Energy–size relationship for breakage of single particles in a rigidly mounted roll mill

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ABSTRACT

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The breakage of single particles in rigidly mounted rolls has been analyzed from a fairly general expression for the thrust profile on the roll surface. The model permits one to estimate an equivalent particle-nip diameter for irregularly shaped particles which is about 90% of the nominal sieve diameter. In agreement with the model, the energy expended in breakage of single particles of quartz, dolomite, limestone and hematite is strongly dependent on the difference between the equivalent diameter and the roll gap. For various combinations of feed size and roll gap tested, a reduction ratio index increases linearly with energy consumed. This seemingly is a consequence of the highly efficient utilization of energy in this mode of size reduction. The slopes of the reduction ratio–energy plots provide a uniform measure of grindability that is least distorted by energy losses terms, such as those which occur in media mills.

INTRODUCTION

The highest utilization of energy in the comminution of solids by mechanical means is achieved in breakage of single particles under slow compression loading (Stairmand, 1975; Schoenert, 1986). Single-particle comminution in rigidly held rolls is only slightly less efficient (Kerber, 1984) and represents the upper limit for the performance of high compression roll mills – a newly invented energy-efficient grinding machine (Schoenert, 1988). Therefore, this mode of size reduction is of considerable theoretical interest and practical importance. In a previous communication (Fuerstenau et al., 1990) it was shown that on a uniform basis of comparison, the process efficiency of single-particle breakage in rolls is considerably higher, up to 5 times greater than

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that of ball milling. The size distributions of broken particles are self-similar when the particle size is rescaled by an appropriate index of fineness, say the median size. The inverse of the median size increases linearly with the net energy expended per unit mass of solids broken. The comminution energy, in turn, depends on the hardness/strength of the material, and in particular on the size of the feed particles and the roll gap. For this reason, our primary aim in this paper is to estimate the roll torque which permits us to correlate the grinding energy with feed size and roll gap, two of the most important process variables. The analysis is based on the extensive experimental data which has been described in the previous paper (Fuerstenau et al., 1990).

ROLL MILL TORQUE AND COMMINUTION ENERGY

As shown in Fig. 1, consider a pair of rolls of diameter D, and length L, turning at speed ω. Let the roll gap be g and the nominal feed size be X_f, while $\bar{X}$ is the equivalent spherical diameter in some sense which is defined more precisely later. Since particles are fed in an orderly queue, along the whole length of the rolls, the roll surface lying on the arc segment A–A′, is subjected to a time-average thrust profile, which is shown schematically on the right-hand side of Fig. 1. Following Schwechten (1987), let this pressure be represented as $P(\alpha)$, where α is the angle between the line joining the roll centers and the line joining any point on the arc A–A′ to the roll center. Clearly $0 \leqslant \alpha \leqslant \alpha_i$ where α_i is one-half of the nip angle. The horizontal component of the force acting on a strip of roll area $(LD/2)\mathrm{d}\alpha$ is:

$$\mathrm{d}F(\alpha) = P(\alpha)\frac{LD}{2}\cos\alpha\,\mathrm{d}\alpha \quad (1)$$

The force F pushing the rolls apart is:

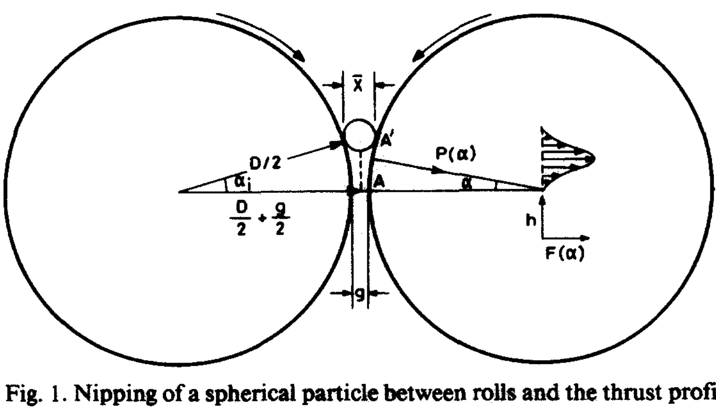

Fig. 1. Nipping of a spherical particle between rolls and the thrust profile on roll surface.

$$F=\frac{LD}{2}\int_{0}^{\alpha_i} P(\alpha)\cos\alpha \mathrm{d}\alpha \tag{2}$$

