

Lei, S.-F. and Hamernik, R.P. (1996). Extraction of frequency domain kurtosis from nonGaussian and nonstationary signals using the wavelet transform. International Symposium on Optical Science, Engineering and Instrumentation: Wavelet Applications in signal and image processing. Proc. SPIE 2825:785-795.

Extraction of frequency domain kurtosis from nonGaussian and nonstationary signals using the wavelet transform

Sheau-Fang Lei and Roger P. Hamernik

Auditory Research Laboratory
State University of New York
Plattsburgh, NY 12901

ABSTRACT

Conventional metrics used to quantify signals in noise/hearing research are primarily derived from time-averaged energy and spectral analyses. Such metrics, while appropriate for Gaussian signals, are of limited value in more complex sound environments. Many of the sounds encountered in industrial/military environments have nonGaussian and nonstationary distributed waveforms. These signals may have the same energy and spectra as those of a continuous Gaussian signal, yet they can produce very different effects on the auditory system. This result has led to efforts to develop additional metrics, incorporating the temporal characteristics of a signal, that could be useful in evaluating hazardous acoustic environments. Previous research suggests that frequency domain kurtosis (FDK) may be useful in such an application. This paper shows that good estimates of FDK can be obtained from an application of the wavelet transform.

The wavelet transform, which has features in common with the cochlear micromechanical analysis of a signal, will reflect the temporal variations of the frequency components in a signal. A signal is decomposed by the wavelet transform on a logarithmic scale (e.g., octave bands), and then the fourth-order kurtosis estimates are computed across the different octave bands from the wavelet transform results. Complex signals whose effects on hearing are known, and which are similar to realistic industrial noises, are used as model signals from which the FDK metric is extracted using the wavelet transform. Animal model experiments have shown that FDK is highly correlated with both the frequency specificity of hearing loss and the severity of trauma. Use of the wavelet transform to obtain an FDK metric lends itself to incorporation into digital analysis systems that may be useful in the assessment of complex noises for hearing conservation purposes.

Keywords: higher-order statistics, wavelet transform, auditory signal processing, noise-induced hearing loss.

1. INTRODUCTION

Conventional metrics [e.g. sound pressure level (SPL), 1 & 1/3 octave band energies, etc.] used to evaluate the potential of a noise environment to produce a noise-induced hearing loss, are primarily derived from time-averaged energy and spectral analysis of the noise. Such metrics adequately quantify a continuous noise having a steady-state waveform (such as a continuous Gaussian noise) and can be used to estimate the hazard to hearing from prolonged exposure to such noises. However, there is evidence^{1, 2, 3, 4} showing that nonGaussian, nonstationary types of noises having the same energy and spectra as a Gaussian noise but different temporal structures, have different audiometric and histological effects on the auditory system. Such studies demonstrate that the conventional metrics, which in a sense are a holdover from the classical view of hearing, the place theory (a spectral representation), are not adequate for the assessment of hearing hazards from noise exposures having nonGaussian, nonstationary characteristics.

The neglect of the temporal features of an exposure, inherent in the use of conventional metrics, is surprising considering the nature of the initial signal processing stage in the cochlea^{5, 6, 7}. From the physiological viewpoint, an acoustic pressure disturbance is transformed into a two dimensional map of neural discharge as a function of time and spatial displacement along the cochlea. It is clear from the physiology that the ear functions not only as a spectrum analyzer, but also as a time-

domain analyzer⁶, that is, temporal information plays a role in the processing of the acoustic waveform, that some would consider more important than spectral information that is traditionally emphasized^{6, 8}. Given that this point of view is viable when considering mechanisms of cochlear signal processing of speech signals, it also seems reasonable to expect that temporal features are important in the over-stimulation responses of the cochlea. This argument has led to efforts to develop additional metrics that can be extracted from the exposure stimulus which incorporate temporal information inherent in the noise waveform. Such metrics should more accurately evaluate the hazard to hearing. In this paper, we suggest that a metric, the frequency domain kurtosis (FDK), obtained from the output of the wavelet transform applied to the analyzed signal, may be useful in such an application⁹.

The use of the wavelet transform (WT) in the analysis of an acoustic (noise) signal (traumatizing noise stimulus), was motivated by its operational similarity with the time-frequency processing of an acoustic signal by the human basilar membrane within the cochlea⁷. The fourth-order kurtosis estimate across frequency can be computed based on the results of the WT applied to the noise waveform.

In section 2 of this paper, the background for the WT and the rationale for its use is discussed. Section 3 presents a brief overview of the kurtosis of nonGaussian and nonstationary signals and presents the computational algorithm for FDK. Section 4 presents the FDK results obtained from complex signals which were used as representative examples of nonGaussian and nonstationary signals. Section 5 presents conclusions about the significance of the FDK metric obtained through the WT. The primary focus of this paper is on the extraction of FDK from a computational modelling perspective. The details relating to the hearing effects of nonGaussian noise environments are beyond the scope of this paper.

