Characteristic curves for multiple-fan ventilation systems

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Abstract — *A subsystem-based approach to the characteristic curves for a multiple-fan ventilation system is presented. The ventilation variables and their relationships associated with a fan branch are considered as a subsystem of the multiple-fan system. For each subsystem, the solution curve is generated by replacing the fan with a hypothetical pressure source or, alternatively, by adding a hypothetical pressure source without removing the fan.*

The projection of the solution curve onto the coordinate planes results in two classes of subsystem characteristic curves: pressure-vs.-quantity and quantity-vs.-quantity. The subsystem characteristic curves are described and illustrated through example ventilation systems. These curves are also discussed using the network equations of a general multiple-fan system.

Introduction

The concept of a mine characteristic curve is of fundamental importance in mine ventilation. The concept is often applied in the analysis of the pressure-quantity relationship for a single-fan ventilation system. For a multiple-fan system, however, the mine characteristic is difficult to define. As a result, the ventilation literature reveals little insight into multiple-fan characteristic curves.

The main purpose of this paper is to extend the concept of the mine characteristic curve to a multiple-fan system, using a subsystem-based approach (Wang, 1984) in which the multiple-fan system is considered as consisting of the subsystems that are associated with a set of fans. Applying the procedure of plotting the mine characteristic curve to the subsystems, one can plot two important classes of subsystem characteristic curves: pressure-vs.-quantity and quantity-vs.-quantity. In addition to a critical discussion of the mine characteristic curve, the subsystem characteristic curves for the multiple-fan system will be described and illustrated through example ventilation systems.

For a unified approach to both blower and exhaust systems, the fan pressure is defined to mean the fan total pressure, assuming that the resistance factor of an outlet branch includes the equivalent resistance for the outlet velocity pressure (Wang, 1983). For a pressure source (or a fan) and its pressure, the same symbol will be used. For example, the pressure source located in branch j and its pressure will be denoted by P_j.

Two fan characteristic curves shown in Fig. 1 were used in computations of the numerical examples. The values for fan pressures were generated by the natural cubic splines approximation method (Burden and Faires, 1985). The curves were assumed to extend horizontally beyond the lower and upper limits shown in Fig. 1. Three example ventilation systems, designated as Examples 1 to 3, are shown in Figs. 2 to 4.

Mine characteristic curves

Mine resistance

To examine the mine characteristic curve and the operating point of a ventilation system, consider Example 1 shown in Fig. 2. It is a single-fan system with three branches. Fan t has a characteristic curve identical to curve A of Fig. 1 and is installed in branch 1. This is a small series-parallel system. Its mine resistance R can be calculated by

$$R = r_1 + [(r_2)^{-1/2} + (r_3)^{-1/2}]^{-2} \quad (1)$$

where r_1, r_2 and r_3 are the resistance factors for branches 1, 2 and 3. It then follows, from the energy and Atkinson's equations, that

$$H - t(q_1) = 0 \quad (2)$$

$$H - R\,|q_1|q_1 = 0 \quad (3)$$

where H is the *mine head* or *system pressure loss*, and the fan pressure t (pressure gain) is a function of air quantity q_1. In Eq. 3, the absolute value of q_1 is used so that H and q_1 have the same sign.

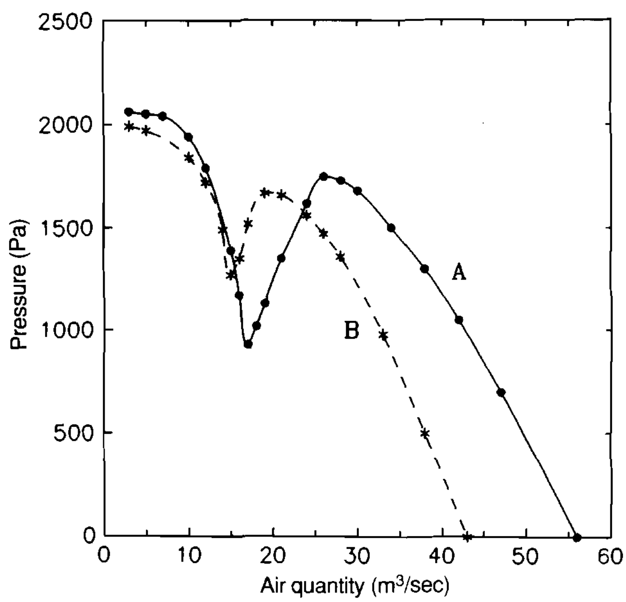

Fig. 1 — Fan characteristic curves A and B.

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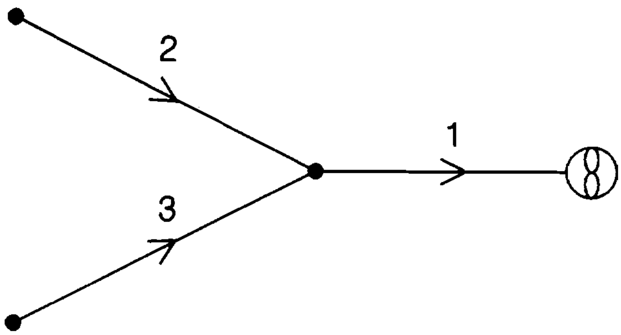

Fig. 2 — Example 1, resistance factors (N•s²/m⁸): r_1 = 0.8, r_2 = 1.1, r_3 = 1.3.

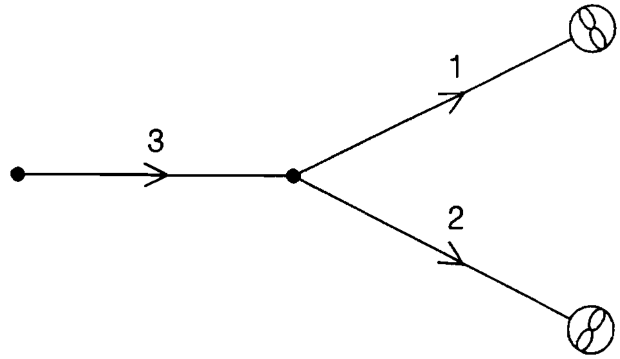

Fig. 3 — Example 2, resistance factors (N•s²/m⁸): r_1 = 0.4, r_2 = 0.3, r_3 = 0.3.

