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The Comminution of Multicomponent Feeds under Batch and Locked-Cycle Conditions: Kinetics, Simulation and Energy Distribution

D.W. FUERSTENAU and K.S. VENKATARAMAN*

Department of Materials Science and Mineral Engineering, University of California, Berkeley, CA 94720 (U.S.A.)

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ABSTRACT

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The results of a detailed investigation of batch and locked-cycle dry grinding of quartz-calcite mixtures in a laboratory ball mill are presented. Batch grinding tests showed that the breakage rate function of the components is time-independent but composition-dependent and is normalizable with specific energy. Locked-cycle tests on the mineral mixtures revealed that approximately 25 2-min cycles are required before the composition of the circuit product and the recycle material attains steady state. The circulating load, after steadily increasing during the first 15 cycles, continued to fluctuate over a narrow range. These findings indicate that industrial-scale grinding circuits may also have long transient periods as the grindability of the ore changes and that circuits might be in transience perpetually.

INTRODUCTION

Natural minerals and rocks are generally very heterogeneous in their physical properties. In the case of an ore or raw coal, while the coarsest minerals are being liberated, the finer particles constituting the mill charge occur as a mixture of liberated minerals. Furthermore, variations in the ore from a mine can cause the mill feed to vary in grindability. In the comminution of heterogeneous materials, an understanding of how different constituents behave by themselves and how they interact with each other may prove useful not only for understanding grinding mill action but also for optimizing the operation.

Closed-circuit grinding involving ball mills and classification devices is the usual method utilized by the mineral industry to prepare an ore for down-

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*Present address: ALCOA Technical Center, Alcoa Center, PA (U.S.A.)

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stream processing. During the past few years, extensive efforts have been directed towards automatic control of grinding circuits in order to optimize the utilization of comminution energy and the recovery of minerals by minimizing overgrinding. Testing the response of an industrial-scale grinding system to variations in operating variables can be an expensive undertaking. On the other hand, laboratory-scale experimentation that simulates continuous closed-circuit grinding can provide detailed information which might be extended to predicting the response of full-scale comminution systems under dynamic conditions.

Over 50 years ago, Bond and his coworkers (Maxson et al., 1934) devised a locked-cycle batch grinding experimental scheme for assessing the grindability of ores. In their procedure, a given amount of ore is dry ground in a batch ball mill for a certain time period, the product is sieved and the oversized material is returned to the mill along with sufficient new feed to maintain the holdup constant. The total charge is ground again and the grinding/sieving process repeated until steady state is achieved at 250% circulating load. This experimental procedure simulates a ball mill operating under plug flow conditions in closed-circuit with a "perfect" classifier at 250% circulating load.

Considerable advances have been made in the last two decades in evolving kinetic models for the analysis of comminution operations. Through such models in conjunction with mill transport models, mill design, operation and control can now be conceived in a rational and meaningful way (Herbst and Fuerstenau, 1980; Austin et al., 1982). Most of the published work to test these models has been carried out with single minerals or more-or-less homogeneous materials. The objective of the work presented in this paper is to verify the applicability of the kinetic models for describing the comminution of multicomponent feed, namely mixtures of calcite and quartz, in a dry batch ball mill and to delineate the behavior of the system during locked-cycle comminution. A study of this nature provides a means for assessing the response of a comminution system to changes in the characteristics of the material actually fed to the mill, since the procedure simulates the grinding of an ore that might be a mixture of soft and hard components.

THE BATCH GRINDING MODEL

The mathematical analysis of the well-known size-discretized and time-continuous batch grinding equation for estimating the model parameters is appropriately treated by Herbst and Fuerstenau (1968), and only the salient features necessary for the analysis of the grinding of mixtures are outlined here. For the comminution of two fully liberated minerals in a mixture, the basic batch grinding equation for expressing the mass balance on material in size class 1 during the course of comminution is given below:

$$\mathrm{d}m_i(t)/\mathrm{d}t = -k_i(t)m_i(t) + \sum_{j=1}^{i-1} b_{ij}k_j(t)m_j(t) \tag{1}$$