2. WAVELET TRANSFORM

2.1 Theoretical Review of WT

Over the past few years, the WT has become an essential tool for researchers and engineers dealing with signal processing problems^{10, 11}. The WT is accomplished mathematically by integration of a signal and a set of basis functions.

$$W(a, b) = \frac{1}{\sqrt{a}} \int x(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt \quad (1)$$

The bar indicates the complex conjugate; a represents a time dilation; b a time translation. This operation decomposes a time varying signal $x(t)$ in terms of a set of basis functions (or wavelets). These wavelets are obtained from a single function $\psi(t)$ (the mother wavelet) by dilations as well as translations. The mother wavelet must satisfy the admissibility condition given by:

$$\begin{aligned} \int_{-\infty}^{\infty} |\psi(t)|^2 dt &< \infty \\ \hat{\psi}(\omega)|_{\omega=0} &= 0 \end{aligned} \quad (2)$$

where $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$. This admissibility condition implies that the mother wavelet has to have a finite energy, and be an oscillatory function. To be useful in practical applications, the mother wavelet is usually nonzero on a finite interval, or decreases rapidly as time approaches infinity. Thus the WT can decompose a signal into a time-frequency representation because the wavelets are localized in time and frequency. The WT is closely analogous to the short-time Fourier transform (STFT). The linearity of the signal decomposition makes these transforms more suitable for the analysis of multicomponent signals than are quadratic time-frequency distributions¹² which suffer from cross-term interferences. Unlike the STFT, the WT, which has a variable time and frequency resolution, can overcome the resolution limitation encountered by the STFT in the joint time-frequency decomposition of a nonstationary signal. This advantage is what enables the WT to decompose a signal spectrum on a logarithmic scale.

Due to the high computational burden of a direct execution of a WT in (1), the discrete WT (DWT) is desirable. The DWT

was not fully developed and used until Daubechies and Mallat^{13, 14} helped complete the framework for the theory and demonstrated the applicability of the mathematical theory to engineering problems. The DWT can be implemented efficiently using Mallat's pyramid algorithm¹⁴. The Mallat algorithm was developed for orthogonal wavelets which are associated with the form of the quadrature mirror filter banks. Therefore, the most commonly used filter pairs for Mallat's algorithm have been Daubechies filters or other restrained filters which lead to an orthogonal decomposition. An alternative approach to implementing the DWT is with the A-trous algorithm, originally proposed by Holschneider et al.¹⁵ for music synthesis. The A-trous algorithm is different from Mallat's algorithm. The A-trous algorithm allows any wavelet to be chosen for analysis without the orthogonal restriction. Therefore the A-trous algorithm gives rise to non-orthogonal decompositions which have greater flexibility in applications using generalized wavelets. The A-trous algorithm derived by Shensa¹⁶ can be implemented with pyramidal calculations having a structure similar to Mallat's algorithm. In this approach, there are two filters which function as bandpass and lowpass filters which are arranged in an iterative sequence, to form a hierarchical filter bank in the pyramidal structure that eventually yields the DWT. The coefficients for the bandpass filter are the discrete samples of the continuous wavelet function chosen for the analyzing mother wavelet provided that the coefficients of the lowpass filter $f(n)$ satisfy the A-trous condition (3).

$$f(2n) = \delta(n) / \sqrt{2} \quad (3)$$

2.2 Auditory filtering modeled as a wavelet transform

An acoustic signal entering the ear causes a mechanical vibration to be transmitted through the conductive structures of the external and middle ear to the fluids of the cochlea; the inner ear. The cochlea, responsible for the transduction of sound waves to neural signals, can be divided functionally into two parts, the basilar membrane (mechanical transduction) and the sensory hair cells (bi-directional mechano-electrochemical transducers). The bi-directional transduction processes in the hair cells, which are responsible for nonlinear effects in the mechanical and neural representation of sound energy, are complicated and many questions are still unanswered or subject to debate¹⁷. The analytical model used in this paper was not chosen based on any specific, detailed model of the cochlea, but rather was suggested by the simplified model of a linear mechanical filtering of acoustic energy by the basilar membrane. The basilar membrane within the cochlea will respond to the stimulus by producing a spatio-temporal pattern of displacements along its entire length. Displacements show a frequency sensitivity from the base to the apex; the point of the maximum displacement for high frequencies occurs near the basal end, and for low frequencies, at the wider apical end. Thus the basilar membrane mechanics separates the incoming stimulus spectrally onto different spatial locations in a tonotopically ordered manner. Thus from the functional view, the cochlea analyzes sound much like performing a spectral decomposition through a parallel bank of auditory filters. The impulse responses of the filters maintain approximately a constant Q-factor scale, related to each other by a dilation. This naturally suggests that this stage of acoustical processing in the cochlea can be modeled as a wavelet transform⁷ of the stimulus, and thus by analogy, if one would like to extract a metric from an environmental noise that will correlate with the cochlear response to that noise, the WT would seem to be an appropriate approach.

The frequency response of each filter in the auditory filterbank can be described as time-invariant, and linear^{17, 18}. The gammatone function is a simple and reasonably accurate linear model that has been used to simulate the impulse response of an auditory filter. The gammatone filter, as indicated by Patterson et al.¹⁸, offers a means of producing an auditory filterbank that fits both physiological and psychophysical data on human frequency selectivity. Holdsworth¹⁹ designed an efficient digital filter by taking advantage of the simple analytic form of the gammatone function. The impulse response of a gammatone filter $g(t)$ is:

$$g(t) = t^{n-1} e^{-2\pi b t} \cos(\omega_c t + \phi) u(t) \quad (4)$$

where n is the filter order, b is the damping factor related to the bandwidth²⁰, ω_c is the radian center frequency and $u(t)$ is the unity step function. The impulse response and frequency spectrum of a fourth order [$n=4$ in (4)] gammatone filter are shown in Figures 1(a) and 1(b). The spectral magnitude of a gammatone filter has an asymmetrically-shaped response in the frequency domain. In particular, the rate of decay (roll-off) of the filter with respect to distance from the center frequency is much higher on the high frequency side than on the low frequency side.

