of m moles at time $t = 0$ into a seminfinite column ($0 < x < \infty$), with no mass transfer out of the column (i.e., $VC - D(\partial C/\partial x) = 0$ at $x = 0$) is

$$C = \frac{m}{\epsilon A(1+k)x}\left\{\left(\frac{2Nt'}{\pi t}\right)^{1/2} e^{-(N/2)[(t'/t)^{1/2}-(t/t')^{1/2}]^2} - Ne^{2N}\,\mathrm{erfc}\left[\left(\frac{N}{2}\right)^{1/2}\left(\left(\frac{t}{t'}\right)^{1/2}+\left(\frac{t'}{t}\right)^{1/2}\right)\right]\right\} \quad (6)$$

where $N = Vx/2D$ and $t' = (1 + k)x/V$. The flux of sorbate in the column, now determined by the combined effects of convection and diffusion, is in general

$$F = \epsilon A\left(CV - D\frac{\partial C}{\partial x}\right) \quad (7)$$

and in particular, from eq 6, the flux becomes

$$F = \frac{m}{t}\left(\frac{Nt'}{2\pi t}\right)^{1/2} e^{-(N/2)[(t/t')^{1/2}-(t'/t)^{1/2}]^2} \quad (8)$$

From eq 4 and 8 the mean residence time in this system is now

$$t_h = t' \quad (9)$$

Again the result is that the mean residence time is unaffected by the rate constant describing mass transfer, in this case the constant describing interparticle diffusion.

Others (*7, 8*) have used as the breakthrough curve under these conditions the results calculated from a unit pulse *concentration* input, i.e., $C_{x=0} = \delta(0)$. It turns out that with this input function the resultant equation for the concentration is identical in form with eq 8, used above to describe the flux following a pulse mole input. Further examination of this result shows that instead of a finite mole input, the boundary conditions now demand that there be an input of an infinite number of moles, with all but an infinitely small fraction being resorbed at the point of input. This is not a realistic model of the results following a finite input into a column with essentially no resorption at the point of input.

STEADY-STATE MODEL

There is a simple geometric construction that shows it is reasonable to expect residence time to be independent of all factors describing rates of mass transfer in the column. Figure 2 shows two breakthrough curves describing mass transfer within two identical columns, the difference being that the lower curve resulted from a unit pulse input at time $t = 0$ whereas the upper curve was from a constant input of one unit per second, also starting at $t = 0$. Because (1) these columns behave as a linear system and (2) the constant input is mathematically the integral of the pulse input, then the output from the constant input must be precisely the integral of the output from the pulse input. From the integral/differential relationship between the two curves, the width of the differential strip, dF, in the upper curve is equal to the area of the differential strip, $F\,dt$, in the lower curve. As the length of the upper differential strip is equal to t, the area of this differential strip must therefore be equal to $tF\,dt$, which was used earlier in eq 4 to define the mean residence time. Therefore, the stippled area in the upper curve, which corresponds exactly to the value of the integral in eq 4, must equal the mean residence time. By a mass balance this must also be equal to the sorbate retained in the column at steady state. The general relationship between accumulated mass and residence time is that if following a constant input of m moles per second of sorbate, M moles accumulate in the column at steady state, then the mean residence time in the column for a pulse input is M/m seconds. This allows us to use steady-state retention in a column, which must be independent of all rate constants, to determine *exactly* the mean residence time of an injected pulse. All that is necessary is that mass transfer within the column be described by linear partial differential equations. This proof is still valid if the carrier gas velocity and adsorption coefficients vary as a function of position within the column.