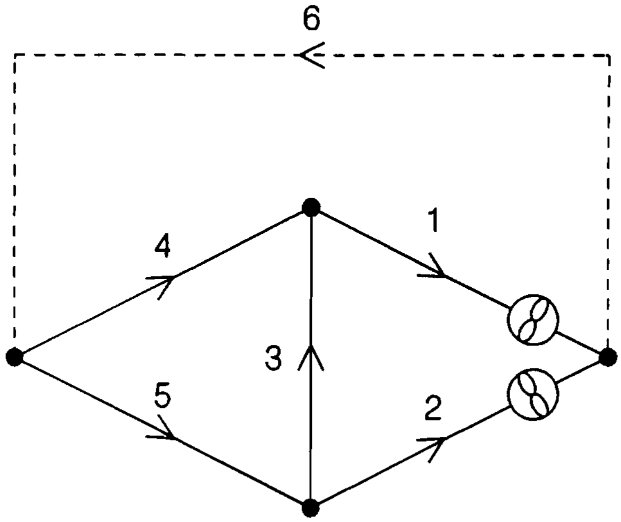

Fig. 4 — Example 3, resistance factors (N•s²/m⁸): r_1 = 1.2, r_2 = 1.1, r_3 = 0.5, r_4 = 1.8, r_5 = 1.2, r_6 = 0.0.

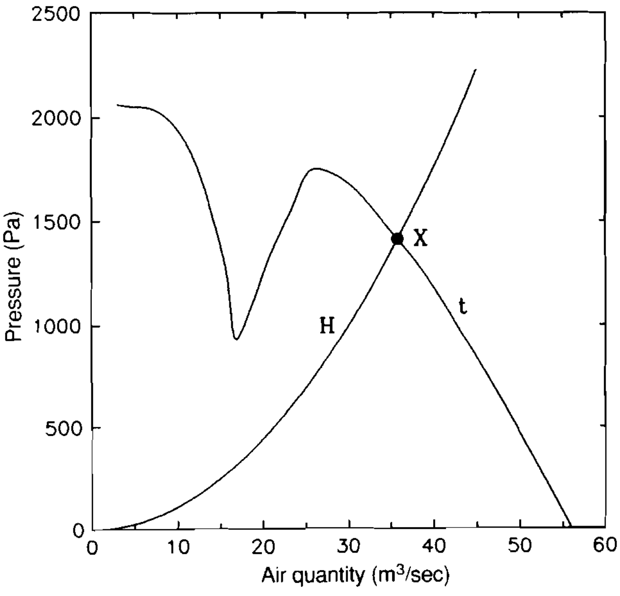

Fig. 5 — Mine characteristic curve H and fan characteristic curve t (Example 1).

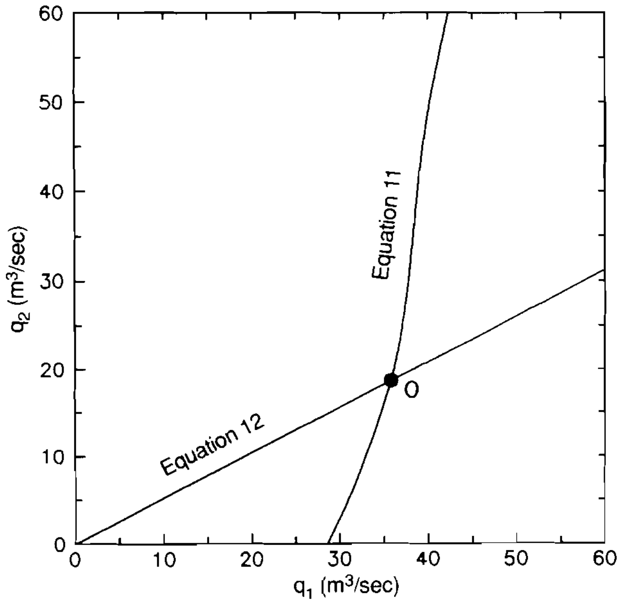

Fig. 6 — Two curves representing network equations (Eqs. 11 and 12, Example 1).

This is a system of two equations in two unknowns, q_1 and H. As plotted in Fig. 5, Eqs. 2 and 3 represent two curves in the q_1-H plane. The curve representing Eq. 2 is identical to the fan characteristic curve t. Another curve, which represents Eq. 3, is known as the *mine characteristic curve* or *system characteristic curve* (curve H in Fig. 5). The intersection of the two curves satisfies both equations and represents a solution of the equations. This intersection or solution point is called the *fan operating point* X.

This approach of using the mine resistance is also applicable to a single-fan system that does not have the series-parallel structure, although the value of mine resistance cannot be obtained with an equation and has to be determined from the solution to a set of network equations (or measured data). In the discussion that follows, the mine characteristic curve is explained from the viewpoint of network analysis.

Network equations

Three cases of Example 1 (Fig. 2) are discussed below.

Case 1: Let us denote the pressure loss, resistance factor, and air quantity of branch j by h_j, r_j and q_j, respectively. Based on Kirchhoff's laws and Atkinson's equation, we have

$$h_1 + h_3 - t(q_1) = 0 \quad (4)$$

$$h_2 - h_3 = 0 \quad (5)$$

$$q_1 - q_2 - q_3 = 0 \quad (6)$$

$$h_j = r_j |q_j| q_j, \quad j = 1, 2, 3 \quad (7)$$

where fan pressure t is a function of q_1.