Corresponding to angle α, the torque arm, h, is given by:

$$h=\frac{D}{2}\sin\alpha \quad ; \quad 0\leqslant h\leqslant H \tag{3}$$

Hence the torque, T, acting on one roll is:

$$T=\frac{LD^2}{4}\int_{0}^{\alpha_i} P(\alpha)\cos\alpha\sin\alpha \mathrm{d}\alpha \tag{4}$$

If a unit mass of material is fed over a time interval, t, then the grinding energy spent in both rolls per unit mass of solids is given by:

$$\begin{aligned} E &= 2\omega tT \\ &= \frac{LD^2}{2}\omega t\int_{0}^{\alpha_i} P(\alpha)\cos\alpha\sin\alpha \mathrm{d}\alpha \end{aligned} \tag{5}$$

In order to evaluate this integral, we must know the nature of the function $P(\alpha)$, that is, the details of the pressure profile on the roll surface. We expect that once the particle is caught, the pressure will rise rapidly along the surface, attain a maximum value, P_0, at some intermediate angle α_0 just as the particle breaks and then fall rapidly. As an example, we can describe this situation by a power function of the following kind:

$$P(\alpha)=\left[\frac{\alpha_i-\alpha}{\alpha_i-\alpha_0}\right]^n P_0 \tag{6}$$

for the rising limb, $\alpha_0\leqslant\alpha\leqslant\alpha_i$ and:

$$P(\alpha)=\left[\frac{\alpha}{\alpha_0}\right]^m P_0 \tag{7}$$

for the falling limb, $0\leqslant\alpha\leqslant\alpha_0$, where m and n are exponents in the power functions. In any case, this class of relationships can be represented by a characteristic pressure P_0 and a dimensionless function $f(y)$, namely:

$$P(\alpha)=P_0 f(y) \tag{8}$$

where the scaled angle y is:

$$y=\frac{\alpha}{\alpha_i} \quad ; \quad 0\leqslant y\leqslant 1 \tag{9}$$

Substitution of eq. 8 into eq. 5 and recalling that for small values of α, $\sin\alpha \simeq \alpha$ and $\cos\alpha \simeq 1$, yields:

$$E=\frac{LD^2}{2}\omega t P_0 \alpha_i^2 \int_0^1 f(y) y \, dy \tag{10}$$

or:

$$E=\frac{LD}{2}\omega t P_0 K^* D\alpha_i^2 \tag{11}$$

where K^*, the integral term, is a dimensionless number. Now it is readily shown from the geometry in Fig. 1 that:

$$D\alpha_i^2 = 2[\bar{X}-g] \tag{12}$$

Combining eqns. 11 and 12 yields:

$$E=\phi(\bar{X}-g) \tag{13}$$

where because our interest is centered on $\bar{X}$ and g, the remaining constants and parameters are for convenience lumped together in ϕ.

RESULTS AND DISCUSSION

Equivalent particle-nip diameter

The equivalent spherical diameter $\bar{X}$ in eq. 13 is not known, but the roll gap g is. Therefore, we first plot E, in kWh per tonne, as a function of three roll gap settings for each of the five narrow size fractions (12×14, 10×12, 9×10, 8×9 and 7×8 Tyler mesh) of quartz feed. The ratio of feed size to roll gap, $\eta = X_f/g$, was kept at 1.53, 1.83 or 2.17. From the least squares straight line plots in Fig. 2, it is possible to compute the equivalent diameter as the ratio of intercept and slope which follows from eq. 13:

$$\bar{X}=\frac{\phi\bar{X}\ (\text{intercept})}{\phi\ (\text{slope})} \tag{14}$$

This procedure was repeated for dolomite, limestone and hematite mineral feeds. Fig. 3 shows the ratio of the equivalent diameter $\bar{X}$ to the lower mesh opening of the feed size fraction X_l for a total of 19 cases for which the data were available. Despite the presence of some unavoidable scatter, it would seem that the equivalent spherical diameter on the whole is about 90% of the nominal diameter as represented by the lower mesh opening of the feed size

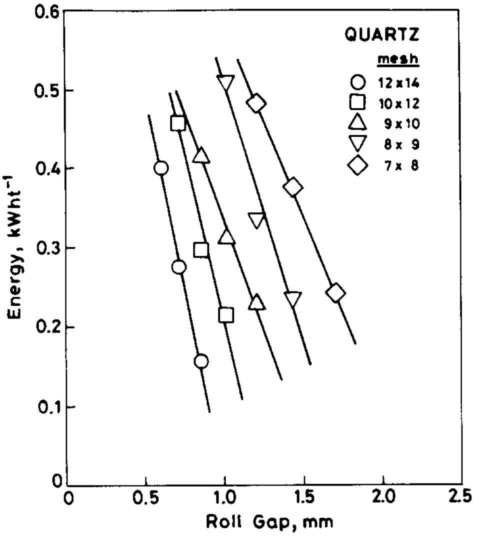

Fig. 2. Energy spent in breaking quartz particles of different size fractions as a function of roll gap.

