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Subsidence Prediction for High Extraction Mining Using Complementary Influence Functions

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ABSTRACT

Surface subsidence caused by high-extraction underground mining is described through complementary influence functions. The concept of complementary functions was introduced recently by Sutherland and Munson. This concept differs from other proposed concepts in that the surface displacement is the result of the combined contributions of mined and unmined zones. This approach eliminates computational difficulties experienced with the conventional influence functions in determining the deflections above the rib side and in the application of influence functions to complex room-and-pillar configurations. The analysis framework is reported here in two forms. The first is for the complete complementary influence function formulation and the second is the degenerate case of the complementary influence functions applied to a longwall geometry. The former is solved analytically; the latter graphically. Both are illustrated with analyses of actual case histories.

CONTENTS

INTRODUCTION

In the first part of this century, mining engineers realized that they needed the capability to predict ground displacements and strains caused by subsurface mine workings. This need to predict surface movements, commonly termed subsidence, is especially critical in Europe because of the extensive surface utilization over economic coal and ore bodies. A similar need is now developing in the United States.

Early considerations of subsidence before the advent of computer analysis led to empirical functions that describe mathematically the observed surface displacements. Two classes of empirical functions are commonly used: profile functions and influence functions[1,2]. Profile functions are direct fits to empirical data and are typically used for subsidence predictions over longwall panels[3]. These functions cannot describe complex mining geometries such as found in room-and-pillar mining. Influence functions are more applicable to these complex mining areas and have been previously developed from the viewpoint of the excavated material. Each unit element of the ground surface above the mined volume is assigned the same response, and integration of the elemental influence over all the elements in the mined area yields the subsidence prediction. These influence functions are widely used, with considerable success. There is, however, a major problem with this method because it significantly overpredicts the subsidence directly over the rib side. Typically the problem is handled by integrating the elements to an imaginary rib location within the mined area rather than to the actual rib location. Solution of the rib side subsidence problem is crucial in developing the capability to predict subsidence over room-and-pillar, as well as longwall, mines.

In recent analyses, numerical computer methods have been used to predict subsidence. Instead of analyzing the mined material, these techniques analyze the behavior of the coal (ore) and overburden layers remaining after mining. Calculations of considerable detail are possible and have shown how elastic bending, breaking and bulking, and void volume are transmitted through the overlying strata to cause surface subsidence[4,5]. Normally, such large scale analyses are beyond the means of mine operators, and a simpler analysis method is required, especially for room-and-pillar configurations.

The motivating concepts for influence functions have been examined recently by Sutherland and Munson[6]. Their analysis illustrated that the material remaining in the seam is as influential on the surface subsidence as that of the void left by the excavation. This realization leads to an influence function concept that is based on two elements: the mined element and an unmined element. In this concept, called complementary influence functions, each of the two classes of elements is assigned a surface response function. The surface displacement is then determined by integrating (appropriately summing) the response for each unit element over all of the elements in the area of interest. This approach has the important property that it eliminates the computational difficulties associated with other formulations and that it can be applied to longwall and room-and-pillar mine plans with equal ease.

In this work, we review the development of the complementary influence functions and then pose them in two forms: the first is the general formulation and the second is the degenerate case of a profile function for longwall geometries. For the former the solution will be analytical, and for the latter it will be graphical. Both forms will be illustrated with analyses of case histories.

COMPLEMENTARY INFLUENCE FUNCTIONS

Response of the Unmined Element

The response of the unmined element is based on the elastic response of the strata overlying the mined volume. This response can take one of several forms (e.g., see Ref. 7, 8, and 9). Here, we will assume the strata respond as if they were an elastic plate supported by the element[7]. This response is given by the function

$$s_s/m = P_s \begin{cases} o & , o \le r \le b , \\ \{I\}\{r_s^2[1 - 2\ln(r_s)]\} & , b \le r \le a + b , \\ 1 & , a + b \le r , \end{cases} \tag{1}$$

where

$$r_s = r/a - r/b$$

s_s is the vertical subsidence (positive down) around the unmined element, m is the mined height, P_s is the proportion of the maximum subsidence attributed to the unmined element, b is the half-width of the element, and a is the radial extent of influence outside the element (see Figure 1). The function {I} that expresses the effects of a varying moment of inertia in the overlying strata. As this solution suggests, the unmined elements hold the surface directly over them at the original ground position, but allow the surface around them to move down according to the elastic solution.

This elastic solution must be modified to account for the variation in the thickness of the elastic beam plate caused by roof failure. In [5], the investigation of the failure zone above a longwall panel illustrated the

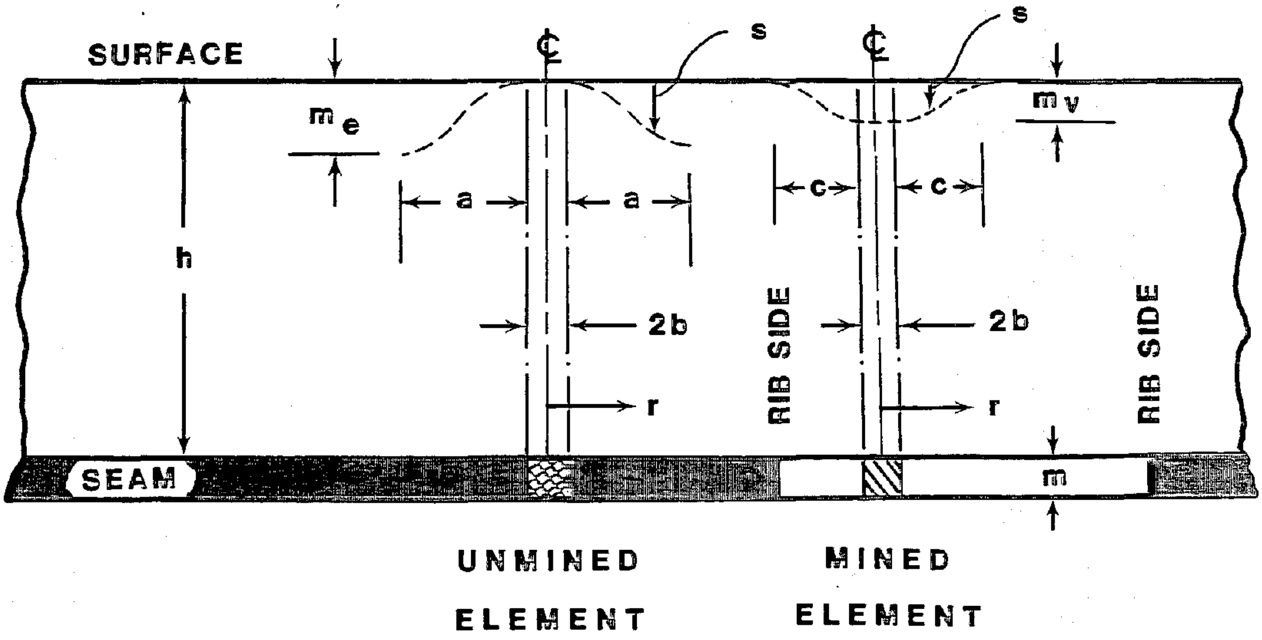

Figure 1. Complementary Elements

occurrance of the thinning of the beam. This thinning can be accommodated in Eq. 1 by using the moment of inertia function {I}. If $t(r_s)$ is the thickness of the plate as a function of the radius r_s and t_o is the center thickness of the plate, then for $b < r < a + b$

$$\{I\} = \left[\frac{t_o}{t(r_s)}\right]^3 \quad . \tag{2}$$

The functional form chosen for $t(r_s)$ is given by

$$t(r_s) = t_o \left[1 + p\ (1 - r_s)^3\right] \quad , \tag{3}$$

where p is a measure of the change in beam thickness.

This elastic solution must also be modified to incorporate the effect of crushing of the unmined element. This modification will be discussed in a later section.

Response of the Mined Element

As a portion of the overlying strata progressively breaks and falls into the mined cavity, a vertically nonuniform distribution of voids is left throughout the caved overburden. Description of the breaking process and of this distribution of void in the panel center has led to the prediction of maximum subsidence[4]. Near the rib side, both the horizontal and the vertical distribution of void is nonuniform. Here the horizontal distribution of void remaining in the overburden is simply related to the probability of lateral migration of a void as it moves to the surface.* By assuming a Gaussian probability distribution for this lateral migration process, the integrated effect at the surface has the form of an error function,

$$s_v/m = P_v \begin{cases} 1 & , \ o \le r \le b \ , \\ \mathrm{erfc}(2r_v) = 1 - \frac{2}{\sqrt{\pi}}\int_o^{2r_v} \exp(-\xi^2)\,d\xi & , \ b \le r \le c + b \ , \\ o & , \ r \le c + b \ , \end{cases} \tag{4}$$

where

$$r_v = r/c - b/c \quad ,$$

s_v is the vertical subsidence due to the mined element, P_v is the proportion of the maximum subsidence due to the mined element, c is the radial extent of influence outside the element, and ξ is an integration parameter.