Substituting the expressions for h_j from Eq. 7 into Eqs. 4 and 5 and eliminating q_3 using Eq. 6, the following network equations are obtained:

$$r_1|q_1|q_1 + r_3|q_1 - q_2|(q_1 - q_2) - t(q_1) = 0 \quad (8)$$

$$r_2|q_2|q_2 - r_3|q_1 - q_2|(q_1 - q_2) = 0 \quad (9)$$

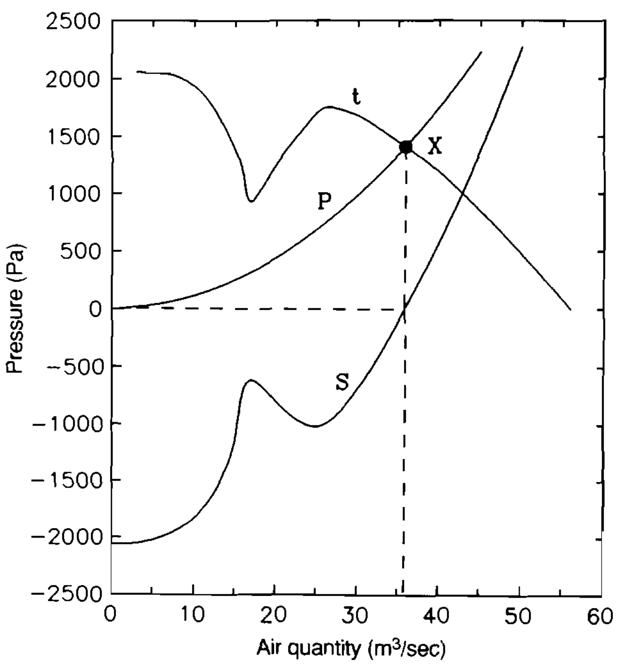

Fig. 7 — Characteristic curves P, S and t (Example 1).

By definition, the system pressure loss H can be expressed as

$$H = h_1 + h_3 = r_1|q_1|q_1 + r_3|q_1 - q_2|(q_1 - q_2) \quad (10)$$

Hence, Eqs. 8 and 9 can be written as

$$H - t(q_1) = 0 \quad (11)$$

$$r_2|q_2|q_2 - r_3|q_1 - q_2|(q_1 - q_2) = 0 \quad (12)$$

This is a system of two independent equations in two unknowns, q_1 and q_2. As plotted in Fig. 6, Eqs. 11 and 12 represent two curves in the q_1 - q_2 plane. The intersection of the two curves satisfies both equations and represents the solution to the network equations. This intersection or solution point is called an *operating point of the ventilation system* or *system operating point* O.

Case 2: In this case, the fan in branch 1 has been replaced by a hypothetical pressure source P. The pressure P of this source will be referred to as the system pressure. Similar to Case 1, the following network equations apply:

$$H - P = 0 \quad (13)$$

$$r_2|q_2|q_2 - r_3|q_1 - q_2|(q_1 - q_2) = 0 \quad (14)$$

There are three unknowns (q_1, q_2 and P) in Eqs. 13 and 14; the number of unknowns is one more than that of the equations. Geometrically, the locus of all points satisfying these equations form a curve (*solution curve*) in the q_1-q_2-P space. Let L denote the solution curve. The projection of L onto the q_1-P plane will be called the *mine characteristic curve* P or *system characteristic curve* P. Furthermore, the projection of L onto the q_1-q_2 plane will be referred to as the *characteristic curve* U. For the single-fan system, the quantity-quantity characteristic curve, such as the curve U, is a straight line.

The characteristic curves P and U belong to the *driving-point characteristic plot* and *transfer characteristic plot*, respectively, of the nonlinear resistive networks (Chua, 1969). They are also referred to as *input-output characteristic plots* (Kuh and Hajj, 1971).

Observe that $t(q_1)$ does not appear in Eqs. 13 and 14. This means that the mine characteristic curve is constructed without the characteristic curve of the fan. Also observe from Eq. 13 that the values for H equal the corresponding values for P; therefore, the solution curve may be plotted in

Table 1 — Data for a single-fan system (Example 1).

q_1 (m³/s)	q_2 (m³/s)	P or H (Pa)	S (Pa)	t (Pa)	Remarks
0.00	0.00	0	-2060	2060	
5.00	2.60	27	-2023	2050	
10.00	5.21	110	-1830	1940	
15.00	7.81	247	-1143	1390	
17.00	8.86	317	-613	930	
20.00	10.42	437	-799	1238	
25.00	13.02	687	-1013	1699	
30.00	15.63	989	-691	1680	
35.00	18.23	1346	-106	1451	
35.84	18.67	1411	0	1411	Op. point
40.00	20.84	1758	547	1181	
45.00	23.44	2224	1381	844	
50.00	26.04	2746	2272	474	
55.00	28.65	3323	3243	80	
60.00	31.25	3954	3954	0	

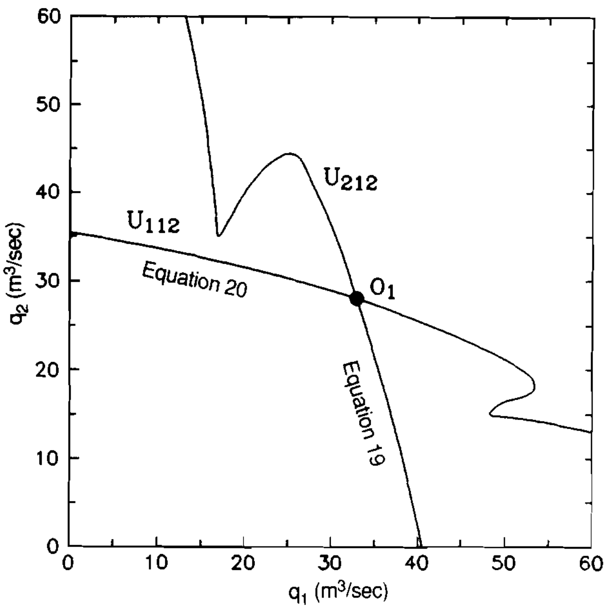

Fig. 8 — Two curves representing network equations (Eqs. 19 and 20; Example 2). These two curves also represent subsystem characteristic curves U_{212} and U_{112}.

the q_1-q_2-H space. In this paper, however, only the q_1-q_2-P space is considered.