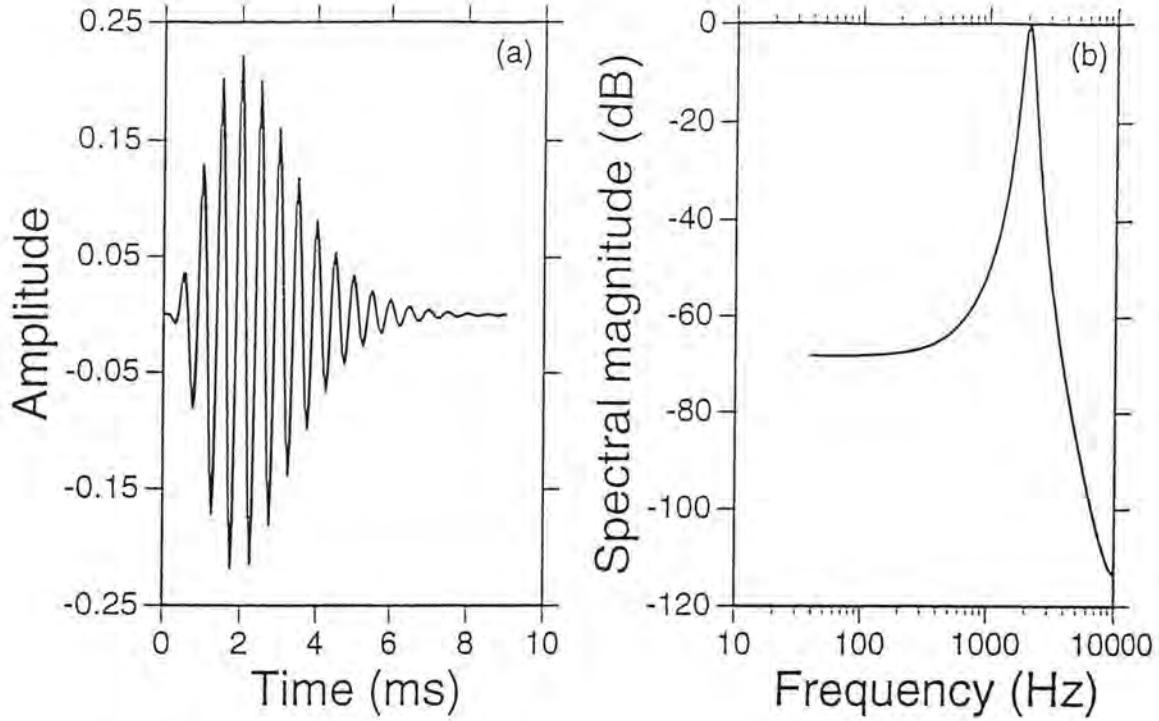


Figure 1. (a) Impulse response of a fourth-order gammatone filter. (b) Frequency response of a fourth-order gammatone filter. $f_c = 2000$ Hz.

A gammatone function, as a candidate for simulating an auditory filter, can be chosen as the analyzing mother wavelet to implement the WT in order to extract the FDK metric. The A-trous algorithm for computing the WT is needed since the gammatone function is not an orthogonal wavelet kernel. For computational implementation, discrete samples taken from the continuous gammatone function are used for the filter coefficients of the bandpass filter. But the gammatone function does not satisfy the admissibility condition (2). However, if the exponentially decaying discrete gammatone function is truncated, then the gammatone function will satisfy the admissibility condition (2) and can serve as the WT kernel. While this introduces some error into the WT, the error is small and can be neglected.

3. FREQUENCY DOMAIN KURTOSIS

3.1 Kurtosis estimates of nonGaussian and nonstationary signals

As indicated in the introduction, the energy and spectral metrics used to characterize a Gaussian noise are inadequate for nonGaussian and nonstationary signals because they fail to describe the irregular amplitude fluctuations of the temporal waveform. Kurtosis, a fourth-order statistic, originally found useful in the measurement of helicopter and jackhammer noises^{21, 22}, is considered a good metric for classifying the impulsive character of continuous nonGaussian noises²². Kurtosis is defined as the ratio of the fourth-order central moment to the squared second-order central moment of the sample distribution. The kurtosis metric, taken from the amplitude distribution of a signal, is dependent on the probability density function (PDF) of the amplitude distribution. The PDF of the amplitude distribution of an impulse has a very different shape from that of a steady-state Gaussian-distributed waveform. Most of the samples of an impulsive signal are concentrated at low amplitude levels while a few samples are at high amplitudes. For a Gaussian-distributed waveform, kurtosis will have a value around 3, while the kurtosis for a highly impulsive type of continuous noise signal can have a much greater value. Kurtosis is sensitive to the peak and background energy levels of a signal which are affected by unsteady multiple peaks, and the nonuniform-interval periodicity within a signal. Kurtosis can serve as a metric which can quantify the extent of the amplitude fluctuations for a nonGaussian and nonstationary signal.

3.2 Frequency domain kurtosis from the wavelet transform

For a nonstationary signal, a joint time-frequency analysis can show how the spectral content of a signal varies in time. Kurtosis estimates can be computed on the contributed amplitude distributions across different frequencies. This metric which is called frequency domain kurtosis is an extension of the kurtosis statistic from the time to the frequency domain.

$$\frac{E \left\{ [X(a_f, t) - E(X(a_f, t))]^4 \right\}}{\left(E \left\{ [X(a_f, t) - E(X(a_f, t))]^2 \right\} \right)^2} \quad (5)$$

$X(a_f, t)$ represents the signal as a function of frequency scale (a_f) and time (t); $E(\cdot)$ is the mean or expected value operator. Dwyer²³ originally used the FDK estimate to obtain additional information about the frequency components of a randomly occurring signal that the traditional power spectral estimate could not obtain. FDK is able to detect nonstationary and nonGaussian transients (such as impulses and unsteady harmonic components) in the presence of a background continuous Gaussian noise. Dwyer's algorithm, however, is based on Fourier analysis (DFT analysis), hence FDK results based on his algorithm yield values on a linear frequency scale. The kurtosis statistic, when applied to the results of the wavelet transform of the input signal will yield the FDK on a logarithmically scaled frequency which is more appropriate for comparison with physiological measurements of hearing.