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*This concept should not be confused with that proposed by Litwiniszyn[10] which considers the propagation of the mined volume to the surface rather than the variation in the residual void.

Thus, the mined element response is based on the horizontal distribution of part of the excavated volume (the bulking) in the overlying strata. This functional form appears similar to other empirical influence function subsidence analyses[1], but is quite different in its action, as will be illustrated.

Application of Complementary Influence Functions to Mines

The application of the complimentary influence functions to a complicated mine plan can be handled best by a computer. The mine plan is constructed with square mined and unmined elements, each with dimensions of 2b. The summation (integration) of the individual responses for each element yields the subsidence prediction for the mine.

Matrix Organization: The two-dimensional array of mined and unmined elements can be considered to be a matrix with each element identified by its matrix coordinate (i,j). The distance between any two elements r_{ijkl} is given by

$$r_{ijkl} = b \left(\sqrt{(i - k)^2 + (j - l)^2} - 0.5 \right) , \qquad (5)$$

where (i,j) are the coordinates of the first element and (k,l) are the coordinates of the second element.

Because the matrix required to analyze a particular mine plan must be somewhat larger than the mine excavations, it will have to be n by m where

$$n = \text{INT} \ [(L + 2r_{max})/b] \quad , \qquad (6)$$

and

$$m = \text{INT} \ [(W + 2r_{max})/b] \quad , \qquad (7)$$

Here L is the length of the area of interest, W is the width of the area of interest, r_{max} is the greater of r_v and r_s, and INT is the greatest interger function.

Normalization: The subsidence contributions of each element are normalized to a value of one. This normalization permits the total subsidence to be determined by

$$s_{ij} = P_v V_{ij} + P_s D_{ij} \qquad (8)$$

where s_{ij} is the normalized subsidence (s/m) at the position (i,j), V_{ij} is the normalized contribution from the mined regions below position (i,j), and D_{ij} is the normalized contribution from the unmined regions below position (i,j).

The Mined Element: The subsidence at surface point (i,j) caused by the mined elements is given by

$$V_{ij} = \frac{1}{n_v} \sum_{k=1}^{n} \sum_{l=1}^{m} [s_v (r_{ijkl})/m] \qquad (9)$$

where V_{ij} is the normalized displacement (positive down). The parameter s_v is given by Eq. (4) if element (k,l) is a mined element and it is zero if element (k,l) is an unmined element. The term n_v is a normalization factor that can be calculated by considering a supercritical longwall panel.* If element (i,j) is the centerline element of the panel, with b and r_v the same as the mine under consideration, then n_v is given by

$$n_v = \sum_{k=1}^{n} \sum_{l=1}^{m} [s_v (r_{ijkl})/m] \quad . \qquad (10)$$

As a practical consideration, only those elements inside a radius of r_v from element (i,j) need be included in the summation in Eq. 8 because the contributions from all elements outside this radius are zero.

The Unmined Element: The subsidence contribution at surface point (i,j) from the unmined elements is treated in a manner different than that of the mined elements. A norm for the subsidence contribution is established and then the variation about the norm is used to account for the number of unmined elements with the range of influence of the position (i,j). The subsidence norm for surface point (i,j) is determined by its nearest unmined neighbor,

$$\overline{D}_{ij} = \begin{cases} 0 & , \text{(i,j) is unmined ,} \\ s_s(y_{min})/m & , \text{(i,j) is mined ,} \end{cases} \qquad (11)$$

where $\overline{D}_{ij}$ is the subsidence norm (positive down), s_s is given by Eqs. 1-3, and y_{min} is the normalized, minimum distance between element (i,j) and an unmined element.

As seen in Eq. 11, $\overline{D}_{ij}$ is only a function of the parameter y_{min}, i.e., the position of its nearest unmined neighbor. Several techniques may be used to determine the effects of varying the number of unmined elements within the

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*For our purposes here, a super-critical panel can be defined to be a panel whose half-width is equal to or greater than the range of influence of either of the two unit elements, i.e., a in Eq. 1 and c in Eq. 4.

a radius of r_s of (i,j). The technique that we have chosen here is to allow the norm to vary by some weighting function. The weighting function chosen here is based on an inverse square relationship normalized to a reference supercritical longwall panel. If we let the weighting parameter r_{ij} be defined for the element (i,j) by

$$\bar{r}_{ij} = \sum_{k=1}^{n} \sum_{l=1}^{m} \begin{cases} [1 - r^2_{ijkl}] & , \text{ if } r_{ijkl} \leq 1 \ , \\ o & , \text{ if } r_{ijkl} > 1 \ . \end{cases} \tag{12}$$

Then the weighting function W_{ij} can be defined as

$$W_{ij} = (1 - y^2_{min}) \left[\frac{\bar{r}_{ij} - \bar{r}_{kl}}{\bar{r}_{ij} + \bar{r}_{kl}} \right] \tag{13}$$

where (i,j) is the element under consideration, (k,l) is the element in the supercritical longwall panel. The reference panel is constructed by using b and r_s from the problem under consideration and placing the element (k,l) a distance of y_{min} away from the side of the panel and a minimum distance of r_s away from the head of the panel.

The weighting function can be either positive or negative. If r_{ij} is greater than r_{kl} [the case when the element (i,j) has more reinforcement from unmined elements than in the equivalent panel], the subsidence contribution is reduced. When r_{ij} is less than r_{kl} [the reinforcement is less], the subsidence contribution is increased. The weighting function is identically zero away from the corners of a supercritical longwall panel and/or when y_{min} is greater than one.

In this form, the total contribution to the subsidence at element (i,j) is given by

$$D_{ij} = \bar{D}_{ij} \ (1 + W_{ij}) \tag{14}$$

Note that D_{ij} is zero if (i,j) is an unmined element.

The Yield Pillar

A pillar that yields under its overburden load can be incorporated directly into this complementary influence function formulation. If the yield pillar is assumed to be a shortened, non-yielding pillar, then the deflection above the pillar is assumed to be H_p. To determine the deflection about this pillar, a simple adaptation of Eq. 1 can be used if only yielding pillars or mined elements are within the radius of influence of the pillar. Namely,

$$D_{ij} = 1 - H_p\ (\bar{D}_{ij}) \quad , \tag{15}$$

where $\bar{D}_{ij}$ is subsidence norm defined by Eq. 11.

In the case where a non-yielding unmined element is within the radius of influence of the yield pillar (i.e., near a panel edge), the deflection between the two elements cannot be computed by Eq. 15, because the extent of the pillar yielding may have been modified by part of the overburden load being carried in the elastic beam above the pillar. Here, we have chosen to characterize this effect by using a variation about the subsidence norm. The variation about the norm is taken to be

$$D_{ij} = 1 - \frac{(1 - \bar{D}_{ij})}{N_{ij}} \tag{16}$$

and the weighting function N_{ij} is based on the nearest unmined neighbor by

$$N_{ij} = 1 + H_p\ (1 - y_{min})^3 \quad . \tag{17}$$

REDUCTION TO PROFILE FUNCTIONS

For longwall mining operations, the simple geometry and the large physical dimensions of the panels implies that the integration of the influence functions is essentially one dimensional. Thus, the influence functions reduce to profile functions. This observation permits the complementary influence functions to be stated as simple complementary profile functions. By nondimensionalizing the geometry of a longwall panel, profile functions are generated for both types of elements. The predicted profile is then obtained by adding the profiles of the mined and the unmined regions together in an appropriate manner.

Graphical Representation

By assuming a super-critical longwall panel, the contributions from the mined and the unmined areas can be integrated separately in non-dimensional form (the details of this procedure were given earlier). For the unmined elements this integration produces a curve that is identical to the assumed strata deformation (Eqs. 1-3). In non-dimensional form, this function depends only on the parameter p (Eq. 3) which describes the change in the thickness of the beam. Carrying out this integration process yields the profiles shown in Figures 2 and 3 for various values of p.