To obtain the data for plotting curves L, P and U, values are assigned to one of the unknowns and solved for the corresponding values for the other unknowns. For the purposes of this paper, the data for the range of q_1 from 0 to 60 m^3/sec were calculated and are listed in Table 1.

Case 3: In Case 3, fan t is reinstalled, and the pressure source P is replaced by another hypothetical pressure source S. Therefore, following Case 2,

$$H - [t(q_1) + S] = 0 \qquad (15)$$

$$r_2|q_2|q_2 - r_3|q_1 - q_2|(q_1 - q_2) = 0 \qquad (16)$$

Again there are two equations in three unknowns, q_1, q_2 and S. Similar to Case 2, the locus of all points satisfying these equations forms a solution curve in the q_1-q_2-S space. This curve is denoted by K, and its projection on the q_1-S plane is called the *characteristic curve* S. The projection of curve K on the q_1-q_2 plane is the same as curve U; hence, no additional symbol will be used. Obviously, when S = 0, Eq. 15 reduces to Eq. 11 of Case 1. Therefore, the intersection of the curve K and the S = 0 plane is a system operating point.

If the data for pressure P or H are available, the corresponding values for curve S can be computed by

$$S = P - t(q_1) = H - t(q_1) \qquad (17)$$

On the other hand, if the data for pressure S are available, the corresponding values for H or P can be computed by

$$H = P = t(q_1) + S \qquad (18)$$

The data for curves K (i.e., data for q_1, q_2 and S) are included in Table 1; the curve S is plotted with curves P and t in Fig. 7.

Subsystem characteristic curves

Two-fan system with two meshes

A two-fan system with two meshes will be used in this section to introduce the concept of subsystem characteristic curves.

Table 2 — Data for a two-fan system with two meshes (Example 2).

Subsystem 1			Subsystem 2			
q_1 (m^3/s)	q_2 (m^3/s)	P_1 (Pa)	q_1 (m^3/s)	q_2 (m^3/s)	P_2 (Pa)	Remarks
0.00	35.43	377	40.51	0.00	492	
2.75	35.00	431	40.00	2.47	543	
26.42	30.00	1234	35.00	21.60	1101	
32.96	28.09	1553	32.96	28.09	1355	Op. point
41.57	25.00	2020	30.00	36.33	1716	
48.43	22.00	2426	28.00	40.71	1914	
52.02	20.00	2639	26.00	44.23	2066	
52.12	17.00	2520	25.00	44.50	2043	
48.31	15.00	2136	23.00	43.46	1892	
55.07	14.00	2644	20.00	39.95	1557	
60.00	13.00	3039	18.00	36.48	1290	
			17.00	35.10	1184	
			16.80	36.22	1237	
			16.00	43.66	1639	
			15.00	50.83	2075	
			13.11	60.00	2683	

Example 2, shown in Fig. 3, is one of the simplest multiple-fan systems that can be constructed. Branches 1 and 2 are designated as fan branches. The fan characteristic curves A and B of Fig. 1 will be assumed for fans t_1 and t_2, respectively. The variables and their relationships associated with a fan branch will be referred to as a *subsystem* of the ventilation system.

Similar to Example 1, we have

$$r_1|q_1|q_1 + r_3|q_1 + q_2|(q_1 + q_2) - t_1(q_1) = 0 \qquad (19)$$

$$r_2|q_2|q_2 + r_3|q_1 + q_2|(q_1 + q_2) - t_2(q_2) = 0 \qquad (20)$$

This is a system of two equations in two unknowns, q_1 and q_2. The two curves representing these equations are plotted in Fig. 8. The intersection of the curves represents an operating point of the ventilation system.

For simplicity, rewrite Eqs. 19 and 20 as

$$H_1 - t_1(q_1) = 0 \qquad (21)$$

$$H_2 - t_2(q_2) = 0 \qquad (22)$$

where H_1 and H_2 are referred to as *subsystem pressure losses* for subsystems 1 and 2, respectively.

Now consider the characteristic curves for subsystem 1 that are associated with branch 1 where fan t_1 is located. Following the procedure for Case 2 of Example 1, replace the fan t_1 with a hypothetical pressure source P_1. Hence,

$$H_1 - P_1 = 0 \qquad (23)$$

$$H_2 - t_2(q_2) = 0 \qquad (24)$$

which is a system of two equations in three unknowns, q_1, q_2 and P_1. As was done in Example 1, denote the solution curve in the q_1-q_2-P_1 space by L_1, but with the subscript 1 referring to subsystem 1. Call the projections of L_1 on the q_1-P_1 and q_1-q_2 planes the *subsystem characteristic curves* P_1 and U_{112}, respectively. Furthermore, the pressure P_1 is called the *subsystem pressure* 1.

The application of the procedure for Case 3 of Example 1 results in

$$H_1 - [t_1(q_1) + S_1] = 0 \qquad (25)$$

$$H_2 - t_2(q_2) = 0 \qquad (26)$$

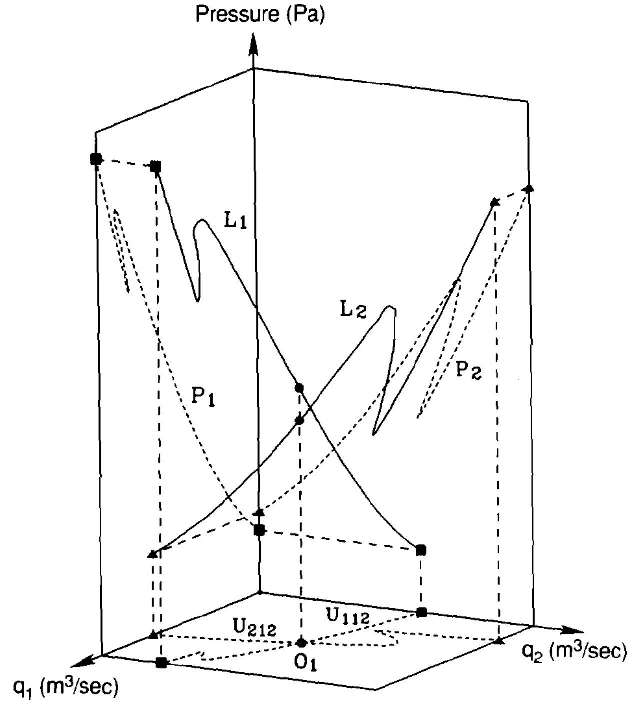

Fig. 9 — Solution curves (L. and L_2) and subsystem characteristic curves (P_1, P_2, U_{112} and U_{212}) in a three-dimensional space (Example 2).