4. FDK RESULTS

4.1 The model nonGaussian and nonstationary signals used for WT analysis

Industrial noise is frequently nonstationary and nonGaussian distributed in its temporal structure. A set of such acoustic waveforms, generically referred to as complex noise, has been used as an experimental stimulus for animal model studies^{24, 25} of noise-induced hearing loss. These stimuli were shown to exacerbate hearing loss when compared to spectrally equivalent Gaussian noises of the same energy. A complex noise $x(t)$, modeled as shown in (6), consists of variable impulsive or other transient components superimposed on a time varying continuous background noise.

$$x(t) = s(t) + n(t)$$

$$s(t) = s_p(t) [1 + \alpha_1 \delta(t - t_1) + \alpha_2 \delta(t - t_2) + \dots] \quad (6)$$

The nonGaussian ensemble $s(t)$ consists of high-level impulses each of which may be reflected several times and interacted with other primary and reflected components prior to arrival at the measured point. The parameters α_i are the attenuated reflected amplitude factors, and t_i are the time delays. These factors and delays are random variables which control the amplitude fluctuations resulting from multipath interference; nonstationary sources or characteristics of the propagating medium. The background noise $n(t)$ can be modeled by a Gaussian random noise. The amplitude distributions of the complex noise samples although quite variable can be synthesized using a phase domain approach²⁵. By manipulating the phase spectrum, within any selected band of frequencies, and then applying the inverse discrete Fourier transform a variety of signals from Gaussian to highly nonGaussian (including purely impulsive) distributed samples with predetermined spectral and statistical characteristics can be synthesized. Therefore complex noise samples, subject to close simulation control, can be created and used as the nonGaussian and nonstationary waveforms in the WT analytical model for obtaining FDK. [To avoid confusion, it should be noted that our use of the term 'signals' refers to 'noise' stimuli that are of interest in animal model experiments on noise-induced hearing loss.]

4.2 FDK results from complex signals analyzed by WT

Five different complex noise signals were chosen to demonstrate the extraction of the FDK metric based on the wavelet transform. The five synthesized signal samples are defined as follows:

Signal I: Combination of Gaussian noise and shaped impulses. The spectrum of the shaped impulses is uniform from .2 to 5 kHz. The probability of impacts occurring in a 32 ms window is set at 0.1.

Signal II: Similar to Signal I, except, rather than shaped impulses, Gaussian distributed noise bursts, derived from the same uniform bandwidth as used for the impacts of Signal I, are used. The probability of a burst occurring in a 32 ms window is set at 0.1.

Signal III: A combination of Gaussian noise and shaped impulses. The spectrum of the shaped impulses is uniform from 1 to 4 kHz. The probability of impacts occurring in a 32 ms window is set at 0.1.

Signal IV: Similar to Signal III, except, rather than shaped impulses, a Gaussian distributed noise burst, derived from the same uniform bandwidth as used for the impacts in Signal III, are used. The probability of a burst occurring in a 32 ms window is set at 0.1.

Signal V: Similar to Signal I, except that the spectra of the impulses and Gaussian noise are interchanged. The spectrum of the shaped impulses is uniform from 5 to 16 kHz, and the Gaussian noise samples are derived from the .2 to 5 KHz uniformly distributed spectrum.

Each time-history waveform from Signals I through V has equal amplitude peaks for the shaped impulses and Gaussian noise bursts. The attenuated reflected amplitude factors have not been considered in this analysis in order to simplify the simulation conditions. However, the nonGaussian and nonstationary characteristics of the waveforms of these signals are still preserved. All five of the above signals have the same energy and time-averaged spectrum uniformly distributed from .2 to 16 kHz but very different temporal patterns as shown in Figure 2. The samples of Signals I through V shown in Figure 2(a) were synthesized by computer using a sampling frequency of 32 kHz with 1024 samples per window. Each sample waveform shown in Figure 2(a) is the result of concatenating 20 windows. These synthetic signals were analyzed by the WT using the gammatone filter as the wavelet kernel. The coefficients of the bandpass filter used in computing the WT are the discrete sampled points taken from the continuous fourth-order gammatone impulse response which has an 8 kHz center frequency. The FDK values are computed for 7 octave bands based on the results of the WT applied to a signal length of 32 s (i.e. 1000 windows). The FDK results for Signals I and III and II and IV are shown in Figures 3(a) and 3(b) respectively. These results clearly highlight the frequency specific differences built into each of the synthetic signals having the same energy and spectra and show that the FDK estimate of a signal is independent of the average spectrum of the signal. The FDK results indicate that the kurtosis estimate across frequencies extracted from the WT can highlight important differences between the nonGaussian and Gaussian components in a signal that are lost in conventional sound level/spectral measurements of a noise environment. Furthermore, the FDK values are higher than 3 (kurtosis for Gaussian conditions) in the octave bands from the region of the spectrum in which the nonGaussian transients were derived.

The four synthesized signals were then run through a conventional electro-acoustic transduction system. The free field acoustic signals were recorded by a 1/4" Bruel & Kjaer 4135 microphone and the digitized samples were temporally filtered by a group of FIR filters whose bandwidths were set to successive octave bands. The FDK estimates were computed on the filtered output signals. The results of the FDK computations using the WT are contrasted with the FDK values for the signals obtained using the filter analysis approach ² in Figure 3. Note that the numerical values of FDK obtained from the WT and the FIR filter approach cannot be directly compared since the WT uses the waveforms from a simulation environment while the results for the FIR filter analysis method were obtained from the calibrated acoustic waveforms measured under field conditions. Nevertheless, it is significant that the FDK from the WT and the FIR filter methods show a similar behavior as a function of frequency.