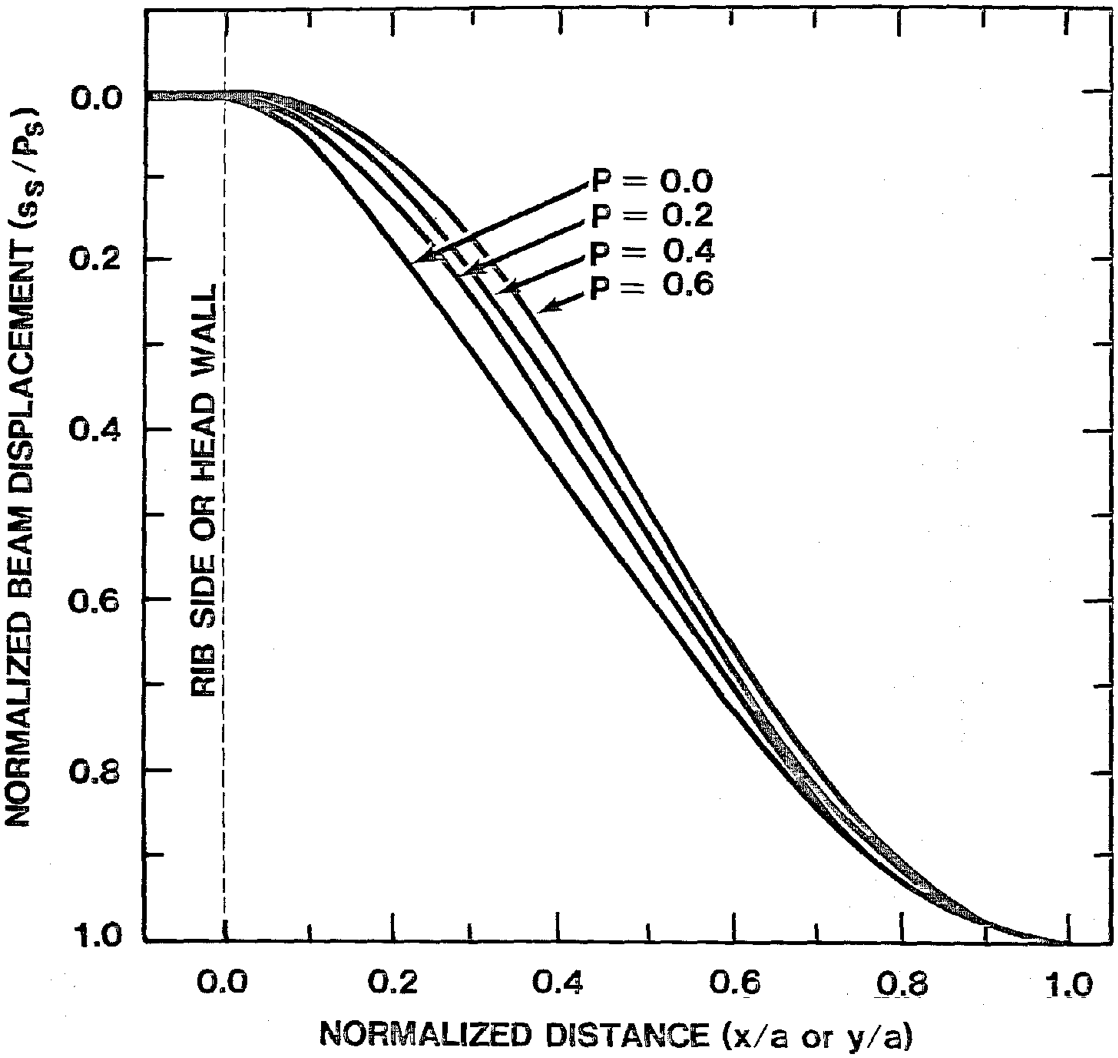

Figure 2. Non-Dimensionalized Longitudinal and Transverse Profile Functions for the Unmined Elements as a Function of the Parameter p for a Longwall Panel

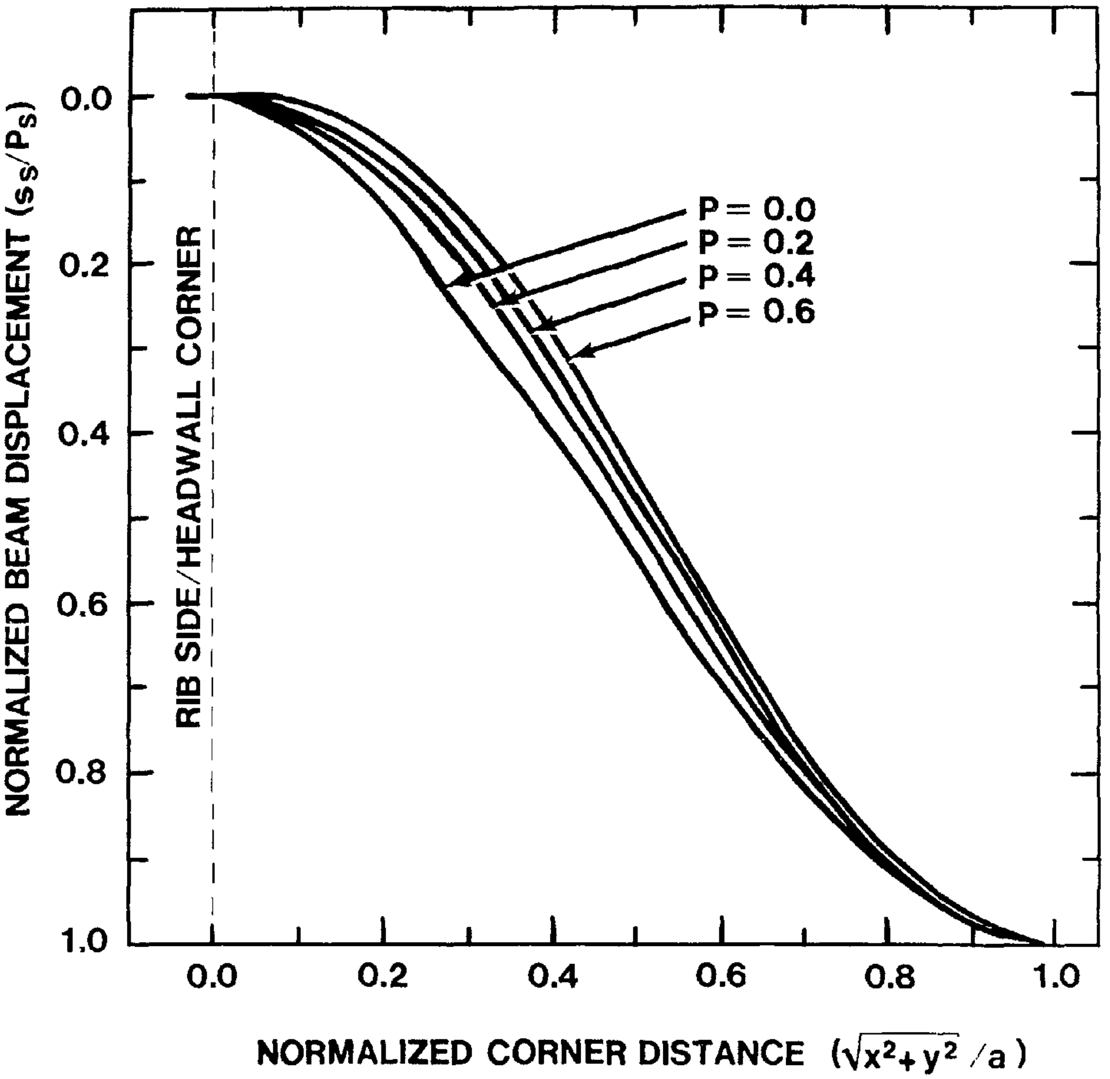

Figure 3. Non-Dimensionalized Corner Profile Functions for the Unmined Elements as a Function of the Parameter p for a Longwall Panel

The profile of the mined element is the weighted sum of the assumed form for the distribution of the void upward (Eq. 4). The resulting profiles are shown in Figure 4.

In Figures 2, 3, and 4, the profiles are shown for longitudinal and transverse profiles (the parameter x measures the distance from the rib side and y the distance from the head wall). Because the predicted profiles are symmetric and only describe prompt subsidence, they describe the profiles along the longitudinal centerline of the panel; the transverse profile across the panel (away from the corners); and the traveling profile away from the panel headwall. Development profiles can be determined by knowing the position of the face with respect to the surface point as a function of time. Also shown in Figures 3 and 4 are the corner profiles. This is the profile along the 45^{o} diagonal through the corners of the longwall panel (i.e., x equals y).

The data shown in Figures 2, 3, and 4 are summarized in tabular form in Tables I, II, and III.

Physical Significance of the Model Parameters

In this form, the parameter set that describes a particular longwall panel are a, P_s, c, and P_v. These parameters have the following interpretations: c is equivalent to the angle of draw; P_v divided by two is the deflection directly above the rib side; P_s plus P_v is the maximum subsidence; and a is a measure of the stiffness of the overlying strata.

Application to a Longwall Panel

The graphical solutions can be applied directly to a supercritical longwall panel in the same manner as any profile function. The steps are:

1. Using the appropriate values of a, P_s, and p replot the profile from Figure 2 or 3 (Table I or II) in dimensional form.
2. Using the appropriate values of c and P_v replot the profile from Figure 4 (Table III) in dimensional form.
3. Align the graphs using the rib side (corner) as the reference location and add the two profiles.