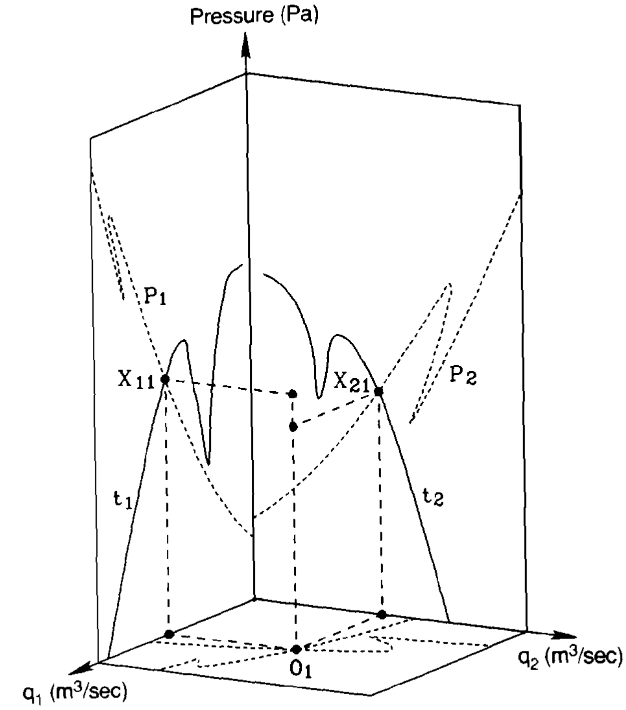

Fig. 10 — Subsystem and fan characteristic curves in a three-dimensional space (Example 2).

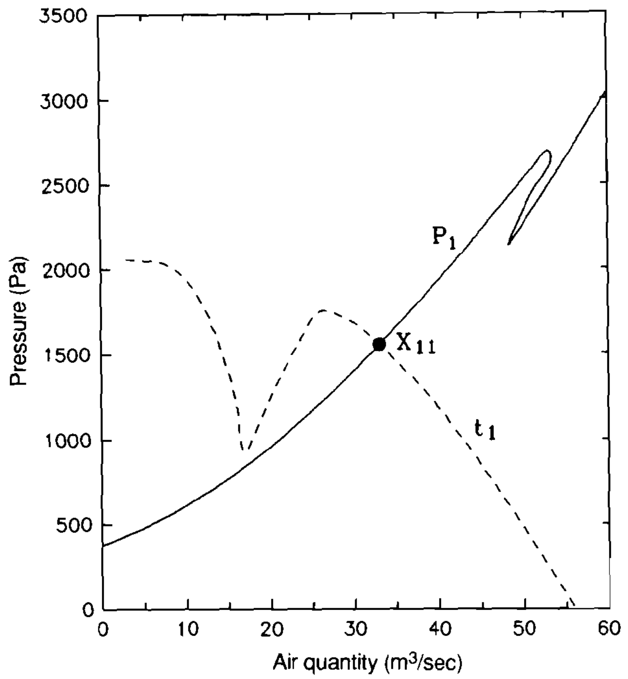

Fig. 11 — Subsystem characteristic curve P_1 (Example 2).

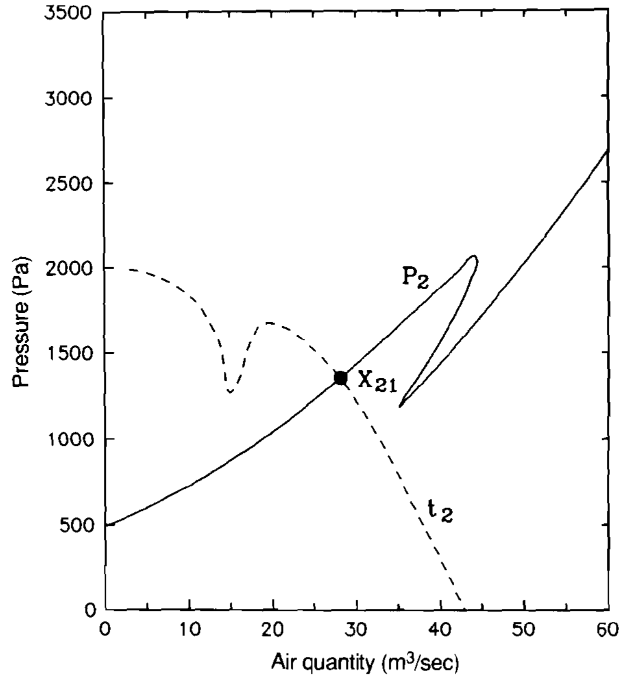

Fig. 12 — Subsystem characteristic curve P_2 (Example 2).

and the solution curve K_1. As was explained for Example 1, the data for curves K_1 and S_1 can be obtained by using the data for curves L_1 and t_1 with the relationship

$$S_1 = H_1 - t_1(q_1). \quad (27)$$

Likewise, for subsystem 2,

$$H_1 - t_1(q_1) = 0 \quad (28)$$

$$H_2 - P_2 = 0 \quad (29)$$

where the unknowns are q_1, q_2 and P_2. The solution curve of Eqs. 28 and 29 in the q_1-q_2-P_2 space is denoted by L_2 and called the solution curve for subsystem 2. The subsystem characteristic curves P_2 and U_{212} represent the projections of L_2 onto the q_2-P_2 and q_1-q_2 planes, respectively.

There are also

$$H_1 - t_1(q_1) = 0 \quad (30)$$

$$H_2 - [t_2(q_2) + S_2] = 0 \quad (31)$$

and the solution curve K_2 for subsystem 2.