where $m_i(t)$ is the mass fraction of material in the *i*th size class at time *t*, $k_i(t)$ is the breakage rate function at time *t* for material in the *i*th size class (which gives the fractional rate at which material is broken from the size class at time *t*) and b_{ij} is the breakage distribution function (which gives the fraction of material that becomes particles in the *i*th size class when material in the *j*th size class is comminuted). As used in the present paper, each parameter in eq. 1 should include a subscript 1 for material 1 and a subscript 2 for material 2 when a mixture of the minerals is being ground. Conventionally, the ascending size classes are taken to be the sieve opening in the Tyler $\sqrt{2}$ series of sieves in descending order. For dry ball milling, the k_i's are generally time-independent.

Under conditions where grinding kinetics are directly proportional to the specific energy input to the mill, Herbst and Fuerstenau (1980) showed that the breakage rate function is given by:

$$k_i = k_j^{\mathrm{E}}[P/H] \tag{2}$$

where *P* is the net input power to the mill, *H* is the material holdup, and k_i^{E} is a set of proportionality constants (specific breakage rate functions) which are independent of mill design and operating variables. For a batch mill drawing constant power, the quantity Pt/H is the specific energy input to the mill. Thus, introduction of eq. 2 into eq. 1 for conditions where the *k*'s are time-independent yields the normalized batch grinding model which permits prediction of mill performance in terms of the specific energy input to the mill:

$$\frac{dm_i(\bar{E})}{\mathrm{d}\bar{E}} = -k_i^{\mathrm{E}}m_i(\bar{E}) + \sum_{j=1}^{i-1} b_{ij}k_j^{\mathrm{E}}m_j(\bar{E}) \tag{3}$$

In this form, the model can be used to estimate energy distribution between the components of a mixture of unlocked mineral during grinding.

EXPERIMENTAL METHODS

Final preparation of monosized calcite and quartz samples involved sieving stage-crushed material on 10 and 14 mesh sieves for 1 h in small batches of approximately 300 g. Grinding experiments were carried out dry in a laboratory stainless steel mill 25.4-cm in diameter and 27.9-cm in length using 2.54-cm diameter stainless steel balls as grinding media. Initially, the 10×14-mesh calcite and quartz feed material was ground alone for 0.5, 1.0, 2.0, 4.0 and 8.0 min under standard mill operating conditions of 54 rpm (60% of mill critical speed), 30 kg of stainless steel balls (roughly 50% of the struck volume of the mill) and 100% filling of void volume of the struck ball charge by the feed material. The torque drawn by the mill was measured and recorded using a BHL torque-sensing system (Yang et al., 1967). After grinding the feed for the specified time, the mill was emptied and the size distribution of the mill contents was determined by a standard wet–dry sieving technique. For the grinding of calcite–quartz mixtures, the two minerals were mixed in the ratio of 1 : 1 by volume and ground for the desired time period under the same operating conditions. The size distribution of the ground mixture was then determined as before and the size distribution of each component in the mixture was obtained by leaching out the calcite in each size fraction with hydrochloric acid and weighing the remaining quartz to obtain the weight of calcite by difference.

The locked cycle experiments were carried out in the same mill system using a 2-min cycle time. The 2-min cycle time is conceptually equivalent to a 2-min residence time in the mill under plug flow conditions. The feed to the mill, which was prepared from the same samples of quartz and calcite used in the batch tests, was minus 14-mesh (1170 μm) material having the natural size distribution produced by a roll crusher. The new feed for each cycle was always in the ratio of 1 : 1 by volume. After a 2-min grind, the product was sieved on a 65-mesh (210 μm) screen for 10 min and the oversize material returned to the mill together with the required amount of fresh feed to maintain the total solids volume at 1150 cm^3 in each cycle. The calcite–quartz content of the product was obtained by the hydrochloric acid leaching procedure.