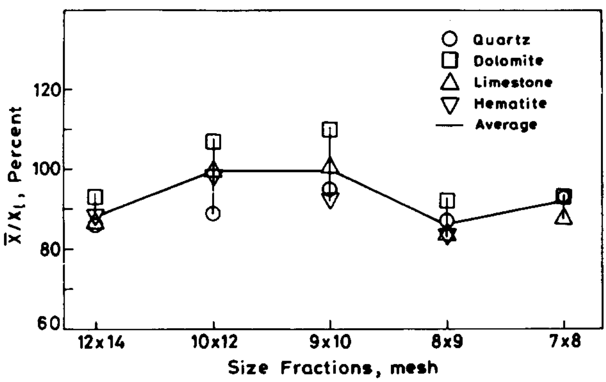

Fig. 3. Equivalent particle-nip diameters of four minerals as percentage of lower mesh opening of the feed size fraction.

interval. In $\bar{X}$ we have in effect an equivalent particle-nip diameter. That it is somewhat smaller than the nominal diameter is reasonable in view of the possibility that not only a particle may align itself such that its longer axis is either vertical or along the length of the roll gap, but also because of the breakage of corners and edges, its size could get reduced before it is effectively nipped. This conclusion is in agreement with the results of Kerber (1984), who compared the size distributions obtained by sieving with those obtained by classification in a set of rolls whose turning direction had been reversed. What is rather interesting is that by using information on the energy expended and roll gap only, we are able to estimate the feed particle size by the proposed model within 90% or so of the nominal size as given by the mesh opening.

Note that there is no reason for the slope ϕ in Fig. 2 to be identical for all the feed sizes. This is because the parameter ϕ comprises factors P_0, K^* and t which change with particle size and feeding rate. Therefore, the model in eq. 13 was tested by normalizing the energy as E/ϕ as a function of $\bar{X}-g$, where ϕ and $\bar{X}$ were computed from the least squares plots such as those shown in Fig. 2. The results given in Fig. 4 to 7 for quartz, dolomite, limestone and hematite minerals, respectively, are quite satisfactory and suggest that in terms of a dimensionless equivalent feed size/roll gap ratio, $\bar{\eta}$, the grinding energy is given by:

$$E=\phi g(\bar{\eta}-1) \tag{15}$$

where $\bar{\eta}$ is equal to $\bar{X}/g$.

Fineness of grind and energy expended

In our previous communication (Fuerstenau et al., 1990) it was shown that a quantitative evaluation of the comminution efficiency may be carried out by examining the degree of fineness attained per unit expenditure of energy and that a convenient and valid measure of fineness is the median size X_{50} of the distribution of comminuted particles. Hence, the extent of grinding achieved may be represented by an index of reduction ratio as follows:

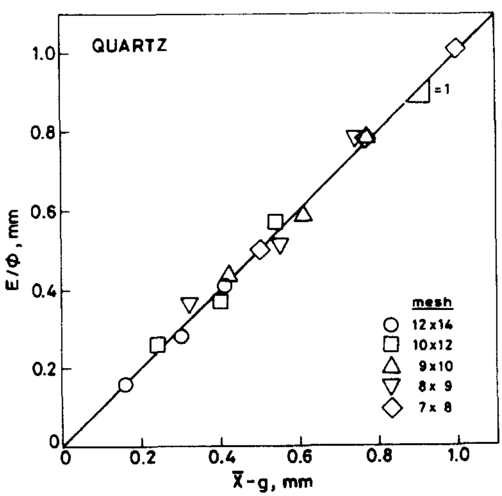

Fig. 4. Normalized energy E/ϕ as a function of difference between equivalent diameter and roll gap for quartz.