For non-critical longwall panels, the exact solution can be obtained only by integrating the influence functions. However, an acceptable approximation to the correct subsidence prediction can be obtained from the above profile functions by comparing two different profile solutions. The upper bound for

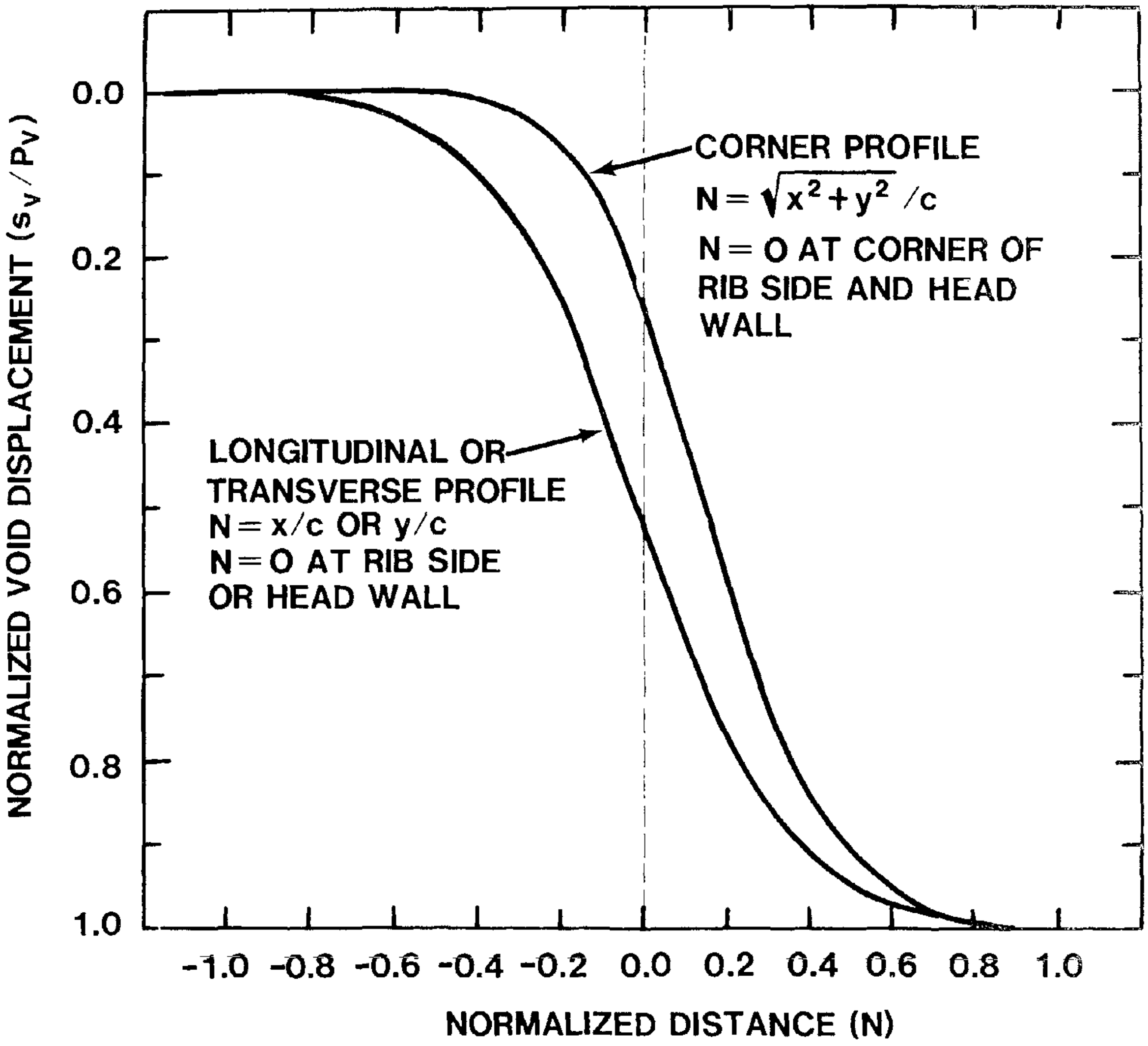

Figure 4. Non-Dimensional Profile Functions for the Mined Elements of a Longwall Panel

Table I

Non-Dimensional Longitudinal and Transverse Profile Functions for the Unmined Elements of a Longwall Panel.

s_S in Percent

Position x/a or y/a				p			
	0.0	0.1	0.2	0.3	0.4	0.5	0.6
0.00	00	00	00	00	00	00	00
0.05	02	01	01	01	01	01	01
0.15	11	09	08	06	06	05	04
0.25	24	21	18	16	15	13	12
0.35	38	35	32	30	28	26	24
0.45	53	50	48	45	43	41	40
0.55	66	65	63	61	60	58	57
0.65	79	78	77	76	75	74	73
0.75	89	88	88	87	87	87	86
0.85	96	96	96	95	95	95	95
0.95	100	100	100	99	99	99	99
1.05	100	100	100	100	100	100	100

TABLE II

Non-Dimension Corner Profile Functions for the Unmined Elements of a Longwall Panel

s_s in Percent

Position	p						
$\sqrt{(x^2+y^2)}/a$	0.0	0.1	0.2	0.3	0.4	0.5	0.6
0.00	00	00	00	00	00	00	00
0.05	01	01	01	01	01	00	00
0.15	09	07	06	05	05	04	03
0.25	20	18	16	14	12	11	10
0.35	33	31	28	26	24	23	21
0.45	48	45	43	41	39	37	36
0.55	62	60	59	57	56	54	53
0.65	75	74	74	73	72	71	70
0.75	87	86	86	86	85	85	84
0.85	95	95	95	95	95	95	94
0.95	99	99	99	99	99	99	99
1.05	100	100	100	100	100	100	100

TABLE III

Non-Dimension Profile Functions for the Mined Elements of a Longwall Panel

s_v in Percent

Position x/c, y/c, or $\sqrt{(x^2+y^2)}/c$	s_v Longitudinal or Transverse	Corner
-1.05	00	00
-0.95	00	00
-0.85	00	00
-0.75	01	00
-0.65	02	00
-0.55	04	00
-0.45	07	00
-0.35	12	02
-0.25	19	04
-0.15	30	09
-0.05	43	18
0.05	57	33
0.15	70	50
0.25	81	65
0.35	88	78
0.45	93	87
0.55	96	93
0.65	98	97
0.75	99	99
0.85	100	100
0.95	100	100
1.05	100	100

the exact solution can be obtained by adding the profiles from both "sides" to one another. The lower bound for the solution is obtained by truncating the side profiles at the center of the overlap. The exact solution lies between these two solutions.

CASE HISTORIES

Old Ben No. 24 - Longwall Panel

Old Ben Coal Company's Mine No. 24 is located approximately 2 miles northwest of Benton in Franklin County, Illinois. The site is approximately 80 miles southeast of St. Louis, Missouri, and 270 miles south-southeast of Chicago, Illinois. The site area lies in the Mt. Vernon Hill Province of southern Illinois and is characterized by gently rolling topography, dissected by streams. The ground surface above the panels is flat and is approximately 117 m above mean sea level.

Principal strata in the mine area are the Bond, Mattoon, Modesto, and Carbondale formations of Pennsylvanian age. Mine No. 24 is producing coal from the Herrin (No. 6) Seam of the Carbondale Formation. This seam occurs at a depth of approximately 189 m in the area of the longwall. The excavation height is nominally 2.1 m [11,12].

The complementary influence functions (profile functions) have been fit to this profile previously by the authors[6] and are repeated here for completeness. The parameter sets used for the model description are 0.48, 0.004, 0.5, 0.48, and 0.08 for a/h, b/h, P_s, c/h and P_v, respectively. Taking {I} equal to 1 yields the result shown in Figure 5. As seen in this plot, the change in the elastic beam thickness above the mined cavity has decreased the displacement above the mined cavity near the rib side and as a result, the fit with constant {I} is rather poor. Taking p equal to 0.3 (Eq. 3), yields a profile that is in much better agreement with the observed profile, see Figure 6.

The application of complementary functions to two-dimensional panel geometry of Old Ben No. 24 is shown in Figure 7. Superimposed on a quadrant of the longwall panel are the predicted subsidence contours for the cases of the influence of the unmined element alone, the mined element alone, and the total subsidence of the complementary functions. As with the subsidence profiles, the contours are in quantitative agreement with field measurements[10].