Listed in Table 2 are the data for both subsystems 1 and 2 excluding those for K_1 and K_2. The solution curves L_1 and L_2, and the associated subsystem characteristic curves P_1, P_2, U_{112}

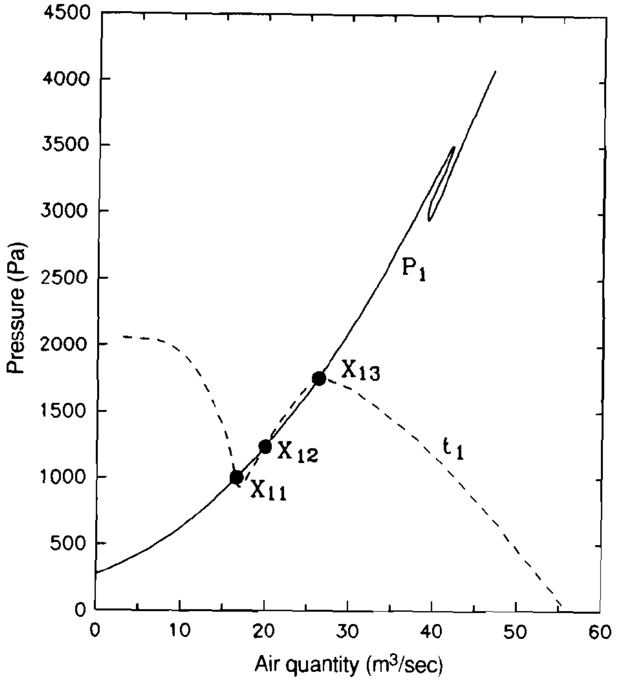

Fig. 13 — Subsystem characteristic curve P_1 (Example 3).

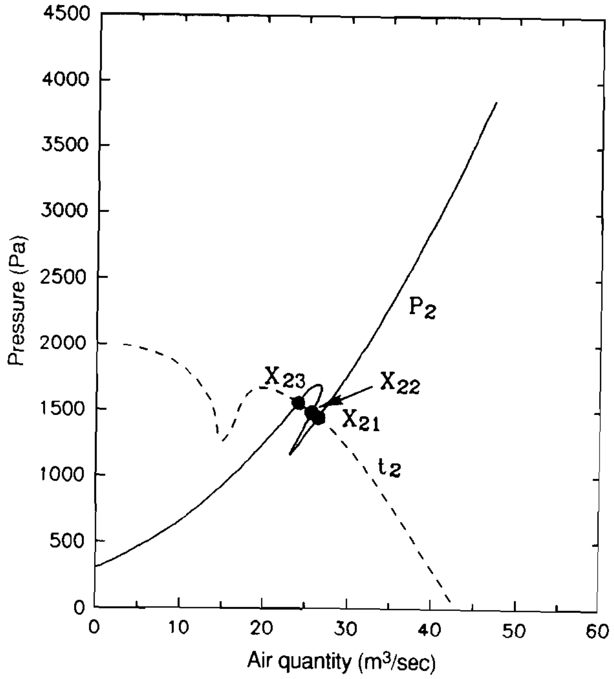

Fig. 14 — Subsystem characteristic curve P_2 (Example 3).

and U_{212}, are plotted in Fig. 9 with the vertical axis as the common axis for the pressure (pressure gain). In this simple example, the system operating point O_1 coincides with the intersection of the subsystem characteristic curves U_{112} and U_{212}. In Fig. 10, the curves L_1 and L_2 of Fig. 9 are removed, and the fan characteristic curves t_1 and t_2 are added to depict the fan operating points X_{11} and X_{21}. Observe from Figs. 9 and 10 that the curves L_1 and L_2 do not intersect in the space except for the special case where the fans t_1 and t_2 operate at the same pressure. The subsystem characteristic curves P_1 and P_2 are also plotted with the fan characteristic curve in Figs. 11 and 12, respectively.

An important property of the subsystem characteristic curves is seen from Figs. 9 to 12. The subsystem characteristic curves P_1 and P_2 do not pass through the origin. The intersection of the subsystem characteristic curve P_1 (or P_2) and the pressure axis (vertical axis) represent a critical pressure C_1 (or C_2), below which the air quantity for the fan branch becomes negative while the characteristic of the fan in the other branch remains unchanged.

The subsystem characteristic curves U_{112} and U_{212} are, respectively, the same as the curves for Eqs. 20 and 19, and they are identified in Fig. 8. Therefore, the intersection of the curves U_{112} and U_{212} is a system operating point. Furthermore, if the solution curves K_1 and K_2 are plotted in the same space with a common pressure axis, they also intersect the system operating point.

Two-fan system with three meshes

Another two-fan system, referred to as Example 3, is shown in Fig. 4. Branch 6 is the return dummy branch, which connects the outlet node to the inlet node. Assume again that the characteristic curves for fans t_1 and t_2 are identical to curves A and B of Fig. 1, respectively. Following the procedure used in Example 2, we obtain

$$r_1|q_1|q_1 + r_4|q_1 - q_3|(q_1 - q_3) +$$
$$r_6|q_1 + q_2|(q_1 + q_2) - t_1(q_1) = 0 \quad (32)$$

$$r_2|q_2|q_2 + r_5|q_2 + q_3|(q_2 + q_3) +$$
$$r_6|q_1 + q_2|(q_1 + q_2) - t_2(q_2) = 0 \quad (33)$$

$$r_3|q_3|q_3 - r_4|q_1 - q_3|(q_1 - q_3) +$$
$$r_5|q_2 + q_3|(q_2 + q_3) = 0 \quad (34)$$

or

$$H_1 - t_1(q_1) = 0 \quad (35)$$

$$H_2 - t_2(q_2) = 0 \quad (36)$$

$$f_3(q_1, q_2, q_3) = 0 \quad (37)$$

where H_1 and H_2 are subsystem pressure losses, and f_3 represents the left side of Eq. 34.

This is a system of three equations in three unknowns, q_1, q_2 and q_3. Each equation represents a surface in the three-dimensional q_1-q_2-q_3 space. The intersection of the three surfaces is an operating point of the ventilation system. Three system operating points, denoted by O_1, O_2 and O_3 have been found. Their numerical values are listed in Table 3.