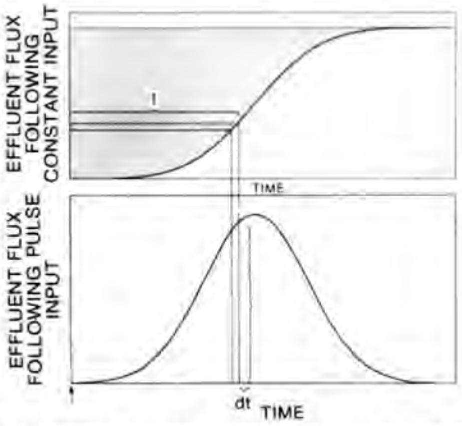

Figure 2. Geometric relationship between breakthrough curves obtained from constant and pulse inputs. See text for procedure used to determine mean residence time from the geometrical relationship between these curves.

Huang and Madey (*3*) give through a series of complex, approximate calculations, results that state that the mean residence time of a pulse in a linear system may be affected by incomplete mass transfer. The general proof given above shows that such a conclusion must be in error.

EDGE EFFECTS

Suppose the chromatographic column were not closed at $x = 0$, but instead were to extend infinitely in both the $+x$ and $-x$ directions. We feel from experience that even if the volume of the column in the upstream direction were infinite, the residence time would remain finite, although somewhat larger than if upstream diffusion were not permitted. Figure 3 shows the differences resulting from permitting upstream diffusion. The simplest way to calculate the effect that this has on the overall residence time is to use the steady-state model developed in the previous section. At steady state the

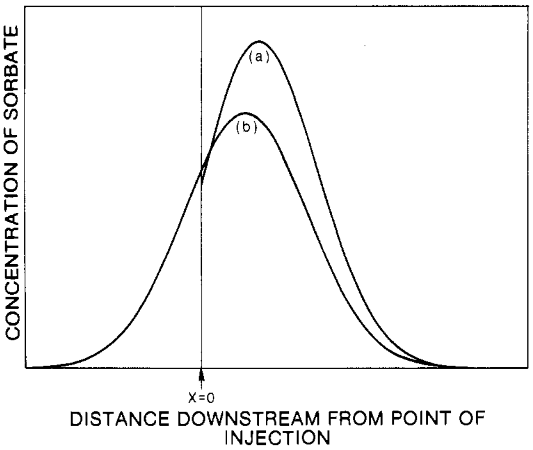

Figure 3. Effect of boundary conditions on spatial distribution of sorbate. Both curves show the combined effect of convection and diffusion on a pulse of sorbate injected earlier at $X = 0$. For curve A an impervious barrier was at $X = 0$, whereas for curve B no such barrier existed.

partial differential equation in x and t that described mass transfer now reduces to the ordinary differential equation

$$D\frac{\mathrm{d}^2C}{\mathrm{d}x^2} - V\frac{\mathrm{d}C}{\mathrm{d}x} = 0 \tag{10}$$

The general solution to this equation is

$$C = a + b\exp\left(\frac{Vx}{D}\right) \tag{11}$$

For positive values of x (i.e. after the point of injection), b must be equal to 0 to keep the solution finite and the obvious solution for this half of the column is

$$C = m/v \tag{12}$$

But for negative values of x, C must decrease to 0 with increasingly negative values of x. Here then

$$C = \left(\frac{m}{v}\right)\exp\left(\frac{Vx}{D}\right) \tag{13}$$

Integration gives the moles of sorbate upstream of the point of injection as

$$M = \frac{m(1 + k)D}{V^2} \tag{14}$$

In the previous section a direct relationship was established between the moles retained at steady state and the residence time for a pulse input. From this relationship the increased residence time for a pulse input is

$$t_{\mathrm{h}} = \frac{(1 + k)D}{V^2} \tag{15}$$

As a sample calculation, if we take $D = 0.1$ cm^2/s, $V = 10$ cm/s, and $k = 1000$, then diffusion upstream will increase the observed residence time for a pulse input by 1.0 s, which is small but perhaps detectable in some systems.

Because Kucera (*5*) assumed an infinite column, we would expect that edge effects would contribute to his calculated mean residence time, and in fact this is what his equations show.