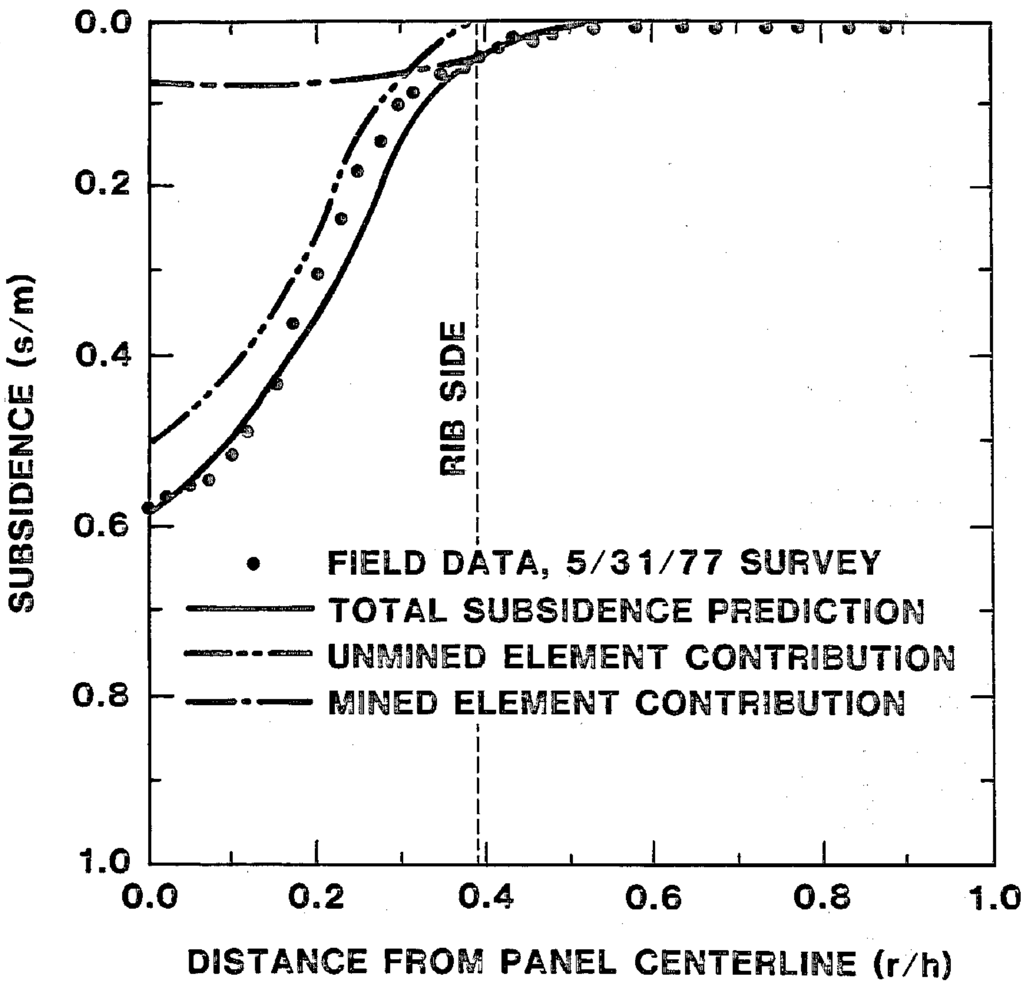

Figure 5. Subsidence Prediction for Old Ben No. 24 (Without Variation in Beam Thickness)

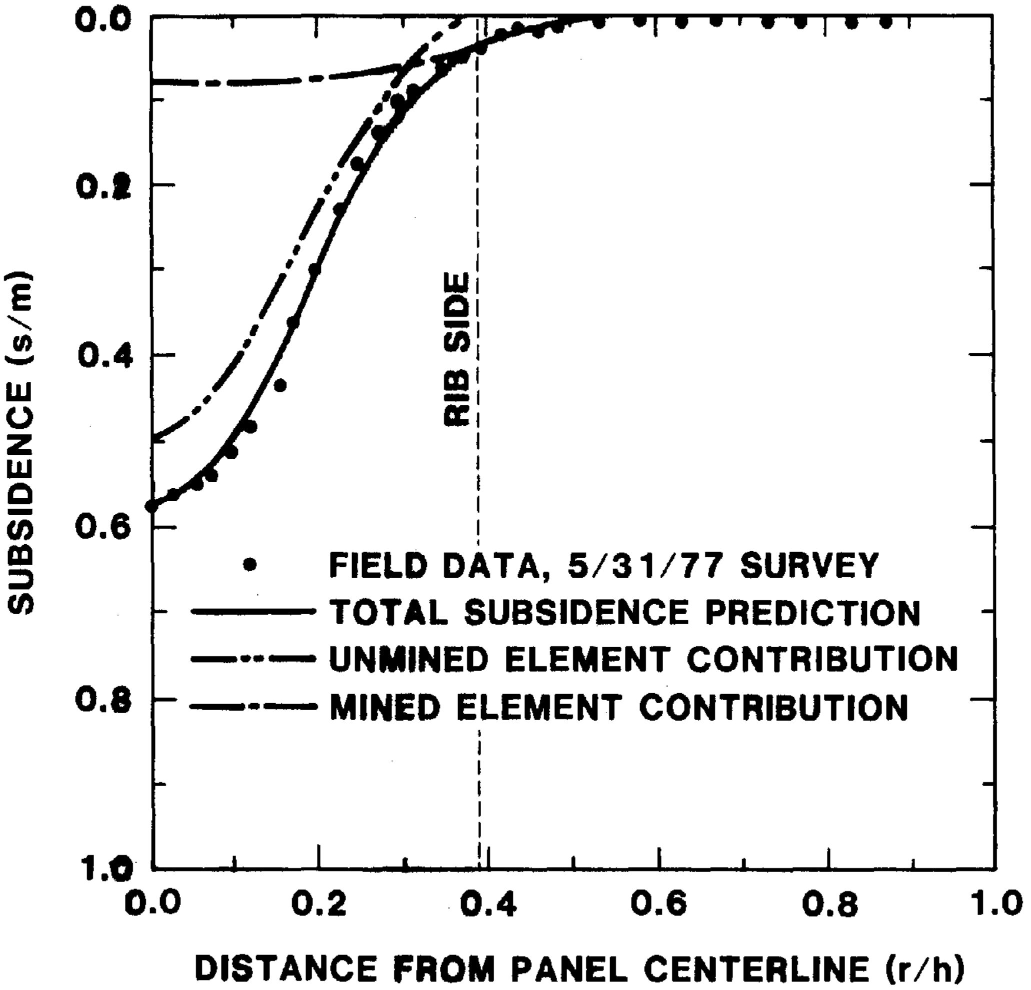

Figure 6. Subsidence Prediction for Old Ben No. 24 (With Variation in Beam Thickness

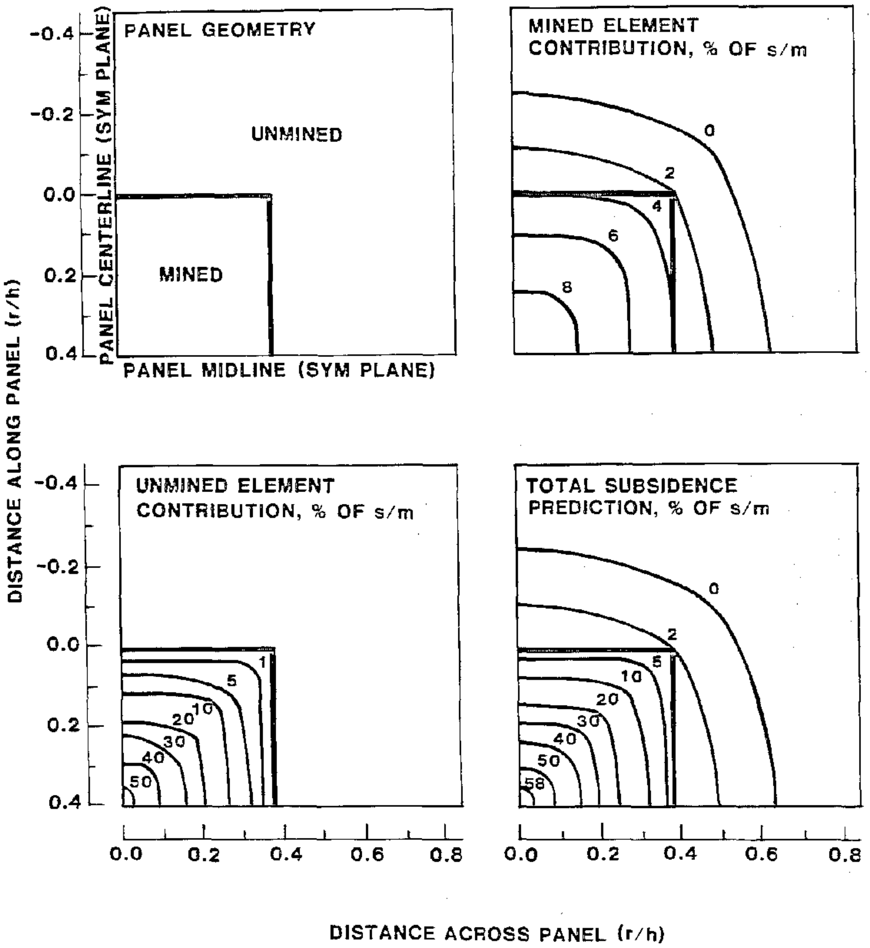

Figure 7. Subsidence Contours for Old Ben No. 24.

Shoemaker - Longwall Panel

The Shoemaker mine is located in northern West Virginia and is operated by the Consolidated Coal Company. The mine is located near Moundsville, Marshall County, West Virginia. The topography is rolling hill country.

Two longwall panels were monitored at this site. These data are exceptionally interesting because the geology produces profiles that vary greatly from one another and, to date, encompass the two extremes in subsidence observed in the U. S. Panel I is 122 m wide and its depth varies from 198 m to 213 m. Panel II, located approximately 1.6 km from Panel I, is 183 m wide and its depth varies from 122 m to 213 m. A nominal 1.7 m thickness of coal was extracted from both panels. The maximum subsidence recorded for Panel I was 0.9 m and for Panel II was 1.0 m. The stratigraphy of both sites is shown in Figure 8[13,5].