RESULTS AND DISCUSSION

Batch grinding

For mono-size feed material under batch grinding conditions, only the disappearance of particles takes place through comminution, without material entering that size class. Fig. 1 presents the first order disappearance plots for the two minerals when ground alone and when ground together as a mixture. These results show that the feed-size breakage (disappearance) rate is indeed first order for both calcite and quartz whether ground alone or as a component in a mixture. However, the rate at which a mineral is comminuted when ground in a binary mixture is different from that when the mineral is ground alone, as indicated by the differences in slopes of the plot. For conditions where the grinding rate depends on the particle environment, such as the presence of other minerals, k_i in Eqs. 1–3 should have the form $k_i(\text{env})$ where $k_i(\text{env})$ represents an environment-dependent but time-independent rate function. How the breakage rate function is affected in multicomponent grinding depends on the relative hardness of the minerals. For example, it can be seen in Fig. 1, that calcite (Mohs hardness $H=3$) grinds faster when ground in the presence of the harder quartz ($H=7$) than when ground alone. On the other

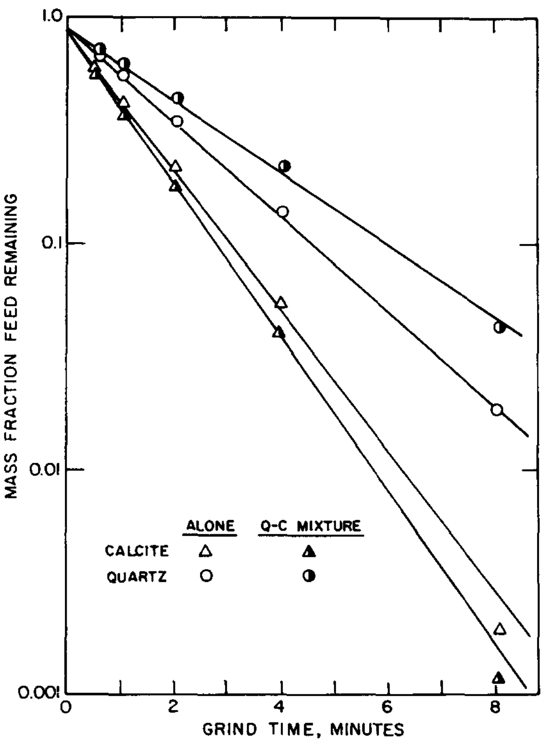

Fig. 1. First-order feed-size disappearance plots for the minerals under different grinding conditions.

hand, quartz grinds more slowly when ground in the presence of the softer calcite than when ground alone. The breakage rate function for calcite was found to be 0.77 min^{-1} when ground alone and 0.83 min^{-1} when ground with quartz; that of quartz was 0.49 min^{-1} when ground alone and 0.44 min^{-1} when ground with calcite. Furthermore, if each of the components of a mixture exhibit first order grinding kinetics, then the disappearance kinetics of the mixture as a whole must exhibit some deviation from first order kinetics.

Breakage distribution functions were determined from plots of the rate of production of fine particles by the method of Herbst and Fuerstenau (1968). Fig. 2 shows the cumulative breakage distribution functions for calcite and quartz when ground alone and when ground together in a mixture. It is evident from this plot that the cumulative feed-size breakage distribution functions (B_{i1}) of each mineral is the same, irrespective of whether that mineral is ground alone or as a component in a mixture. This implies that how the mineral distributes itself into daughter fragments during comminution is independent of the grinding environment. It has been observed that the B_{i1}'s are normalizable, regardless of mill size, mill speed and ball charge (Austin et al., 1984). The additional observation that B_{i1}'s are also normalizable in the case

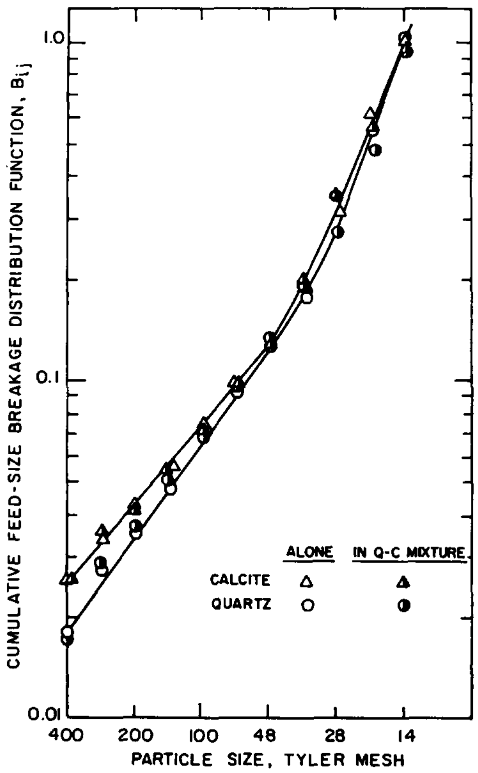

Fig. 2. Cumulative feed-size breakage distribution functions for the minerals under different grinding conditions.