Theoretically the opposite effect—a decrease in residence time—could occur if at the effluent end of the column a detector were placed which acted as a perfect sink for the sorbate. For such a case it turns out that the decrease in residence time is exactly the same magnitude as the increase in residence time given by eq 15.

IRREVERSIBLE CHEMICAL REACTION

The above relationships do not hold if there is an irreversible chemical reaction taking place with the sorbate as it passes through the column. Such a chemical reaction reduces the measured residence time by selectively removing those molecules that otherwise would be retained the longest in the column, thereby biasing the result toward those molecules having shorter residence times. This effect can be illustrated by an example. For a column filled with a homogeneous spherical packing which not only adsorbs but also reacts irreversibly in a first-order reaction with the sorbent, the residence time of an injected pulse is reduced by a percent equal to (*9*)

$$R = 100\left\{1 - \frac{1 + \frac{3k}{2}\left(\frac{\coth\phi}{\phi} - \operatorname{csch}^2\phi\right)}{(1 + k)\left[1 + \frac{12\beta D}{V^2}\left(\frac{\coth\phi}{\phi} - \frac{1}{\phi^2}\right)\right]^{1/2}}\right\} \tag{16}$$

where R is the percent reduction in residence time (dimensionless), D_{p} is the intraparticle diffusion coefficient (cm^2/s), d is the particle diameter (cm), β is the first-order reaction constant (s^{-1}), $\phi = (\beta d^2/4D_{\mathrm{p}})^{1/2}$, and D, V, and k are defined as before. A similar result was derived by Galan et al. (*4*). Note that the percent reduction in residence time does not depend on column length, although it does depend on other factors including the reaction rate and the carrier gas velocity.

The simplest procedure for determining the "true" residence time in the presence of an irreversible first-order chemical reaction is to multiply the effluent at each time, t, by a correction factor, $e^{\lambda t}$, such that the "corrected" effluent has the same mass as the input. Specifically, after finding the value of λ, such that the measured flux, following an input of m moles, when integrated with $e^{\lambda t}$, gives

$$m = \int_0^\infty e^{\lambda t}F(t)\,\mathrm{d}t \tag{17}$$

the mean residence time in the absence of chemical reaction is then estimated as

$$t_{\mathrm{h}} = \int_0^\infty e^{\lambda t}tF(t)\,\mathrm{d}t/m \tag{18}$$

This correction procedure has the effect of removing the bias against the slower moving molecules.

STATE OF INJECTED SORBATE

It can make a difference if the sorbate is injected into the stationary rather than the mobile phase. As an example, Perry et al. (*10*) using the same differential equation for mass transfer as Young, but assuming instead that the injected material is added to the stationary phase, calculate the flux to be

$$F = \frac{mN}{t'}e^{-N[1+(t/t')]}I_0\left[2N\left(\frac{t}{t'}\right)^{1/2}\right] \tag{19}$$

One noteworthy difference between this equation and eq 3 is that now there is no initial rapid movement in the mobile phase (no $\delta(0)$ term). Instead there is an exponentially decreasing factor at $x = 0$ representing the transfer of the injected sorbate into the mobile phase. The mean retention time, as calculated from eq 4 and 19, is

$$t_{\mathrm{h}} = t' + t'/N \tag{20}$$

The second term results from the finite time required for the injected sorbate to pass into the mobile phase.

For large values of N, Perry et al. suggest using the approximation

$$F \approx \frac{m(N/\pi)^{1/2} e^{-N[1-(t/t')^{1/2}]^2}}{2t^{1/4}(t')^{3/4}} \tag{21}$$

which under the same conditions should be an equally good approximation to Young's equation. But in a short column where this approximation is not valid, differences brought about by the state of the injected sorbate might be noted.