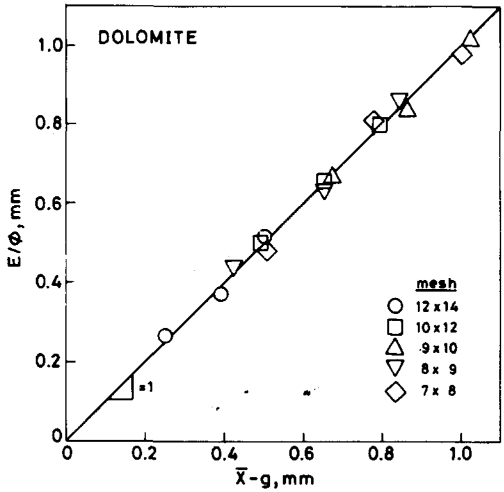

Fig. 5. Normalized energy E/ϕ as a function of difference between equivalent diameter and roll gap for dolomite.

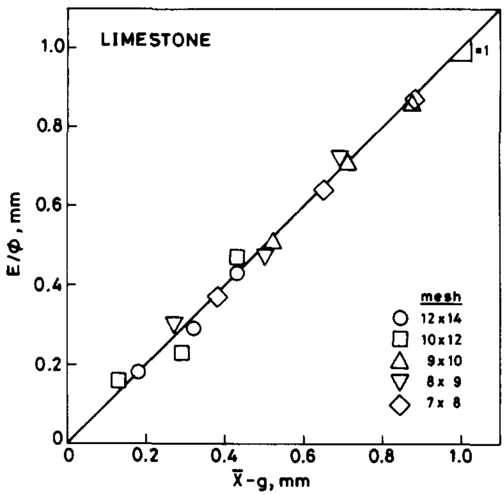

Fig. 6. Normalized energy E/ϕ as a function of difference between equivalent diameter and roll gap for limestone.

$$R_r = \frac{X_f}{X_{50}} \tag{16}$$

which is approximately equal to 1.1 $\bar{X}/X_{50}$.

Figs. 8–11 show plots of the reduction ratio as a function of the net energy consumed, E, in kWh per tonne for quartz, dolomite, limestone and hematite,

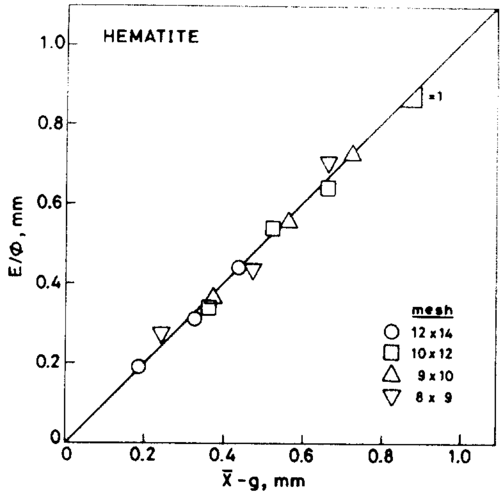

Fig. 7. Normalized energy E/ϕ as a function of difference between equivalent diameter and roll gap for hematite.

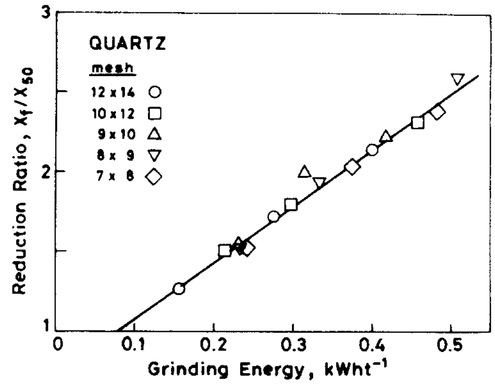

Fig. 8. Linear relationship between reduction ratio of quartz and energy consumed by rolls.

in that order. Here X_{50} was obtained by the Lagrangian interpolation formula. It is evident that in spite of widely different combinations of feed size and roll gap employed, for each of the four minerals there exists, to a degree of approximation, a simple linear relationship between E and the reduction ratio:

$$\frac{X_f}{X_{50}} = jE + c \tag{17}$$

where j is the slope and c the intercept on the X_f/X_{50} axis. The only exceptions are one data point each in the limestone and hematite plots, which are apparently gross outliers, and for reasons that are not clear a separate straight line for the 12×14 mesh hematite feed which incidentally gives a correlation coef-

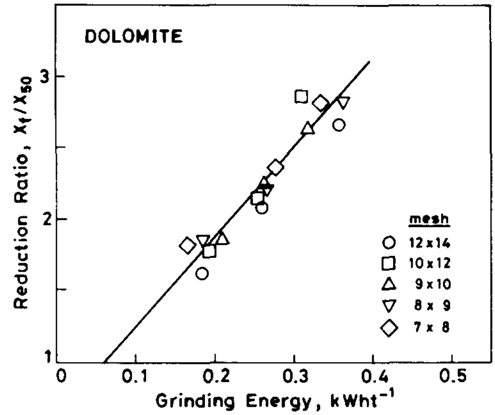

Fig. 9. Linear relationship between reduction ratio of dolomite and energy consumed by rolls.