of mixture grinding should be of use in predictive mill design for the grinding of complex ores.

The concept of reduced breakage rate functions has considerably simplified the computations involved in tumbling mill simulation and scale up, since this is a unique parameter for a particular mineral and mill system, independent of mill operating conditions and dependent only on the energy expended during grinding. Since ores are inherently heterogeneous, it is imperative that the constancy of k_i^E for minerals should be observed in the case of mixture grinding also, before we can take advantage of the simplified computations found in the case of homogeneous grinding. Fig. 3 presents plots in accordance with eq. 3 for the two minerals, both when they are ground alone and together as a mixture. From these results, it is clear that the k_1^E value is both time- and environment-independent. That is, when energy is taken into account, the breakage kinetics are independent of whether the mineral is comminuted homogeneously or heterogeneously. This observation should have utility in the design, scale up and simulation of tumbling mill systems.

The nonfeed-size k_i^E's can be estimated from the simple power law relation (Herbst and Fuerstenau, 1980):

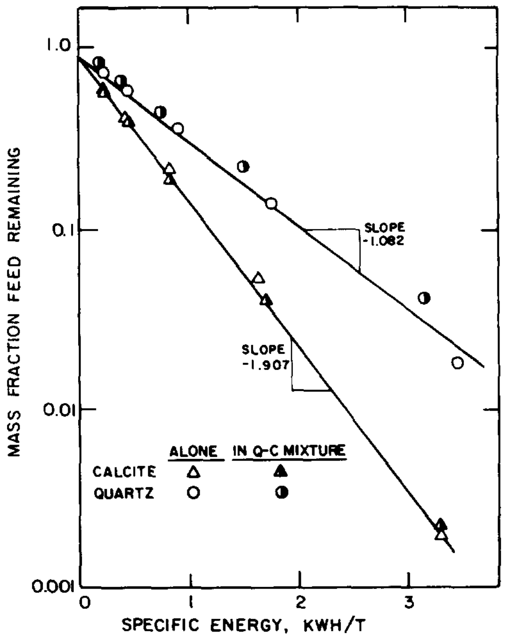

Fig. 3. Normalizability of breakage rate functions with specific energy for calcite and quartz.

$$k_1/k_i = k_1^{\mathrm{E}}/k_i^{\mathrm{E}} = (x_1/x_i)^{\alpha} \quad i=2,3,... \tag{4}$$

where x is the geometric mean particle size and α is the distribution modulus of the Gaudin–Schuhmann plot. Using the k_i^{E} values obtained with eq. 4, the energy required to produce the experimentally obtained size distribution of the components in mixture grinding was computed by simulation with the program ESTMILL. Fig. 4 illustrates the agreement between the simulated size distributions of calcite and quartz ground in calcite–quartz mixtures and those obtained experimentally. In this figure, the results are presented for the 0.5, 2.0 and 8.0 min grinds. One can readily visualize the nature of the size distribution of each component after a given grinding period. The specific energy consumptions needed to simulate each product size distribution given in Fig. 4 were the following: 0.29, 0.94 and 3.43 kWh/T for calcite and 0.18, 0.68 and 2.86 kWh/T for quartz after 0.5, 2 and 8 min of grinding, respectively.

The energy consumed by calcite and by quartz when they are ground together as a multicomponent feed for all the experiments was obtained from the simulation technique. For the grinding periods used in this investigation, calcite consumes a greater portion of the energy applied to the system. This is in agreement with earlier work of Somasundaran and Fuerstenau (1963) which showed preferential energy consumption of limestone over quartz at shorter grind times but a reversal in the order after longer grind times.