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Potassium Hydroxide Eluent for Nonsuppressed Anion Chromatography of Cyanide, Sulfide, Arsenite, and Other Weak Acids

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Potassium hydroxide solution was found to be an effective eluent for nonsuppressed anion chromatography. Polyvalent ions, SCN^-, and ClO_4^-, which are strongly retained by an anion-exchange resin, could not be quantitatively measured, because potassium hydroxide was too weak an eluent. However, this method was applicable to the determination of 15 inorganic monovalent anions (F^-, Cl^-, ClO_3^-, Br^-, BrO_3^-, I^-, IO_3^-, NO_3^-, NO_2^-, CN^-, HS^-, CNO^-, BF_4^-, silicate, arsenite). The main advantage is that weak acids of $pK_a > 7$ (cyanide, sulfide, arsenite, silicate, and phenol compounds) which cannot be detected by conventional ion chromatography using a conductivity detector can be determined by this method. The detection limits for cyanide, sulfide, and arsenite were 0.1 ppm, 0.1 ppm, and 0.2 ppm (as As), respectively.

Since ion chromatography (IC) was introduced by Small et al. (*1*) in 1975, it has been applied to the analysis of anions and cations in various fields (*2–6*). To improve problems with column efficiency and conductivity detection, other systems for the anion analyses, such as ion-exchange chromatography or reversed-phase chromatography with UV (*7, 8*), electrochemical (*9–12*), and atomic absorption spectrometric (*13–16*) detectors, have been developed recently. However, IC using a conductivity detector is still a powerful method for anion analysis which has often proved difficult and tedious using conventional analytical methods. IC has been applied to the determination of organic (*17*) and oxo acids (*18, 19*) in addition to some common inorganic anions (F^-, Cl^-, Br^-, NO_2^-, NO_3^-, etc.). With suppressed IC, weak acids of $pK_a > 7$ could not be detected because the conductance of the effluent was measured in a neutral or acidic solution (*20*). Nonsuppressed IC using a basic eluent permitted detection of weak acids but the determination of cyanide, sulfide, and arsenite has been shown to be inadequate.

The authors previously reported the nonsuppressed IC of anions using a potassium hydroxide eluent (*3, 17, 21, 22*). This method has two advantages. First, it is sensitive because of the large ion equivalent conductance of hydroxide ion, and second, weak acids such as phenol (*17*) and silicic acid (*21, 22*) can be determined since the separation and detection are carried out in a basic solution. Although this eluent is a weak eluent relative to a carbonate eluent or organic acid eluents, it can quantitatively elute monovalent anions. For example, silicic acid which is essentially a tetravalent ion could be determined, because it dissolved as the monovalent ion ($H_3SiO_4^-$) in potassium hydroxide eluent (*22*).

In this paper, the applicability of this method to the analysis of some weak acids (cyanide, sulfide, and arsenite) is discussed.

EXPERIMENTAL SECTION

Apparatus. A Toyo Soda Model nonsuppressed ion chromatograph HLC-601 equipped with an anion exchange column (50 mm × 4.6 mm i.d.) packed with TSKgel IC-Anion-PW (particle size 10 ± 1 μm; capacity 0.03 ± 0.005 mequiv/g) was used. HLC-601 consisted of a computer-controlled pump, conductivity detector, a sample injector (100 μL), and an oven. Two separation columns were connected, if necessary. The flow rate was maintained at 1 mL/min under a pressure of 15–25 kg/cm^2. The separator columns and a conductivity detector were set in an oven regulated at 30 °C.

Reagents. The eluent was prepared daily by dissolving analytical grade potassium hydroxide in distilled deionized water and deaerating it. Stock solutions (1000 ppm) of cyanide and sulfide were prepared weekly by dissolving the analytical grade potassium cyanide and sodium sulfide in water, respectively. Their working standard solutions were prepared daily by diluting the stock solutions with water. A silicate standard solution was prepared according to the previous reports (*21, 22*). Stock solutions (1000 ppm) of the other inorganic anions were prepared by dissolving their potassium or sodium salts, dried under vacuum at 110 °C overnight if necessary, in water. Standard solutions of heavy metal ions were prepared by dissolving the analytical grade reagents of their nitrate or sulfate salts in water. Working standard so-

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