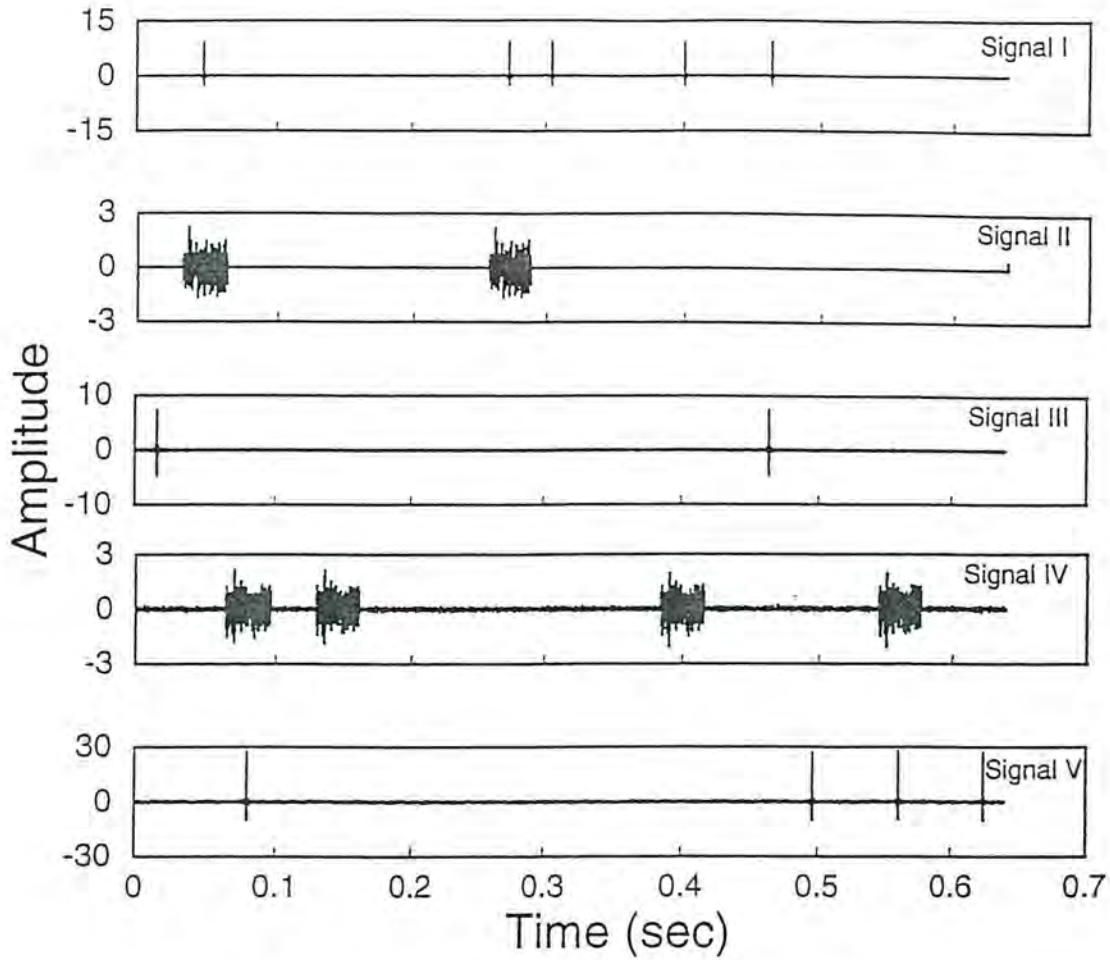


Figure 2(a) Temporal waveforms of Signals I through V.

The FDK values shown in Figure 3 for Signal I are higher than those of Signal II since the transient in Signal I is a shaped impact while the transient in Signal II is a Gaussian noise burst. Kurtosis for an impact is higher than that of a Gaussian noise burst since the impact shows more “impulsiveness” in its amplitude distribution. The same is true when the FDK values of Signal III and Signal IV are compared. Figure 4 further highlights the frequency specificity of the FDK metric obtained from the WT by comparing Signal I and V in which the nonGaussian and Gaussian components are derived from complementary octave bands in the uniform spectral distribution. The FDK statistic can thus indicate not only nonGaussian components in a signal but also their locations in the frequency domain.

Figure 5 shows the FDK values for Signal I using three different probabilities for controlling the occurrence of the impulses. The energy and spectrum are still maintained at the same levels for the different probabilities. With less probability of occurrence of the impulses, fewer impulses will be present in a given length of the temporal waveform, and thus there will be more relative fluctuations in the amplitude distribution because of the longer interpeak interval, resulting in a larger kurtosis estimate.

Comparisons from all the above FDK results, while computed under a limited set of different conditions, indicate that the FDK estimate obtained from the WT provides a frequency specific quantification of the extent of nonstationary fluctuations relative to Gaussian conditions, and that this metric is sensitive to factors such as relative amplitude peaks, transient shapes, and interpeak intervals, etc. Factors such as these contribute to the temporal pattern of the signal under consideration and their contributions to the nonGaussian character of the signal are to some extent preserved/quantified in the FDK metric.