There are two fans and, therefore, two subsystems. The system of equations associated with curves L_1 and L_2 are, respectively, given by

$$H_1 - P_1 = 0 \quad (38)$$

Table 3 — Three operating points for Example 3.

Operating Point	q_1 (m^3/s)	q_2 (m^3/s)	q_3 (m^3/s)	t_1 (Pa)	t_2 (Pa)
1	16.59	26.47	-2.74	1002	1446
2	20.03	25.68	-0.52	1241	1485
3	26.29	24.07	3.61	1755	1557

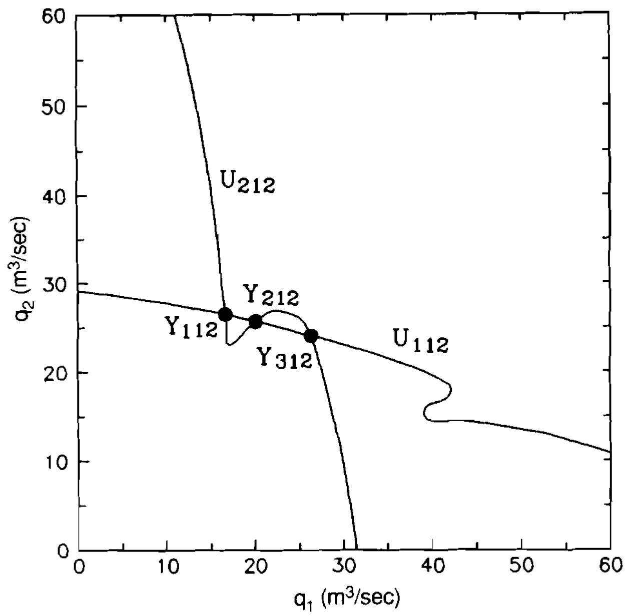

Fig. 15 — Subsystem characteristic curves U_{112} and U_{212} (Example 3).

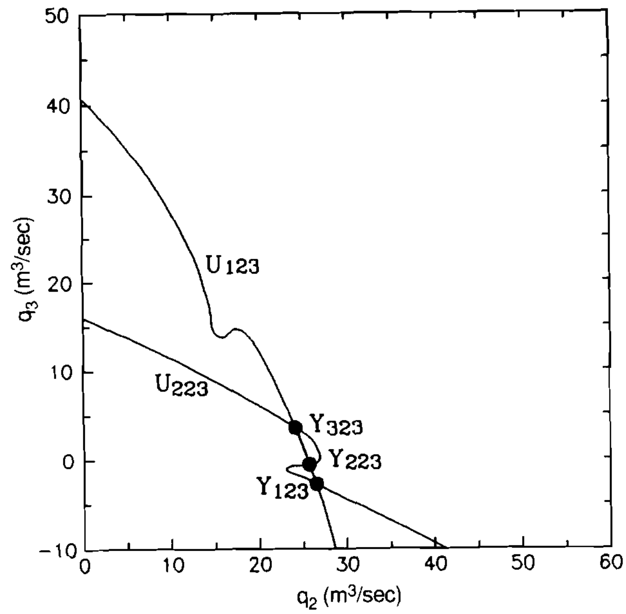

Fig. 16 — Subsystem characteristic curves U_{123} and U_{223} (Example 3).

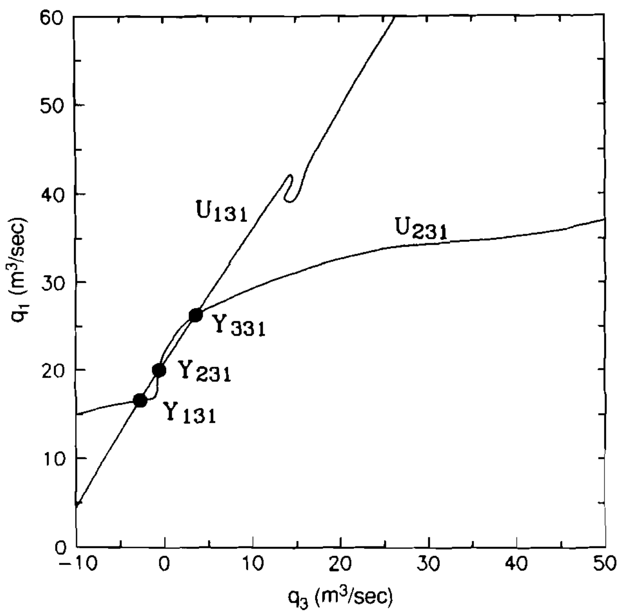

Fig. 17 — Subsystem characteristic curves U_{131} and U_{231} (Example 3).

$$H_2 - t_2(q_2) = 0 \quad (39)$$

$$f_3(q_1, q_2, q_3) = 0 \quad (40)$$

and

$$H_1 - t_1(q_1) = 0 \quad (41)$$

$$H_2 - P_2 = 0 \quad (42)$$

$$f_3(q_1, q_2, q_3) = 0 \quad (43)$$

The solution curves L_1 and L_2 for the subsystems are represented in the four-dimensional q_1-q_2-q_3-P_1 and q_1-q_2-q_3-P_2 spaces, respectively, and are difficult to visualize. Consequently, we shall concentrate on their projections on the planes. These are designated as the subsystem characteristic curves P_1, P_2, U_{112}, U_{123}, U_{131}, U_{212}, U_{223} and U_{231}. The characteristic curves U_{kij} refer to the projections of the curves L_k onto the q_i-q_j plane.

These characteristic curves are plotted in Figs. 13 to 17. Notice that three operating points are identified in each of Figs. 13 to 17. In Figs. 13 and 14, X_{kj} denotes the j-th operating point of fan t_k; in Figs. 15 to 17, Y_{kij} denotes the projection of the system operating point O_k on the q_i-q_j plane.