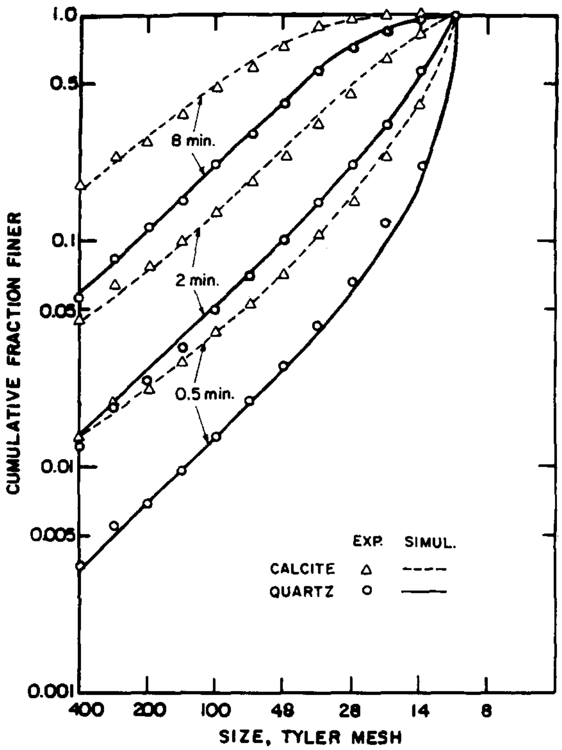

Fig. 4. Simulated and experimental size distributions for calcite and quartz ground together in calcite-quartz mixtures.

The following equation was used to obtain the total energy by simulation (E_{T_s}):

$$E_{T_s} = m_1 E_{1s} + m_2 E_{2s} \tag{5}$$

where m_1 and m_2 are the mass fractions of components 1 and 2, respectively, E_{1s} and E_{2s} are simulated values of specific energies (kWh/T of component) required to give the experimentally observed size distribution for each of the components. E_{T_s} is the simulated energy in kWh/T of the mixture. In Table I, the values of E_{T_s} obtained using eq. 5 can be compared with the experimentally measured energy. Study of this table shows that the values tally quite satisfactorily for these complicated systems. Therefore, it appears that k_i^E can be a very useful parameter in the design calculations for tumbling mill systems, even for heterogeneous ores.

Locked-cycle grinding

In discussions under batch grinding, we showed earlier that in the dry grinding of mineral mixtures, even though the breakage kinetics of the components

TABLE I

Energy consumed by components during the batch ball mill grinding of calcite–quartz mixtures

Grind time (min)	Energy (kWh) consumed by each component in the mixture		Total energy consumed to comminute the mixture (kWh/T)	
	calcite	quartz	simulation	measured
0.5	0.14	0.09	0.23	0.20
1.0	0.26	0.16	0.42	0.40
2.0	0.47	0.34	0.81	0.81
4.0	0.91	0.64	1.55	1.61
8.0	1.74	1.42	3.16	3.25

remain first-order, the absolute values of the breakage rate functions are dependent on whether a mineral is comminuted alone or as part of a mixture with other minerals that have very different hardnesses and grindabilities. In the closed-circuit grinding of complex ores, because of the different grindabilities of the components, the coarse material returned to the mill from the classifier will have compositions different from that of the feed. Typically, the mill circuit goes through a transience during the startup and when there are significant fluctuations in feed composition/grindabilities. During this transience, the composition of the different product streams of the circuit undergo continuous changes before reaching a steady state. We investigated the evolution of the transience during the startup in the dry, locked-cycle, ball mill grinding of a 1 : 1 mixture (by volume) of calcite and quartz.