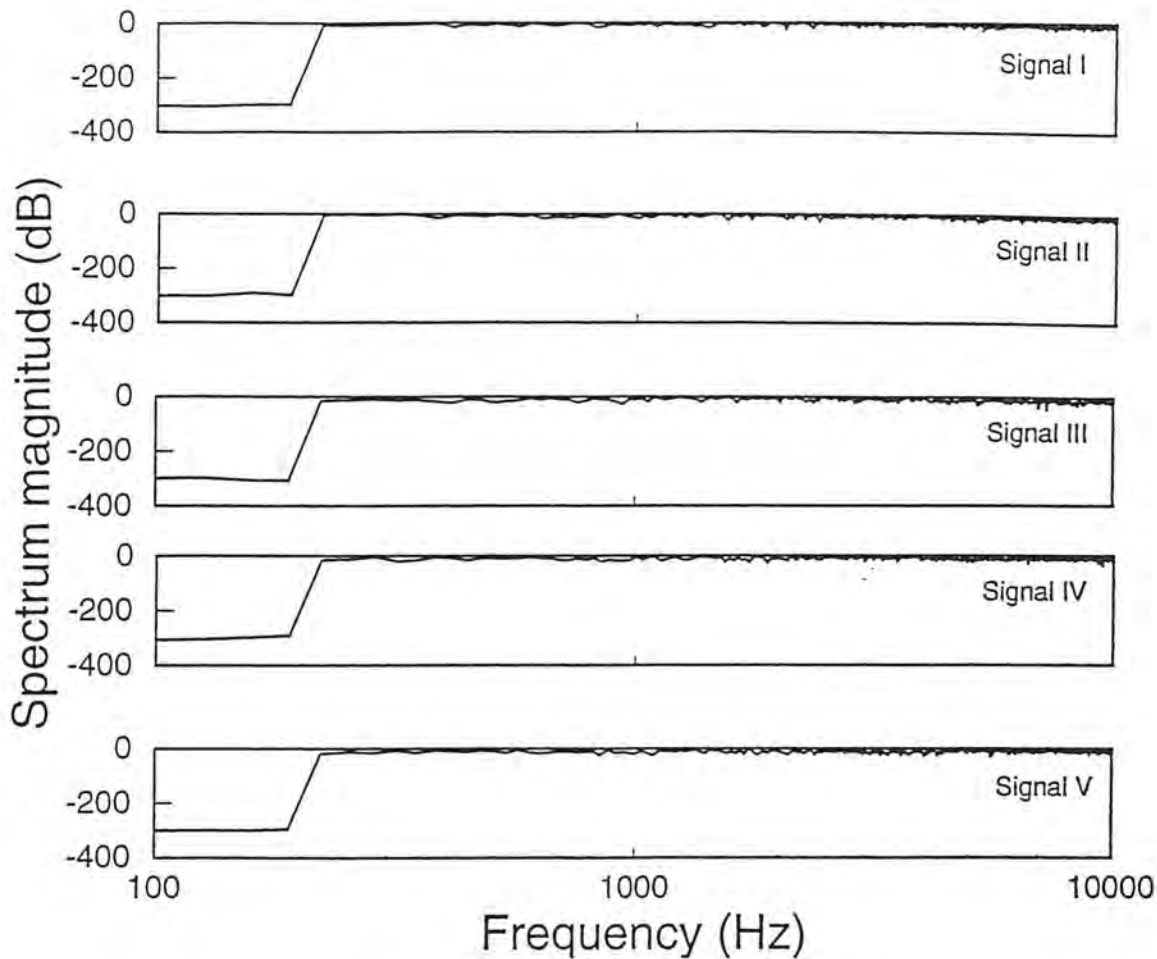


Figure 2(b) Frequency spectra of Signals I through V.

5. CONCLUSIONS

The FDK results identify and quantify the frequency specific amplitude fluctuations of the nonGaussian components in a nonstationary continuous signal and thus can augment the conventional metrics used in the assessment of nonGaussian environments for the purpose of hearing conservation criteria. Such information is completely neglected when metrics based on conventional methods such as time-averaged energy and spectral analysis are used. With the rapid advancement of digital technology and computer systems, the software for computing the FDK metric, based on the WT of the sampled noise environment, can be integrated into the hardware and software environments of a noise monitoring system.

ACKNOWLEDGEMENTS

We gratefully acknowledge the National Institute for Occupational Safety and Health to support this research (grant number K01-OH00139).

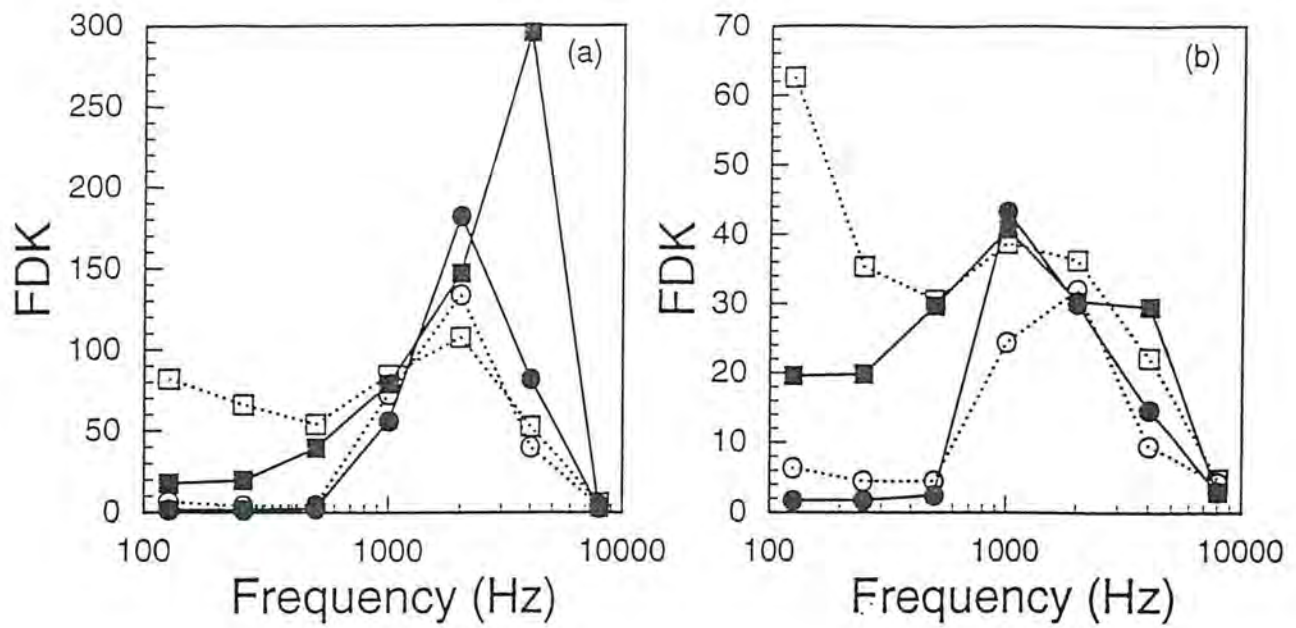


Figure 3 (a) FDK values of Signals I (■, □) and III (●, ○); (b) FDK values of Signals II (■, □) and IV (●, ○). Solid symbols - wavelet transforms and open symbols - filter analysis approach.