Multiple-fan systems

In this section, a general multiple-fan system is used to discuss the subsystem characteristic curves. The system considered is restricted to the ventilation system where the fan branches and the return dummy branch form a cutset of the ventilation network. Some important properties of such a system were discussed by Wang (1984).

Let B, N and F be the numbers of branches, nodes and fans in the multiple-fan ventilation network. Then the number of fundamental meshes M is given by

$$M = B - N + 1 \quad (44)$$

Choose a spanning tree such that the return branch is a tree branch and the fan branches are chords. Next label the branches according to the following sequence:

- the fan branches (chords) are designated 1, . . ., F;
- if M > F, the other chords are designated F+1, . . ., M;
- the return branch is labeled B; and
- the remaining branches are labeled M+1, ..., B-1.

We then form the fundamental meshes and label each mesh with the branch number of the chord contained therein.

This gives the following set of equations corresponding to Eqs. 32 to 34:

$$f_i(q_1, \ldots, q_M) - t_i(q_i) = 0, \quad i = 1, \ldots, F \quad (45)$$

$$f_i(q_1, \ldots, q_M) = 0, \quad i = F+1, \ldots, M \quad (46)$$

where t_i is the pressure of fan t_i, and

$$f_i = \sum_{j=1}^{B} b_{ij} r_j \left| \sum_{k=1}^{M} b_{kj} q_k \right| \sum_{k=1}^{M} b_{kj} q_k, \quad i = 1, \ldots, M \quad (47)$$

In Eq. 47, b_{ij} is an element of the fundamental mesh matrix; its value equals 1, -1 or 0 (Wang, 1982). This is a system of M equations in M unknowns $q_1, \ldots, q_M$. The solution point of this system of equations is called an operating point of the ventilation system. Let N_o be the number of system operating points, and denote the system operating points by O_i ($i = 1, \ldots, N_o$).

For the solution curve L_k of subsystem k ($k = 1, \ldots, F$), we have

$$f_i(q_1, \ldots, q_M) - t_i(q_i) = 0, \quad i = 1, \ldots, F; i \neq k \quad (48)$$

$$f_i(q_1, \ldots, q_M) - P_1 = 0, i = k \quad (49)$$

$$f_i(q_1, \ldots, q_M) = 0, \quad i = F+1, \ldots, M \quad (50)$$

For another type of solution curve K_k of subsystem k ($k = 1, \ldots, F$), we have

$$f_i(q_1, \ldots, q_M) - t_i(q_i) = 0, \quad i = 1, \ldots, F; i \neq k \quad (51)$$

$$f_i(q_1, \ldots, q_M) - [t_i(q_i) + S_i] = 0, \quad i = k \quad (52)$$

$$f_i(q_1, \ldots, q_M) = 0, \quad i = F+1, \ldots, M \quad (53)$$

As were discussed in the last section, the characteristic curves for the subsystems k ($k = 1, \ldots, F$) include the following:

- P_k, the projection of the solution curve L_k on the q_k-P_k plane;
- S_k, the projection of the solution curve K_k on the q_k-S_k plane;
- U_{kij}, the projection of solution curve L_k (or solution curve K_k) on the q_i-q_j plane ($i,j = 1, \ldots, M$; $i \neq j$) and
- t_k, the fan characteristic curve for subsystem k.

The j-th intersection of curves P_k and t_k is called the operating point of fan t_k and is denoted by X_{kj}. The projection of the system operating point O_k on the q_i-q_j plane is referred to as the operating point Y_{kij}. On the given q_i-q_j plane, the subsystem characteristic curves U_{kij} ($k = 1, \ldots, F$) all intersect at the operating point Y_{nij} ($n = 1, \ldots, N_o$).

The operating point of the system is also represented by $S_k = 0$ on the curve K_k of the subsystem k. Therefore, curves K_k ($k = 1, \ldots, F$) all intersect at a system operating point in the coordinate system having a common pressure axis for all S_k. In addition, the projection of curve K_k on the q_i-q_j plane is the same as the curve U_{kij}.

Concluding remarks

The mine characteristic curve, which applies to the single-fan system, has been critically discussed. The solution point of the network equations in terms of air quantities has been considered as the system operating point. This concept is rarely expressed in mine ventilation literature.

Two types of solution curves L_k and K_k for subsystem k have been defined using the network equations of the general multiple-fan system and illustrated by examples of two-fan systems. One of the examples has three system operating points, a total of six fan operating points. The subsystem characteristic curves discussed include P_k, U_{kij} and S_k. The subsystem pressure P_k at $q_k = 0$ is defined as the critical pressure C_k for the subsystem k. This pressure for the single-fan system is zero.

For a single-fan system (i.e., $F = 1$), Eqs. 45 and 46 become

$$f_1(q_1, \ldots, q_M) - t_1(q_1) = 0 \quad (54)$$

$$f_i(q_1, \ldots, q_M) = 0, \quad i = 2, \ldots, M \quad (55)$$

Clearly, the single-fan system represented by the above equations is a special case of the multiple-fan system. On the other hand, the multiple-fan system represented by Eqs. 45 and 46 is a generalization of the single-fan system. However, instead of the single mine characteristic curve P_1 in the q_1-P_1 plane, there are F subsystem characteristic curves P_k in the corresponding q_k-P_k planes ($k = 1, \ldots, F$).

A point (q_k*, P_k*) on the subsystem curve P_k represents the pressure gain P_k* which must be provided in branch k to maintain air quantity q_k* (and corresponding quantities in other branches), while the other fans remain operational with the same fan characteristic curves which are used in the construction of curve P_k. To determine the corresponding operating point of the fan t_d, we first read the air quantity q_d* from the subsystem characteristic curve U_{kkd} at q_k*. Then the fan operating point (q_d*, t_d*) is read from the fan characteristic curve $t_d(q_d)$ at $q_d = q_d^*$.

The subsystem characteristic curves for the multiple-fan system have been presented with a generalized system of notations. Much more work, however, needs to be done, especially the development of an efficient technique for tracing the subsystem characteristic curves for large ventilation systems. It is hoped that further research will lead to the application of computer graphics in displaying the characteristic curves in mine ventilation planning.♦

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