The transverse profiles for the two panels are shown in Figure 9. The two panels behave quite differently. For Panel I, the profile is spread comparatively wide, whereas for Panel II, the profile is very narrow. As discussed in [5], this major discrepancy cannot be attributed to the slightly different mine geometries of each panel. To explain this major variation between the two profiles, Sutherland and Schuler[5] did a detailed finite element analysis of the two panels. These analyses indicated that the caved zone above Panel I extended to the bottom of the massive shale layer (depth of approximately 130 m in Figure 8) while the caved zone above Panel II penetrated into the shale layer to the limestone partings (at a depth of approximately 65 m). Thus the subsidence profile in Panel I is dictated by an elastic beam of overlying strata that is approximately twice as thick as the beam in Panel II.

To model the two profiles using complementary influence functions (profile functions) the profile for Panel II was fitted to the parameter set first. The resulting parameter set is 0.5, 0.05, 0.45, 1.0, 0.21 and 0.5 for a/w, b/w, P_s, c/w, P_v, and p, respectively. The comparison of the measured and predicted profiles is shown in Figure 9.

Since the elastic beam is approximately twice as thick in Panel I as in Panel II, the stiffness of Panel I should be increased by a factor of eight. Changing P_s to 0.05 from 0.45 and increasing P_v to 1.16 to obtain the correct maximum subsidence yields the profile shown in Figure 9 (no other parameters were changed).

Sundust - Room-and-Pillar Panel

The Sundust mine is located in southwestern Pennsylvania, approximately 20

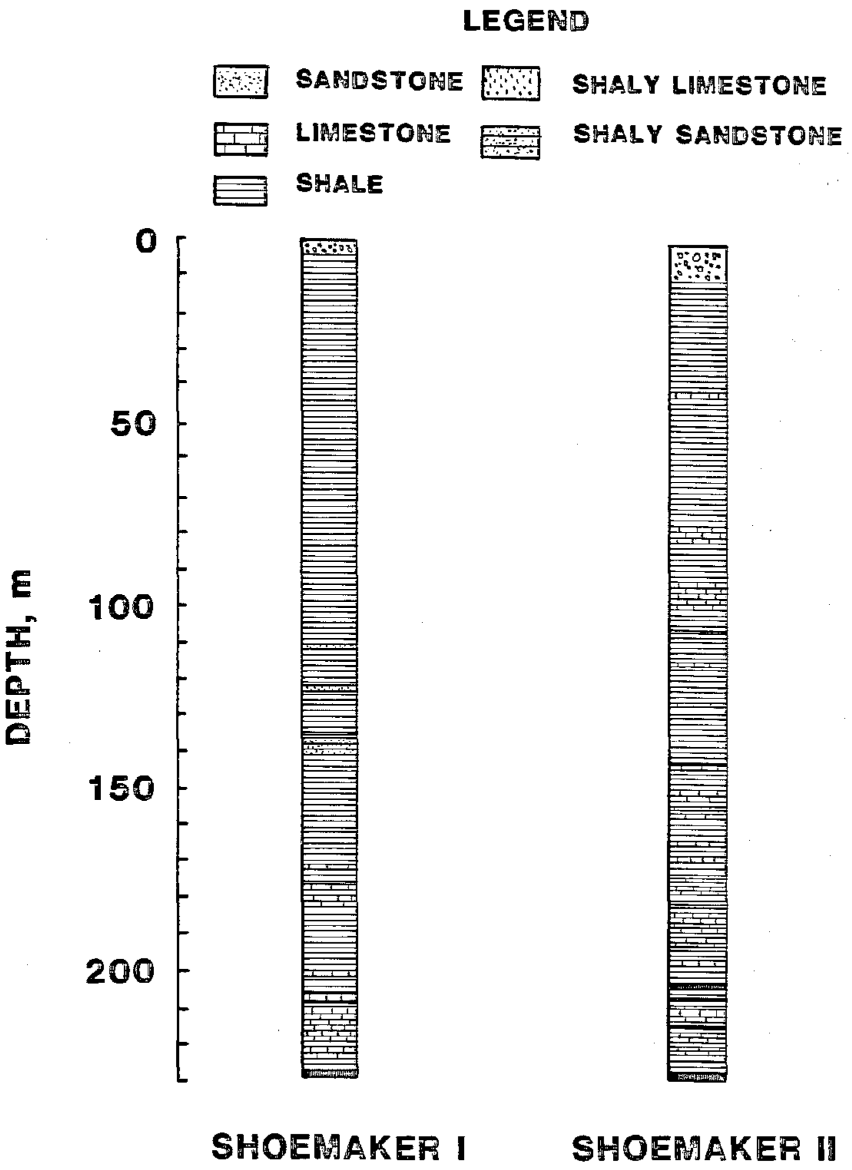

Figure 8. Stratigraphic Logs from the Shoemaker Mine

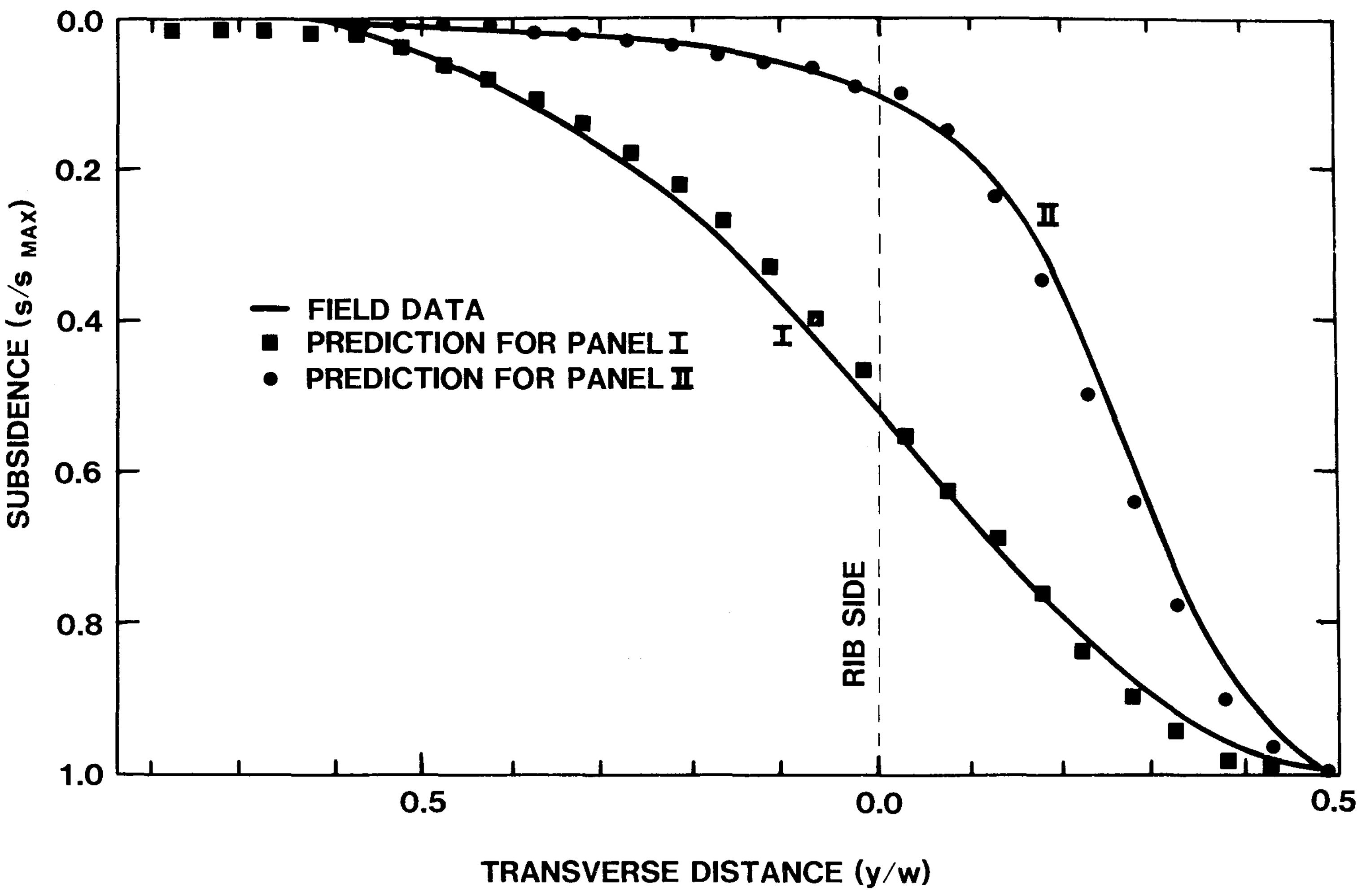

Figure 9. Subsidence Predictions for Panels I and II of the Shoemaker Mine

miles south-southwest of Pittsburgh and 30 miles north-northeast of Wheeling, West Virginia. The topographic relief was moderate to high (rolling pasture land) and typical of the hilly terrain of the Appalachian Plateau Province. The surface slope ranged from about 4 percent to 18 percent. The overburden ranged from 107 m to 122 m. Its geological characteristics were 80 percent shale, 11 percent sandstone, 4 percent limestone, and 4 percent soil. The mined height was nominally 2 m.

