AN EXAMINATION OF THREE ICON-BASED SOLVERS FOR ORDINARY DIFFERENTIAL EQUATIONS

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INTRODUCTION

The purpose of this study is to examine the characteristics of three icon-based solvers for numerically solving ordinary differential equations. Icon-based solvers are attractive software packages because they allow the user to build a model using icons that represent mathematical operations. By interconnecting the mathematical operations, the software keeps track of the connections and associated variable names thereby allowing the users to focus on the development of the model. A group of mathematical operations that perform a repetitive task can be assigned to an icon and that icon can be repeated as necessary. Thus, the user is able to build more complex models from simpler models.

The ordinary differential equations of interest in this study are those derived from mass and energy balances for a thermodynamic control volume. The report first presents a general form for the ordinary differential equations. This is followed by a description of the software packages and the icons used to solve the equation. Next, results are given to verify the accuracy of the solvers. Finally, a recommendation is made on which solver should be used for development of a simulation model for an animal confinement building.

ORDINARY DIFFERENTIAL EQUATION

The mathematical simulation of thermal and flow processes in buildings leads to a set of first-order ordinary differential equations (Groszczyk, 1999). The general form of the equations is

$$A\frac{dy}{dt} = B(y - y_i) + Q \tag{1}$$

where y is the dependent variable (concentration, temperature), t is the independent variable (time), y_i is a parameter (supply value of y), A and B are constants (capacitance of control volume and flow capacitance), and Q is a source term (concentration, thermal load). In general, there are several differential equations that must be solved simultaneously but Equation (1) is general enough and exhibits all the necessary characteristics to be considered for the current study.

Equation (1) can be written in the form of

$$\frac{dy}{dt} = \frac{1}{A}\left[B\,(y - y_i) + Q\right] \tag{2}$$

The initial condition for Equation (2) is

$$\text{at } t = 0,\ y = y_o \tag{3}$$

The analytical solution for Equation (2) subject to the initial condition in Equation (3) is

$$y(t) = e^{\,B\,t/A}\left(y_o - y_i + \frac{Q}{B}\right) + \left(y_i - \frac{Q}{B}\right) \tag{4}$$

The next step is to solve Equation (2) using icon-based solvers and to verify that the solvers give accurate results.

SOLVERS

Three icon-based solvers are used to solve Equation (2) subject to the initial condition in Equation (3). The solvers are Simulink (Simulink, 1999), Xmath (Xmath, 1999), Easy5 (Easy5, 1999) and are available on a UNIX workstation connected to the Iowa Computer-Aided Engineering Network. Simulink is part of Matlab and can be purchased with a student version of Matlab. All solvers consist of a workspace where the model is constructed, a set of libraries that contain built-in icons for various mathematical operations and special functions, a set of equation solvers, and several ways to display and record numerical values of the variables.

Figures 1, 2, and 3 show diagrams of the icons for Simulink, Xmath, and Easy5. The arrangement and types of icons are nearly the same. The description of the diagrams starts with the output of the integrator. First, y_i is subtracted from the output of the integrator y using a summation icon (circle). The value of y_i is assigned in a constant icon (square) in Simulink and Xmath. In Easy5, it is defined in a gain icon (triangle). Second, the output from the summation icon is multiplied by B using a gain icon. The value of B is assigned in the gain icon. Third, the output of the gain icon is added to Q using a summation icon. The value of Q is assigned in the same way as y_i. Fourth, the output from the summation icon is divided by A using a product icon in Simulink or using a divider icon in Xmath and Easy5. The value of A is assigned in the same way as y_i. The loop is completed by connecting the output of the product icon to the input of the integrator icon. The integrator performs the left hand side of Equation (2). The results can be shown by using display icons in the three solvers. Also, the three solvers permit the numerical results to be written to a file that is used for further processing.

RESULTS AND DISCUSSION

To verify the accuracy of the results from the three solvers, two cases are simulated. For each solver, a step size of 0.1 is chosen and the duration of the simulation is 1.

In Case 1, the numerical values assigned to the parameters in Equation (2) are

$$A = 1,\ B = 1,\ Q = 1,\ y_i = 2,\ y_o = 2 \qquad (5)$$

The analytical solution Equation (2) becomes

$$y = e^{t} + 1 \qquad (6)$$

The results for y by using the analytical solution, Simulink, Xmath, and Easy5 are shown in Table 1. The results of analytical solution and Simulink are also plotted in Figure 4.