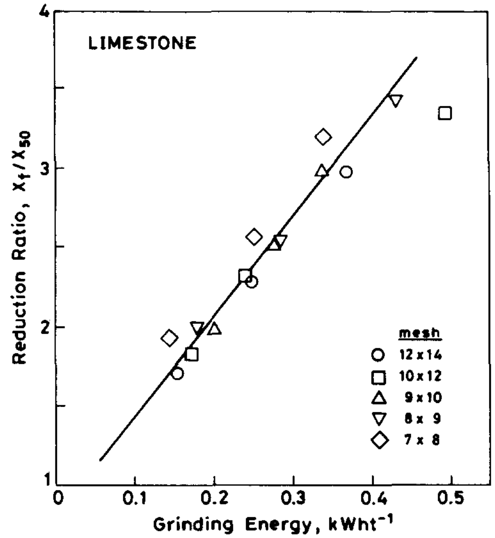

Fig. 10. Linear relationship between reduction ratio of limestone and energy consumed by rolls.

ficient $r = +0.999$ in the linear regression analysis. Eq. 17 can also be written as:

$$\frac{X_f}{X_s} = j[E - E_0] + 1 \quad (18)$$

where E_0 may be interpreted as the combined term due to experimental errors, friction losses, particle shape effect, work done in breaking edges and corners before the particle is nipped, etc. Table I gives values of j, E_0 and r for the four minerals.

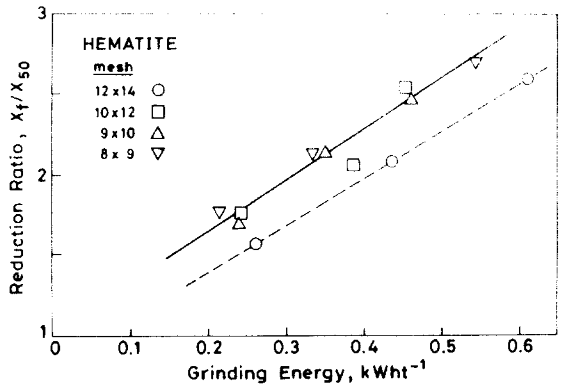

Fig. 11. Linear relationship between reduction ratio of hematite and energy consumed by rolls.

TABLE I

Values of slope, intercept and correlation coefficient for the regression of reduction ratio on comminution energy in the roll mill

Material	Slope, j (tonne/kWh)	Intercept, E_0 (kWh/tonne)	Correlation coefficient r
Quartz	3.58	0.08	+0.990
Dolomite	6.19	0.06	+0.949
Limestone	6.05	0.02	+0.973
Hematite*[1]	3.15	−0.01	+0.989

*[1]Excluding 12×14 mesh feed.

The slope j is an overall relative measure of the grindability of the mineral. That the grindability of quartz is little more than one-half of limestone and dolomite is consistent with their relevant mechanical properties. For example, according to Yashima et al. (1987) the Young's modulus of elasticity of quartz is 8.71×10^{10}Pa and that of limestone and marble is 6.80×10^{10} and 5.34×10^{10}Pa, respectively. It is difficult to extend this line of reasoning to hematite, presumably because of its specular morphology. For the same reason perhaps the E_0 value of this mineral is also way out of line as compared to the other three solids. Nevertheless, the coefficient of correlation values which range from +0.95 to +0.99 suggest that the relationship in eq. 18 holds remarkably well under a wide range of operating conditions and for quite diverse kinds of feed materials.