The mine plan for the room-and-pillar panel under consideration is shown in Figure 10. The panel is 131 m wide and 610 m long. Near the center of the panel, a block of twelve pillars was left intact to support a surface structure. The mine plan called for "complete" pillar extraction using a pocket-and-wing method. Actual extraction could not be determined but it is estimated to be 80 percent. Adjacent to this panel was another room-and-pillar panel (through point C in Figure 10). The extraction ratio was 50 percent in the area. The surface above the other side of the panel (point C' in Figure 10) is a marshy area. Surveys were conducted along lines A-A', B-B', and C-C'[13]. A large fissure was open near the beginning of the panel (see Figure 10) by the subsidence process[14].

The prediction of the response of this panel using complementary influence functions employed the yield pillar concept described above. The basic parameter set used in these calculations was 0.81, 0.0227, 0.81, 0.65, and 0.04 for a/w, b/w, P_s, c/w, and P_v, respectively. The change in beam pillar thickness, p, was taken as 0.3. As a detailed mine plan was not available for the panel under consideration or for the adjacent panel, pillars were distributed throughout the panel such that the extraction ratios averaged 80 percent and 50 percent, respectively. The height of the support pillar was 0.45 m in the main panel and 0.5 m in the adjacent panel. The twelve large pillars near the center of the panel were treated as non-yielding pillars.

As seen in Figures 11, 12, and 13, the predicted profiles are in good agreement with the measured values of the subsidence. The minor discrepancies in the predictions probably can be attributed to the uncertainty in the pillar locations and the assumed extraction ratios.

CONCLUSIONS

A new approach has been developed for the use of influence functions in the prediction of mine subsidence. In this approach, complementary influence functions are developed for the response of both mined and unmined elements. The surface displacement is then determined by integrating (appropriately summing) the response for each unit element over its area of influence. Both

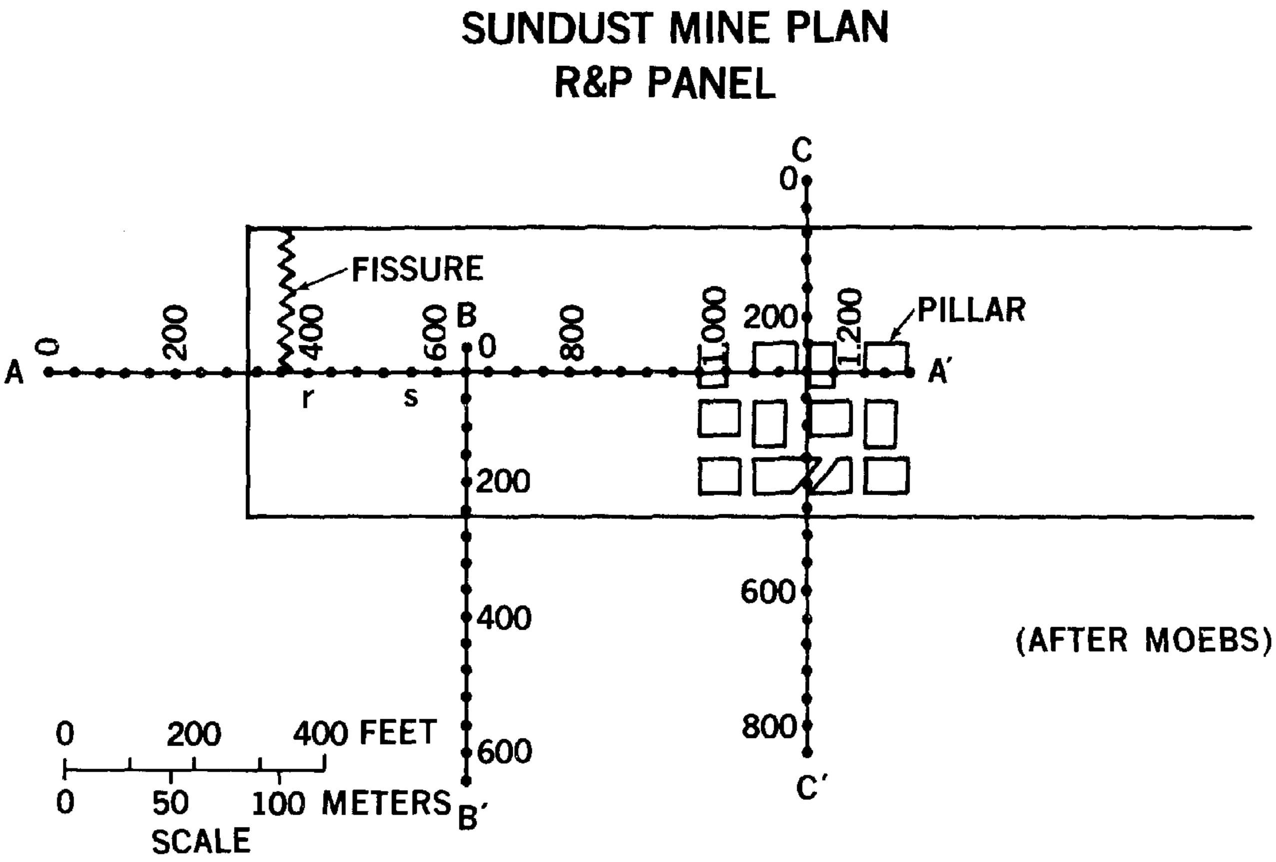

Figure 10. Sundust Mine Plan

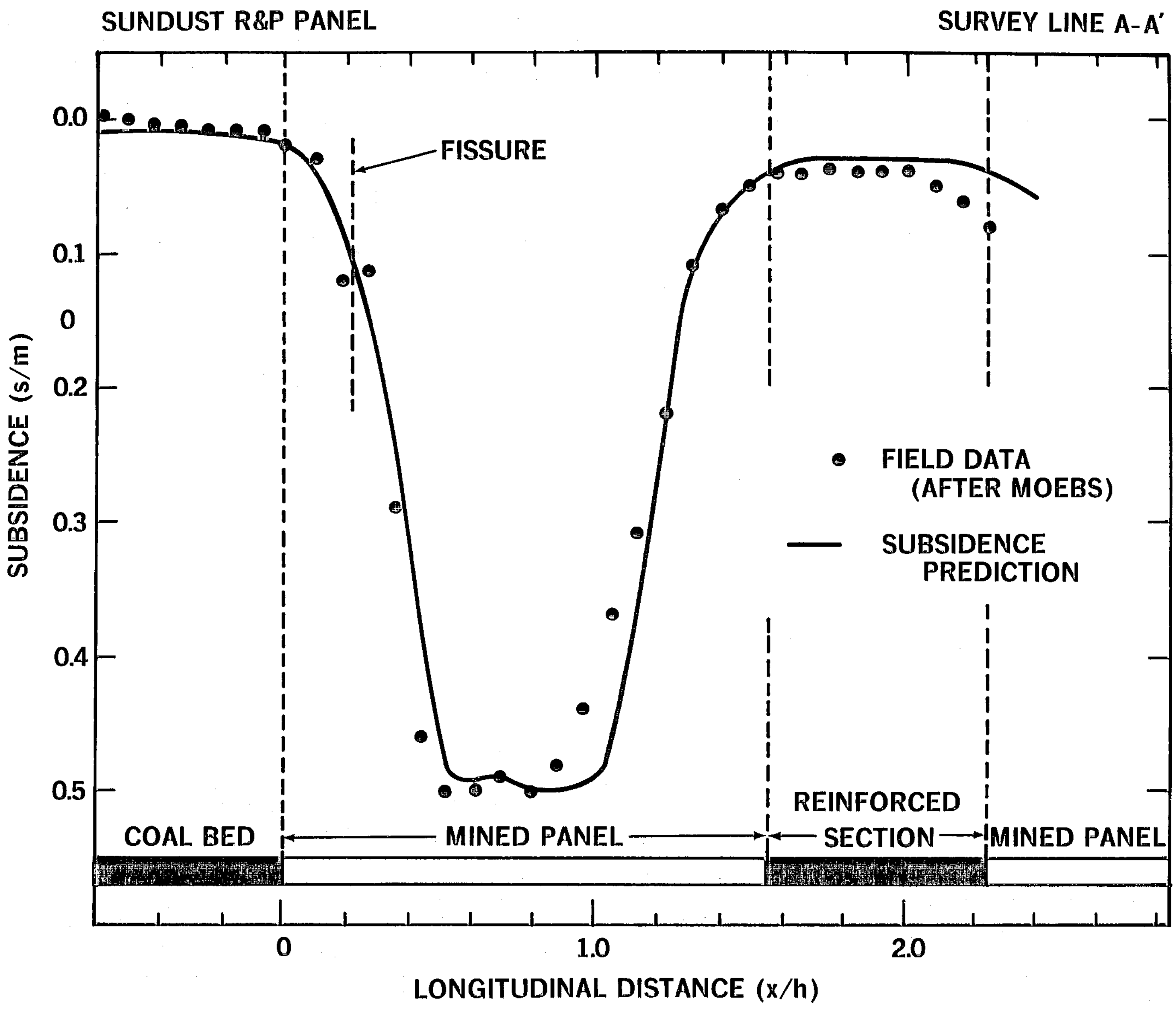

Figure 11. Longitudinal Subsidence Prediction for Line A-A' of the Sundust Room-and-Pillar Panel