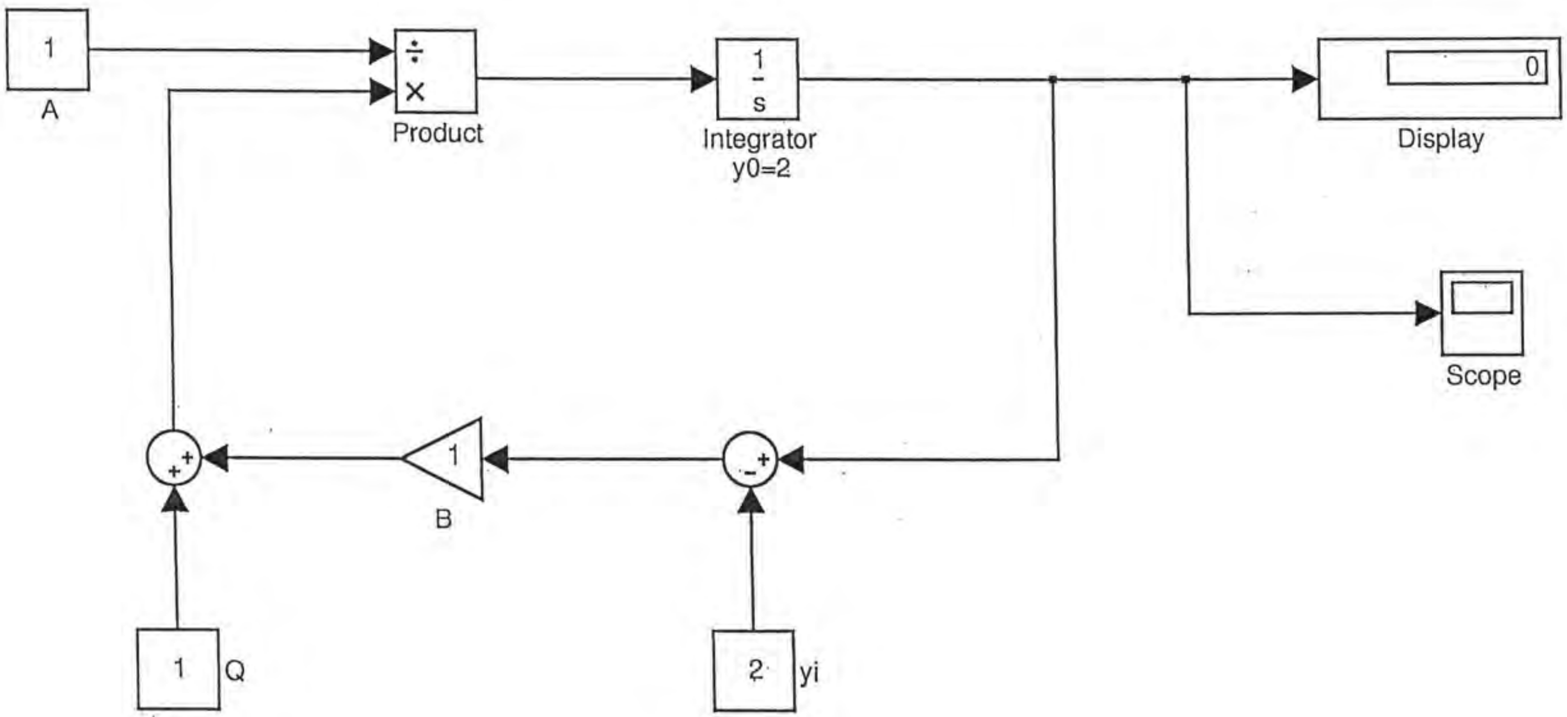

Figure 1 Schematic diagram of icons for Simulink.