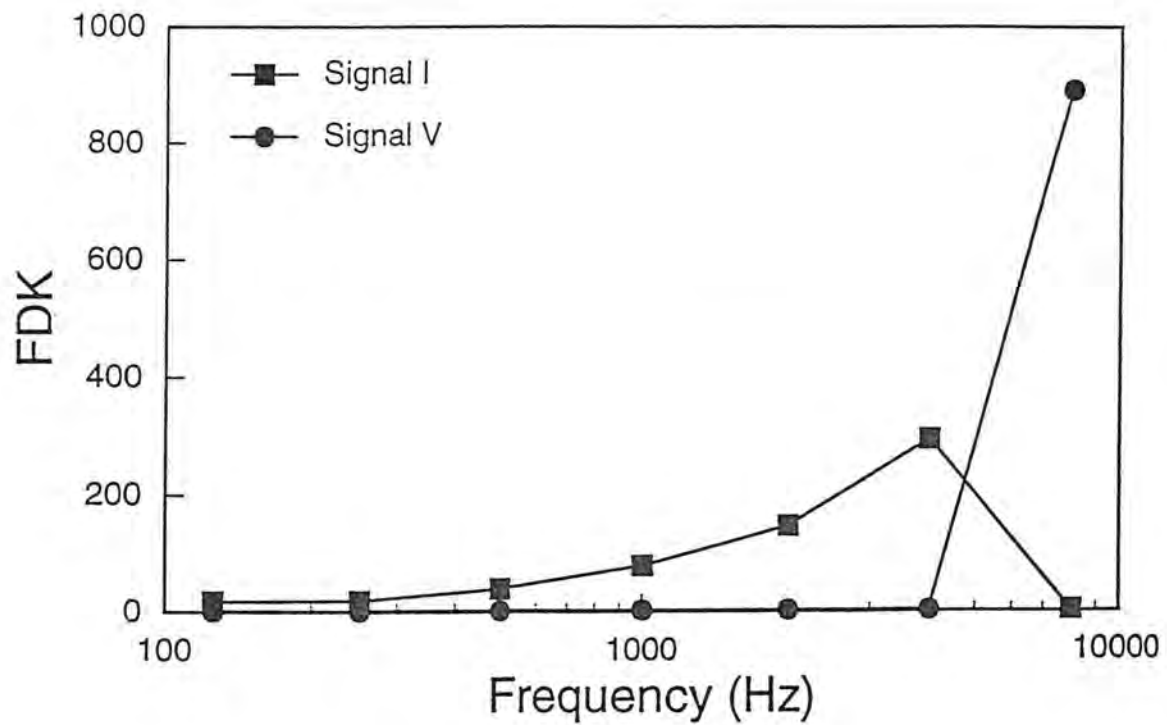


Figure 4. FDK values of Signal I and Signal V.

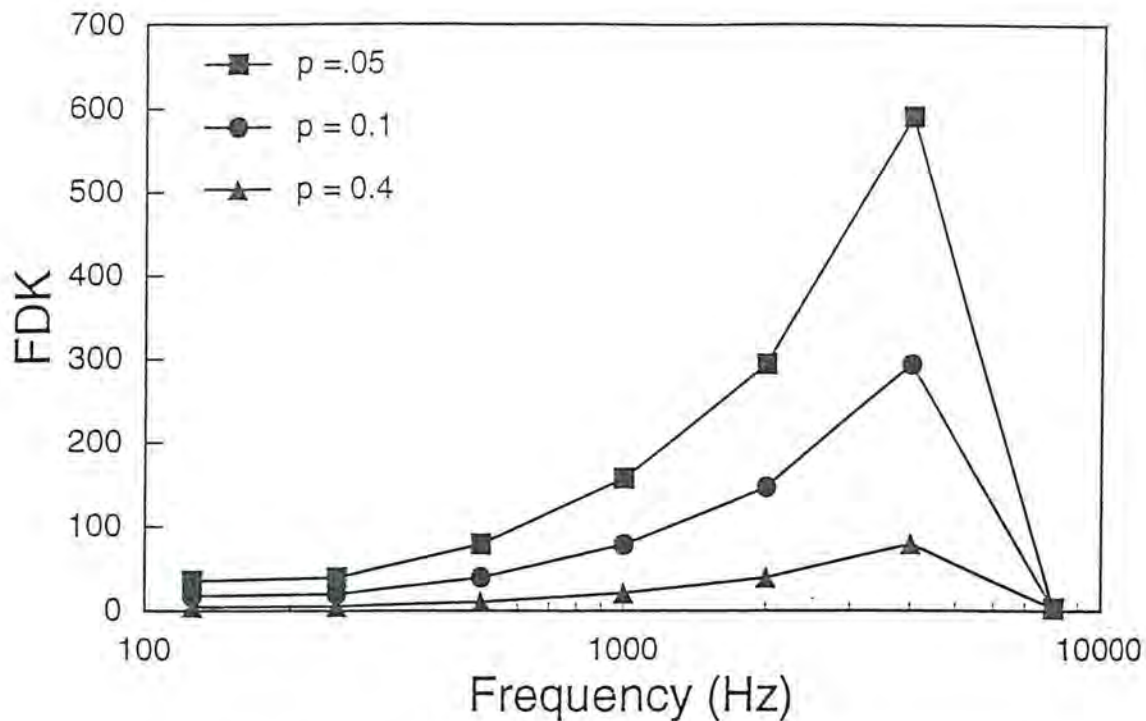


Figure 5. FDK values of Signal I with different probabilities of impacts occurring.