Fig. 5 presents the mineral composition of the recycle material and the circuit product as a function of the cycle number. As can be seen from these results, the quartz content of both the recycle and the product asymptotically increases as a function of cycle number. This change is very slow, requiring about 25 cycles to reach steady state. We see also that during the course of the locked-cycle tests, the circuit product is initially richer in the relatively soft calcite and poorer in the harder quartz, but finally attains a composition identical to that of the new feed. The composition of recycle material starts with that of the feed and becomes increasingly richer in the harder component, quartz. This is what one would expect intuitively. The harder quartz grinds at a slower rate with smaller k_i values (lower grindability) than the softer calcite (higher k_i values). The system attains steady state when the overall composition of the circuit product is the same as that of the fresh feed. At this condition the relatively slower grinding rate of the harder mineral is eventually compensated for by the relatively larger proportion of the harder component in the mill holdup.

A very assiduous experimental campaign is required to quantify the effect of changing feed composition on the breakage kinetic parameters. However, with

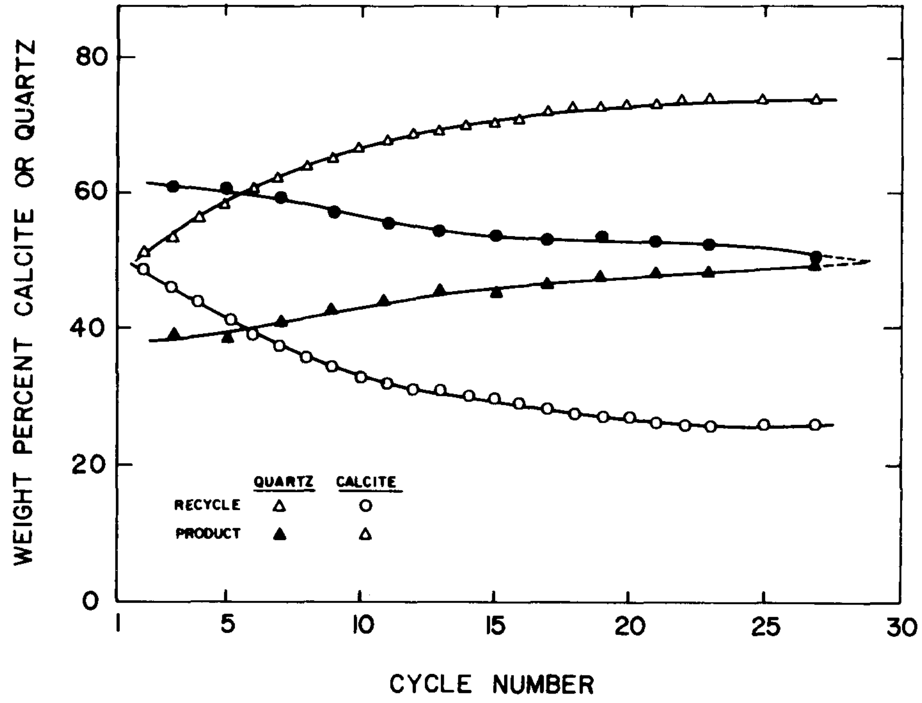

Fig. 5. Composition of the recycle material and the circuit product as a function of cycle number in the locked-cycle experiments.

the results given here, we can surmise that in the grinding of multicomponent feeds, the breakage rate function of the individual components will vary with changes in the composition of the mill holdup, depending on the relative hardness/grindability of the individual mineral components. Therefore, with composition slowly changing with cycle number as shown in Fig. 5, the breakage rate parameters also will change accordingly.

This observation is of considerable significance. In any control strategy for closed-circuit grinding in ball mills, accurate numerical values for these parammeters should be incorporated into the mathematical models used for the control. In most plants, despite best efforts to blend the feed material, any grinding circuit could be expected to process material whose composition or physical characteristics change with time. Very rarely does any grinding circuit operate under a true steady state condition for long periods of time. Therefore, any disturbance in the feed composition would lead to a transient state during which time the composition of the mill contents could continuously change with time. During this period, since the kinetic parameters also change, we may have to incorporate into the control models changes in the kinetic parameters, based on some function of changing composition of the mill contents.

There is yet another aspect to the problem. In our locked-cycle experiments, with a 2-min cycle time, nearly 40 min were required for the system to reach steady state (Fig. 5). It is not unreasonable that disturbances in feed composition occur, say, every 30 or 40 min, which leads to the inference that grinding circuits may be in perpetual transience. This is not a comforting speculation either for the mill operator or for those engaged in the design of control schemes for grinding circuits. However, an awareness of this inherent limitation should be helpful for analyzing complex grinding problems.