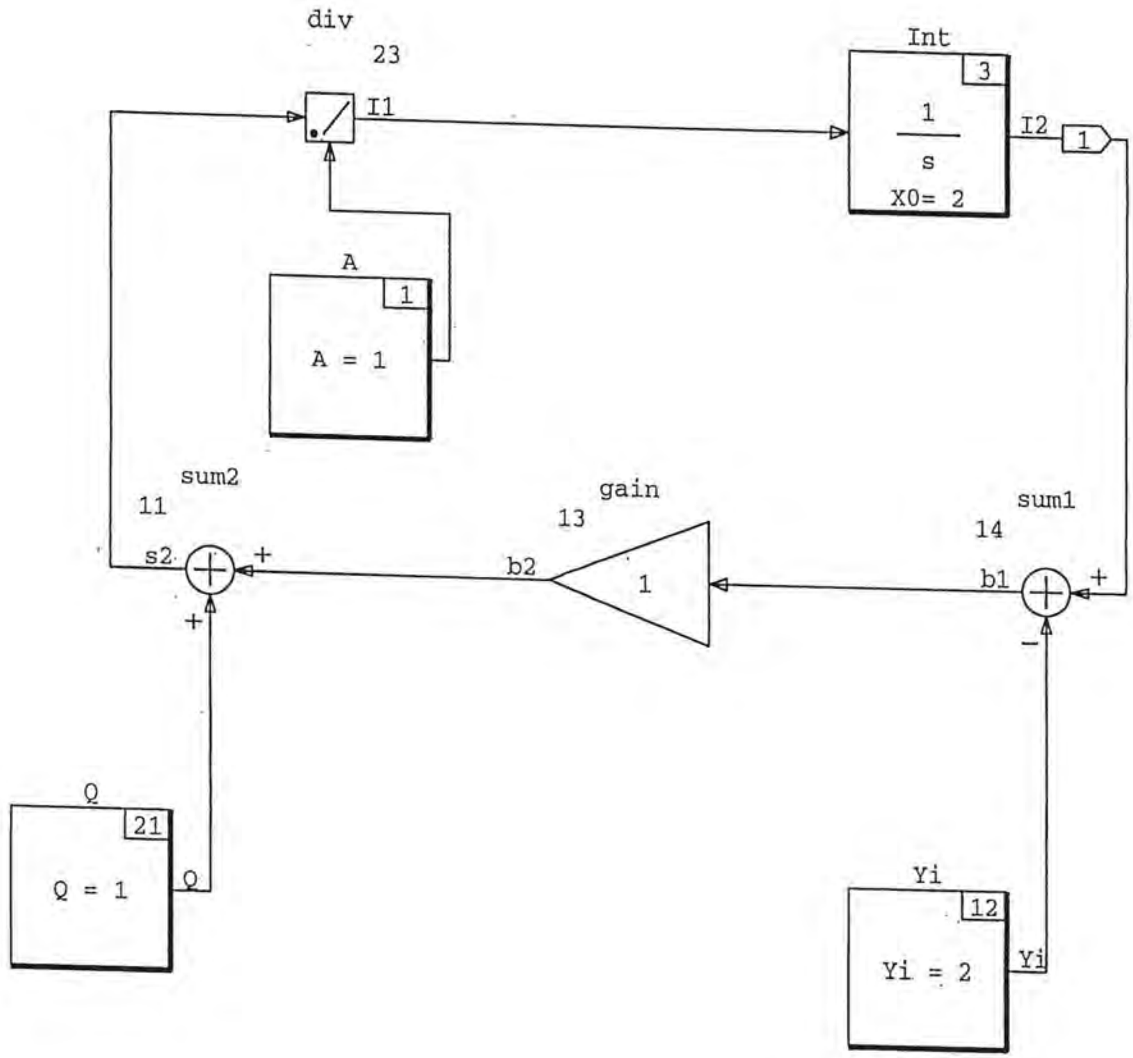

Figure 2 Schematic diagram of icons for Xmath.