"Unbroken" feed particles

A somewhat intriguing aspect of single-particle crushing in rolls is that a variable fraction of particles in the product, ranging from 20% or more down

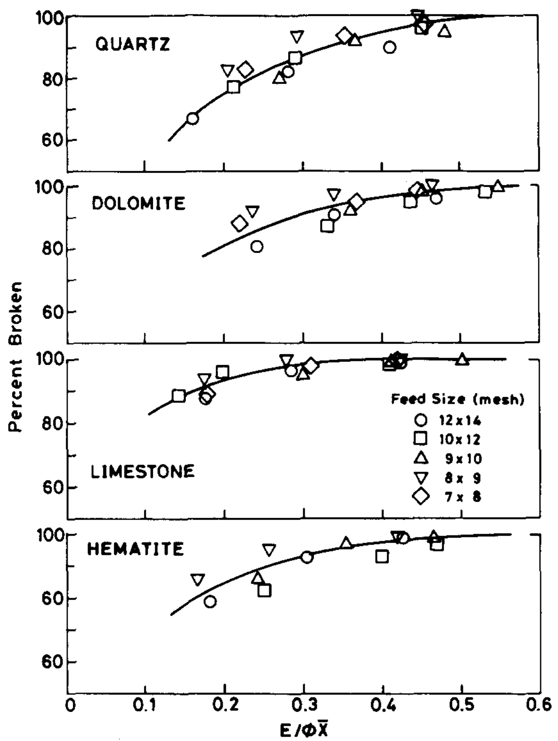

Fig. 12. Unique relationship between the percent of feed broken out of feed size interval and a dimensionless number $E\phi/\bar{X}$.

to 1% and less, is retained in the original feed size interval. This is inspite of the fact that the values of η employed are 1.53, 1.83 and 2.17. One possible explanation is that the feed particles, especially those in the lower end of the feed size interval, and large fragments, especially those formed by breakage of feed particles lying in the upper end of the feed size interval, can slip through the roll gap by aligning their long axis either vertically down or horizontally along the roll length. It has been determined, primarily by trial-and-error analysis, that the fraction of feed broken as defined above is a unique function of a dimensionless parameter θ:

$$\theta = 1 - \frac{1}{\bar{\eta}} = \frac{\bar{X} - g}{\bar{X}} \tag{19}$$

Hence, this parameter can be looked upon as the hypothetical maximum strain that a particle can suffer. In view of eq. 13, θ is also equal to:

$$\theta = \frac{E}{\phi \bar{X}} \tag{20}$$

Fig. 12 shows that for all four minerals tested, there is a strong relationship between the amount of material broken out of the feed size interval and parameter θ. However, it would seem that the explanation for unbroken material may not lie exclusively in geometrical terms. For example, for the same value of θ the different amounts broken in case of quartz, dolomite and limestone correlate well with the intercepts E_0 in Table I, that is, the higher is the E_0, the less in the amount broken out. In other words, friction between the roll surface and the solids, as well as breakage of corners and edges, could also be playing an unknown but not insignificant role in this phenomenon.

CONCLUDING REMARKS

It is reiterated that the analysis presented in this paper is of an exploratory nature. It is primarily meant for the purpose of verifying the postulates of an equivalent spherical diameter and a time-average thrust profile on the rolls which can be represented by a normalized type expression. The results are encouraging when tested against rather extensive experimental data. Eq. 13 has an interesting interpretation: the term ϕ has units of force and $\bar{X} - g$ is the distance that the opposing points on the roll surface at A′ move horizontally inwards as they reach point A at the level of the roll gap (see Fig. 1). However, the actual force acting on the rolls and pushing them apart is given by eq. 2. Along with the torque measurements, instrumentation of the rolls for monitoring this force would conceivably provide a better insight into the structure of the thrust profile on the roll surface.

From eq. 15, it is evident that the energy expended is not a simple function of $\bar{\eta}$ only, but depends, apart from the time factor which is imbedded in the feeding rate, directly on g and indirectly on particle size through its effect on ϕ. Undoubtedly, considerable scope exists for calculating the thrust profile from first principles of the force balance along the arc segment A′–A.

Of major importance is the empirical fact that the reduction ratio index is proportional to the energy expended irrespective of the feed size and roll gap combination used. At the same time, the slope j in eq. 17 or 18 provides a relative measure of the grindability of different minerals for the purpose of comparison on a consistent basis which is not cluttered with energy dissipation sinks that exist in the commercial mills, such as friction losses in turning the grinding charge in a ball mill.

Finally it is shown that the amounts of nominally unbroken particles depend only on the parameter θ, the hypothetical maximum strain, defined in eq. 19. The exact implications of this result, however, are not clear at present.

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