Fig. 6 presents the percentage circulating load ratio [(recycle/fresh feed) ×100] as a function of cycle number. This figure clearly shows that the circulating load ratio fluctuates in this experimental system. In the case of the locked-cycle grinding of homogeneous materials, steady state is rapidly attained and the circulating load ratio does not fluctuate (Gumtz and Fuerstenau, 1970). The circulating load ratio plot in Fig. 6 is more complex to explain. After oscillating between a narrow range of 450 to 480%, the circulating load ratio appears to reach a steady state value of about 475%. Since the recirculating ratio is the ratio of recycle material to the fresh feed, small changes in the relative amounts exaggerate the ratio considerably. Also, any extraneous noise could have contributed to the oscillations. The minus 14-mesh feed was taken from drums by a riffling method in order to have representative samples of the minerals. However, as with any sampling system, it is impossible to completely avoid segregation, which could have contributed to the oscillations over the narrow range.

A recursive technique was used to estimate the values of the k_i's for calcite and quartz in each of the locked-cycle experiments. Fig. 7 presents the change in the estimated value of k_1, that is the top size (1160×830 μm or 14×20 mesh), with the cycle number for both calcite and quartz. The trend in the change in the estimated values of k_1 with cycle number is consistent with the explanation offered in the preceding paragraphs. That is, the breakage rate function of the softer mineral increases and that of the harder component decreases as the mixture in the mill becomes richer in the harder component. The most important observation is that the k_1 values are strongly composition-dependent. These are computed values, and therefore can only be indicative of the trend to be expected. More detailed experiments are necessary to corroborate this observation.

From the grinding kinetic studies and the results given in Figs. 5 and 6, we can analyze what determines the composition of the circuit at steady state. At steady state, the circulating load is 475% and the composition of the recycle is 73% quartz and 27% calcite. At the beginning of the experiment, the mill feed (which always occupied 100% interstitial volume of the static bed of the grinding media) consisted of a 1 : 1 by volume mixture of minus 1120 μm (minus 14-mesh) calcite and quartz. The total mill feed at the beginning of the test consisted of 3.088 kg (1.524 kg calcite and 1.564 kg quartz) of the two-component mixture. Since the densities of calcite and quartz are nearly the same

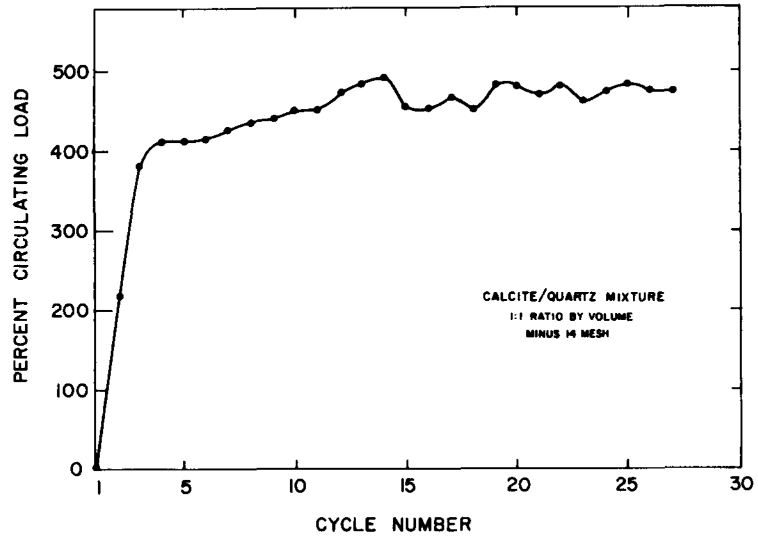

Fig. 6. Evolution of the circulating load as a function of the cycle number in the locked-cycle experiments.