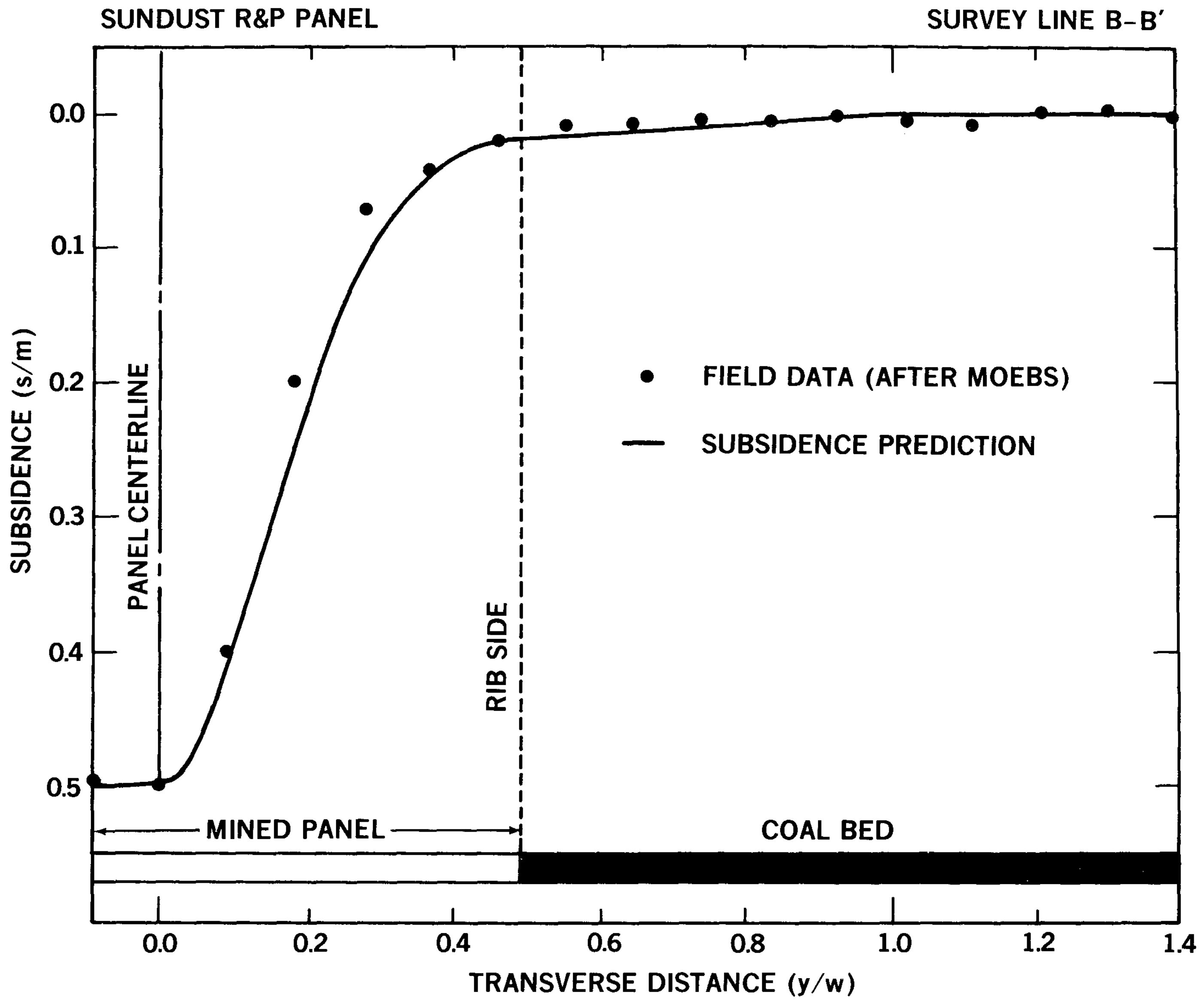

Figure 12. Transverse Subsidence Prediction for Line B-B' of the Sundust Room-and-Pillar Panel

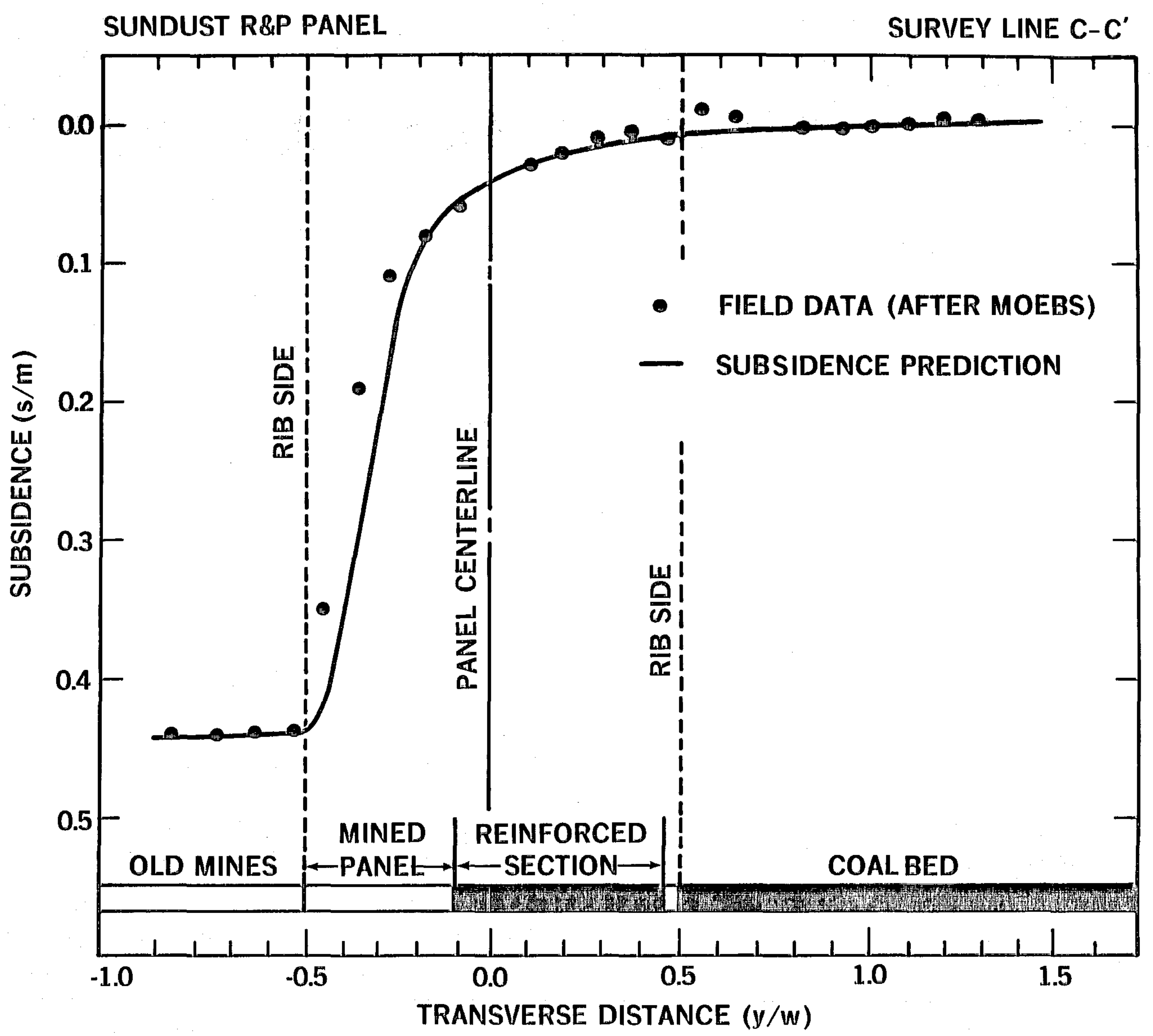

Figure 13. Transverse Subsidence Prediction for Line C-C' of the Sundust Room-and-Pillar Panel

elements contribute significantly to the subsidence prediction. Development of complementary influence functions represents a significant advancement in the subsidence analysis of complicated room-and-pillar mines using empirical techniques. Comparisons between field data and predictions for two longwall panels and a room-and-pillar panel illustrate the capabilities of this technique.

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