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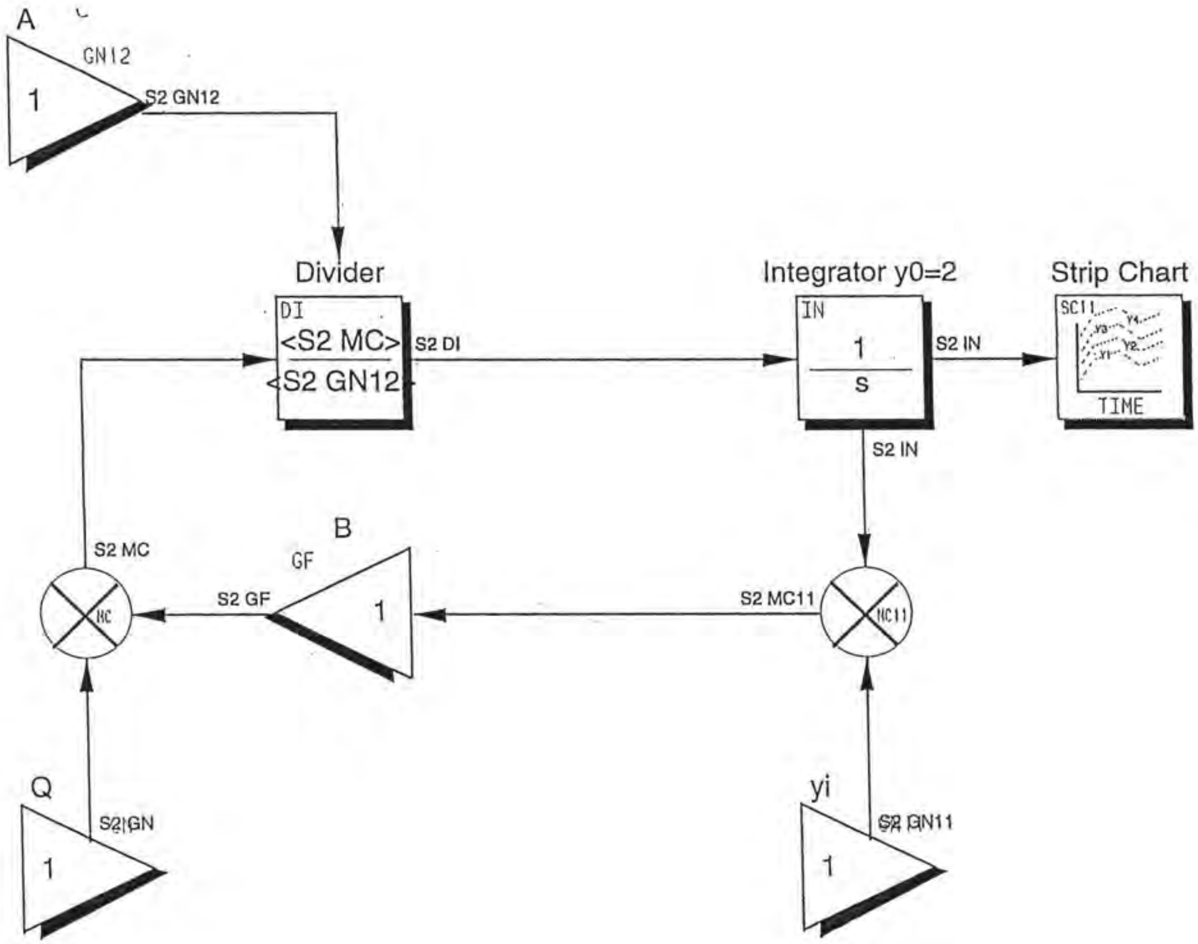

Figure 3 Schematic diagram of icons for Easy5.

Table 1 Results of analytical solution, Simulink , Xmath, and Easy5.

t	Analytical solution y_a	Relative error: $(y - y_a) / y_a * 100\ \%$		
		Simulink	Xmath	Easy5
Case 1				
0.00	2.0000	0	0	0.0000
0.10	2.1052	0	0	0.0048
0.20	2.2214	0	0	0.0225
0.30	2.3499	0	0	0.0213
0.40	2.4918	0	0	0.0281
0.50	2.6487	0	0	0.0264
0.60	2.8221	0	0	0.0283
0.70	3.0138	0	0	0.0265
0.80	3.2255	0	0	0.0310
0.90	3.4596	0	0	0.0318
1.00	3.7183	0	0	0.0346
Case 2				
0.00	3.0000	0	0	0.0000
0.10	3.4316	0	0	0.0204
0.20	3.9330	0	0	0.0280
0.30	4.5155	0	0	0.0288
0.40	5.1923	0	0	0.0289
0.50	5.9787	0	0	0.0284
0.60	6.8923	0	0	0.0290
0.70	7.9537	0	0	0.0302
0.80	9.1870	0	0	0.0305
0.90	10.6198	0	0	0.0301
1.00	12.2845	0	0	0.0285