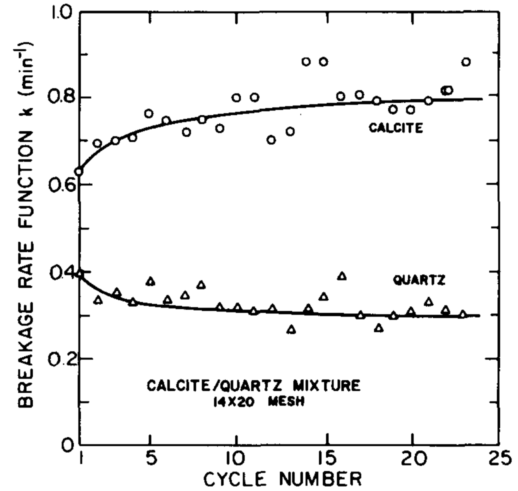

Fig. 7. Computed values of the breakage rate functions for the 14 × 20 mesh calcite and quartz size fractions in the locked cycle experiments.

(namely 2.73 and 2.66 g/cm^3, respectively), the total quantity of the calcite–quartz mixture in the mill was 3.088 kg in each run. However, the composition of the mill contents continuously became richer in the harder-to-grind quartz as the system approached steady state.

At steady state the quantity of plus 210-μm material of each component in the mill feed is the sum of the component in the fresh feed and in the recycle stream. This sum for calcite was 0.887 kg, and for quartz 2.088 kg, giving a total of 2.975 kg. This quantity is less than the 3.088 kg of total feed at the beginning of the test. The difference is the amount of minus 210-μm material in the 1 : 1 volume mixture of the fresh feed. Since the circulating load ratio is large, and the recycled material is narrowly sized, one can analyze the product composition in terms of the plus 210-μm material in the mill at the beginning of each cycle.

The absolute rate (kg/min) at which materials break out of a size class i is given by the product of the breakage rate function k_i (min^{-1}) for the size class and the holdup H (kg) (Malghan and Fuerstenau, 1975). The breakage rate functions of material in the top size range, 1160×830 μm (10×14 Tyler mesh), are 0.31 min^{-1} for quartz and 0.79 min^{-1} for calcite. The *ratio* of the rates at which particles of the two components break out of the 1120×848 μm (14×20 mesh) size class is therefore:

$$\frac{k_c H_c}{k_q H_q} = \frac{0.79 \times 0.887}{0.31 \times 2.088} = 1.08$$

We see that this ratio is almost the same as the weight ratio 1.04 for the two components in the fresh feed. Thus, the breakage rate functions of materials constituting a mixed feed can be used to estimate the composition of the holdup in a mill that is operating in closed circuit at steady state.

SUMMARY AND CONCLUSIONS

When minerals are comminuted in ball mills in the presence of other minerals, the breakage rate functions are environment-dependent but time-independent. However, the breakage rate functions are still normalizable with respect to specific energy whether a mineral is ground alone or as a component of a mixture. It is possible to resolve the fraction of the total net energy input to the mill into that consumed by each component of a multicomponent feed by simulation using the reduced breakage rate function concept. For the grind times used in these experiments, the breakage rate function of the softer mineral (calcite) increased when ground in the presence of a harder mineral (quartz). Stated in another way, calcite consumes a greater proportion of the grinding energy than does quartz when the two are ball milled together.

Locked-cycle tests show that in the grinding of mixtures a relatively long time is required to attain steady state, during which time the composition of the mill contents continuously change. During this period, the breakage rate functions must also change continuously. Twenty-five 2-min cycles were found to be necessary for the recycle composition to attain a constant value. The composition of the total charge inside the mill at steady state is controlled by the relative grindabilities of the minerals in the mixture, that is by the value of breakage rate function of each mineral in the mixture. Any mathematical model used in automatic control schemes for the closed-circuit grinding of heterogeneous materials should explicitly take into account how the kinetic parameters of the component minerals vary with feed composition. Furthermore, with disturbances in the feed composition occurring typically every 30 or 40 min, it is possible that grinding circuits could be perpetually in a transient condition.

The circulating load (recycle ratio) was followed as a function of cycle number. Surprisingly, during the locked-cycle grinding of mixtures, the circulating load continued to escillate. One would not have expected this unusual behavior since grinding systems behave linearly. This, too, may be a phenomenon that further complicates control of the circuit in large-scale mills.

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