REFERENCES

- [1] R.P. Hamernik, W.A. Ahroon, K.D. Hsueh, S.-F. Lei, and R.I. Davis, "Audiometric and histological differences between the effects of continuous and impulsive noise exposures," *J. Acoust. Soc. Am.*, 93(4):2088-2095, April 1993.
- [2] S.-F. Lei, W.A. Ahroon, and R. P. Hamernik, "The application of frequency and time domain kurtosis to the assessment of hazardous noise exposures," *J. Acoust. Soc. Am.*, 96(3):1435-1444, Sept. 1994.
- [3] D.E. Dunn, R.R. Davis, C. J. Merry, and J. R. Franks, "Hearing loss in the chinchilla from impact and continuous noise exposure," *J. Acoust. Soc. Am.*, 90:1979-1985, Oct. 1991.
- [4] J.H. Patterson, I.M. Lomba-Gautier, D.L. Curd, R.P. Hamernik, *The role of peak pressure in determining the auditory hazard of impulse noise*. USAARL report 86-7, 1986.
- [5] R.F. Lyon and C. Mead, "An analog electronic cochlea," *IEEE Trans. ASSP*, Vol. ASSP-36(7):1119-1134, 1988.
- [6] R.F. Lyon, "A computational Model of Filtering, Detection, and Compression in the Cochlea," *Proc. of the IEEE International conf. ASSP*, p. 1282-1285, Paris, France, May 1982.
- [7] X. Yang, K. Wang, and S.A. Shamma, "Auditory Representations of Acoustic signals," *IEEE Trans. Information Theory*, Vol. 38(2):824-839, March 1992.
- [8] M. Slaney and R.F. Lyon, "On the importance of time-a temporal representation of sound," *Visual Representations of Speech Signals*, M. Cooke, S. Beete, and M. Crawford, eds., John Wiley & Sons, Sussex, England, 1993.

- [9] S.-F. Lei and R.P. Hamernik, "Wavelet transform and frequency domain kurtosis: Application to assessment of hearing hazard from noise exposures," 1995 IEEE ASSP workshop on applications of signal processing to audio and acoustics, p. 1.4.1-3, New Paltz, NY, Oct. 1995.
- [10] M. Vetterli, and C. Herley, "Wavelets and filter banks: theory and design," IEEE Trans. Signal Processing, 40(9):2207-2232, Sept. 1992.
- [11] I. Daubechies The Wavelet transform, time-frequency localization and signal analysis, IEEE Trans. Information Theory, 36(5):961-1005, Sept. 1990.
- [12] F. Hlawatsch and G.F. Boudreaux-Bartels, "Linear and Quadratic time-frequency signal representations," IEEE SP magazine, p.21-67, April 1992.
- [13] I. Daubechies, "Orthonormal bases of compactly supported wavelets," Comm. Pure Applied Math., 41:909-996, 1989.
- [14] S.G. Mallat A theory of multiresolution signal decomposition: the wavelet representation IEEE Trans. Pattern Anal. Machine Intell., 2:674-693, July 1989.
- [15] M. Holschneider, R. Kronland-Martinet, J. Morlet, and Ph. Tchamitchian, "A real-time algorithm for signal analysis with the help of the wavelet transform," *wavelets: Time-Frequency Methods and Phase Space*, J.M. Combes, A. Grossman, and Ph. Tchamitchian, eds. Berlin:Springer, IPITI, p. 286-297, 1989.
- [16] M. J. Shensa, "The discrete wavelet transform: wedding the A-trous and Mallat Algorithms," IEEE Trans. Signal Processing, 40(10):2464-2482, Oct. 1992.
- [17] M. Cooke, *Modelling Auditory Processing and Organisation* , Cambridge University Press, NY, 1993.
- [18] R.D. Patterson, K. Robinson, J. Holdsworth, D. McKeown, C. Zhang, and M. Allerhand, "Complex sounds and Auditory Images," *Auditory Physiology and Perception, Advances in Biosciences*, Y. Cazals, L. Demany, and K. Horner, eds., p. 429-446, Pergamon Press, 1992.
- [19] J. Holdsworth, I. Mimmo-Smith, R.D. Patterson and P. Rice, "Implementing a Gammatone filter bank," Personal communication, 1988.
- [20] B.R. Glasberg and B.C.J. Moore, "Derivation of auditory filter shapes from notched-noise data," Hearing Research, 47:103-108, 1990.
- [21] *International Standards Organization, ISO 3891, DAD1* , "Procedure for describing aircraft noise heard on the ground: Measurement of noise from helicopters for certification," Geneva, 1978.
- [22] J. Erdreich, "A distribution based definition of impulse noise," J. Acoust. Soc. Am., 79(4):990-998, April 1986.
- [23] R. F. Dwyer, "Use of the kurtosis statistic in the frequency domain as an aid in detecting random signals," IEEE Journal of Oceanic Engineering, Vol. OE-9(2):85-92, April 1984.
- [24] K.D. Hsueh and R.P. Hamernik, "A generalized approach to random noise synthesis: Theory and computer simulation," J. Acoust. Soc. Am., 87(3):1207-1217, March 1990.
- [25] K.D. Hsueh and R.P. Hamernik, "Performance characteristics of a phase domain approach to random noise synthesis," Journal of Noise Control Engineering, p. 18-32, Jan. 1991.