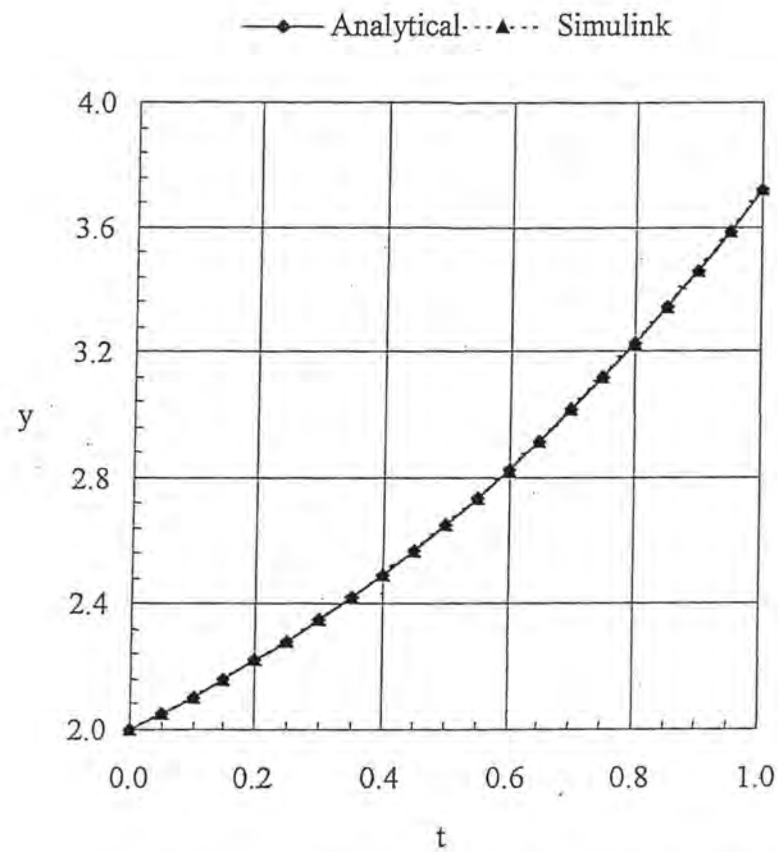

Figure 4 Plot of analytical and Simulink solution for Case 1.

Because there is no difference of the results from the two solutions, the two lines coincide in Figure 4. The results in Table 1 show that the analytical solution, Simulink, and Xmath have the same values. The results of Easy5, however, are slightly higher than the others but the relative error is less than 0.04 %.

In Case 2, the numerical values assigned to the parameters in Equation (2) are

$$A = 2, B = 3, Q = 5, y_i = 2, y_o = 3 \tag{7}$$

The analytical solution becomes

$$y(t) = \frac{8}{3} e^{3t/2} + \frac{1}{3} \tag{8}$$

The results for y by using the analytical solution, Simulink, Xmath, and Easy5 are shown in Table 1. The results of analytical solution and Simulink are also plotted in Figure 5. and plotted in Figure 5. Just like Case 1, the analytical solution, Simulink, and Xmath have the same values. The results of Easy5, however, are slightly higher than the others and the relative error is less than 0.04 %.

SUMMARY

This study examines the procedures for using three icon-based software packages for solving a first-order ordinary differential equation. It is shown that Simulink, Xmath, and Easy5 can all be used to solve the ordinary differential equations. Simulink and Xmath gave results similar to those of the analytical solution. The results for Easy5 are slightly higher. It is also observed that Simulink, and Xmath run faster than Easy5. Finally, Simulink is more convenient for the user to construct the model than Xmath and Easy5. It is recommended that Simulink be used for future studies.

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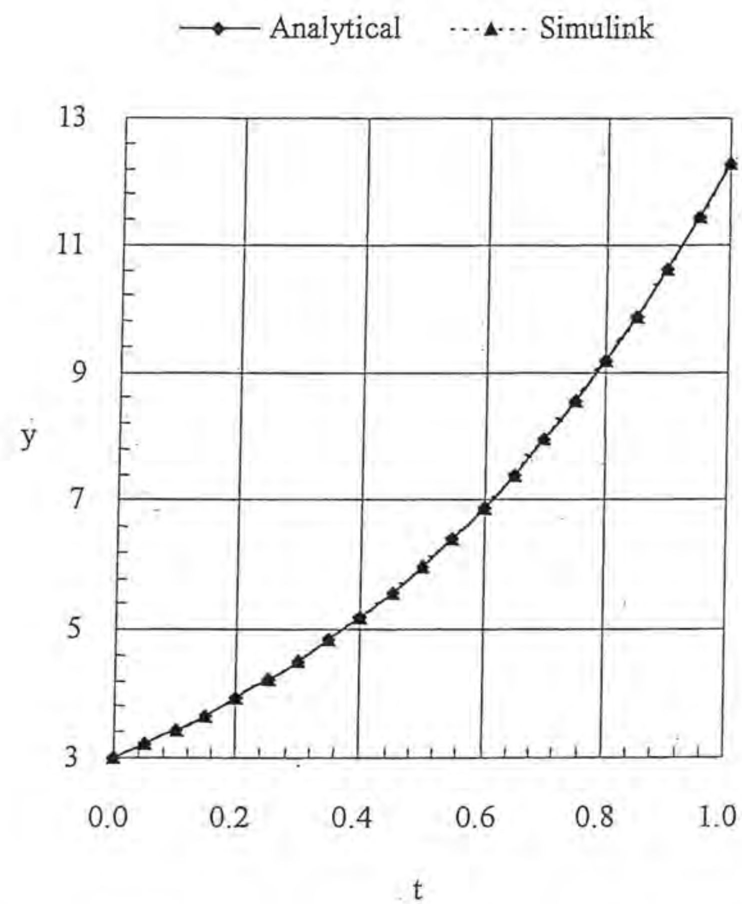

Figure 5 Plot of analytical and Simulink solutions for Case 2.

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