Nasopharyngeal Bacterial Interactions in Children

Technical Appendix

Figure. Number of children by bacterial colonization status in a stationary population (A) and cases enrolled during a specified time period (B).

\[ N \times p_1 \]
\[ N \times p_2 \]
\[ N \times [1 - ((p_1 + p_2) - (p_1 \times p_2))] \]

A.

B.

<table>
<thead>
<tr>
<th>Bacterium 2</th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>( N \times p_1 \times p_2 \times t \times c \times t )</td>
<td>( N \times (p_1 - (p_1 \times p_2)) \times t \times c \times t )</td>
</tr>
<tr>
<td>Negative</td>
<td>( N \times (p_1 \times p_2) \times t \times c \times t )</td>
<td>( N \times (1 - ((p_1 + p_2) - (p_1 \times p_2))) \times t \times c \times t )</td>
</tr>
</tbody>
</table>

\( N \): the population size

\( p_1 \): the prevalence of bacterium 1 colonization

\( p_2 \): the prevalence of bacterium 2 colonization
Suppose the colonization of bacterium 1 and of bacterium 2 occur independently in the population.

The odds ratio between bacterium 1 and bacterium 2 in the population (OR$_{pop}$) will be

$$\text{OR}_{pop} = \frac{N \times p_1 \times p_2 \times N \times [1 - ((p_1 + p_2) - (p_1 \times p_2))]}{N \times \{p_1 - (p_1 \times p_2)\} \times N \times \{p_2 - (p_1 \times p_2)\} \times p_1 \times p_2 - (p_1 \times p_2)^2 \times (p_1 + p_2) - (p_1 \times p_2)^2} \times \frac{p_1 \times p_2 - (p_1 \times p_2) \times (p_1 + p_2) - (p_1 \times p_2)^2}{p_1 \times p_2 - (p_1 \times p_2) \times (p_1 + p_2) - (p_1 \times p_2)^2} = 1$$
The OR between bacterium 1 and bacterium 2 in the enrolled cases will be

\[
\text{OR}_{\text{case}} = \text{OR}_{\text{pop}} \times \frac{r_c \times t \times r_n \times t}{r_c \times t \times r_n}
\]

\[
= \text{OR}_{\text{pop}} \times \frac{r_n}{r_c}
\]

which is the reciprocal of risk ratio for enrollment (= developing the disease; \(r_c/r_n\)).